# CIS 3715 Final Project Report

Leomar Durán April 2022

# Part I Final Report

## Project title and student names

- Project title: Using satellite imagery to train a model for identifying the type of landmarks.
- Student names (1)
  - Leomar Durán

#### 1 Introduction section

#### 1.1 Motivation

The sciences of geomatics and land surveying interest me as hobbies. I really enjoy the idea of collecting data about the terrain, whether it be rural or urban, and working with that data to find solutions to problems or even just for fun.

#### 1.2 The Problem

#### Overview

An image of a terrain is given to a computer which will make a decision on the fly based on the type of terrain. We will train a model that will be used by the computer to identify this terrain.

This sort of decision might be involved in deciding if the terrain would be appropriate for developing a building thereon. A preliminary sweep by a machine may save on costs of having an engineer waste time looking for land to develop. Another example of making this decision may be helpful for automatic landing software that will be used to safely land aircraft on stable terrain. A third example is that combined with time series data, we can predict different types of weather-related and other natural phenomena, such as draughts, floods and earthquakes. The rain shadow affect is an important scenario where terrain plays a major role in weather.

#### Data science fundamentals

This problem involves multi-class classification using the linear and ridge regressions specifically. We will classify the terrain according to features such as whether the area is barren land, forested land (trees), grassland, and land with no special features for 4 disjoint classes.

We will evaluate the results using accuracy, recall, precision and the F1 score.

#### Project objective and constraints

For this project, we hope to train a model to learn different 4 disjoint classes of terrain, and then classify a test sample.

The algorithm that we pick has to deal well with the curse of dimensionality, as there will be  $(28 \times 28)$ px/examples  $\times$  4 channels/px = 3136 channels/examples.

Ideally the solution would also perform well for multiple clases, but this is less of an issue than dimensionality.

#### 1.3 Related works

One of the historical approaches to this problem is that by Basu, Ganguly, Mukhopadhyay, *et al.* [1] themselves, who used a combination of computer vision and neural networks.

Computer vision: Use machine learning and neural networks to teach computers to see [2] compares computer vision with human sight as well as artificial intelligence, making the analogy that computer vision is to seeing as artificial intelligence is to thinking.

However, Computer vision: Use machine learning and neural networks to teach computers to see [2] also clarifies the disadvantages of computer vision to traditional machine learning models. Specifically, "[c]omputer vision needs lots of data. It runs analyses of data over and over until it discerns distinctions and ultimately recognize images." That is to say that computer vision has high time and spatial complexity. Because of the amounts of data required, it will require much storage, and the same compounded by the number of iterations that must be performed, training a computer vision model will require much more time. IBM explains that the scale needed for time and storage is such that few organizations have the necessary infrastructure, and as a result, many use a service such as IBM's to perform computer vision[2].

When it comes to neural networks, common issues include overfitting and underfitting[3][4]. Overfitting is when a model is too specific to the training data. As a result when the model is tested against the testing data, small differences can create large errors compared to the expected output[5]. Underfitting results from a model that is too simple and results in high errors in comparison to the expected results for both testing and training data[3]. In order to avoid both, Lawrence and Giles [4] suggests the more complex technique of backpropagation.

Rolf, Proctor, Carleton, *et al.* [6] provides another method of training a model on satellite data. Specifically, they used a hybrid system. First, there is a 18-layer convolutional neural network[7]. However, rather than producing a single output, it produces 2<sup>9</sup> features. There is a second convolutional neural network with a ReLu activation function, which produces 2<sup>13</sup> features[6]. The values are then concatenated and placed in a linear regression with a ridge regularization function[6].

Sharma [8] explains the two main issues with the convolutional neural network, two of which form the basis of this model. One issue named are that the convolutional neural network is sensitive to variation in images, such as different in phase of the object being imaged, differences in lighting, and differences in positioning. The other issue is that convolutional neural networks are sensitive to even low levels of additive Gaussian white noise.

To satisfy what we have learned from past systems, we will work from the bottom up using a simpler system that will not overfit, iterating until we find the lowest system that will have good performance. This will also solve the issue of noise because this problem is complicated by overfitting. As for position and phase of the image, this is a much different problem to tackle and outside of the scope of this project. However, if the image were more more regular and you could expect a guideline, it could be used to properly orient the image using a rotation matrix.

# 2 Approach

#### 2.1 Idea

Our primary idea is that we want to avoid overfitting or complex models. As explained in 1.3, overfitting is a very common issue with neural networks, which is usually resolved through backpropagation. However, we want to avoid complexity where possible. This will be our primary motivator in starting from the ground up.

Our progress is also available at the project repository.

#### 2.2 Proposed work

We are given The SAT-4 airborne dataset[1]. This data is hosted by the Louisiana State University's Division of Computer Science and Engineering<sup>1</sup> and can be downloaded directly from the Google Drive<sup>2</sup> along with the SAT-6 airborne dataset, or by itself from Kaggle<sup>3</sup>.

The dataset consists of 400,000 example tiles taken from satellite imagery originally from the National Agriculture Imagery Program (NAIP) dataset. Each example has features representing the pixels of a  $(28 \times 28)$ px image multiplied by the channels for red, green, blue and near infrared (NIR). According to Basu, Ganguly, Mukhopadhyay, *et al.* [1], these tiles represent "different landscapes like rural areas, urban areas, densely forested, mountainous terrain, small to large water bodies", so these as a disjoint set of landscapes would make for appropriate labels.

