Midterm Project: Fit Distributions to Monthly Returns

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1. Intro

For this project, monthly log returns of stock prices from BlackRock (BLK) and Goldman Sachs (GS) were analyzed from the past ten years. The adjusted closing price was used to calculate the monthly log returns to account for any corporate actions within each company. The purpose of this project is to create synthetic returns that mimic the statistical properties of the original monthly returns. This is created utilizing each company's individual returns and their relationship to each other. The exact statistical analysis will be provided further down.

BlackRock and Goldman Sachs are both global financial institutions that offer a variety of financial services. BlackRock is considered to be the world's largest asset manager while Goldman Sachs is considered a superior investment bank. It is also important to note that Goldman Sachs (founded in 1869) has been around for over 100 years and BlackRock (founded in 1988) is fairly newer in comparison. However, since they both cater to the same industry I would say their stock performance could be fairly related. Also, Goldman Sachs does have an asset management business that rivals BlackRock. Therefore, their monthly log returns should have some sort of relationship.

Download monthly stock prices for both 'BLK' and 'GS' from Yahoo Finance. Select and transform adjusted prices into log returns. Note: Time series format from initial download was removed as well.

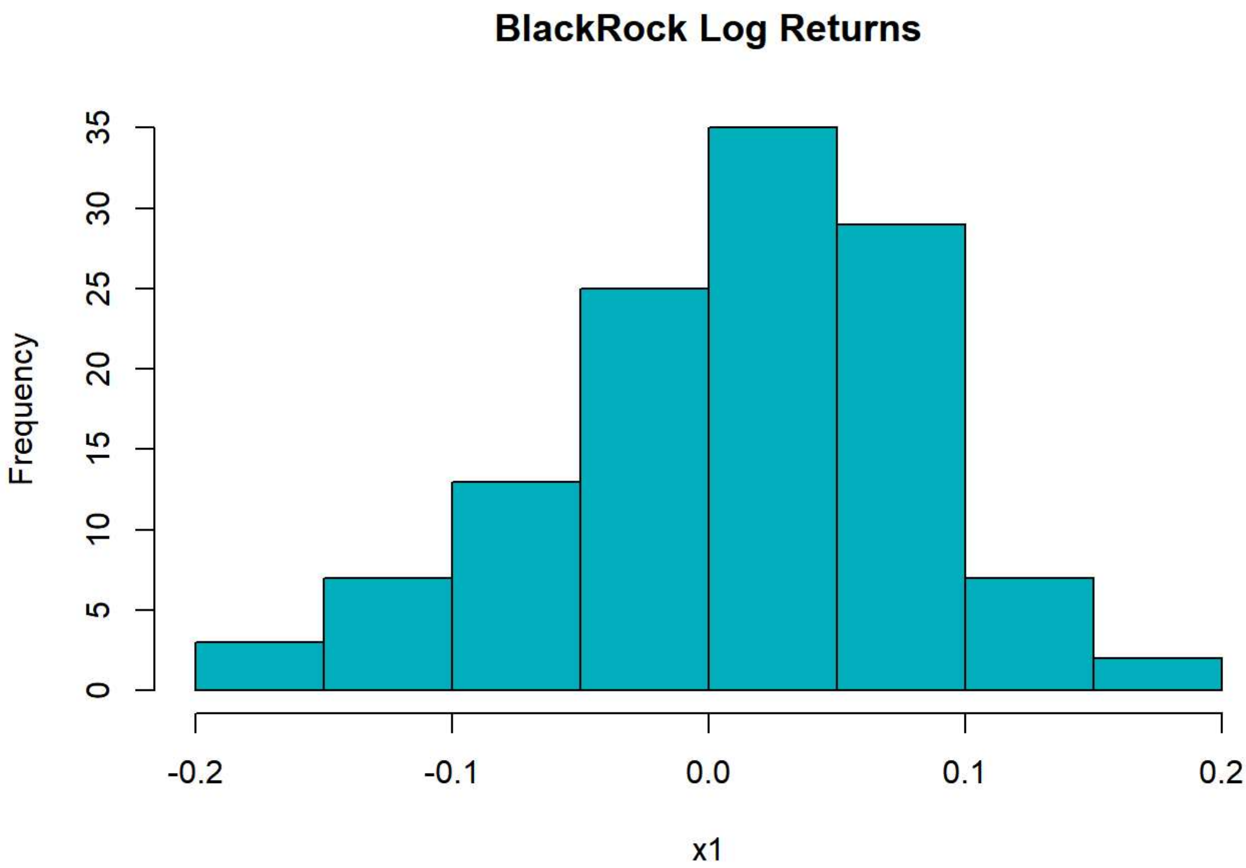
```
getSymbols("BLK;GS",
           from = "2012/08/01",
           to = "2022/10/01",
           periodicity = "monthly")
```

```
## [1] "BLK" "GS"
```

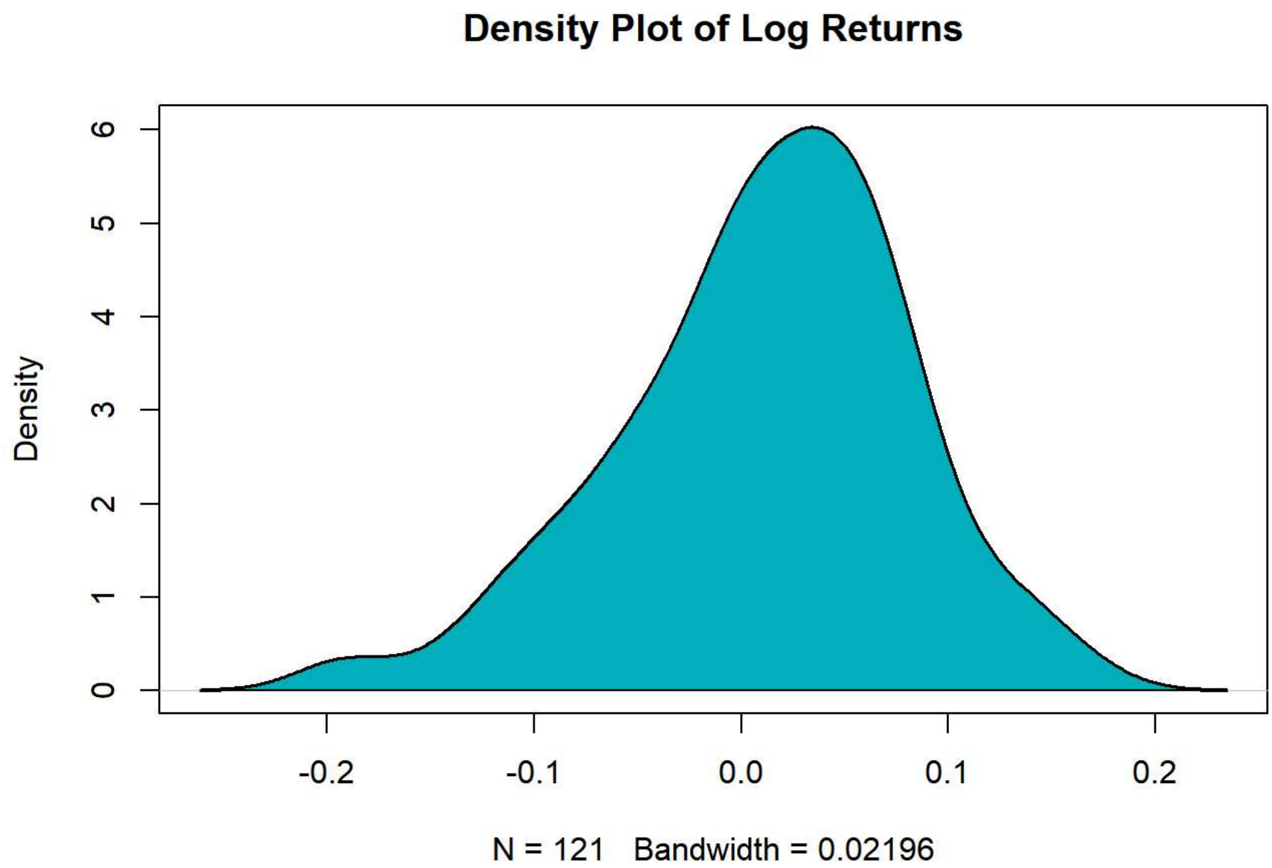
```
BLK_adj<-diff(log(BLK$BLK.Adjusted))
BLK_adj<-BLK_adj[-1,]
BLK_adj<-data.frame(BLK_adj)
x1 <- BLK_adj$BLK.Adjusted

GS_adj <-diff(log(GS$GS.Adjusted))
GS_adj<-GS_adj[-1,]
GS_adj<-data.frame(GS_adj)
x2 <- GS_adj$GS.Adjusted
```

