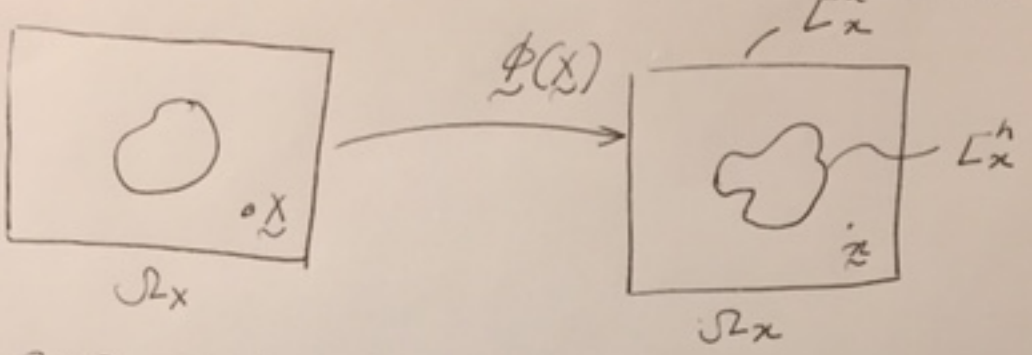
Formulate the forward Problem in the current config.



* BVP Find $\underset{\sim}{u}(\underset{\sim}{x}), p(\underset{\sim}{x})$, s.t.

$$\begin{bmatrix} \nabla_x \cdot \underset{\sim}{T} = \underset{\sim}{0} & \text{in } \Omega_x \\ \underset{\sim}{u} = \underset{\sim}{g} & \text{on } \Gamma_x^u \\ \underset{\sim}{T} \cdot \underset{\sim}{n} = \underset{\sim}{h} & \text{on } \Gamma_x^h \end{bmatrix}$$

→ given in forward problem!

$$\underset{\sim}{T} = -p\,\underset{\sim}{1} + 2\,\underset{\sim}{F}\,\frac{\partial W}{\partial \underset{\sim}{C}}\,\underset{\sim}{F}^T$$

(W: strain energy density)

$$\underset{\sim}{F} = \frac{\partial \underset{\sim}{x}}{\partial \underset{\sim}{X}}, \quad \underset{\sim}{C} = \underset{\sim}{F}^T \underset{\sim}{F}$$

* Weak Form: Find $[\underset{\sim}{u}, p] \in \mathcal{S} \times L_2$, s.t. $\forall [\underset{\sim}{w}, q] \in \mathcal{V} \times L_2$ ($= \det(\underset{\sim}{F})$)

$$\int_{\Omega_x} \nabla^s \underset{\sim}{w} : \underset{\sim}{T}\, d\underset{\sim}{x} + \int q\,(J-1)\, d\underset{\sim}{x} = \int_{\Gamma_h} \underset{\sim}{w} \cdot \underset{\sim}{h}\, d\underset{\sim}{x}$$

* To evaluate $\underset{\sim}{T}$, we need $\underset{\sim}{F}$, $\underset{\sim}{C}$ fcts. The starting point is $\underset{\sim}{F}$. However what we know is $\partial \underset{\sim}{u} / \partial \underset{\sim}{x}$.

* Now.

$$\frac{\partial \underset{\sim}{u}}{\partial \underset{\sim}{x}} = \frac{\partial \underset{\sim}{u}}{\partial \underset{\sim}{X}} \frac{\partial \underset{\sim}{X}}{\partial \underset{\sim}{x}} = \frac{\partial}{\partial \underset{\sim}{X}} [\underset{\sim}{u}(\underset{\sim}{X})] \underset{\sim}{F}^{-1}$$

$$= \frac{\partial}{\partial \underset{\sim}{X}} [\underset{\sim}{x} - \underset{\sim}{X}] \underset{\sim}{F}^{-1} = [\underset{\sim}{F} - \underset{\sim}{1}] \underset{\sim}{F}^{-1}$$

$$= \underset{\sim}{1} - \underset{\sim}{F}^{-1}$$

$$\Rightarrow \underset{\sim}{F}^{-1} = \underset{\sim}{1} - \frac{\partial \underset{\sim}{u}}{\partial \underset{\sim}{x}} \Rightarrow \boxed{\underset{\sim}{F} = [\underset{\sim}{1} - \nabla_x \underset{\sim}{u}]^{-1}}$$

→ very important

Using this we can compute everything we want. That is $\underset{\sim}{F}, J, \underset{\sim}{C}$ etc.