Our proposed solution is a multi-class logistic regression. However, we intended to work from the ground up starting with linear regression and testing until we found something that worked good enough without overfitting as was a worry in past works. However, we had an issue with logistic regression.

This issue is that since the dataset has multiple classes, the specific encoding that was used in the data for those classes was one-hot encoding. Thus the labels were vectors. However, the scikit-learn package's logistic regression is designed to work specifically with scalar labels. Because of this the ridge regression seemed like the last more natural choice for this dataset.

#### Design and implementation challenges

A challenge to this solution is the size of the dataset. Because of its size (about 3 Gbyte), we expect long processing times. One possible solution to this challenge may be to reduce the datasize from 400,000 to a more managable number such as 20,000.

Another issue that we will run into is deciding the best way to split the classes for the multiclass classification.

#### Anticipated project outcomes and impacts

An anticipated outcome is a model that can identify the types of terrains accurately from the given dataset.

<sup>1&</sup>lt;http://csc.lsu.edu/~saikat/deepsat/>

<sup>2&</sup>lt;https://drive.google.com/u/0/uc?export=download&confirm=sWVM&id=0B0Fef71\_vt3PUkZ4YVZ5WWNvZWs>

<sup>&</sup>lt;sup>3</sup><https://www.kaggle.com/datasets/crawford/deepsat-sat4>

# 3 Results

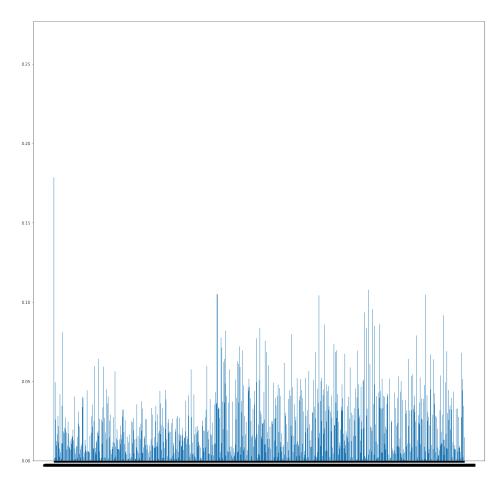


Figure 1: The weights including bias learned.

Fig. 1 shows the weights that were learned. The bias is most important in this model. Most of the weights are equally important, but there are a few that are more important, and many more that are not very important and could be reduced. I did not perform any feature reduction however.

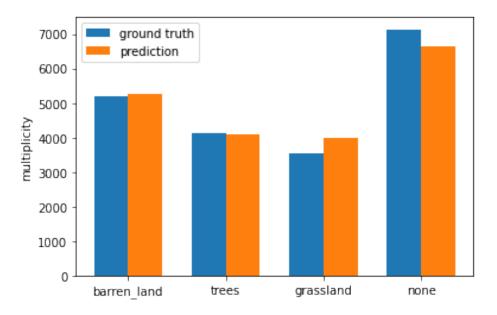


Figure 2: Bar plot of the multiplicities of the classes in ground truth and prediction.

In Fig. fig:multiplicity bar plot, we see a visual representation of the multiplicities of the terrain type classes. It seems like a close fit. However, all this is, is a visual representation of a count of each of the classes.

| dividend   | divisor        | quotient |
|------------|----------------|----------|
| MAE train  | MAE test       | 0.8367   |
| RMSE train | RMSE test      | 0.8431   |
| MAE test   | MAD train      | 0.7619   |
| RMSE test  | $\sigma$ train | 0.7358   |

Table 1: Error ratios.

Table 1 shows us the results of the linear regression in numbers. Specifically, we found that the linear regression was slightly overfit by the two error ratios of 0.8367 and 0.8431. These represent an overfit because the errors of the training are less than the errors against the test data. Thus the ratio is less than 1.

This table also shows us that the mean absolute error is at about 0.76 mean absolute deviations around the median and that the root mean square error is at about 0.74 standard deviations, which mean that the model has a moderate predictive power since it is close to 1.

In their normal forms, mean absolute error and root mean square error give us a measure of the predictive power of the model[9]. Then we can use these last two normalizations because the more familiar standard deviation is also known as the root mean square. In fact, the root mean square error is the same formula as the root mean square, but using the corresponding predicted

value for each label instead of the mean. Likewise we use the mean absolute deviation about the median to normalize the mean absolute error.