a. Preliminary Analysis

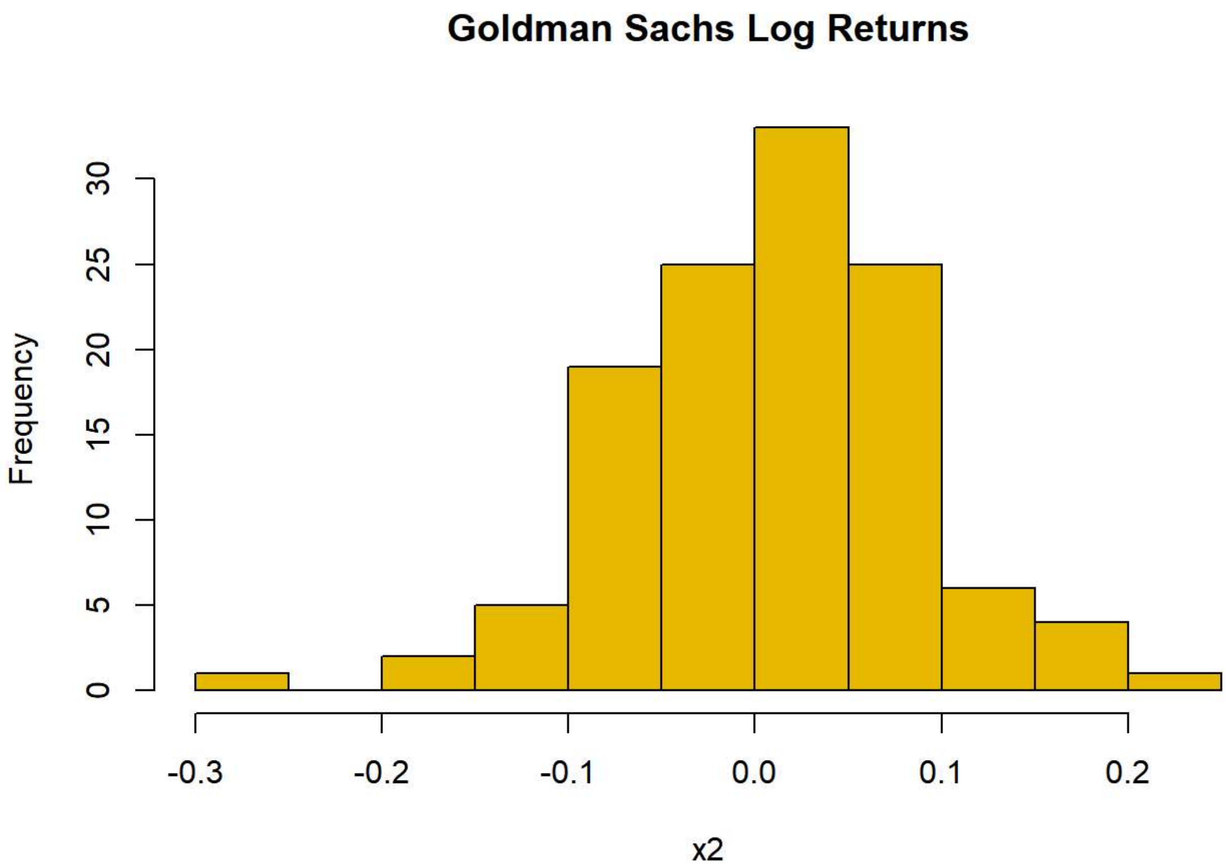
```
BLK_hist<-hist(x1,main = "BlackRock Log Returns", col = "#00AFBB")
```



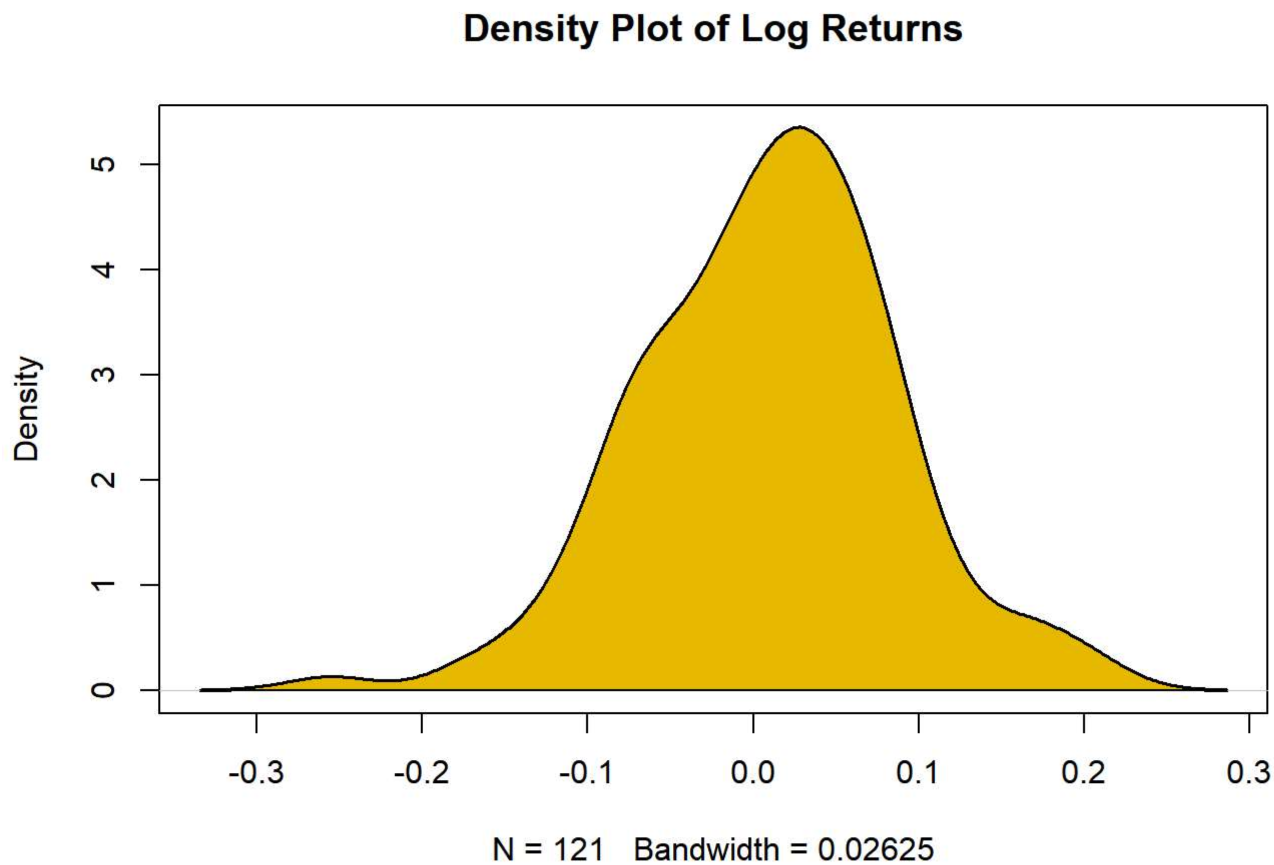

```
BLK_density<-plot(density(x1),lwd = 2,"Density Plot of Log Returns") + polygon(density(x1), col
 = "#00AFBB")
```




```
GS_hist<-hist(x2,main = "Goldman Sachs Log Returns", col = "#E7B800")
```




```
GS_density<-plot(density(x2),lwd = 2,"Density Plot of Log Returns") + polygon(density(x2), col =
"#E7B800")
```



The histograms and density plots of each company indicate that both returns are not perfectly symmetric and suffer from some slight degree of left skewness which is expected. I would predict that a skewed distribution would match their returns better.

2. Statistical Analysis

To create synthetic returns we first have to find the best fitting distribution model for each company's marginal distributions. This is done by fitting the returns to the Normal, Skewed Normal, Student's T, and Skewed Student's T distributions using a log-likelihood optimization method. From this we can find the optimal parameters specific to each type of distribution. To determine which distribution fits the returns best, Akaike's Information Criteria (AIC) and Bayesian Information Criteria (BIC) estimates are compared to analyze each distributions strength . The distribution that has the lowest value of AIC will be selected (AIC is better for predication). Visualizations of the fittings are added below as well to validate the selection. From the results below (Table 1., Table 2.), the best fitting distribution for both BlackRock and Goldman Sachs is the Skewed Student T distribution.

Next, we take the optimized parameters (Table 3.) from the Skewed Student T distribution to transform both stocks original monthly returns into uniform marginal distributions (Uniform Distribution Figures). This is to set up the modeling of a copula which is used to describe the dependence between random variables. Several copulas are fitted and analyzed. To be specific the Gaussian, T, Frank, Joe, and Gumbel Copulas are used. The AIC is used again to select the copula which gives the best representation of the stocks relationship. Table 5. indicates that the frank copula was the best fit. Now we can use the frank copula to extract and mimic the marginal distribtions by randomly drawing a sample size equal to the original monthly log returns. This sample data (cdf) is synthetic and is then transformed back into a fitted Skewed Student T distribution for comparison to the original returns. The simulated returns should be close to the actual data if the stocks have joint dependence.

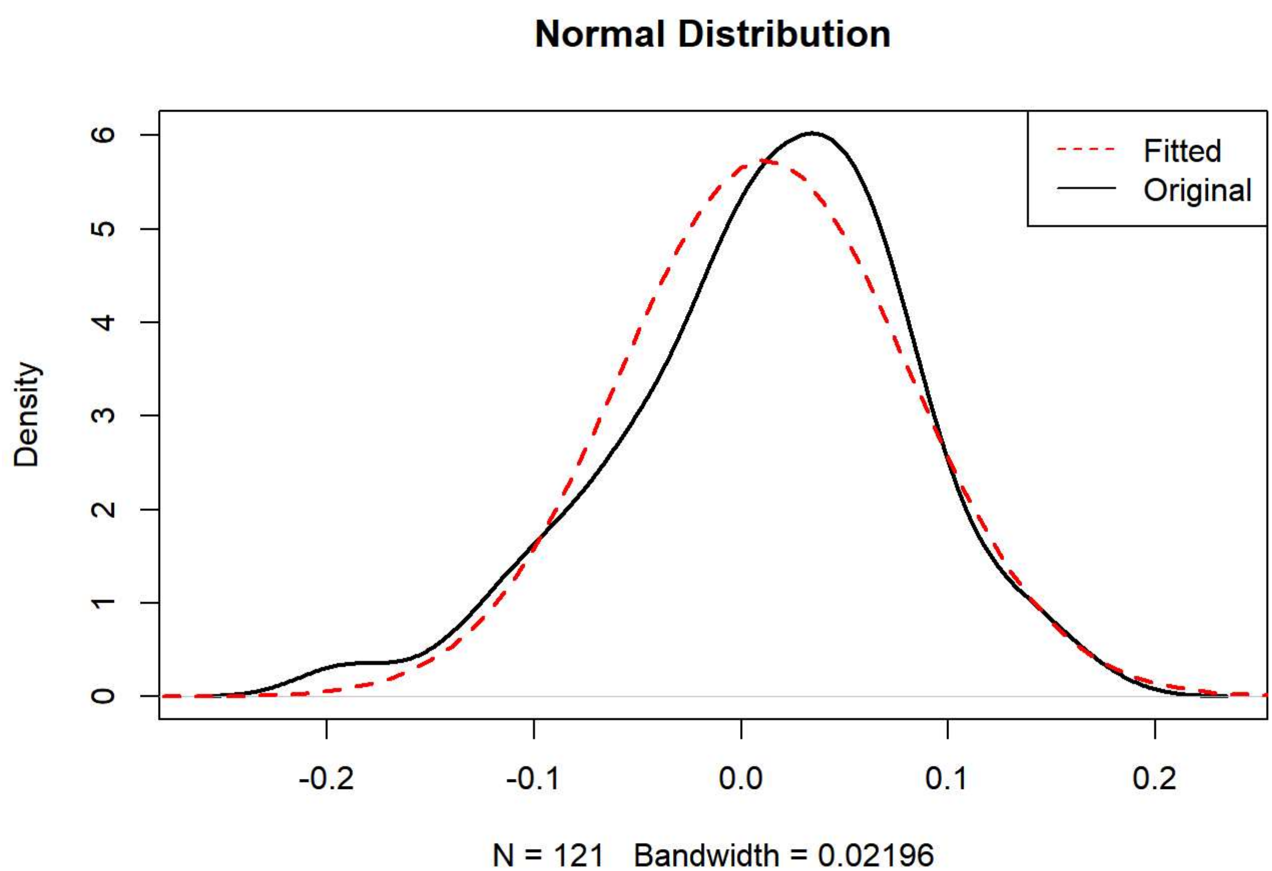
a. Marginals

BlackRock:

