

**AN EVALUATION OF APOLLO
POWERED DESCENT GUIDANCE**

A Thesis
Presented to the
Faculty of
San Diego State University

In Partial Fulfillment
of the Requirements for the Degree
Master of Science in Aerospace Engineering
with a Concentration in
Guidance, Navigation, and Controls

by
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Powered Descent Guidance

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ABSTRACT OF THE THESIS

An Evaluation of Apollo
Powered Descent Guidance
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Master of Science in Aerospace Engineering with a Concentration in Guidance, Navigation,
and Controls

San Diego State University, 2018

This is my abstract which describes my whole thesis.

Many extraterrestrial missions require a powered descent phase. Because this phase is late in the mission its fuel efficiency has an outsized effect on payload capacity. This thesis presents a strategy for optimizing fuel use using well tested guidance algorithms that reduces fuel consumption over conventional strategies by x%.

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I would like to thank Dr. Lu for his serendipitous arrival at SDSU and consequent advice and instruction. With his mentoring I have been able to launch an enjoyable and fulfilling career in GN&C, something I could not have achieved otherwise.

CHAPTER 1

INTRODUCTION

You can save yourself 12 minutes now by not reading this document; later on the average payback is 105-fold when you run into problems.

1.1 HISTORY

In the early 1990s Richard Frost put together a \LaTeX style file for SDSU theses based on \LaTeX 2.09. Joe Mahaffy wrote an example thesis to guide students in the use of \LaTeX code. Since that time \LaTeX has been upgraded to $\text{\LaTeX}2\epsilon$ and the formatting requirements for SDSU have also changed. In 2004, Jiri Lebl and Mike O’Sullivan worked on a revision, upgrading to $\text{\LaTeX}2\epsilon$. This $\text{\LaTeX}2\epsilon$ class file basically has almost nothing in common with the old \LaTeX style; it is a modification of the standard report class.

The class file `sdsu-thesis.cls` and this example are/have been maintained by

- Aug 2010 — *current*: Peter Blomgren, <blomgren.peter@gmail.com>.

In September 2010, the “short” example was merged into the “long” example to form this document; this seemed like a reasonable thing to do.

While every effort is/has been made to make sure that the template and thesis style conforms to the *SDSU Thesis Manual*, **no guarantees can be made**. Depending on the thesis reviewer and his/her interpretation of what is Really ImportantTM in the thesis manual, and his/her level of caffeineation some theses fly through, and some get stuck even though they use the same template and style file.

PLEASE let the maintainer know of any and all feedback you get from the thesis reviewer so that the template and/or style file can be updated as necessary. THANK YOU!!!

1.2 PURPOSE

This document illustrates some of the typesetting tasks that are commonly encountered in a thesis containing mathematics¹. All theses must follow the guidelines of the SDSU Thesis Manual for formatting. Most formatting issues will be automatically handled by the \LaTeX class file included with the source file for this document, but there may be some special circumstances that will require some tinkering with spacing, pagebreaks, etc.

¹<http://en.wikibooks.org/wiki/LaTeX> provides a wealth of information regarding \LaTeX , check it out.

\LaTeX is a remarkably powerful package, but it does take some effort to learn. The best plan is to start with something already written and learn from the example. The current document was produced for this purpose. This document illustrates how the student should format the chapters and sections of the thesis, prepare the bibliography, and include other appropriate items commonly found in a technical document. The title page, signature page, acknowledgments page, abstract, and everything else are all formatted according to specifications of the SDSU Thesis Manual of 2004, once you enter the text to include.

For a general reference it is recommended that the student obtain the user's guide and reference manual of Leslie Lamport [11]. The student should obtain copies of the files used to generate this document, then examine the ASCII files used to generate the document and the \LaTeX output. The files for this example thesis are the following.

- `Makefile`: Contains "recipes" for building the thesis; type `type make`, and `make thesis.pdf`.
- `abstract.tex`: Contains the abstract.
- `append.tex`: Contains all the text for appendices.
- `body.tex`: Contains all the text for chapters.
- `sdsu-thesis.cls`: Defines the layout and formatting.
- `thbib.bib`: Contains a bibliographical database.
- `thesis.tex`:
 1. Contains information for the title page, and other front-matter.
 2. Includes the files `abstract.tex`, `body.tex`, and `append.tex`.
 3. Defines the bibliographical style (`siam` in this example) and creates the bibliography using the file `thbib.tex`.
- `cos.eps`, `mapping.eps`, `plot2.eps`, and `somb.eps`: Encapsulated postscript files that are included by `body.tex` (in the subdirectory `Figures/.`)

Positioning and captioning of figures and tables should agree with the thesis manual. Occasionally, \LaTeX does not break when you want it too, so you have to add a `\newpage` command to get the correct break, such as when a section header starts at the bottom of a page or when paragraphs have widows or orphans.

1.3 FORMAT OF THE THESIS

The Department of Mathematics and Statistics wants the student to use a standard technical format. This implies that equations, theorems, definitions, tables, etc. should be numbered $N.M$, where N is the chapter number and M is successively increased through the chapter. \LaTeX does this automatically. Numbering of the equations is on the right side of the

page (default in \LaTeX). The student may use *italics*, SMALL CAPS, or **bold fonts** to highlight important phrases. The code for creating theorems, definitions etc., is illustrated in this example thesis. Positioning and captioning of figures and tables should agree with the thesis manual. Occasionally, \LaTeX does not break when you want it too, so you have to add a `\newpage` command to get the correct break, such as when a section header starts at the bottom of a page or when paragraphs have widows or orphans.