| regulariza     | ation $\lambda$ | accuracy | recall   | precision | $F_1$ score |
|----------------|-----------------|----------|----------|-----------|-------------|
| [dB]           | <1>             |          |          |           |             |
| none           |                 | 0.58665  | 0.449 40 | 0.60650   | 0.503 64    |
| <del>-10</del> | 0.3162          | 0.587 15 | 0.449 22 | 0.607 58  | 0.50374     |
| <b>-9</b>      | 0.3548          | 0.587 20 | 0.449 29 | 0.60778   | 0.503 81    |
| -8             | 0.3981          | 0.58730  | 0.44943  | 0.60784   | 0.503 92    |
| -7             | 0.4467          | 0.58735  | 0.44935  | 0.60794   | 0.50388     |
| -6             | 0.5012          | 0.587 35 | 0.449 27 | 0.60794   | 0.503 82    |
|                | 0.5623          | 0.587 55 | 0.449 08 | 0.608 11  | 0.503 65    |
| -4             | 0.631           | 0.58760  | 0.44908  | 0.60837   | 0.50370     |
| -3             | 0.7079          | 0.58735  | 0.44864  | 0.60788   | 0.503 16    |
| -2             | 0.7943          | 0.58745  | 0.44885  | 0.60786   | 0.50330     |
| -1             | 0.8913          | 0.587 55 | 0.44868  | 0.608 03  | 0.503 18    |
| 0              | 1.000           | 0.58775  | 0.44874  | 0.608 64  | 0.503 34    |
| 1              | 1.122           | 0.58775  | 0.448 56 | 0.60862   | 0.503 21    |
| 2              | 1.259           | 0.588 00 | 0.44887  | 0.609 07  | 0.503 55    |
| 3              | 1.413           | 0.588 10 | 0.44870  | 0.609 57  | 0.503 51    |
| 4              | 1.585           | 0.588 50 | 0.44878  | 0.61041   | 0.50378     |
| 5              | 1.778           | 0.588 50 | 0.44852  | 0.61078   | 0.503 60    |
| 6              | 1.995           | 0.588 45 | 0.448 17 | 0.61071   | 0.503 26    |
| 7              | 2.239           | 0.588 55 | 0.447 99 | 0.61074   | 0.503 09    |
| 8              | 2.512           | 0.58875  | 0.44799  | 0.61139   | 0.503 28    |
| 9              | 2.818           | 0.589 10 | 0.44790  | 0.61228   | 0.503 43    |
| 10             | 3.162           | 0.589 25 | 0.44791  | 0.61235   | 0.503 36    |

Table 2: Scores with different values of the regularization hyperparameter  $\lambda$ .

Table 2 shows us the scores for the different regularization hyperparameters  $\lambda$  of a ridge regression with the first being the original linear regression with no regularization. The models have somewhat bad recall with moderately fair precision. A visitual representation is provided by Fig.3 below.

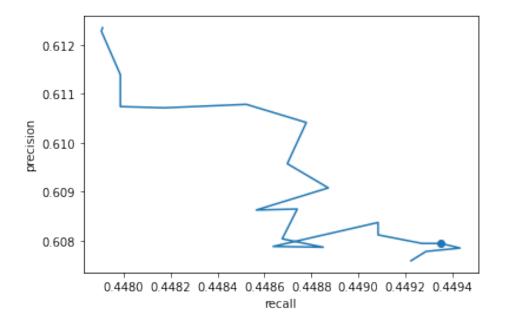


Figure 3: Precision versus recall by regularization hyperparameter  $\lambda$  with linear regression as a disc.

Hyperparameter  $\lambda = -8 \, \text{dB} = 0.3981$  the  $F_1$  score, representing the harmonic mean of recall and precision, maximizes at  $F_1 = 0.503\,92$ .

#### 4 Conclusion

Overall, the project was not as successful as I had planned. The ridge regressions gave very low recall and a somewhat low precision.

I believe that the two issues that gave me the most difficulty were the large number of examples (400,000) and the labels which were each a 4-vector one-hot encoded.

The sampling program took a while to write because of the memory issues, and because I was having a hard time keeping up with other classes, which I neglected the project as a result.

As for the one-hot encoding, I was not sure how best to handle it. I considered after the project that label or ordinal encoding may be options. However that carries with it the connotation of ranking, which I'm not sure how the terrain type would rank, and it may depend on application, which is outside the scope of this project.

It may have also been interesting to attempt clustering to see if the examples fit the labels provided, or even if 4 labels was the optimal number.

As for the outcomes, I believe that we were able to prove that a somewhat reliable model is possible without resorting to computer vision or a neural network (convolutional or otherwise). I also got to learn many new techniques, to teach myself features of pandas and sklearn with which I was not familiar, and to explore the limits of those with which that I am familiar.

### 5 Acknowledgements

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# CIS 3715 Final Project Report Appendix

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# Part II Jupyter notebook of the project

# 1 Using satellite imagery to train a model for identifying the type of landmarks

#### 1.1 Preprocess data

Now we may work with the data.

Start by importing necessary modules and setting up important constants.

```
[52]: import pandas as pd
                                                                      # for the dataframes
      from math import *
                                                                      # for sqrt
      from statistics import *
                                                                      # for mean
      import numpy as np
                                                                      # for linear algebra
      import matplotlib.pyplot as plt
                                                                      # for various plots
      from sklearn.linear_model import LinearRegression, Ridge
                                                                      # for the learning_
       \rightarrowmodels
      from sklearn.metrics import \
          mean_absolute_error, mean_squared_error
                                                                      # for evaluating
       \rightarrowmodels
      from sklearn.metrics import \
          accuracy_score, f1_score, recall_score, precision_score # for further_
       \rightarrow evaluating models
```

```
[78]: # constants
      X_TRAIN_FILENAME = r'dataset/X_train_sat4_samp20000.csv' # filename of the_
      → training dataset input
      Y_TRAIN_FILENAME = r'dataset/y_train_sat4_samp20000.csv'
                                                                 # filename of the_{\square}
      → training dataset output
      X_TEST_FILENAME = r'dataset/X_test_sat4_samp20000.csv'
                                                                # filename of the
      → testing dataset input
      Y_TEST_FILENAME = r'dataset/y_test_sat4_samp20000.csv'
                                                                  # filename of the_
      → testing dataset output
      R2\_TOLERANCE = 0.81
                                                                  # use 0.81 for r^2
      → for strong correlations
      ACCURACY_TOLERANCE = 0.05
                                                                  # maximum allowed
       →error for accuracy
```

