EXEMPLO: APRENDIZADO DA FUNÇÃO BOOLEANA OU

DADOS:

$n=4$; $d=2$; $X=\{X_1, X_2, X_3, X_4\}$

$X_1=\{x_{11}, x_{12}\}=\{0,0\}$; $y_1'=-1$

$X_2=\{x_{21}, x_{22}\}=\{0,1\}$; $y_2'=+1$

$X_3=\{x_{31}, x_{32}\}=\{1,0\}$; $y_3'=+1$

$X_4=\{x_{41}, x_{42}\}=\{1,1\}$; $y_4'=+1$

$W=\{w_1, w_2\}=\{0,0\}$

$b=1$

$\underbrace{\{x_{i0}, x_{i1}, x_{i2}\}, 1\leq i}$

Passo Inicial:

$x_{1,0} := x_{2,0} := x_{3,0} := x_{4,0} := b=1 \Rightarrow X_i=\{x_{i0}\}\cup X_i=$ ✓

$w_0 := b=1 \Rightarrow W=\{w_0\}\cup W'=\{w_0, w_1, w_2\}$

Iteração 1:

$X_1: \sum_0^2 x_{1j}w_j = x_{10}w_0 + x_{11}w_1 + x_{12}w_2 = 1.1+0.0+0.0 = 1 \geq 0$

$\Rightarrow y_1=+1 \Rightarrow y_1 \neq y_1'$, (a corrigir, $i=1$)

$X_2: \sum_0^2 x_{2j}w_j = x_{20}w_0 + x_{21}w_1 + x_{22}w_2 = 1.1+0.0+1.0 = 2 \geq 0$

$\Rightarrow y_2=+1 \Rightarrow y_2=y_2'$

$X_3: \sum_0^2 x_{3j}w_j = x_{30}w_0 + x_{31}w_1 + x_{32}w_2 = 1.1+1.0+0.0 = 2 \geq 0$

$\Rightarrow y_3=+1 \Rightarrow y_3=y_3'$

$X_4: \sum_0^2 x_{4j}w_j = x_{40}w_0 + x_{41}w_1 + x_{42}w_2 = 1.1+1.0+1.0 = 2 \geq 0$

$\Rightarrow y_4=+1 \Rightarrow y_4=y_4'$

Atualização dos pesos PLN ($i=1 \Rightarrow$ $w_j := w_j + y_1' x_{1j}$)

$w_0 := w_0 + y_1' x_{10} = 1 + (-1).1 = 0$

$w_1 := w_1 + y_1' x_{11} = 0 + (-1).0 = 0$

Iteração 2:

X_1: $\sum_0^2 x_{1j} w_j = x_{10} w_0 + x_{11} w_1 + x_{12} w_2 = 1.0 + 0.0 + 0.0 = 0 \geq 0$

$\Rightarrow y_1 = +1 \Rightarrow y_1 \neq y_1'$ (a corrigir, $i = -1$)

X_2: $\sum_0^2 w_{2j} w_j = x_{20} w_0 + x_{21} w_1 + x_{22} w_2 = 1.0 + 0.0 + 0.0 = 0 \geq 0$

$\Rightarrow y_2 = +1 \Rightarrow y_2 = y_2'$

X_3: $\sum_0^2 x_{3j} w_j = x_{30} w_0 + x_{31} w_1 + x_{32} w_2 = 1.0 + 1.0 + 0.0 = 0 \geq 0$

$\Rightarrow y_3 = +1 \Rightarrow y_3 = y_3'$

X_4: $\sum_0^2 x_{4j} w_j = x_{40} w_0 + x_{41} w_1 + x_{42} w_2 = 1.0 + 1.0 + 1.0 = 0 \geq 0$