Bibliographical citations are relatively easy. Here is one [1] and another citation [13] and we can't forget Milnor [15]. Look at `thbib.bib` to see how to create the database for the bibliography.

You type new paragraphs by just leaving an empty line between them.

1.4 PROCESSING \LaTeX FILES

The best way to learn \LaTeX is to take advantage of someone else's work from which you can model your document. This pseudo-thesis should give you a good working example to create your own document. The key commands to create any document are the following:

```
\documentclass[options]{class}
\begin{document}
    Insert any text you want in here.
\end{document}
```

where *class* is some class type. For SDSU Thesis you would normally use the `sdsu-thesis` class. When you're writing your thesis and want a draft printout you can also add options such as `savepaper` which will single space your document, and use larger margins. Most postscript viewers will allow you to print only a subset of pages as well. The standard for the final thesis is 1 1/2 space. If you want double spaced then uncomment the `doublespace` option.

README files for the different operating systems accompany this distribution. Here we explain how to use the command line to process \LaTeX on a Unix/Linux system (or with modifications Mac OS X).

To process a \LaTeX document that you have named `filename.tex`, you simply type `latex filename`. For example, this document is generated by its driver file with `latex thesis`. If it is the first time and you have a bibliography, then you need the following sequence of commands:

```
latex filename
bibtex filename
```

```
latex filename
latex filename
```

Unless you have added new references, the `bibtex filename` can be omitted. You will need to execute the `latex filename` command twice if there are any renumbering of items, like equations. When there are errors you can usually hit the carriage return and work through them². Other alternatives include typing either `x` or `q` to allow it to proceed. The errors will be kept in a file called `filename.log`. Be sure to pay attention to comments the \LaTeX produces as it gives you some warnings, such as when you need to make another run due to changes in the numbering of references or equations.

After you have performed the above procedure, you will have a file named `filename.dvi` (or `thesis.dvi` in our case) which is a device independent file. There are several means of viewing your output. If you are working in an Xwindow environment, then the simplest procedure is to type `xdvi filename.dvi`, which will open a window for viewing the \LaTeX document. It will not include any postscript figures, but it is automatically updated each time you `latex` your document. I highly recommend starting with this environment. (If you are on rohan and accessing saturn, then you will need the environment setup described below for ghostview.)

The second procedure for either viewing with ghostview or printing involves the conversion of the `.dvi` file to a postscript file. (You may want to examine `man dvips` for assistance.) The simplest way to convert the `.dvi` file to a postscript file is to type the following:

```
dvips -o filename.ps filename.dvi
```

This creates the postscript file, `filename.ps`. If you do not need the entire document, then you can type:

```
dvips -px -ly -o filename.ps filename.dvi
```

where `x` is the number of the first page and `y` is the number of the last page.

Then to get a hard copy you should use the standard printing commands of your system. If you are on your own Linux system at home, usually `lpr filename.ps` will print the file.

²Do not panic if you get lots of errors; fix the *first* one! The following errors are often due to things being out-of-context due to the first error. **ALWAYS fix the first error before worrying about the others!!! ALWAYS fix the first error before worrying about the others!!!**

In case your computer system has a different paper size set up as default then “letter” you can force a letter paper size by adding `-t letter` as an option to `dvips`. This can happen if you are running in a different language than American English.

The makefile simplifies many of the sequences of commands that you might use. For example, just type `make` to create a postscript file or `make view` to create a postscript and view it using a postscript viewer. Also, `make clean` will remove all the `.log` `.aux` `.ps` `.dvi` files.

CHAPTER 2

MISCELLANEOUS COMMANDS: AN INTRODUCTION TO EQUATIONS, THEOREMS, FIGURES AND TABLES

In this chapter we see how equations, theorems, figures and tables are created, enumerated and referenced. We also play around with lengths of chapter and section headings. For example, this chapter begins with a long chapter heading that must conform to the thesis manual. Later on there is a very long section heading. These examples show how the sdsu thesis class file automatically handles formatting.

2.1 BASIC MATH

You can have fun formulas, such as $x = 7y^x$. If you want the equations displayed you can use two dollar signs, \$\$ to enclose the mathematics, or you can use

```
\begin{equation*}
  math stuff
\end{equation*}
```

as in

```
\begin{equation*}
  \int_{\partial\Omega} \omega = \int_{\Omega} d\omega.
\end{equation*}
```

which produces

$$\int_{\partial\Omega} \omega = \int_{\Omega} d\omega.$$

There are several other ways to display equations. The code for this one (which you can see in `body.tex`) aligns all the equal signs.

$$(x + 2)^3 = (x + 2)(x + 2)^2 \tag{2.1}$$

$$= (x + 2)(x^2 + 4x + 4) \tag{2.2}$$

$$= x^3 + 6x^2 + 12 * x + 8 \tag{2.3}$$

Notice that this last set of equations is numbered, but the previous one is not. The `*` in the \LaTeX code eliminates the numbering.

2.2 EQUATIONS

Enumeration of equations, theorems, definitions, tables, is handled automatically by \LaTeX . Each of these items may be given a label using $\text{\label{\<labelname>}}$. The item can then be referred to by $\text{\ref{\<labelname>}}$. Below we demonstrate how to create and label an equation. Our first is a general differential equation,

$$\dot{x} = f(t, x), \quad x(0) = x_0. \quad (2.4)$$

To see that the numbering is going fine we insert a matrix system as follows:

$$\dot{y} = \begin{bmatrix} a_1 & 0 & \cdots & 0 \\ 0 & a_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & a_n \end{bmatrix} y. \quad (2.5)$$

The numbering is valuable when one wants to refer to the Equations (2.4) and (2.5). Note that when referring to Equation (2.4) you must capitalize the word equation. Also, when you enter a specific equation, figure, or table, *e.g.*, Eqn. (2.4), then you should type a ~ between the word Eqn., Fig., or Table and its labeling number to prevent inappropriate division of the label at the end of a line.