Read in the files and do a high level inspection.

```
[3]: # read in the training data

X_train = pd.read_csv(X_TRAIN_FILENAME, header=None, index_col=None)

y_train = pd.read_csv(Y_TRAIN_FILENAME, header=None, index_col=None)
```

```
[4]: # data shape constants
  (N_XAMPS, N_FEATS) = X_train.shape
  (_, N_LBLS) = y_train.shape
  # colors for grapphing
```

```
CHANNELS = (r'r', r'g', r'b', r'maroon')
     N_CHANNELS = len(CHANNELS)
     # number of pixels
     NF_PIXELS = N_FEATS/N_CHANNELS
     F_WIDTH = sqrt(NF_PIXELS)
     F_HEIGHT = NF_PIXELS/F_WIDTH
     # round number of pixels
     NI_PIXELS = int(NF_PIXELS)
     I_WIDTH = int(F_WIDTH)
     I_HEIGHT = int(F_HEIGHT)
     print(r"{} images of ({}x{})px x {} channels".format(N_XAMPS, I_WIDTH, I_HEIGHT,_
      →N_CHANNELS))
    20000 images of (28x28)px x 4 channels
[5]: # combine training data features, labels
     df_train = pd.concat([X_train, y_train], axis=1)
[6]: # print shapes of X, y
     print("X_train shape\t{}".format(X_train.shape))
     print("y_train shape\t{}".format(y_train.shape))
     print("combined shape\t{}".format(df_train.shape))
    X_train shape
                    (20000, 3136)
    y_train shape
                    (20000, 4)
    combined shape (20000, 3140)
[7]: # print some basic information about the dataset
     print('\n===data frame information===')
     df_train.info()
     # print its parameters
     print('\n===data frame parameters===')
     df_train.describe()
    ===data frame information===
    <class 'pandas.core.frame.DataFrame'>
    RangeIndex: 20000 entries, 0 to 19999
    Columns: 3140 entries, 0 to 3
    dtypes: int64(3140)
    memory usage: 479.1 MB
    ===data frame parameters===
[7]:
                    0
                                  1
                                                              3
     count 20000.000000 20000.000000 20000.000000 20000.000000 20000.000000
     mean
              127.778700
                            123.954900
                                          110.979800
                                                        158.810900
                                                                      127.608900
```