Normal Distribution

```
## Fitting normal distribution to returns
BLK_n <- fitdistr(x1,"normal")
BLK_para_n <- BLK_n$estimate
## calculating AIC & BIC
AIC_BLK_n <- 2*(2)-(2*(BLK_n$loglik))
BIC_BLK_n <- -2*(BLK_n$loglik)+(log(length(x1))*2)
```

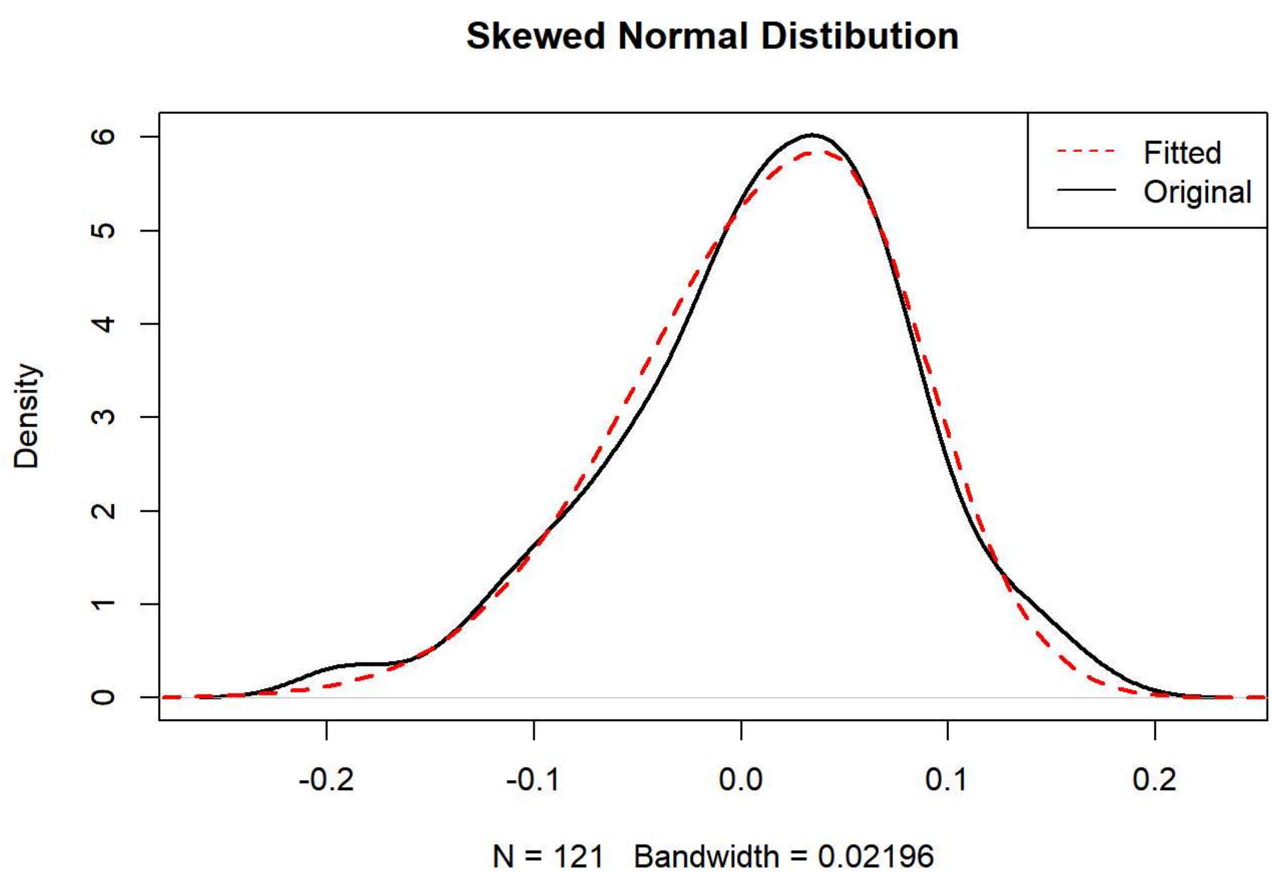
```
x = seq(-20,20,by=0.01 )
plot(density(x1),lwd =2, main = "Normal Distribution", col = "black")
lines(x,dnorm(x,mean = BLK_para_n[1],sd = BLK_para_n[2]),lwd = 2,lty = 2,col="red")
legend(x = "topright",legend = c("Fitted","Original"),col=c("red","black"), lty = c(2,1))
```



Skewed Normal Distribution

```
BLK_sn <- snormFit(x1)
BLK_para_sn <- BLK_sn$par

AIC_BLK_sn <- 2*(2)-(2*(-BLK_sn$objective))
BIC_BLK_sn <- -2*(-BLK_sn$objective)+(log(length(x1))*2)
```

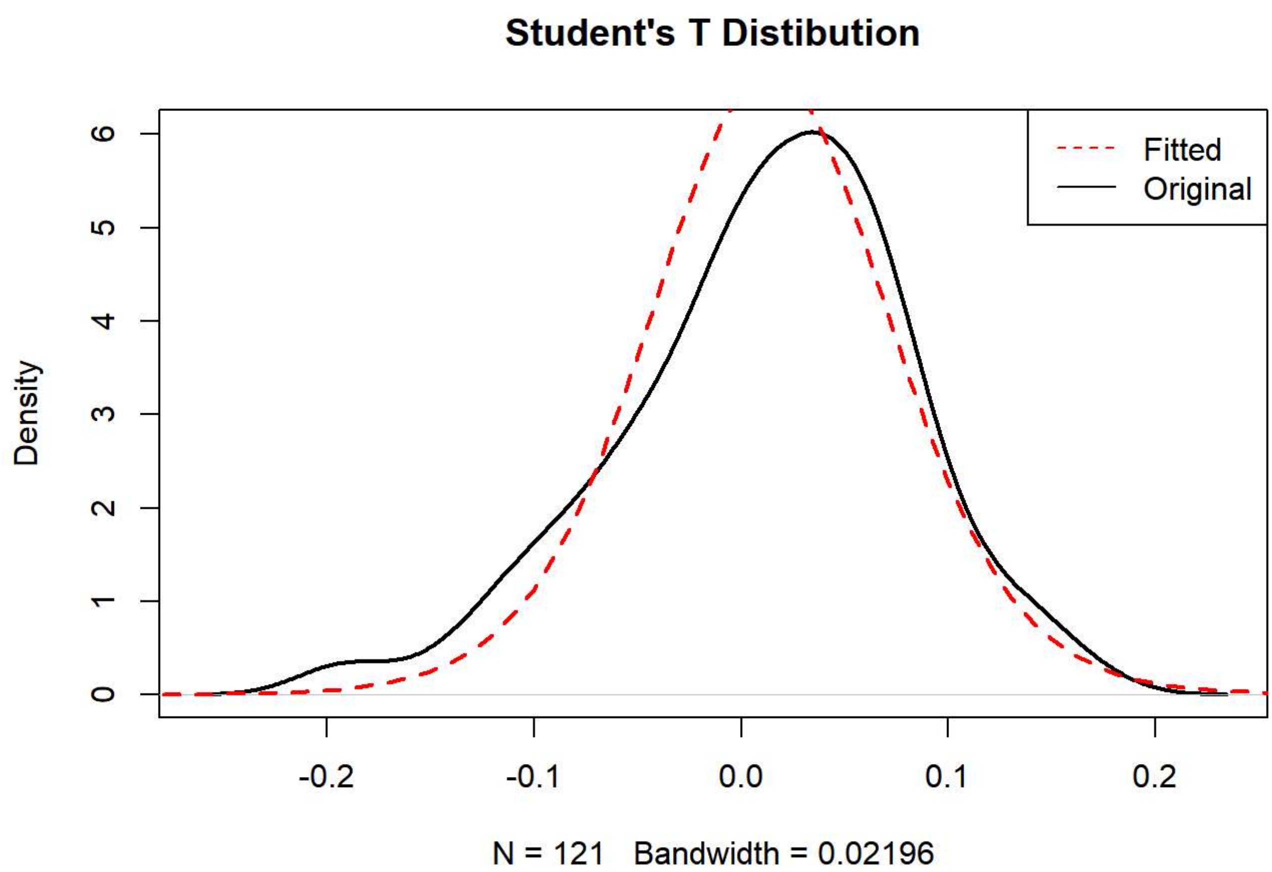
```
plot(density(x1),lwd =2, main = "Skewed Normal Distibution", col = "black")
lines(x,dsnorm(x,mean = BLK_para_sn[1],sd = BLK_para_sn[2],xi = BLK_para_sn[3]),lwd = 2,lty = 2,
col="red")
legend(x = "topright",legend = c("Fitted","Original"),col=c("red","black"), lty = c(2,1))
```