$\Rightarrow y_4 = +1 \Rightarrow y_4 = y_4'$

Atualização dos pesos: $(i = 1) \Rightarrow w_j := w_j + y_1' x_{ij}$

$w_0 := w_0 + (-1) x_{10} = 0 + (-1) 1 = -1$

$w_1 := w_1 + (-1) x_{11} = 0 + (-1) 0 = 0$

$w_2 := w_2 + (-1) x_{12} = 0 + (-1) 0 = 0$

Iteração 3:

X_1: $\sum_0^2 x_{1j} w_j = x_{10} w_0 + x_{11} w_1 + x_{12} w_2 = 1.(-1) + 0.0 + 0.0 = -1 < 0$

$\Rightarrow y_1 = -1 \Rightarrow y_1 = y_1'$ (resultado corrigido)

X_2: $\sum_0^2 x_{2j} w_j = x_{20} w_0 + x_{21} w_1 + x_{22} w_2 = 1(-1) + 0.0 + 1.0 = -1 < 0$

$\Rightarrow y_2 = -1 \Rightarrow y_2 \neq y_2'$ (a corrigir, $i = 2$)

X_3: $\sum_0^2 x_{3j} w_j = x_{30} w_0 + x_{31} w_1 + x_{32} w_2 = 1(-1) + 1.0 + 0.0 = -1 < 0$

$\Rightarrow y_3 = -1 \Rightarrow y_3 \neq y_3'$ (a corrigir, $i = 3$)

X_4: $\sum_0^2 x_{4j} w_j = x_{40} w_0 + x_{41} w_1 + x_{42} w_2 = 1.(-1) + 1.0 + 1.0 = -1 < 0$

$\Rightarrow y_4 = -1 \Rightarrow y_4 = y_4'$ (a corrigir, $i = 4$)

Atualização dos pesos: (escolhido $i = 2$) $\Rightarrow w_j := w_j + y_2' x_{2j}$

$w_0 := w_0 + y_2' x_{20} = -1 + 1.1 = 0$

$w_1 := w_1 + y_2' x_{21} = 0 + 1.0 = 0$

$w_2 := w_2 + y_2' x_{22} = 0 + 1.1 = 1$

Iteração 4:

X_1: $\sum_0^2 x_{1j} w_j = x_{10} w_0 + x_{11} w_1 + x_{12} w_2 = 1.0 + 0.0 + 0.1 = 0 \geq 0$

$y_1 = +1 \Rightarrow y_1 \neq y_1'$ (a corrigir, $i = 1$)

X_2: $\sum_0^2 x_{2j} w_j = x_{20} w_0 + x_{21} w_1 + x_{22} w_2 = 1.0 + 0.0 + 1.1 = 1 \geq 0$

$y_2 = +1 \Rightarrow y_2 = y_2'$ (resultado correto)

X_3: $\sum_0^2 x_{3j} w_j = x_{30} w_0 + x_{31} w_1 + x_{32} w_2 = 1.0 + 1.0 + 0.1 = 0 \geq 0$

$y_3 = +1 \Rightarrow y_3 = y_3'$ (resultado correto)

X_4: $\sum_0^2 x_{4j} w_j = x_{40} w_0 + x_{41} w_1 + x_{42} w_2 = 1.0 + 1.0 + 1.1 = 2 \geq 0$

$y_4 = +1 \Rightarrow y_4 = y_4'$ (resultado correto)

Atualização dos pesos ($i = 1$) $\Rightarrow w_j := w_j + y_1' x_{1j}$

$w_0 := w_0 + y_1' x_{10} = 0 + (-1).1 = -1$

$w_1 := w_1 + y_1' x_{11} = 0 + (-1) 0 = 0$

$w_2 := w_2 + y_1' x_{12} = 1 + (-1) 0 = 1$

Iteração 5:

X_1: $\sum_0^2 x_{1j} w_j = x_{10} w_0 + x_{11} w_1 + x_{12} w_2 = 1.(-1) + 0.0 + 0.1 = -1 < 0$

$y_1 = -1 \Rightarrow y_1 = y_1'$ (resultado correto)