| 25% 98.00000 199.00000 110.000000 123.000000 123.000000 175% 159.000000 122.000000 125.0 | std<br>min | 42.825413<br>0.000000 |           |             |              |            |
|--|------------|-----------------------|-----------|-------------|--------------|------------|
| 50%         124.000000         122.000000         110.000000         166.000000         123.000000           75%         159.000000         148.000000         132.000000         185.000000         159.000000           max         255.000000         255.000000         255.000000         253.000000         244.000000           s         5         6         7         8         9         Count           count         20000.00000         20000.000000         20000.000000         20000.000000         20000.000000         200000.000000           std         38.08994         35.786297         37.861817         42.864149         37.943326           min         2.00000         0.000000         140.00000         98.00000         1000000         1000000         1000000         9.000000         9.000000         9.000000         166.00000         123.00000         1200000         9.000000         148.00000         122.00000         9.000000         148.00000         123.00000         123.00000         128.00000         123.00000         148.00000         123.00000         124.00000         123.00000         125.00000         255.00000         255.00000         255.00000         20000.00000         20000.00000         20000.00000         20000.00000         2   |            |                       |           |             |              |            |
| 75%  |            |                       |           |             |              |            |
| max         255.000000         255.000000         253.000000         244.000000           count         5         6         7         8         9         \           count         20000.00000         20000.000000         20000.000000         20000.000000         20000.000000           mean         123.83680         110.863550         158.690150         127.607600         123.907450           std         38.08994         35.786297         37.861817         42.864149         37.943326           min         2.00000         0.000000         0.000000         0.000000         1.000000         9.000000           25%         99.00000         89.000000         140.000000         98.000000         199.00000         199.00000         185.00000         199.00000         199.00000         185.000000         123.00000         199.00000         185.00000         123.000000         123.00000         123.00000         123.000   |            |                       |           |             |              |            |
| count         5         6         7         8         9         Count           count         20000.00000         20000.000000         20000.000000         20000.000000         20000.000000           mean         123.83680         110.863550         158.690150         127.607600         123.907450           std         38.08994         35.786297         37.861817         42.864149         37.943326           min         2.00000         0.000000         1.000000         0.000000         1.000000           25%         99.00000         189.000000         140.00000         98.000000         122.000000           50%         122.00000         199.000000         166.000000         123.000000         122.000000           75%         148.00000         132.000000         185.000000         158.00000         148.000000           max         255.00000         255.000000         252.000000         246.00000         255.00000           max         250.00000         258.000000         25000.0000         2000.00000         2000.00000           mean         111.151800         158.804500         128.173150         124.216950         124.216950           std        35.980267         37.691749         4   |            |                       |           |             |              |            |
| count         20000.00000         20000.00000         20000.000000         20000.000000         20000.000000         20000.000000         20000.000000         20000.000000         20000.000000         20000.000000         20000.000000         123.907450         std         38.08994         35.786297         37.861817         42.864149         37.943326         min         2.00000         0.000000         0.000000         0.000000         1.000000         25%         99.00000         89.00000         140.00000         98.00000         99.00000         1.000000         25%         99.00000         190.00000         166.00000         123.00000         122.00000         122.00000         122.00000         122.00000         148.00000         122.00000         148.00000         123.00000         148.00000         123.00000         124.00000         124.00000         124.00000         125.000000         255.000000         255.000000         255.000000         225.000000         246.00000         225.000000         225.000000         225.000000         226.00000         233.333         \         3133         \$3133         \$3133         \$3133         \$3133         \$3133         \$3133         \$3134         \$3134         \$3135         \$3134         \$3134         \$3134         \$3134         \$3134         \$3134   | max        | 200.000000            | 200.0000  | 200.0000    | 200.0000     | 211.000000 |
| mean         123.83680         110.863550         158.690150         127.607600         123.907450           std         38.08994         35.786297         37.861817         42.864149         37.943326           min         2.00000         0.000000         0.000000         0.000000         1.000000           25%         99.00000         89.00000         140.000000         98.00000         199.00000           50%         122.00000         109.00000         166.00000         123.00000         122.00000           75%         148.00000         132.00000         185.00000         158.00000         148.00000           max         255.00000         255.00000         252.00000         246.00000         225.00000           mean          3130         3131         3132         3133         \           count          20000.00000         20000.00000         20000.00000         20000.00000         20000.00000           std          35.980267         37.691749         42.905264         37.907691           min          0.00000         0.00000         0.00000         2.000000           25%          89.00000         140.00000 <th< td=""><td></td><td></td><td></td><td></td><td></td><td></td></th<>  |            |                       |           |             |              |            |
| std         38.08994         35.786297         37.861817         42.864149         37.943326           min         2.00000         0.000000         0.000000         0.000000         1.000000           25%         99.00000         89.000000         140.00000         98.000000         19.000000           50%         122.00000         109.000000         140.00000         123.000000         122.00000           75%         148.00000         132.00000         185.00000         158.00000         148.00000           max         255.00000         255.00000         252.000000         246.00000         255.00000           count          3130         3131         3132         3133         \           count          20000.00000         20000.00000         20000.00000         20000.00000         20000.00000           mean          111.151800         158.804500         128.173150         124.216950           std          35.980267         37.691749         42.905264         37.907691           min          0.000000         140.000000         124.000000         120.000000           25%          89.00000         166.000000  |            |                       |           |             |              |            |
| min         2.00000         0.000000         0.000000         0.000000         1.000000           25%         99.00000         89.00000         140.00000         98.00000         99.00000           50%         122.00000         109.00000         166.00000         123.00000         122.00000           75%         148.00000         132.00000         185.00000         158.00000         148.00000           max         255.00000         255.00000         252.00000         246.00000         255.00000            3130         3131         3132         3133         \           count          20000.00000         20000.00000         20000.00000         20000.00000         20000.00000           mean          111.151800         158.804500         128.173150         124.2169   |            |                       |           |             |              |            |