Student's T Distribution

```
BLK_t=fitdistr(x1,"t")
BLK_para_t <- BLK_t$estimate

AIC_BLK_t <- 2*(2)-(2*(BLK_t$loglik))
BIC_BLK_t <- -2*(BLK_t$loglik)+(log(length(x1))*2)
```

```
plot(density(x1),lwd =2, main = "Student's T Distibution", col = "black")
lines(x,dstd(x,mean = BLK_para_t[1],sd = BLK_para_t[2], nu = BLK_para_t[3]),lwd = 2,lty = 2,col=
"red")
legend(x = "topright",legend = c("Fitted","Original"),col=c("red","black"), lty = c(2,1))
```



Skewed Student’s T Distribution

```
BLK_st=sstdFit(x1)
BLK_para_st <- BLK_st$estimate

AIC_BLK_st <- 2*(2)-(2*(-BLK_st$minimum))
BIC_BLK_st <- -2*(-BLK_st$minimum)+(log(length(x1))*2)
```

```
plot(density(x1),lwd =2, main = "Skewed Student's T Distibution", col = "black")
lines(x,dsstd(x,mean = BLK_para_st[1],sd = BLK_para_st[2], nu = BLK_para_st[3],xi = BLK_para_st[
4]),lwd = 2,lty = 2,col="red")
legend(x = "topright",legend = c("Fitted","Original"),col=c("red","black"), lty = c(2,1))
```

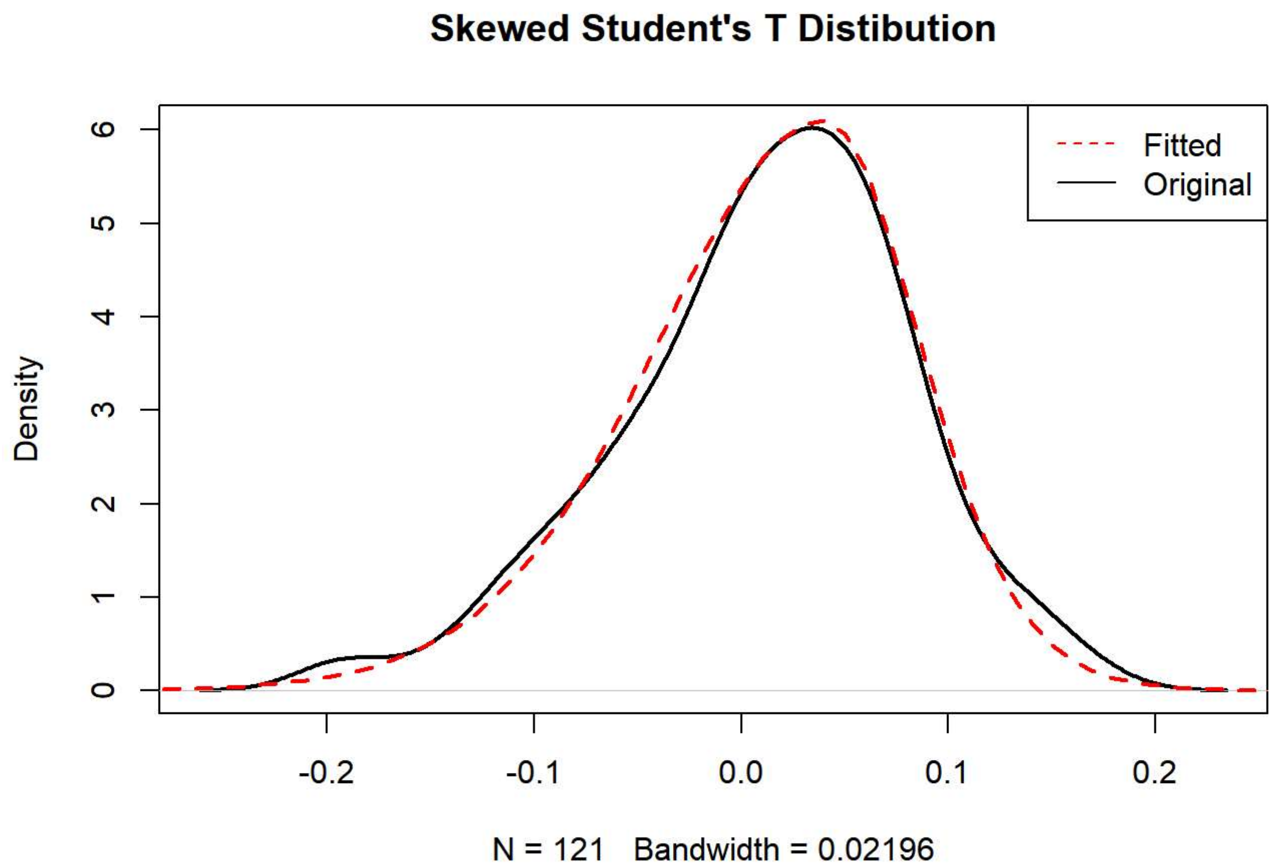


Table 1.

```
## creating table to store estimators
BLK_goodness<-matrix(nrow = 4, ncol = 2)
BLK_goodness[1,]=c(AIC_BLK_n,BIC_BLK_n)
BLK_goodness[2,]=c(AIC_BLK_sn,BIC_BLK_sn)
BLK_goodness[3,]=c(AIC_BLK_t,BIC_BLK_t)
BLK_goodness[4,]=c(AIC_BLK_st,BIC_BLK_st)
BLK_goodness<-data.frame(BLK_goodness)
names<-c("Normal", "Skewed Normal", "Student T", "Skewed Student T")

row.names(BLK_goodness) <- names
names(BLK_goodness) <- c("AIC","BIC")
BLK_goodness<-round(BLK_goodness,2)

htmlTable(BLK_goodness, caption = "BlackRock Fitting Results")
```

BlackRock Fitting Results

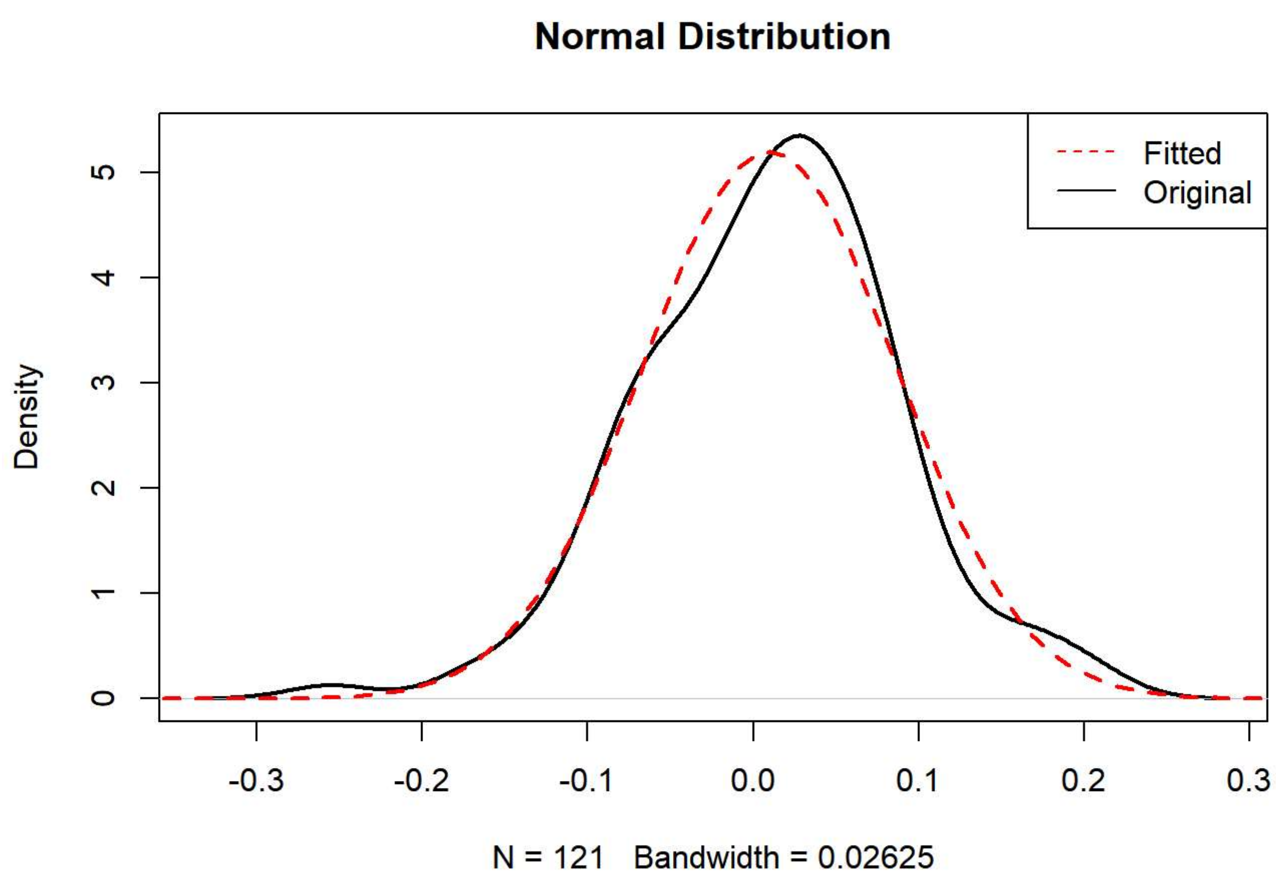
	AIC	BIC
Normal	-297.88	-292.29
Skewed Normal	-302.8	-297.21
Student T	-298.44	-292.85
Skewed Student T	-303.1	-297.51

Goldman Sachs:

Normal Distribution

```
GS_n<-fitdistr(x2,"normal")
GS_para_n <- GS_n$estimate

AIC_GS_n <- 2*(2)-(2*(GS_n$loglik))
BIC_GS_n <- -2*(GS_n$loglik)+(log(length(x2))*2)
```

