2.3.2

(1) 可逆矩阵推演

$$Ab = \begin{bmatrix} 4 & -5 & -9 & 1 \\ -2 & -2 & 8 & 1 \\ -1 & -3 & 3 & 1 \end{bmatrix}$$

$$\longrightarrow \begin{bmatrix} 1 & -\frac{5}{4} & -\frac{9}{4} & \frac{1}{4} \\ 0 & -\frac{9}{2} & \frac{7}{2} & \frac{3}{2} \\ 0 & -\frac{17}{4} & \frac{3}{4} & \frac{5}{4} \end{bmatrix}$$

$$\longrightarrow \begin{bmatrix} 1 & 0 & -\frac{29}{9} & -\frac{1}{6} \\ 0 & 1 & -\frac{7}{9} & -\frac{1}{3} \\ 0 & 0 & -\frac{23}{9} & -\frac{1}{6} \end{bmatrix}$$

$$\longrightarrow \begin{bmatrix} 1 & 0 & 0 & \frac{1}{23} \\ 0 & 1 & 0 & -\frac{13}{46} \\ 0 & 0 & 1 & \frac{3}{46} \end{bmatrix}$$

(2) 奇异矩阵推演

$$Ab = \begin{bmatrix} 0 & -1 & -6 & 1 \\ 7 & 1 & 6 & 1 \\ -4 & -1 & -6 & 1 \end{bmatrix}$$

$$\longrightarrow \begin{bmatrix} 1 & \frac{1}{7} & \frac{6}{7} & \frac{1}{7} \\ 0 & -1 & -6 & 1 \\ 0 & -\frac{3}{7} & -\frac{18}{7} & \frac{11}{7} \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 & 0 & \frac{2}{7} \\ 0 & 1 & 6 & -1 \\ 0 & 0 & 0 & \frac{8}{7} \end{bmatrix}$$

2.4 (选做)

$$A=\begin{bmatrix} I & X \\ Z & Y \end{bmatrix}$$

假设 A 为 $m\times m$ 方阵，Y 为 $n\times n$ 方阵.

因为：矩阵 Y 首列全为 0，所以经初等行变换可得，Y 的秩

$$R(Y)<n$$

又因为：Z 为全零矩阵，所以 A 的秩

$$R(A)<m$$

所以 A 为奇异矩阵.

3.3.1 (选做)

证明：
$$\frac{\partial E}{\partial m}=\left[\frac{1}{2}\sum_{i=1}^{n}(y_i-mx_i-b)^2\right]'=\frac{1}{2}\sum_{i=1}^{n}2(y_i-mx_i-b)(-x_i)$$

$$=\sum_{i=1}^{n}-x_i(y_i-mx_i-b)$$

$$\frac{\partial E}{\partial b}=\left[\frac{1}{2}\sum_{i=1}^{n}(y_i-mx_i-b)^2\right]'=\frac{1}{2}\sum_{i=1}^{n}2(y_i-mx_i-b)(-1)$$

$$=\sum_{i=1}^{n}-(y_i-mx_i-b)$$

3.3.2 (选做)

$$E=\frac{1}{2}\sum_{i=1}^{3}(y_i-mx_i-b)^2$$

$$=\frac{1}{2}\left[(1-m-b)^2+(2-2m-b)^2+(3-2m-b)^2\right]$$

$$=\frac{1}{2}(14-22m-12b+9m^2+10mb+3b^2)$$

$$=7-11m-6b+\frac{9}{2}m^2+5mb+\frac{3}{2}b^2$$

$$\begin{cases} \dfrac{\partial E}{\partial m} = -11 + 9m + 2b = 0 \\ \\ \dfrac{\partial E}{\partial b} = -6 + 2m + 3b = 0 \end{cases} \Longrightarrow \begin{cases} m = \dfrac{3}{7} \\ \\ b = -\dfrac{1}{7} \end{cases}$$

3.3.3（选做）

证明：
$$X^T X = \begin{bmatrix} x_1 & x_2 & \cdots & x_n \\ 1 & 1 & \cdots & 1 \end{bmatrix} \begin{bmatrix} x_1 & 1 \\ x_2 & 1 \\ \vdots & \vdots \\ x_n & 1 \end{bmatrix} = \begin{bmatrix} x_1^2 + x_2^2 + \cdots + x_n^2 & x_1 + x_2 + \cdots + x_n \\ x_1 + x_2 + \cdots + x_n & 1 \end{bmatrix}$$

$$X^T X h = \begin{bmatrix} (x_1^2 + x_2^2 + \cdots + x_n^2)m + (x_1 + x_2 + \cdots + x_n)b \\ (x_1 + x_2 + \cdots + x_n)m + b \end{bmatrix}$$

$$X^T Y = \begin{bmatrix} x_1 & x_2 & \cdots & x_n \\ 1 & 1 & \cdots & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} x_1 y_1 + x_2 y_2 + \cdots + x_n y_n \\ y_1 + y_2 + \cdots + y_n \end{bmatrix}$$

$$X^T X h - X^T Y = \begin{bmatrix} (x_1^2 + x_2^2 + \cdots + x_n^2)m + (x_1 + x_2 + \cdots + x_n)b - (x_1 y_2 + x_2 y_2 + \cdots x_n y_n) \\ (x_1 + x_2 + \cdots + x_n)m + b - (y_1 + y_2 + \cdots + y_n) \end{bmatrix}$$

$$= \begin{bmatrix} \sum_{i=1}^{n} x_i (m + b - y_i) \\ \sum_{i=1}^{n} x_i m + b - y_i \end{bmatrix} = \begin{bmatrix} \dfrac{\partial E}{\partial m} \\ \dfrac{\partial E}{\partial b} \end{bmatrix}$$

得证.