| 25%   99.00000   89.00000   140.00000   98.00000   99.000000   |            |                       |           |             |              |            |
| 50%   122.00000  |            |                       |           |             |              |            |
| 75%  |            |                       |           |             |              |            |
| max         255.00000         255.000000         252.000000         246.000000         255.000000           count         20000.000000         20000.000000         20000.000000         20000.000000         20000.000000           mean         1111.151800         158.804500         128.173150         124.216950           std         35.980267         37.691749         42.905264         37.907691           min         0.000000         0.000000         0.000000         2.000000           25%         89.00000         140.00000         99.00000         100.00000           50%         110.00000         166.00000         124.00000         122.00000           75%         133.00000         185.00000         159.00000         148.00000           max         255.00000         254.00000         248.00000         251.00000           mean         111.20020         158.91350         0.263800         0.201600         0.178600           std         35.64033         37.59736         0.440703         0.401205         0.383027           min         0.00000         140.00000         0.000000         0.000000         0.000000         0.000000           50%         110.00000         <  |            |                       |           |             |              |            |
| Count   Cou    | 75%        | 148.00000             | 132.00000 | 0 185.00000 | 0 158.000000 | 148.000000 |
| count  | max        | 255.00000             | 255.00000 | 0 252.00000 | 0 246.000000 | 255.000000 |
| count  |            |                       | 0400      | 0404        | 0400         | 0400 \     |
| mean          1111.151800         158.804500         128.173150         124.216950           std          35.980267         37.691749         42.905264         37.907691           min          0.000000         0.000000         0.000000         2.000000           25%          89.00000         140.00000         99.000000         100.00000           50%          110.00000         166.00000         124.00000         122.00000           75%          133.00000         185.00000         159.00000         148.00000           max          255.00000         254.00000         248.00000         251.00000           max          255.00000         254.00000         2000.00000         2000.00000           std         3134         3135         0         1         2         \           count         20000.00000         20000.00000         20000.00000         20000.00000         20000.00000         20000.00000           std         35.64033         37.59736         0.440703         0.401205         0.383027           min         0.00000         3.0000         0.00000         0.00000         0.  |            |                       |           |             |              |            |
| std        35.980267       37.691749       42.905264       37.907691         min        0.000000       0.000000       2.000000         25%        89.00000       140.00000       99.00000       100.00000         50%        110.00000       166.00000       124.00000       122.00000         75%        133.00000       185.00000       159.00000       148.00000         max        255.00000       254.00000       248.00000       251.00000         3134       3135       0       1       2       \         count       20000.00000       20000.000000       200000.00000       20000.00  |            |                       |           |             |              |            |
| min          0.000000         0.000000         2.000000         2.000000           25%          89.000000         140.000000         99.000000         100.000000           50%          110.000000         166.000000         124.000000         122.000000           75%          133.000000         185.000000         159.000000         148.000000           max          255.000000         254.000000         248.000000         251.000000           count         20000.00000         20000.00000         20000.000000         20000.000000         20000.000000           mean         111.20020         158.91350         0.263800         0.201600         0.178600           std         35.64033         37.59736         0.440703         0.401205         0.383027           min         0.00000         3.00000         0.000000         0.000000         0.000000           25%         89.00000         140.00000         0.000000         0.000000         0.000000           50%         110.00000         185.00000         1.000000         0.000000         0.000000           mean         0.356000         245.00000         1.000000         1.000000         <   |            |                       |           |             |              |            |
| 25% 89.00000 140.000000 99.000000 100.000000 50% 110.000000 166.000000 124.000000 122.000000 75% 133.000000 185.000000 159.000000 148.000000 max 255.00000 254.000000 248.000000 251.000000  **The count of the co                 |            |                       |           |             |              |            |
| 50%        110.000000       166.000000       124.000000       122.000000         75%        133.000000       185.000000       159.000000       148.000000         max        255.00000       254.000000       248.000000       251.000000         count       20000.00000       20000.00000       20000.000000       20000.000000       20000.000000       20000.000000       20000.000000         mean       111.20020       158.91350       0.263800       0.201600       0.178600         std       35.64033       37.59736       0.440703       0.401205       0.383027         min       0.00000       3.00000       0.000000       0.000000       0.000000         25%       89.00000       140.00000       0.000000       0.000000       0.000000         50%       110.00000       185.00000       1.000000       0.000000       0.000000         max       255.00000       245.00000       1.000000       1.000000       1.000000         std       0.478827       10.00000       1.000000       1.000000       1.000000         50%       0.000000       0.000000       1.000000       1.000000       1.000000   |            |                       |           |             |              |            |
| 75%          133.000000         185.000000         159.000000         148.000000           max          255.000000         254.000000         248.000000         251.000000           3134         3135         0         1         2         \           count         20000.00000         20000.00000         20000.00000         20000.00000         20000.00000         20000.00000         20000.00000         20000.000000         20000.00000         0.201600         0.178600         std         35.64033         37.59736         0.440703         0.401205         0.383027         0.383027         min         0.00000         3.00000         0.000000   |            |                       |           |             |              |            |
| max          255.00000         254.00000         248.000000         251.000000           count         3134         3135         0         1         2         \           count         20000.00000         20000.00000         20000.00000         20000.00000         20000.00000         20000.00000         20000.00000         20000.00000         20000.00000         20000.00000         20000.00000         0.178600         std         35.64033         37.59736         0.440703         0.401205         0.383027         0.383027         0.00000         0.000000         0.000000         0.000000         0.000000         0.000000         0.000000         0.000000         0.000000         0.0000000         0.0000000         0.000000         0.0000000  |            |                       |           |             |              |            |