To display an equation without numbering, one uses the `math displaystyle` mode which works as follows:

$$\dot{y} = g(y),$$

which is an autonomous equation in y . The y at the end of the last sentence is in standard math mode. Further information on equations is provided in Appendix A.

2.3 THEOREMS, ETC.

The student needs to highlight important results such as theorems, hypotheses, or definitions. In this section we investigate how \LaTeX handles definitions, theorems, corollaries, etc.

Definition 2.1. *A linear differential equation is asymptotically stable if and only if all eigenvalues, λ , of the operator matrix have negative real part.*

We follow this with a couple of theorems and a corollary.

Theorem 2.1. *If the matrix A in the linear differential equation,*

$$\dot{y} = Ay, \quad y(0) = y_0, \quad (2.6)$$

is symmetric, then the solution of (2.6) is non-oscillatory.

Corollary 2.1. *If the matrix A in (2.6) is symmetric and has negative eigenvalues, then the solution is non-oscillatory and asymptotically stable.*

In order to check how the numbering proceeds we insert here another theorem.

Theorem 2.2. *If the matrix H in the linear differential equation,*

$$\dot{y} = Hy, \quad y(0) = y_0, \quad (2.7)$$

is antisymmetric, then the solution of (2.7) is oscillatory.

The `thesis.tex` also defines environments for `lemma` and `proposition` though you can add more if you wish. For example sometimes it is useful to add an `example` style environment. See the preamble of the document for more information.

2.4 OPTIMAL POWERED DESCENT GUIDANCE LAW

Derivation of an optimal powered descent guidance law begins with a formulation of the State Equation 2.8

$$\dot{x} = f(x) \quad (2.8)$$

The state equations for the 3-dimensional powered descent guidance problem are as follows

$$\dot{r} = V \quad r(t_0) = r_0 \quad (2.9)$$

$$\dot{V} = g(r) + a_T \quad V(t_0) = V_0 \quad (2.10)$$

with terminal constraints at a fixed final time t_t

$$r^*(t_f) = r_f^* \quad (2.11)$$

$$V(t_f) = V_f^* \quad (2.12)$$

where a_T is the thrust acceleration vector. a_T is limited such that

$$0 < a_{min} \leq \|a_T\| \leq a_{max} \quad (2.13)$$

2.4.1 Performance Index

Fuel consumption is related to the thrust acceleration vector by engine parameters represented by some positive constant k

$$\dot{m} = -k\|a_T\| \quad (2.14)$$

A fuel optimal guidance law should therefore use the performance index

$$J = \int_{t_0}^{t_f} \|\mathbf{a}_T\| dt \quad (2.15)$$

Choosing to minimize the square of the total acceleration $\mathbf{a} = \mathbf{g} + \mathbf{a}_T$ gives a performance index

$$J = \frac{1}{2} \int_{t_0}^{t_f} (\mathbf{g} + \mathbf{a}_T)^T (\mathbf{g} + \mathbf{a}_T) dt \quad (2.16)$$

For a constant gravitational acceleration \mathbf{g} , this performance index attempts to minimize $\|\mathbf{a}_T\|^2$. It is not fuel optimal as in Equation 2.15, but it does provide a cost to large thrust accelerations and might be expected to give good fuel performance.

2.4.2 Guidance solution

Choosing the guidance command $\mathbf{u} = \mathbf{g} + \mathbf{a}_T$ and applying optimal control theory results in the following

$$H = \mathbf{p}_r^T \mathbf{V} + \mathbf{p}_V^T \mathbf{u} - \frac{1}{2} \mathbf{u}^T \mathbf{u} \quad (2.17)$$

$$\dot{\mathbf{p}}_r = -\frac{\partial H}{\partial \mathbf{r}} = 0 \implies \mathbf{p}_r = -\mathbf{c}_2$$

$$\dot{\mathbf{p}}_V = -\frac{\partial H}{\partial \mathbf{V}} = -\mathbf{p}_r \implies \mathbf{p}_V = \mathbf{c}_1 + \mathbf{c}_2 t$$

$$\frac{\partial H}{\partial \mathbf{u}} = 0 \implies \mathbf{u} = \mathbf{p}_V = \mathbf{c}_1 + \mathbf{c}_2 t$$

For convenience, let $\tau = t_f - t$

$$\mathbf{u} = \mathbf{k}_1 + \mathbf{k}_2 \tau \quad (2.18)$$

where \mathbf{k}_1 and \mathbf{k}_2 are constant vectors.