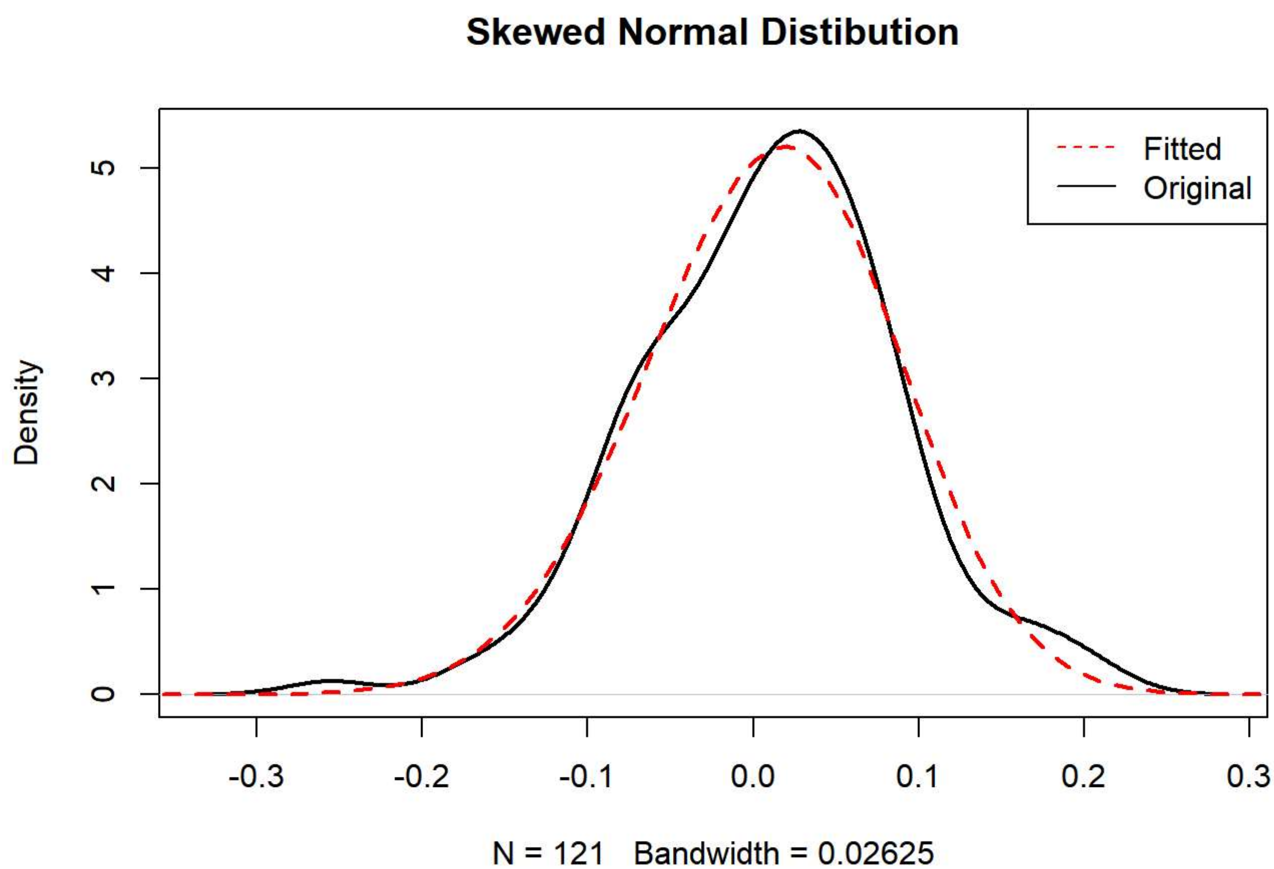
```
plot(density(x2),lwd =2, main = "Normal Distribution", col = "black")
lines(x,dnorm(x,mean = GS_para_n[1],sd = GS_para_n[2]),lwd = 2,lty = 2,col="red")
legend(x = "topright",legend = c("Fitted","Original"),col=c("red","black"), lty = c(2,1))
```



Skewed Normal Distribution

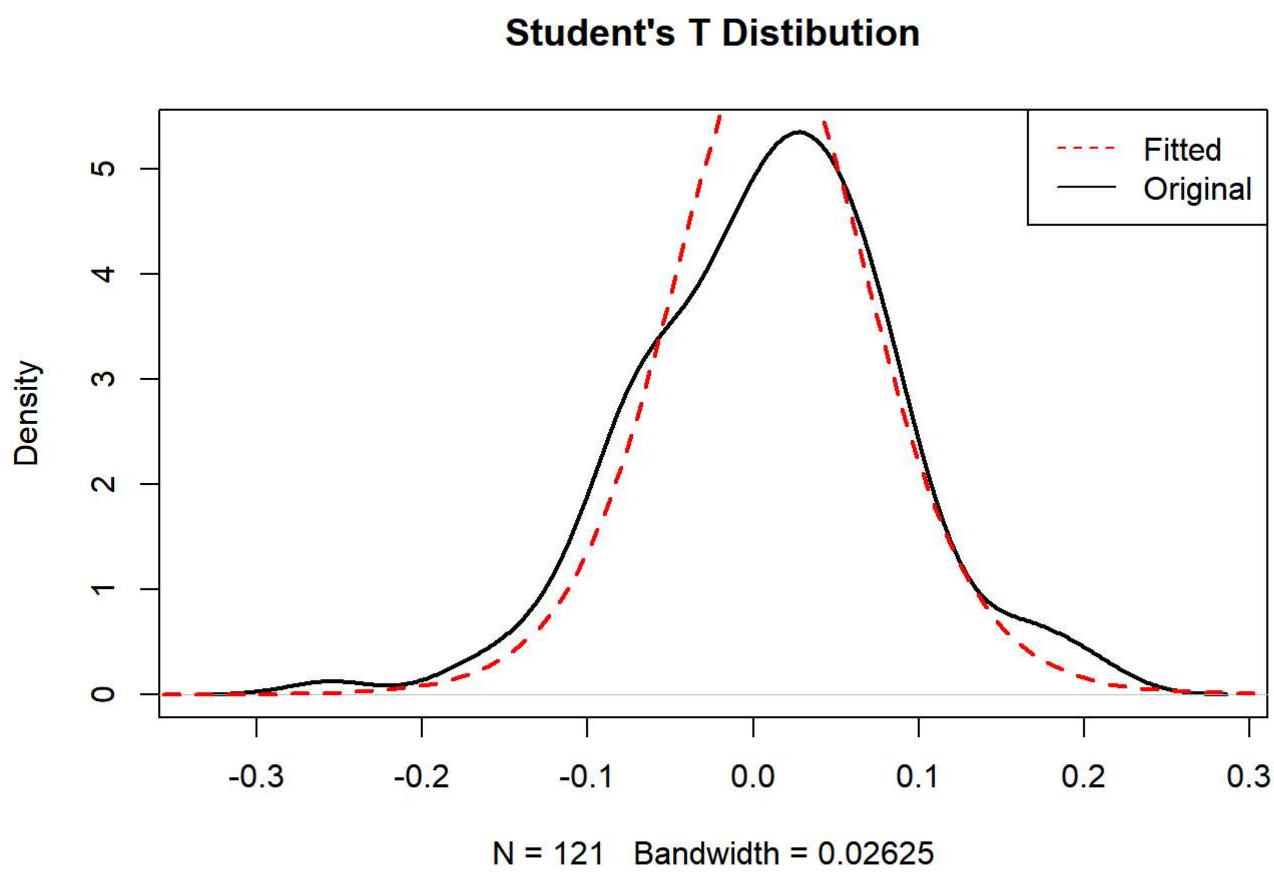
```
GS_sn <- snormFit(x2)
GS_para_sn <- GS_sn$par


AIC_GS_sn <- 2*(2)-(2*(-GS_sn$objective))
BIC_GS_sn <- -2*(-GS_sn$objective)+(log(length(x2))*2)
```

```
plot(density(x2),lwd =2, main = "Skewed Normal Distibution", col = "black")
lines(x,dsnorm(x,mean = GS_para_sn[1],sd = GS_para_sn[2],xi = GS_para_sn[3]),lwd = 2,lty = 2,col
="red")
legend(x = "topright",legend = c("Fitted","Original"),col=c("red","black"), lty = c(2,1))
```



Student T Distribution

```
plot(density(x2),lwd =2, main = "Student's T Distibution", col = "black")
lines(x,dstd(x,mean = GS_para_t[1],sd = GS_para_t[2], nu = GS_para_t[3]),lwd = 2,lty = 2,col="re
d")
legend(x = "topright",legend = c("Fitted","Original"),col=c("red","black"), lty = c(2,1))
```



Skewed Student T Distribution

```
GS_st=sstdFit(x2)
GS_para_st <- GS_st$estimate

AIC_GS_st <- 2*(2)-(2*(-GS_st$minimum))
BIC_GS_st <- -2*(-GS_st$minimum)+(log(length(x2))*2)
```

```
plot(density(x2),lwd =2, main = "Skewed Student's T Distibution", col = "black")

lines(x,dsstd(x,mean = GS_para_st[1],sd = GS_para_st[2], nu = GS_para_st[3],xi = GS_para_st[4]),
lwd = 2,lty = 2,col="red")

legend(x = "topright",legend = c("Fitted","Original"),col=c("red","black"), lty = c(2,1))
```