| Count         3134         3135         0         1         2         \           count         20000.00000         20000.00000         20000.000000         20000.000000         20000.000000         20000.000000         20000.000000         20000.000000         20000.000000         20000.000000         20000.000000         0.178600         x<   | 75%        | 133.0                 | 00000 185 | .000000 159 | .000000 148. | 000000     |
| count         20000.00000         20000.00000         20000.000000         20000.000000         20000.000000           mean         111.20020         158.91350         0.263800         0.201600         0.178600           std         35.64033         37.59736         0.440703         0.401205         0.383027           min         0.000000         3.00000         0.000000         0.000000         0.000000           25%         89.00000         140.00000         0.000000         0.000000         0.000000           50%         110.00000         166.00000         0.000000         0.000000         0.000000           75%         133.00000         185.00000         1.000000         0.000000         0.000000           max         255.00000         245.00000         1.000000         1.000000         1.000000           std         0.478827         0.000000         0.000000         0.000000         0.000000           50%         0.000000         0.000000         0.000000         0.000000         0.000000   | max        | 255.0                 | 00000 254 | .000000 248 | .000000 251. | 000000     |
| count         20000.00000         20000.00000         20000.000000         20000.000000         20000.000000           mean         111.20020         158.91350         0.263800         0.201600         0.178600           std         35.64033         37.59736         0.440703         0.401205         0.383027           min         0.000000         3.00000         0.000000         0.000000         0.000000           25%         89.00000         140.00000         0.000000         0.000000         0.000000           50%         110.00000         166.00000         0.000000         0.000000         0.000000           75%         133.00000         185.00000         1.000000         0.000000         0.000000           max         255.00000         245.00000         1.000000         1.000000         1.000000           std         0.478827         0.000000         0.000000         0.000000         0.000000           50%         0.000000         0.000000         0.000000         0.000000         0.000000   |            |                       |           |             |              |            |
| mean         111.20020         158.91350         0.263800         0.201600         0.178600           std         35.64033         37.59736         0.440703         0.401205         0.383027           min         0.00000         3.00000         0.000000         0.000000         0.000000           25%         89.00000         140.00000         0.000000         0.000000         0.000000           50%         110.00000         166.00000         0.000000         0.000000         0.000000           75%         133.00000         185.00000         1.000000         0.000000         0.000000           max         255.00000         245.00000         1.000000         1.000000         1.000000           std         0.478827         0.000000         0.000000         0.000000         0.000000           50%         0.000000         0.000000         0.000000         0.000000         0.000000         0.000000  |            |                       |           |             |              |            |
| std       35.64033       37.59736       0.440703       0.401205       0.383027         min       0.000000       3.00000       0.000000       0.000000       0.000000         25%       89.00000       140.00000       0.000000       0.000000       0.000000         50%       110.00000       166.00000       0.000000       0.000000       0.000000         75%       133.00000       185.00000       1.000000       0.000000       0.000000         max       255.00000       245.00000       1.000000       1.000000       1.000000         std       0.478827       0.000000         50%       0.000000       0.000000  | count      |                       |           |             |              |            |
| min         0.00000         3.00000         0.000000         0.000000         0.000000           25%         89.00000         140.00000         0.000000         0.000000         0.000000           50%         110.00000         166.00000         0.000000         0.000000         0.000000           75%         133.00000         185.00000         1.000000         0.000000         0.000000           max         255.00000         245.00000         1.000000         1.000000         1.000000           std         0.478827         0.000000         0.000000         0.000000         0.000000         0.000000           50%         0.000000         0.000000         0.000000         0.000000         0.000000         0.000000  |            |                       |           |             |              |            |
| 25% 89.00000 140.00000 0.000000 0.000000 0.0000000 50% 110.00000 166.00000 0.000000 0.000000 0.000000 75% 133.00000 185.00000 1.000000 0.000000 0.000000 max 255.00000 245.00000 1.000000 1.000000 1.000000  Scount 20000.000000 mean 0.356000 std 0.478827 min 0.000000 25% 0.000000 50% 0.000000   | std        |                       |           |             |              |            |
| 50% 110.00000 166.00000 0.000000 0.000000 0.000000 75% 133.00000 185.00000 1.000000 0.000000 0.000000 0.000000 max 255.00000 245.00000 1.000000 1.000000 1.000000 1.000000 mean 0.356000 std 0.478827 min 0.000000 0.000000 0.000000 0.000000 0.000000   | min        | 0.00000               | 3.00000   | 0.000000    | 0.000000     | 0.00000    |
| 75% 133.00000 185.00000 1.000000 0.000000 0.0000000 max 255.00000 245.00000 1.000000 1.000000 1.000000 1.000000 std 0.478827 min 0.000000 0.000000 0.000000 0.000000 0.000000  | 25%        | 89.00000              | 140.00000 | 0.000000    | 0.000000     | 0.00000    |
| max     255.00000     245.00000     1.000000     1.000000       3     count     20000.000000       mean     0.356000       std     0.478827       min     0.000000       25%     0.000000       50%     0.000000   | 50%        | 110.00000             | 166.00000 | 0.000000    | 0.000000     | 0.00000    |
| 3 count 20000.000000 mean 0.356000 std 0.478827 min 0.000000 25% 0.000000 50% 0.000000   | 75%        | 133.00000             | 185.00000 | 1.000000    | 0.000000     | 0.00000    |
| count       20000.000000         mean       0.356000         std       0.478827         min       0.000000         25%       0.000000         50%       0.000000   | max        | 255.00000             | 245.00000 | 1.000000    | 1.000000     | 1.000000   |
| count       20000.000000         mean       0.356000         std       0.478827         min       0.000000         25%       0.000000         50%       0.000000   |            |                       |           |             |              |            |
| mean       0.356000         std       0.478827         min       0.000000         25%       0.000000         50%       0.000000  |            |                       |           |             |              |            |
| std       0.478827         min       0.000000         25%       0.000000         50%       0.000000  |            |                       |           |             |              |            |
| min 0.000000<br>25% 0.000000<br>50% 0.000000   |            |                       |           |             |              |            |
| 25% 0.000000<br>50% 0.000000   |            |                       |           |             |              |            |
| 50% 0.000000   |            |                       |           |             |              |            |
|  |            | 0.000000              |           |             |              |            |
| 75% 1.000000   | 50%        | 0.000000              |           |             |              |            |
|  | 75%        | 1.000000              |           |             |              |            |
| max 1.000000   | max        | 1.000000              |           |             |              |            |
|  |            |                       |           |             |              |            |

```
[8 rows x 3140 columns]
```