Integrating the equations of motion with $\dot{\mathbf{V}} = \mathbf{u}$ then gives

$$\int \dot{\mathbf{V}}(t) dt = \mathbf{k}_1(t - t_0) + \frac{1}{2} \mathbf{k}_2(t - t_0)^2 + \mathbf{V}(t_0) \quad (2.19)$$

$$\int \dot{\mathbf{r}}(t) dt = \frac{1}{2} \mathbf{k}_1(t - t_0)^2 + \frac{1}{6} \mathbf{k}_2(t - t_0)^3 + \mathbf{V}(t_0)(t - t_0) + \mathbf{r}(t_0) \quad (2.20)$$

Setting $t = t_f$ and letting $t_{go} = t_f - t_0$ satisfies the terminal constraints from Equations 2.11 and 2.12, resulting in 6 linear equations in 6 unknowns

$$\mathbf{k}_1 t_{go} + \frac{1}{2} \mathbf{k}_2 t_{go}^2 = \mathbf{V}_f^* - \mathbf{V}_0 \quad (2.21)$$

$$\frac{1}{2} \mathbf{k}_1 t_{go}^2 + \frac{1}{6} \mathbf{k}_2 t_{go}^3 = \mathbf{r}_f^* - \mathbf{r}_0 - \mathbf{V}_0 t_{go} \quad (2.22)$$

These equations can be separated into sets of two per vector component. Define an inertial guidance frame $\mathbf{e} = (\hat{x}, \hat{y}, \hat{z})^T$ such that guidance vector \mathbf{u} is composed of components in \mathbf{e} , $\mathbf{u} = (u_x, u_y, u_z)^T$. For the equations in \hat{x} we have

$$\begin{bmatrix} t_{go} & \frac{1}{2}t_{go}^2 \\ \frac{1}{2}t_{go}^2 & \frac{1}{6}t_{go}^3 \end{bmatrix} \begin{pmatrix} k_{1x} \\ k_{2x} \end{pmatrix} = \begin{pmatrix} V_{fx}^* - V_{0x} \\ r_{fx}^* - (r_{0x} + V_{0x}t_{go}) \end{pmatrix} \quad (2.23)$$

Solving the two-equation system is accomplished by inverting the A matrix, leading to a coefficient matrix E

$$E = \begin{bmatrix} -2/t_{go} & 6/t_{go}^2 \\ 6/t_{go}^2 & -12/t_{go}^3 \end{bmatrix} \quad (2.24)$$

The coefficients in \hat{x} are then

$$\begin{pmatrix} k_{1x} \\ k_{2x} \end{pmatrix} = E \begin{pmatrix} V_{fx}^* - V_{0x} \\ r_{fx}^* - (r_{0x} + V_{0x}t_{go}) \end{pmatrix} \quad (2.25)$$

It can be shown that the equations in \hat{y} and \hat{z} take the same form. This 2×2 E matrix is the origin of the name *E-Guidance*, the guidance law used in the Apollo lunar landing missions.

2.4.3 Time-to-go

The E-Guidance solution depends upon a reliable estimate of remaining time-to-go (t_{go}). The Apollo mission's guidance used an estimate that updated continuously using Newton's method, but it was intended to only operate until start of the terminal descent phase at which point guidance switched to a manual vertical descent operation. Updating the t_{go} estimate continuously is attractive since it should be robust; if conditions have to change during the mission a closed-loop (continuously updating) solution will adjust and a new, realistic t_{go} will feed into the guidance solution. This quality was important to the Apollo Guidance solution because it relied upon pilot inputs to define the landing location visually, which meant allowing for landing site redesignations mid-mission. If t_{go} was not recomputed after site redesignation, the guidance law would impose unrealizable thrust acceleration commands.

For the purposes of this study, live landing site redesignation was not considered. Without the possibility of landing site redesignation, an open-loop t_{go} solution lends the guidance law more stability in that the performance is less dependent upon specific assumptions and conditions imposed by the t_{go} algorithm. For instance, one closed-loop t_{go} algorithm is implemented in Algorithm 1.

Each guidance update uses the previous update's t_{go} minus clock time as its initial guess t_{go0} , and the max iterations may be limited to some reasonable number.

Algorithm 1 Fixed-Point-Iteration t_{go}

```

1: procedure FPI( $t_{go1}$ )
2:    $tol \leftarrow c$ 
3:   while  $|t_{go0} - t_{go1}| \geq tol$  do
4:      $t_{go0} \leftarrow t_{go1}$ 
5:      $\Delta V \leftarrow \sqrt{(\mathbf{V} - \mathbf{V}_0 + \mathbf{g} \cdot t_{go})^T (\mathbf{V} - \mathbf{V}_0 + \mathbf{g} \cdot t_{go0})}$ 
6:      $t_{go1} \leftarrow \frac{m_0}{\dot{m}} \left( e^{\frac{-\Delta V}{v_{ex}}} - 1 \right)$   $\triangleright \dot{m} < 0$ 
7:   end while
8:   return  $t_{go1}$ 
9: end procedure

```

This algorithm requires an assumption about a fixed mass flow rate \dot{m} which is not guaranteed by the guidance law. Adjustment of this mass flow rate estimate is very particular to the initial conditions of the mission, resulting in a necessarily conservative t_{go} to account for initial condition dispersion.

One attractive open-loop option is the time to perform a gravity turn landing at maximum thrust. The equation for a gravity turn time-to-go, $t_{go_{GT}}$, was presented first in Cherry 1964 [5]. After engine ignition and initiation of the Powered Descent Guidance, the live time-to-go is computed as $t_{go_{GT}}$ minus elapsed clock time.

2.5 FIGURES OR HOW TO GET INTO REAL TROUBLE IF YOU TAKE ADVANTAGE OF WHAT L^AT_EX CAN DO

This section shows how to display figures and refer to them in the text. L^AT_EX does have the ability to insert postscript files using the `graphicx` package. Make sure to include `\usepackage{graphicx}` in your preamble, that is between the L^AT_EX commands `\documentclass` and `\begin{document}`. See http://en.wikibooks.org/wiki/LaTeX/Importing_Graphics for information about importing graphics into your document.