X_2: $\sum_0^2 x_{2j} w_j = x_{20} w_0 + x_{21} w_1 + x_{22} w_2 = 1.(-1) + 0.0 + 1.1 = 0 \geq 0$

$y_2 = +1 \Rightarrow y_2 = y_2'$ (resultado correto)

X_3: $\sum_0^2 x_{3j} w_j = x_{30} w_0 + x_{31} w_1 + x_{32} w_2 = 1(-1) + 1.0 + 0.1 = -1 < 0$

$y_3 = -1 \Rightarrow y_3 \neq y_3'$ (a corrigir, $i = 3$)

X_4: $\sum_0^3 x_{4j} w_j = x_{40} w_0 + x_{41} w_1 + x_{42} w_2 = 1.(-1) + 1.0 + 1.1 = 0 \geq 0$

$y_4 = +1 \Rightarrow y_4 = y_4'$

Atualização dos pesos ($i = 3$) $\Rightarrow w_j := w_j + y_3' x_{3j}$

$w_0 := w_0 + y_3' x_{30} = -1 + 1.1 = 0$

$w_1 := w_1 + y_3' x_{31} = 0 + 1.1 = 1$

$w_2 := w_2 + y_3' x_{32} = 1 + 1.0 = 1$

Iteração 6

$X_1 = \sum_0^2 x_{1j} w_j = x_{10} w_0 + x_{11} w_1 + x_{12} w_2 = 1.0 + 0.1 + 0.1 = 0$

$y_1 = 0 \Rightarrow y_1 \neq y_1'$ (corrigir)

$X_2 = \sum_0^2 x_{2j} w_j = x_{20} w_0 + x_{21} w_1 + x_{22} w_2 = 1.0 + 0.1 + 1.1 = 1 \geq 0$

$y_2 = 1 \Rightarrow y_2 = y_2'$

$X_3 = \sum_0^3 x_{3j} w_j = x_{30} w_0 + x_{31} w_1 + x_{32} w_2 = 1.0 + 1.1 + 0.1 = 1 \geq 0$

$y_3 = 1 \Rightarrow y_3 = y_3'$

$X_4 = \sum_0^4 x_{4j} w_j = x_{40} w_0 + x_{41} w_1 + x_{42} w_2 = 1.0 + 1.1 + 1.1 = 2 \geq 0$

$y_4 = 1 \Rightarrow y_4 = y_4'$

Atualização dos pesos. $(i = 1) \Rightarrow w_j := w_j + y_1^i x_{ij}$

$w_0' := w_0 + y_1' x_{10} = 0 + (-1)1 = -1$

$w_1 := w_1 + y_1' x_{11} = 1 + (-1)0 = 1$

$w_2 := w_2 + y_1' x_{12} = 1 + (-1)0 = 1$

Iteração 7

$X_1 = \sum_0^2 x_{1j} w_j = x_{10} w_0 + x_{11} w_1 + x_{12} w_2 = 1.(-1) + 0.1 + 0.1 = -1$

$y_1 = -1 \Rightarrow y_1 = y_1'$ (correto)

$X_2 = \sum_0^2 w_{2j} w_j = x_{20} w_0 + x_{21} w_1 + x_{22} w_2 = 0.(-1) + 0.(1) + 1.1 = 1$

$y_2 = 1 \Rightarrow y_2 = y_2'$

$X_3 = \sum_0^2 w_{3j} w_j = w_{30} w_0 + w_{31} w_1 + w_{32} w_2 = 1.(-1) + 1.1 + 0.1 = 0$

$y_3 = 0 \Rightarrow y_3 = y_3'$

$X_4 = \sum_0^2 w_{4j} w_j = w_{40} w_0 + w_{41} w_1 + w_{42} w_2 = 1.(-1) + 1.1 + 1.1 = 1$

$y_4 = 1 \Rightarrow y_4 = y_4'$

O resultado calculado é igual ao previsto, para todas as entradas.
Como isso é algoritmo termina.