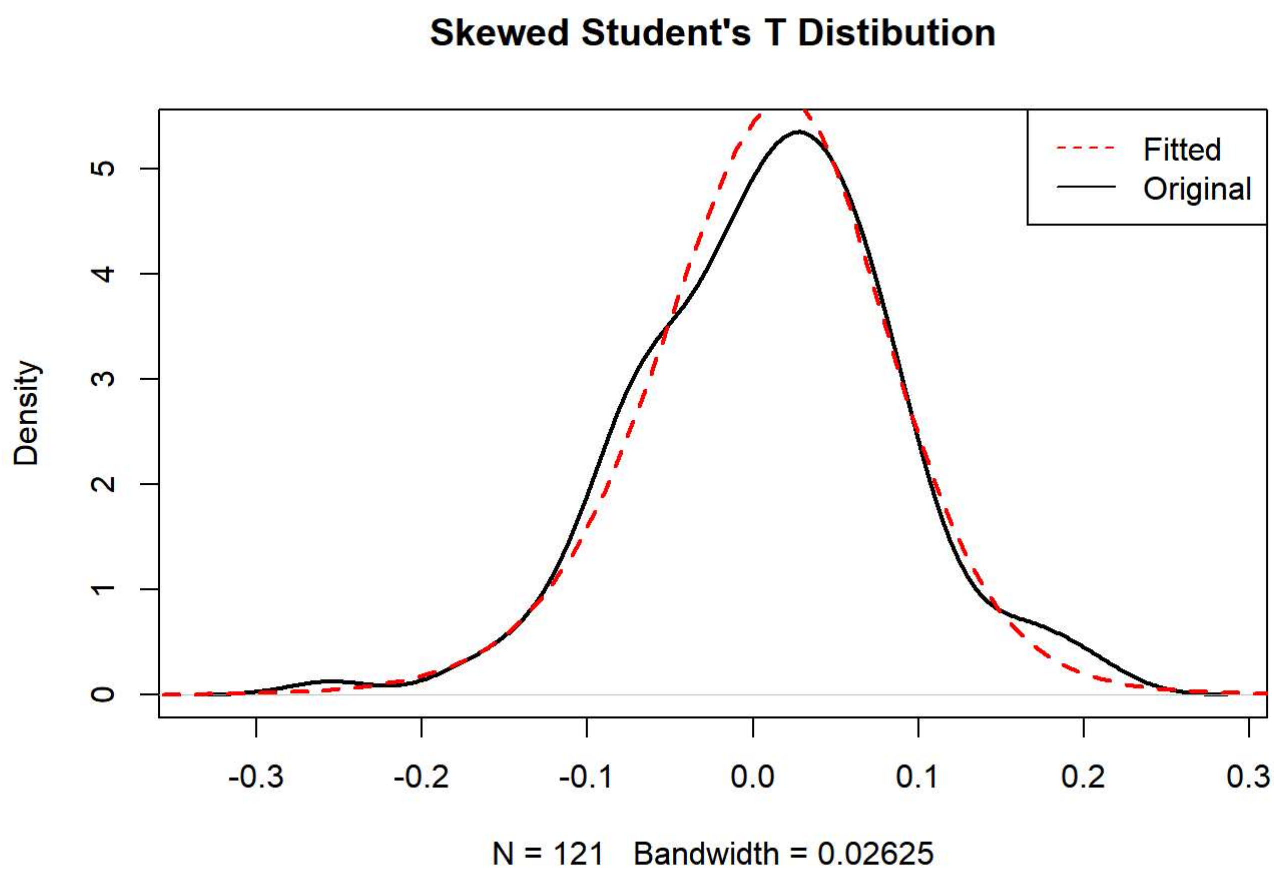


Table 2.

Goldman Sachs Fitting Results

	AIC	BIC
Normal	-274.03	-268.44
Skewed Normal	-274.56	-268.97
Student T	-275.79	-270.2
Skewed Student T	-276.26	-270.67

Table 3.

Skewed T Optimal Parameters

	BLK	GS
Location: mean	0.0112	0.0096
Scale: sd	0.0695	0.0769
Shape: nu	19.2244	9.9449
Skewness: xi	0.7578	0.9151

b. Copulas

```
###### uniform marginal distributions for skewed t
u1<-psstd(x1,mean = BLK_para_st[1],sd = BLK_para_st[2],nu = BLK_para_st[3],xi = BLK_para_st[4])

u2<-psstd(x2,mean = GS_para_st[1],sd = GS_para_st[2],nu = GS_para_st[3],xi = GS_para_st[4])
cor(x1,x2)
```

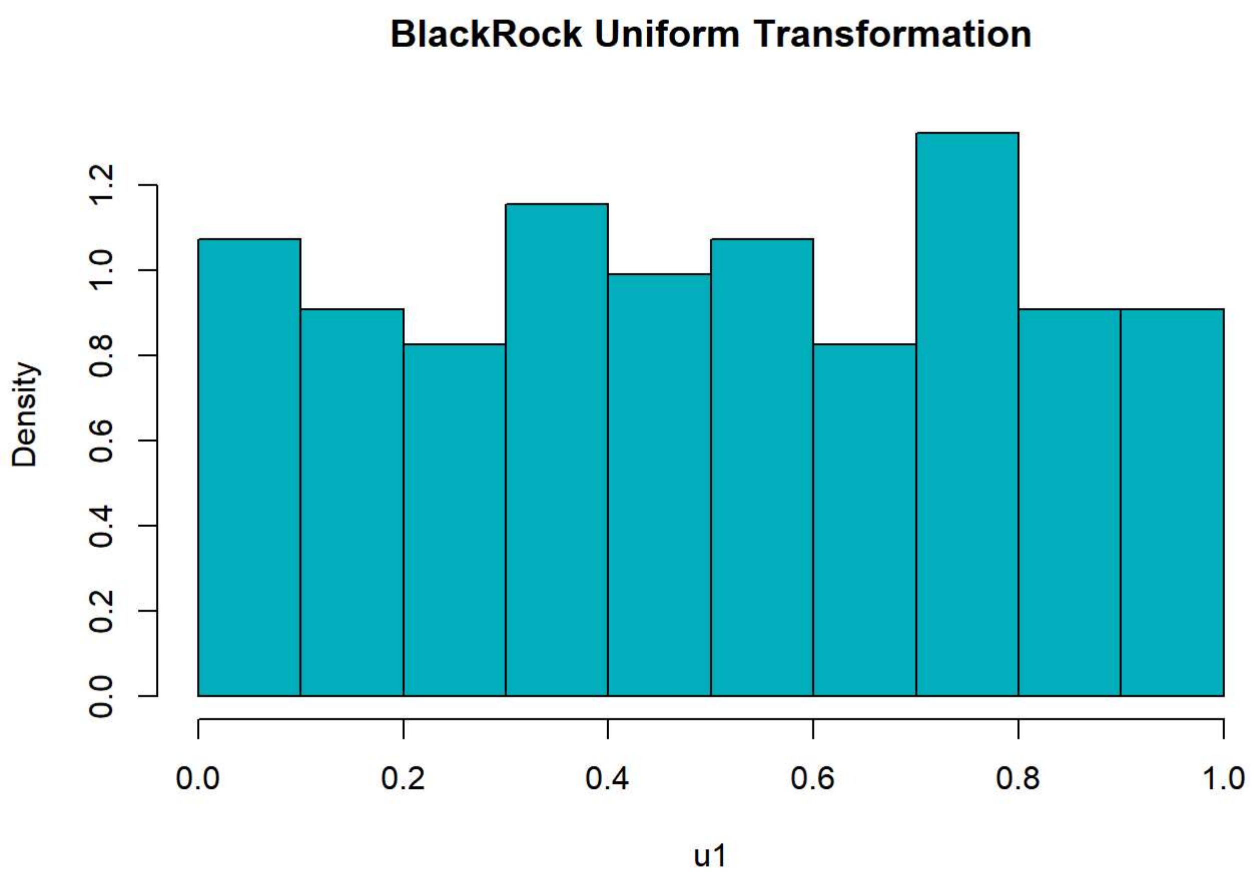
```
## [1] 0.6753745
```

```
cor(u1,u2)
```