To insert a figure that is formatted in encapsulated postscript, which must include a Bounding Box line which is named `fname.ps` you do the following:

```

\begin{figure}[ht]
  \includegraphics[width=\linewidth]{fname.eps}
  \caption{Insert a caption here. \label{figlabel}}
\end{figure}

```

to produce the figure. The `[ht]` argument to the figure command is a *suggestion* to L^AT_EX to put

the figure [h]ere, or at the [t]op of the page; [p] for a separate page is also possible. Avoid putting tables and figures at the [b]ottom of the page as this is frowned upon by the thesis manual; the preference is to put tables and figures right after they are first referenced, *i.e.* [h]ere, but at the [t]op of the following page is acceptable in cases where it does not fit [h]ere. You can make the suggestion stronger by saying [h!] for “[h]ere!,” but the internal rules may still override your suggestion. “\linewidth” above can be replaced by some number of inches (or other size L^AT_EX size measure such as pt, em, or ex). This will left justify the figure. Centering is a little more complicated. We place everything in a minipage environment:

Style note

NEVER put anything in the margin like this!!!

```
\begin{figure}[ht]
  \centering
  \begin{minipage}{xin}
    \includegraphics[width=\linewidth]{fname.ps}
    \caption{Insert a caption here. \label{figlabel} }
  \end{minipage}
\end{figure}
```

To demonstrate how the department would like to see figures in the thesis the following is provided. If you are examining these files with xdv, you will only see a blank spot. However, both printed and ghostview methods described in the previous chapter will allow viewing. Suppose that we create a figure to graph the curve

$$y = \sin(\omega t), \quad (2.26)$$

where ω is the circular frequency. Figure ?? is a graph of Equation (2.26), and figure ?? is an illustration of a mapping in the complex plane. The interval of time viewed is $t \in [-5, 5]$. The figure reference should be denoted by either Fig. ?? or by Figure ?? with specific figures capitalized as noted here.

When you have a collection of figures and large figures, you may want to delay insertion of them until the end of the chapter. At the end of this chapter we are including a full page figure (Fig. ??) to demonstrate this L^AT_EX command. Note that if you cannot obtain postscript figures or are having too much trouble using the technique described above, then you can use the \vspace command to provide an empty space in the manuscript, then use the old-fashioned technique of taping in your figure and photocopying it.

2.6 TABLES

The Department of Mathematical Sciences does not have specific requirements on the exact layout of a table. However, the tables should be easily readable and properly labeled according to the regulations in the SDSU Thesis Manual. In this section we want to demonstrate how L^AT_EX handles tables. More complicated examples can be found in

Lamport's book [11, 12]. We begin with a small table, given by Table 2.1 which inserts nicely into the text. Note that the same centering trick as was employed for figures is done here and we set the width of the minipage environment to 1.9 inches.

Table 2.1. A Small Table for Listing Some Parameters Used in Some Numerical Procedure. LONG CAPTION— The Department of Mathematical Sciences does not have specific requirements on the exact layout of a table. However, the tables should be easily readable and properly labeled according to the regulations in the SDSU Thesis Manual.

Trial	a	b	c	ω
1	5	10	15	π
2	10	20	15	2π

The manual however allows for the caption to be a little wider if the table is really small and so we can use a wider minipage and then center the table inside there. See for example Table 2.2 where we used width of 3.5 inches.

Style note

Table 2.2. Another Small Table for Listing Some Parameters Used in a Numerical Procedure.

Trial	a	b	c	ω
1	5	10	15	π
2	10	20	15	2π

Note that you can use the center environment instead of `\centering` but that might add a little bit of unwanted whitespace. With `\centering` on the other hand, you might have to put braces around the text you wish to center and sometimes need to add a `\par`. If you use it inside a minipage, table or figure environment, you don't have to really worry about that. Note however that without the use of minipage you cannot center the caption as it automatically left aligns itself to conform with the thesis manual.

Tables can also be left aligned see for example Table 2.3. Here we don't use the `minipage` environment, but we must then add linebreaks so that the table caption does not go wider then the table itself. We need to add then two titles, one for the list of tables and one for the caption here. The former will not have line breaks and the latter will.

**Table 2.3. Another Such
Table but Left Aligned**

Trial	a	b	c	ω
1	5	10	15	π
2	10	20	15	2π

Sometimes a table might not fit onto a single page, in this case you must not use the `table` environment, but instead the `longtable` environment. Do note that `longtable` automatically centers so you need not worry about that. See Table 2.4 for some absolutely random numbers. To use `longtable` environment you must include the `longtable` package in your preamble. **see the note in `thesis.tex` on how to fix the `longtable` entries in the “List of Tables” if they are incorrect.**

Table 2.4. A Table of Some Totally Random Numbers

First	Second	Third
$16883.20050 \times 64.19591$	23174^{2905}	(5112, 5468, 27117)
$7216.3398 \times 12239.16770$	19961^{9127}	(16136, 21997, 26051)
$15977.29588 \times 5732.19698$	14995^{26728}	(28634, 14278, 17183)
$24699.2338 \times 8803.18474$	19221^{28853}	(18539, 6044, 19259)
$21444.11156 \times 24727.15793$	18372^{28126}	(28032, 2375, 15319)
$4391.18511 \times 4548.30442$	1720^{1369}	(3406, 21419, 16364)
$30135.17285 \times 30643.14550$	9216^{213}	(23353, 27690, 19435)
$19438.13461 \times 25479.5929$	2137^{3868}	(30657, 17930, 22240)
$26015.13194 \times 24615.8566$	17585^{10358}	(13114, 15259, 12079)
$14483.18666 \times 730.30848$	16033^{18015}	(28723, 30583, 27231)
$28936.21168 \times 22153.15603$	7838^{2847}	(8315, 13767, 4984)
$12183.11656 \times 22915.1655$	4903^{3341}	(26271, 13469, 20927)
$3861.26584 \times 3418.15940$	8299^{22084}	(16670, 6379, 5349)
$1917.2334 \times 3164.29148$	31271^{24332}	(18534, 14106, 32170)
$21381.22421 \times 13170.26365$	1836^{24826}	(16512, 3492, 29730)

