

$$\begin{cases} \nabla \cdot \bar{\mathbf{v}} = 0 & \text{in } \Omega \\ \underline{u} = \underline{g} & \text{on } \Gamma_g \\ \bar{\mathbf{v}} \cdot \underline{n} = \underline{h} & \text{on } \Gamma_h \\ J-1 = 0 & \text{in } \Omega \end{cases} \quad \begin{aligned} A(\bar{\mathbf{w}}, \bar{\mathbf{v}}; \beta) + R(\bar{\mathbf{w}}, \bar{\mathbf{v}}; \beta) &= (\underline{w}, \underline{h})_{\Gamma_h} \\ A(\bar{\mathbf{w}}, \bar{\mathbf{v}}; \beta) &= \int_{\Omega} w_{ij} \bar{v}_{ij} \, dv + \int_{\Omega} (J-1) \underline{g} \, dv \\ R(\bar{\mathbf{w}}, \bar{\mathbf{v}}; \beta) &= \sum_{n \in \Gamma} \int_{n \in \Gamma} \underline{p}_{,i} \underline{g}_{,i} \, dv \end{aligned} \quad \begin{aligned} \bar{\mathbf{w}} &\equiv [\underline{w}, \underline{g}] \\ \bar{\mathbf{v}} &\equiv [\underline{u}, \underline{p}] \end{aligned}$$

$$\bar{\mathbf{v}} = \frac{1}{J} \mathbf{F} \mathbf{S} \mathbf{F}^T, \quad \mathbf{S} = -\mathbf{P} \mathbf{J} \mathbf{C}^{-1} + 2 \frac{\partial W}{\partial \mathbf{C}}, \quad W = \frac{\mu}{2r} (e^{r(J^{-\frac{2}{3}} \mathbf{I} - 3)} - 1)$$

$$\mathbf{x} = \bar{\mathbf{x}} + \underline{u} \Rightarrow \frac{\partial \mathbf{x}}{\partial \bar{\mathbf{x}}} = \frac{\partial \bar{\mathbf{x}}}{\partial \bar{\mathbf{x}}} + \frac{\partial \underline{u}}{\partial \bar{\mathbf{x}}} \Rightarrow \mathbf{I} = \mathbf{F}^{-1} + \frac{\partial \underline{u}}{\partial \bar{\mathbf{x}}} \Rightarrow \bar{\mathbf{F}} = \left(\mathbf{I} - \frac{\partial \underline{u}}{\partial \bar{\mathbf{x}}} \right)^{-1}$$

$$\begin{aligned} \lim_{\epsilon \rightarrow 0} \frac{d}{d\epsilon} A(\bar{\mathbf{w}}, \bar{\mathbf{v}} + \epsilon \Delta \bar{\mathbf{v}}; \beta) &= \int_{\Omega} w_{ij} (\delta_{ik} \bar{v}_{jl} + c_{ijkl}) \Delta u_{k,l} \, dv \\ &+ \int_{\Omega} \underline{g} \bar{J} \bar{F}_{ij}^{-1} \Delta u_{ij} \, dv \\ &- \int_{\Omega} w_{ij} \Delta p \, dv \end{aligned}$$

$$\lim_{\epsilon \rightarrow 0} \frac{d}{d\epsilon} R(\bar{\mathbf{w}}, \bar{\mathbf{v}} + \epsilon \Delta \bar{\mathbf{v}}; \beta) = \sum_{n \in \Gamma} \int_{n \in \Gamma} \underline{g}_{,i} \Delta p_{,i} \, dv$$

$$c_{ijkl} = C_{IJKL} \cdot F_{iI} F_{jJ} F_{kK} F_{lL} \cdot \frac{1}{J}$$

$$\begin{aligned} C_{IJKL} &= 2 \frac{\partial S_{IJ}}{\partial C_{KL}} = 4 \frac{\partial^2 W}{\partial C_{IJ} \partial C_{KL}} - 2 \frac{\partial}{\partial C_{KL}} (\mathbf{P} \mathbf{J} \mathbf{C}_{IJ}^{-1}) \\ &= 2\mu e^{r(J^{-\frac{2}{3}} \mathbf{I} - 3)} \left(r L_{IJ} L_{KL} - \frac{1}{3} L_{IJ} C_{KL}^{-1} + D_{IJKL} \right) - 4p \frac{\partial^2 J}{\partial C_{IJ} \partial C_{KL}} \end{aligned}$$

$$L_{IJ} = J^{-\frac{2}{3}} \left(-\frac{1}{3} \mathbf{I}, C_{IJ}^{-1} + \delta_{IJ} \right)$$

$$D_{IJKL} = -\frac{1}{3} J^{-\frac{2}{3}} \left(C_{IJ}^{-1} \delta_{KL} - \frac{1}{2} \mathbf{I}, (C_{IK}^{-1} C_{JL}^{-1} + C_{IL}^{-1} C_{JK}^{-1}) \right)$$

$$\frac{\partial^2 J}{\partial C_{IJ} \partial C_{KL}} = \frac{1}{4} J \left(C_{IJ}^{-1} C_{KL}^{-1} - C_{IK}^{-1} C_{JL}^{-1} - C_{IL}^{-1} C_{JK}^{-1} \right)$$