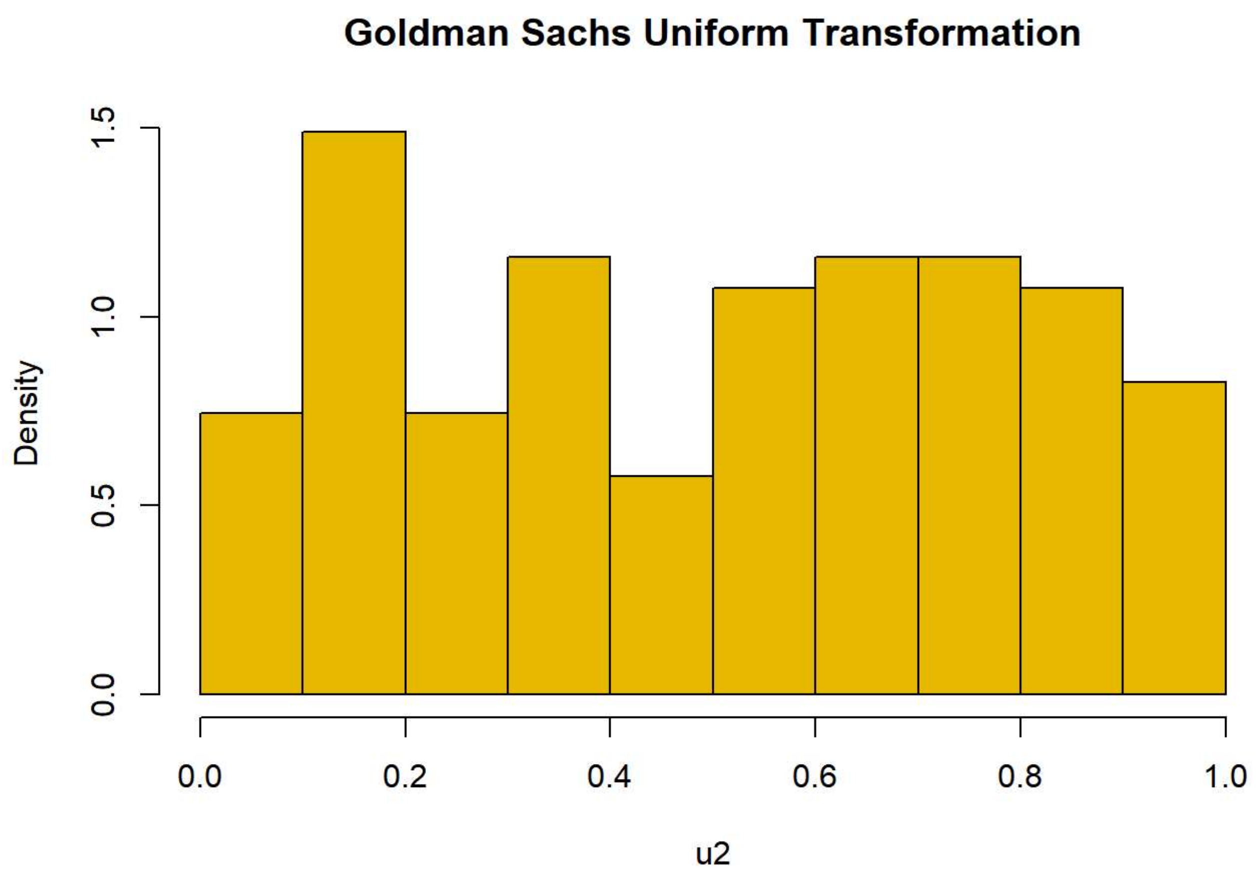
```
## [1] 0.6924305
```

Uniform Distribution Figures

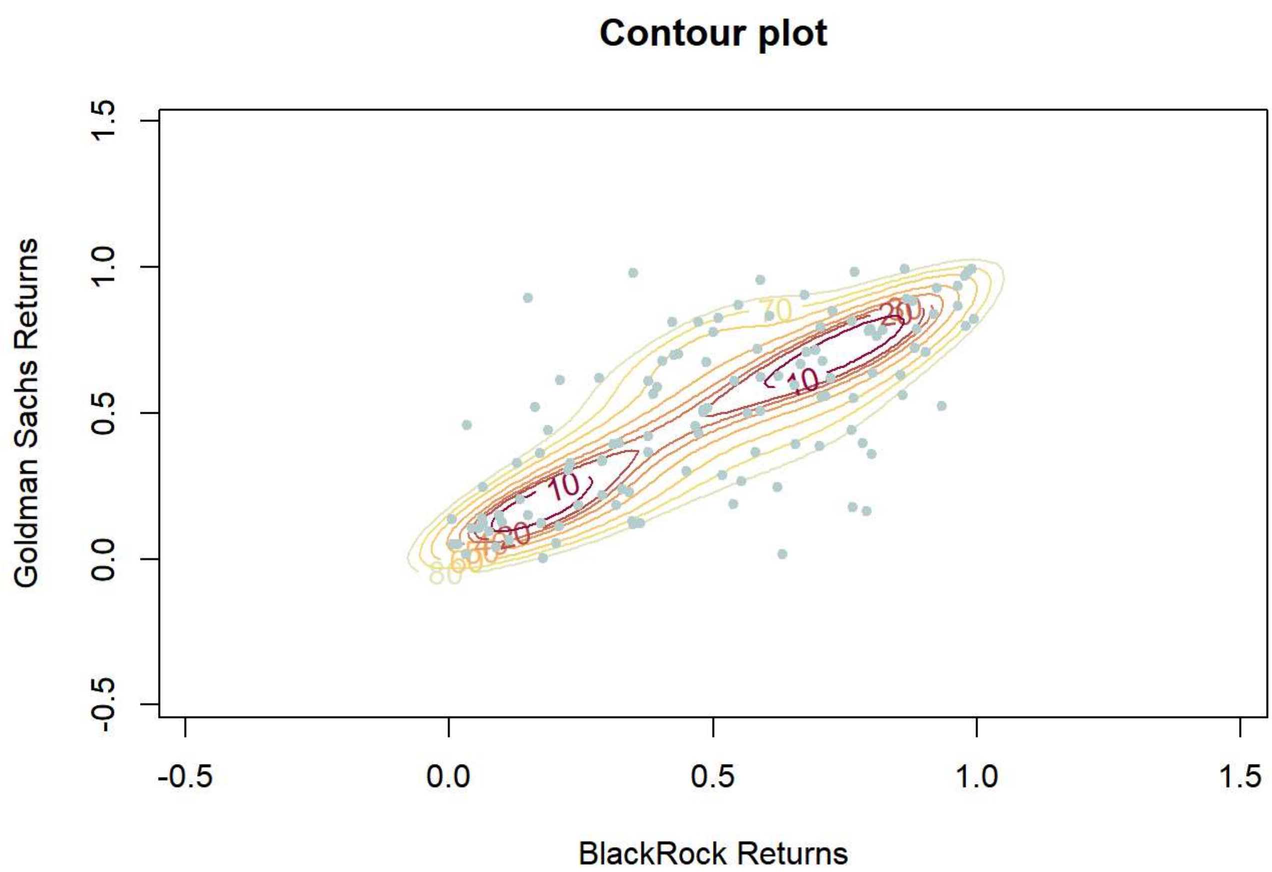
```
hist(u1,probability = TRUE,col = "#00AFBB",main = "BlackRock Uniform Transformation")
```




```
hist(u2,probability = TRUE,col = "#E7B800",main = "Goldman Sachs Uniform Transformation")
```




```
U.hat<-cbind(u1,u2)
### Non-Parametric Density estimation plot
fhatU=kde(x=U.hat,H=Hscv(x=U.hat))
plot(fhatU,cont=seq(10,80,10), main = "Contour plot",xlab = "BlackRock Returns", ylab = "Goldman
Sachs Returns")
points(u1,u2, col = "lightcyan3",pch =20)
```




```
## Stock Correlations
tau=cor.test(as.numeric(u1),as.numeric(u2),method="kendall")$estimate
r=cor.test(as.numeric(u1),as.numeric(u2),method="pearson")$estimate
## estimator for spearman's rho used in copulas
omega=sin((tau*pi)/2)
```

Table 4.

Correlation Table of Marginal Returns

	Value
Kendall	0.51
Pearson	0.69
Spearman	0.72

Marginal relationship appears strong between BlackRock and Goldman Sachs.

Gaussian Copula

```
Cgauss <- fitCopula(copula=normalCopula(dim=2),data=U.hat,method="ml",start=c(omega))
guass_para<-coef(Cgauss)

guass_loglike <- loglikCopula(param=guass_para,u=U.hat,copula=normalCopula(dim=2))
guass_AIC <- -2*guass_loglike + 2*length(guass_para)
```

T - Copula

```
Ct<-fitCopula(copula=tCopula(dim=2),data=U.hat,method="ml",start=c(omega,10))
t_para<-coef(Ct)

t_loglik<-loglikCopula(param=t_para,u=U.hat,copula=tCopula(dim=2))
t_AIC<- -2*t_loglik + 2*length(t_para)
```

Frank Copula

```
Cfr <- fitCopula(copula=frankCopula(1,dim=2),data=U.hat,method="ml")
fr_para<-coef(Cfr)

fr_loglik <- loglikCopula(param=fr_para,u=U.hat,copula=frankCopula(dim=2))
fr_AIC <- -2*fr_loglik + 2*length(fr_para)
```

Joe Copula

```
Cjoe <- fitCopula(copula=joeCopula(dim=2),data=U.hat,method="ml")
joe_para<-coef(Cjoe)

joe_loglik <- loglikCopula(param=joe_para,u=as.matrix(U.hat),copula=joeCopula(dim=2))
joe_AIC <- -2*joe_loglik + 2*length(joe_para)
```

Gumbel Copula

```
Cgum <- fitCopula(copula=gumbelCopula(dim=2),data=U.hat,method="ml",start=1)
gum_para<-coef(Cgum)

gum_loglik <- loglikCopula(param=gum_para,u=as.matrix(U.hat),copula=gumbelCopula(dim=2))
gum_AIC <- -2*gum_loglik + 2*length(gum_para)
```

Table 5.