(table continues)

Table 2.4 (Continued)

First	Second	Third
$19854.29763 \times 10431.8013$	856^{4247}	(11431, 16797, 12547)
$748.699 \times 18926.6097$	2617^{21261}	(9262, 31765, 19764)
$826.17531 \times 1102.229$	6144^{23524}	(13399, 32510, 25360)
$5457.16254 \times 28852.2419$	3340^{25847}	(12851, 11353, 26704)
$17098.22785 \times 10733.29645$	23533^{11432}	(15804, 29630, 14049)
$4297.6124 \times 13047.24061$	6951^{30578}	(25163, 7180, 3955)
$15919.20579 \times 3697.8512$	26036^{19951}	(4596, 28456, 23292)
$30444.8539 \times 1877.24380$	25637^{24662}	(2345, 22515, 15427)
$13777.5551 \times 12290.27827$	9848^{18414}	(8106, 1141, 25365)
$5916.26304 \times 32545.9871$	9456^{20356}	(13568, 17968, 13625)
$752.22564 \times 9313.24044$	20240^{17852}	(25921, 11852, 10721)
$17816.14197 \times 468.475$	27975^{6019}	(12765, 23034, 15867)
$31180.31140 \times 17008.23777$	4288^{10545}	(23555, 14160, 20001)
$11143.27728 \times 5201.24768$	28480^{27765}	(1313, 19756, 15238)
$19165.12910 \times 27090.29887$	30726^{8520}	(30355, 31201, 3727)
$3607.11199 \times 26761.19474$	9611^{25133}	(3715, 620, 29421)
$14260.24175 \times 10813.1493$	2551^{5774}	(6694, 27319, 1486)
$1691.28633 \times 21243.16929$	15030^{1385}	(11252, 12149, 32111)
$19772.9737 \times 30544.23499$	13344^{8975}	(17492, 50, 18586)
$9857.3765 \times 19207.6510$	18025^{10614}	(17324, 19518, 13165)

A larger table, given by Table 2.5 and reproduced from another document, then you may need to allow an entire page for the table. This is done by typing the command `\begin{table}[p]`. This test example is included in the minipage environment to show how a footnote¹ can be added to a table. Several problems have been noted before on how \LaTeX handles the location of the table in the text.

2.7 POTENTIAL PITFALLS

2.7.1 Tables and Figures

There is a conflict between the `\usepackage{subfig}`, `\usepackage{caption}` and the `sdsu-thesis.cls` class specification. The long table captions show up correctly

¹We also need to see how a regular footnote appears in the text, so one was inserted here. Multiple lines are easily handled by \LaTeX .

Table 2.5. Computations for Products of the *RRN* Genes at Different Growth Rates

$\tau(\text{min})$	100	60	40	30	24
C period	67	50	45	43	42
D period	30	27	25	24	23
V_0	0.437	0.577	0.815	1.15	1.63
\bar{c}^a	11.1	16.8	22.1	28.1	31.4
\bar{c}_{85}^b	1.73	2.68	3.65	4.81	5.57
\bar{c}_{57}^c	1.36	1.98	2.43	2.87	2.96
$\bar{c}_{85}(\times 100)/\bar{c}^d$	15.6	15.9	16.5	17.1	17.7
$\bar{c}_{57}(\times 100)/\bar{c}^e$	12.3	11.8	11.0	10.2	9.44
$\bar{c}_{85}/\bar{c}_{57}$	1.27	1.35	1.50	1.68	1.88
r^f	3.75	10.27	22.56	38.42	56.98
c_{max}^g	11.28	17.04	22.33	28.36	31.77
c_{max}/c_{min}^h	1.041	1.036	1.027	1.024	1.026

^a $\times 1000$ ribosomes/ μm^3 .

^b $\times 1000$ ribosomes/ μm^3 , representing the average concentration of the product of the *rrn* gene located at 85'.

^c $\times 1000$ ribosomes/ μm^3 , representing the average concentration of the product of the *rrn* gene located at 57'.

^dPercentage of \bar{c} produced by the *rrn* gene located at 85'.

^ePercentage of \bar{c} produced by the *rrn* gene located at 57'.

^fInitiations/min/gene.

^g $\times 1000$ ribosomes/ μm^3 , representing the maximum concentration during the cell cycle.

^hRatio of maximum to minimum concentration during the cell cycle.

(bold and left aligned with table). Use `\usepackage{subfigure}` instead and all captions, as well as the list of tables page show up ok.

If you insist on `\usepackage{subfig}`, make sure to **first** issue the command `\usepackage[bf,labelsep=period,textfont=bf]{caption}` where the first "bf" makes the labels "Figure n" bold; `labelsep=period` says "use '.' instead of ':'"; and `textfont=bf` makes the caption text bold. This may solve your subfig problems.