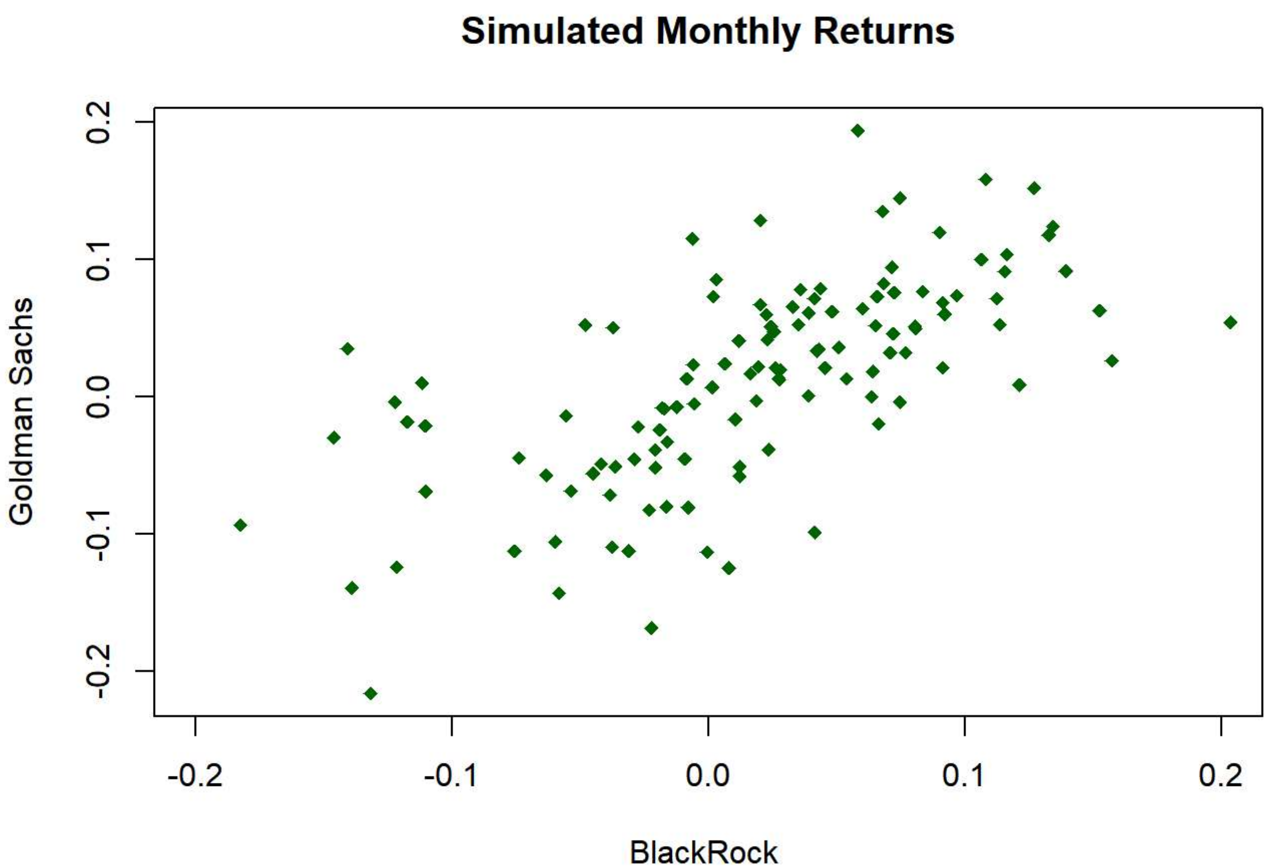
Copula Results

	AIC
Guassian	-75.61
T	-76.13
Frank	-78.08
Joe	-53.72
Gumbel	-71.81

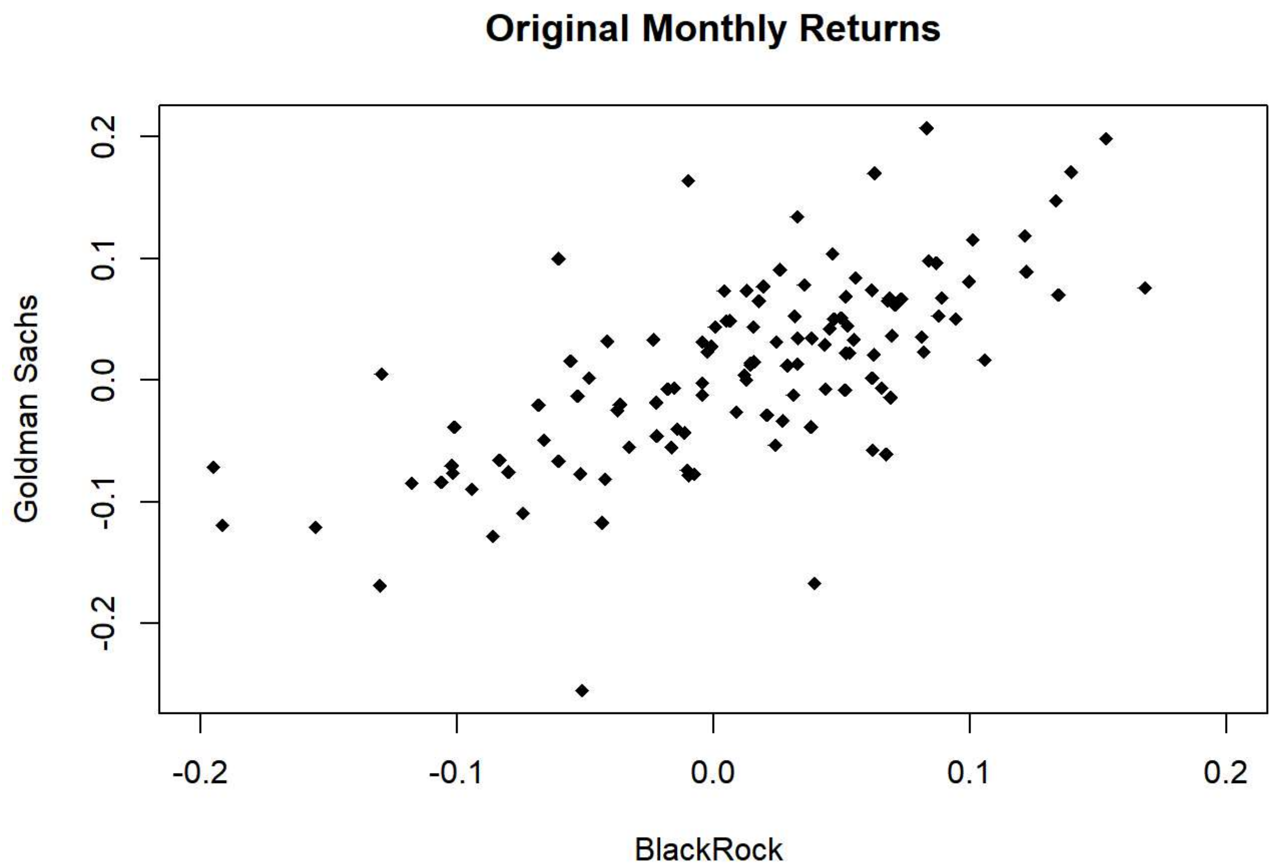
c. Simulated Sample

```
#generate a random sample with size n from the fitted frank copula
Sim_prob<- rCopula(copula=frankCopula(param = fr_para),n=length(x1))
Sim_prob <-data.frame(Sim_prob)
names(Sim_prob)<-c("BLK","GS")

#transform marginals into the estimated skewed t
Sim_BLK <- qsstd(Sim_prob[,1],mean = BLK_para_st[1],sd = BLK_para_st[2],nu = BLK_para_st[3],xi = BLK_para_st[4])

Sim_GS <- qsstd(Sim_prob[,2],mean = GS_para_st[1],sd = GS_para_st[2],nu = GS_para_st[3],xi = GS_para_st[4])
```

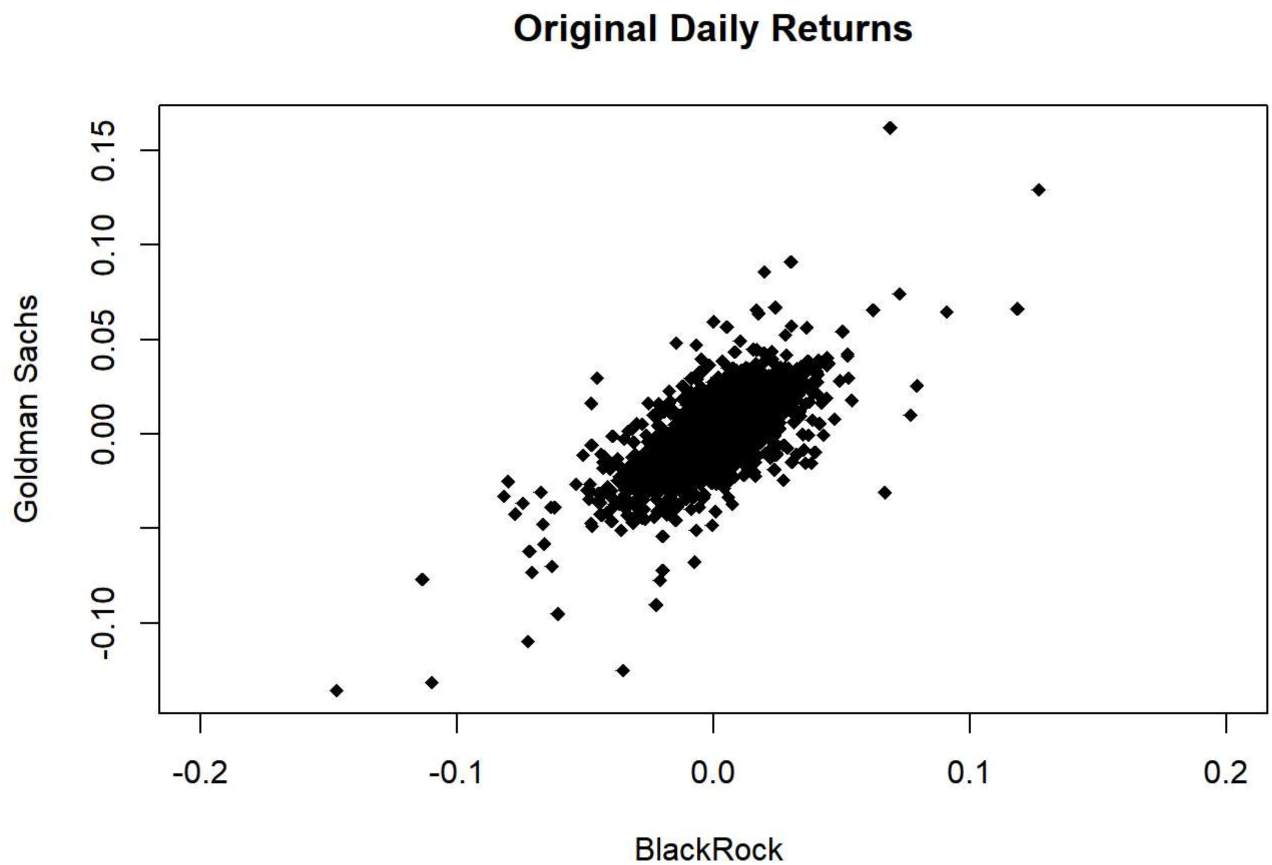
```
plot(Sim_BLK,Sim_GS, main = "Simulated Monthly Returns" ,xlim = c(-0.2,0.2), pch =18, col = "darkgreen", xlab = "BlackRock ", ylab = "Goldman Sachs")
```




```
plot(x1,x2, main = "Original Monthly Returns",xlim = c(-0.2,0.20) ,pch =18, col = "black", xlab = "BlackRock ", ylab = "Goldman Sachs")
```




```
plot(xd1,xd2, main = "Original Daily Returns" , pch =18,xlim = c(-0.2,0.20), col = "black", xlab
= "BlackRock ", ylab = "Goldman Sachs")
```



3. Conclusion

The results of the simulated monthly returns appear to be close to the actual monthly log returns. Therefore, Blackrock and Goldman Sachs have a dependent relationship since their simulated monthly log returns maintains their joint relationship. However, the simulated log returns and original log returns do not appear to closely match the distribution of daily log returns (clustered more in the center). This is not surprising considering that the daily log returns appear to have smaller values in comparison due to the short time period between its returns. Its variance would most likely be smaller as a result as well. It is important to note also that the positive relationship between BlackRock and Goldman Sachs returns is still maintained graphically even when daily.