Table captions ("table titles" [16]) go ABOVE the table, must be in *headline style* where "all major words are capitalized," and there is no period at the end of the caption; in figure captions only the first word is capitalized, and there is a period at the end. — **THE STYLE DOES NOT CURRENTLY ENFORCE THIS, YOU HAVE TO DO IT MANUALLY.**

Style note

Charts, graphs, diagrams, maps, photographs, and other graphic illustrations should all be labeled as *Figures* [16, §4.6.9, and §4.10.4]. Figure captions are capitalized sentence style in the text; therefore, the List of Figures entries should be in sentence style.

All tables and figures must be referenced in text *prior* to their appearance. Those references should be by number.

2.7.1.1 CENTERED TABLES FIGURES

It is not as simple as adding `\centering` into the figure or table environment as that will center the caption on the page rather than left align it with the left edge of the figure or table. So the way to solve this is to figure out the width of the figure or table and add it in a minipage and center that. For example if our table is 2 inches wide when typeset, then we could do

```
\begin{table}[ht]
  \centering
  \begin{minipage}{2in}
    \caption{Caption goes here}
    ... here is your table ...
  \end{minipage}
\end{table}
```

2.7.2 Margins

It is believed that the `sdsu-thesis.cls` template complies with the SDSU thesis manual: 1.25 inch left, 1 inch top, bottom and right. But your *printout* may not give the right measurement, if your printer/printer-driver scales the document. You may have to turn off scaling and/or tweak the settings in the `sdsu-thesis.cls` file.

Someone said: “*Some laser printers don’t do the margins correctly, for example my printer shifts the page a bit. You can correct this with the \hoffset and \voffset lengths as:*

```
\hoffset -0.0625in
\voffset 0.15625in”
```

2.7.3 Bad Pagebreaks

Sometimes LaTeX does not do exactly what you want with respect to pagebreaks. To solve this you can manually add a `\pagebreak` command where it should break, or you could add `\enlargethispage{12pt}` to make a page slightly larger if needed; though I’m not sure how the thesis reviewer will look on such transgressions, so do that at own risk.

Bad pagebreaks in the table of contents (or list of tables/figures): If you get a bad pagebreak in a table of contents you can force a pagebreak by:
`\addtocontents{toc}{\protect\pagebreak}` you add this at the point in your document that corresponds to that place in the table of contents. For list of tables and list of figures, replace ‘toc’ in the line above with ‘lot’ or ‘lof.’

2.7.4 Bad Linebreaks

Bad linebreaks in chapter, section (subsection, etc...), or table/figure caption titles: This classfile tries to make all titles conform to the requirements of the thesis manual, but it is possible that it gets things wrong and you may want to add linebreaks (the `\\` command) yourself. However, the table of contents title should not have any linebreaks. The way you do it is to add an optional argument to `\chapter`, `\section`, `\caption` as in:

`\chapter[Title for Table of Contents]{Title With\\Linebreaks}`

Note that for `\caption`’s in figures and tables you might have to do this whenever you have a small figure or table as the table/figure environment cannot make the caption only as long as the figure since it doesn’t know how large the figure is until it typesets everything. See example above and more examples in the long-example directory. You can also solve the `\caption` issue with `minipage` in the same way we do centering, see section 2.7.1.1.

2.7.5 Vertical Space

This classfile tries to make all the vertical space as required, but sometimes you may need to modify what it does, or you just need to insert some vertical space. You use the `\vspace` and `\vspace*` commands (see L^AT_EX manual). You can use positive or negative length there and `\vspace*` makes sure the space appears even if there is a pagebreak in between. For example to add 2 inches of space you can add `\vspace{2in}`.

2.7.6 Bold Math in the Thesis: $x = \pi$

Math in section titles need to be **bold**, but cannot be bold in the Table of Contents.

CHAPTER 3

SECTIONING — THE MIDDLE

Middle chapter. Here we put the middle things, that is, things that are in the middle and not in the beginning or in the end. Here we also test all the section, subsection, and other headings.

Note that CHAPTER TITLES need to be in ALL CAPS — YOU have enter the chapter titles in ALL CAPS!!! *Style note*

3.1 A SECTION

Some section text. Note that there should ALWAYS be some text in between two sectioning levels; a \section directly followed by a \subsection will not go through the review.

3.1.1 A Subsection With a Very Long Title To See How That Will Look When Printed

Some subsection text.

3.1.1.1 A SUBSUBSECTION

Some subsubsection text.

3.1.1.1.1 A Subsubsubsection

Some subsubsubsection text. If you are using this, you are probably over-organizing things.

3.1.1.1.1.1 A Paragraph. Some paragraph text. You never really get this deep — don't be ridiculous.

CHAPTER 4

REFERENCING

Below a list of references are provided in the acceptable format for Master's thesis submission. References are to be numbered and should appear either alphabetically or in the order of appearance in the text. (\LaTeX does the former for the student.) For students using \LaTeX these are obtained using the plain style with BibTeX . The Department of Mathematics and Statistics will accept either the plain style or the SIAM style. (For the SIAM style, get a copy of the SIAM.BST file from your graduate adviser or the Mathematical Sciences computer system.) There are references for journal articles [1], books and booklets [4, 22], inbooks, incollections, and inproceedings [3, 6, 20]. *Note that when you have more than one citation in a single bracket they must be in increasing numerical order!* Other sources may be proceedings [2], technical reports (techreport) [18], theses (mastersthesis, or PhDthesis) [7], or unpublished material [19]. This should provide a fairly comprehensive list for any material that the student may encounter. For additional assistance, see the graduate adviser in your area of concentration. \LaTeX source codes are available for copying.

Style note

If you cite a website [17] and you can't find the year on the website, you should put "n.d." (not dated) at the end. (this is true for other reference also.) It must also has the word "accessed" and the month and year you access the website. You can change how things with no author(s) are sorted in the bibliography by supplying a key entry (see `thbib.bib`), *e.g.* this news release [21] will be sorted under "U," the leading letter of the publishing agency (as preferred by the thesis publisher).

This [8] is an example of a patent. **Notice:** how the month and year fields in `thbib.bib` have been abused to force the "correct" format.

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[9] [5] [10]

APPENDIX A
MORE INFORMATION ON EQUATIONS

MORE INFORMATION ON EQUATIONS

To demonstrate how an appendix should be inserted into the thesis we have provided two appendices. This first appendix illustrates some more advanced techniques to improve the appearance of your equations. Below is a system of partial differential equations from a model for cellular control by an external nutrient. The equations are complicated and \LaTeX tends to allow them to run into each other. To prevent this we have used the `\vrule` command to separate them. Note this is an ordinary \TeX command and is not in L. Lamport's book [11]. Furthermore, we have some complicated boundary conditions that we needed to align, so we used the `array` command, but to get the equations looking right we also needed the `\dfrac` command instead of the `\frac` command. The equations for our model are as follows:

$$\begin{aligned}
 \dot{U}_1(t) &= \tilde{f}(W_1(t-T)) - U_1(t) + \gamma_1 U_2(R\sigma, t), \\
 \dot{W}_1(t) &= -\hat{b}_3 W_1(t) + \gamma_3 W_2(R\sigma, t), \\
 \frac{\partial U_2}{\partial t} &= D_1 \nabla^2 U_2 - U_2 - \tilde{f}(W_1(t-T)) - \gamma_1 U_2(R\sigma, t), \\
 \frac{\partial V_2}{\partial t} &= D_2 \nabla^2 V_2 - b_2 V_2 + c_0 (U_2 + U_1(t)), \\
 \frac{\partial W_2}{\partial t} &= D_3 \nabla^2 W_2 - b_3 W_2 + (\hat{b}_3 - b_3) W_1 - \gamma_3 W_2(R\sigma, t) \\
 &\quad + k \left[\left[\left(\frac{D_3}{r^2} \right) \frac{d}{dr} \left(r^2 \frac{dh}{dr} \right) - b_3 h \right] V_2(R, t) - h \dot{V}_2(R, t) \right],
 \end{aligned} \tag{A.1}$$

for $t > 0$ and $R\sigma < r < R$ and with the boundary conditions:

$$\begin{aligned}
 \frac{\partial U_2(R\sigma, t)}{\partial r} &= \beta_1 U_2(R\sigma, t), & \frac{\partial U_2(R, t)}{\partial r} &= 0, \\
 \frac{\partial V_2(R\sigma, t)}{\partial r} &= 0, & \frac{\partial V_2(R, t)}{\partial r} &= 0, \\
 \frac{\partial W_2(R\sigma, t)}{\partial r} &= \beta_3 W_2(R\sigma, t), & \frac{\partial W_2(R, t)}{\partial r} &= 0.
 \end{aligned}$$

Notice that the system is numbered only once by (A.1) and that this is centered as best we can on one line. All other lines have the `\nonumber` command.

A.1 THEOREMS

The appendix can also include technical theorems and lemmas which are call in the same manner as before. For example,

Theorem A.1. *The system of equations (A.1) can exhibit periodic solutions for certain parameter values.*

Proof. The argument uses Hopf bifurcation techniques and is very complicated. See Mahaffy *et al* [14]. □

APPENDIX B
LISTS AND QUOTATIONS

LISTS AND QUOTATIONS

The thesis will rarely use list environments, but they are valuable for résumés. For more information on creating a résumé you may want to see the author of this document (you also need to learn quite a bit about `\parbox` commands). To create a list you will want to use one of `itemize`, `enumerate`, or `description`. For example:

continuous A function f is **continuous** at x if and only if for every $\varepsilon > 0$ there exists a $\delta(x) > 0$ such that whenever $|y - x| < \delta$, $|f(y) - f(x)| < \varepsilon$.

uniformly continuous A function f is **uniformly continuous** if and only if for every $\varepsilon > 0$ there exists a $\delta > 0$ such that whenever $|y - x| < \delta$, $|f(y) - f(x)| < \varepsilon$ independent of x and y .

equicontinuous A family of functions f_n is **equicontinuous** at a point x if and only if for every $\varepsilon > 0$ there exists a $\delta > 0$ such that whenever $|y - x| < \delta$, $|f_n(y) - f_n(x)| < \varepsilon$ for all functions f_n .

L^AT_EX provides an environment for block quotations. To agree with the thesis manual follow the format below for a quotation exceeding four lines. From Lewis Carroll's *Hunting of the Snark* we hear the Bellman tell his crew:

The Bellman himself they all praised to the skies—
Such a carriage, such ease and such grace!
Such solemnity, too! One could see he was wise,
The moment one looked in his face!

He had bought a large map representing the sea,
Without the least vestige of land:
And the crew were much pleased when they found it to be
A map they could all understand.

“What’s the good of Mercator’s, North Poles and Equators,
Tropics, Zones, and Meridian Lines?”
So the Bellman would cry: and the crew would reply,
“They are merely conventional signs!”

“Other maps are such shapes, with their islands and capes!
But we’ve got our brave Captain to thank”
(So the crew would protest) “that he’s bought us the best—
A perfect and absolute blank!”