We can see from these summaries that all 3140 columns, features and labels, are int64, and thus numerical features.

```
[8]: # calculate the number of missing values
n_missing = df_train.isnull().sum()

# print the number missing for each column,
# but ignore 0s since there are so many columns
print(r'===# missing values per column====')
print(n_missing[n_missing != 0])

===# missing values per column===
```

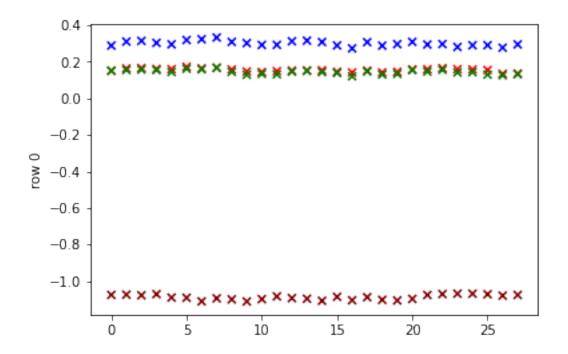
===# missing values per column=== Series([], dtype: int64)

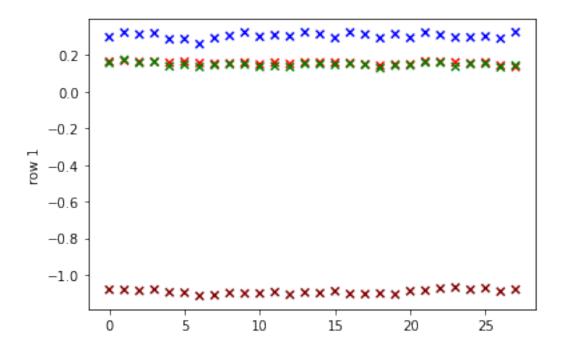
Additionally, we find that there are no columns with missing values. Let's inspect the distributions of each feature and the label.

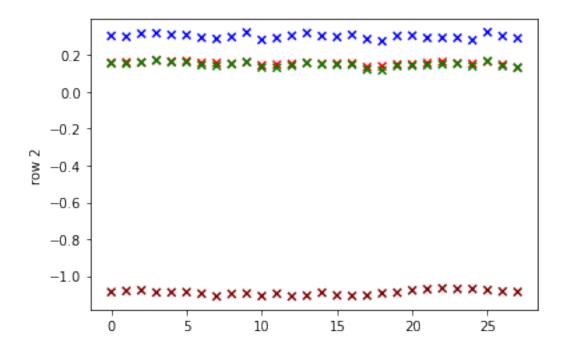
Since there are so many features (3136), we will plot the skewness of each feature by pixel row, rather than a histogram.

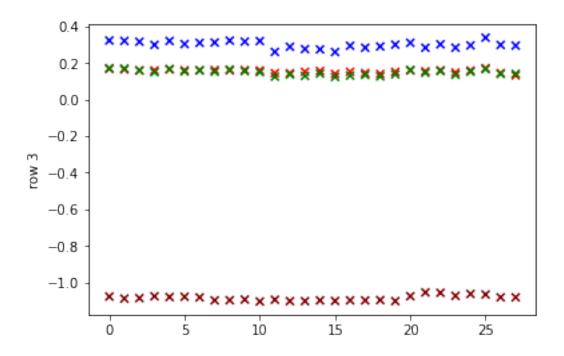
```
[9]: # centralize the data frame
    central_X = X_train - X_train.mean()
    # calculate standard deviations
    X_std = X_train.std()
    # calculate the skews of each column
    skews = N_XAMPS*(central_X**3).sum()/((N_XAMPS - 1)*(N_XAMPS - 2)*X_std**3)
```

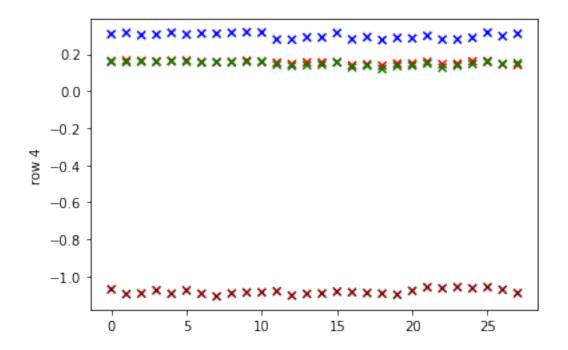
```
[10]: # let's scatter graph the features
      # 28x28, color code RGB
      # x's in scatter graph
      scatter_x = sum(([k]*N_CHANNELS for k in range(I_WIDTH)), [])
      # colors for the scatter graph
      scatter_colors = (CHANNELS*I_WIDTH)
      # its size
      ROW_SIZE = (N_CHANNELS*I_WIDTH)
      # loop through pixel rows
      for i_row in range(0, I_HEIGHT):
          # offset in data frame for this row
          offset = (i_row*ROW_SIZE)
          # create the row as a list
          row = list(skews[X_train.columns[offset:(offset + ROW_SIZE)]])
          # plot the row
          plt.scatter(scatter_x, row, c=scatter_colors, marker='x')
          plt.ylabel("row {}".format(i_row))
          plt.show()
      # next i_row
```

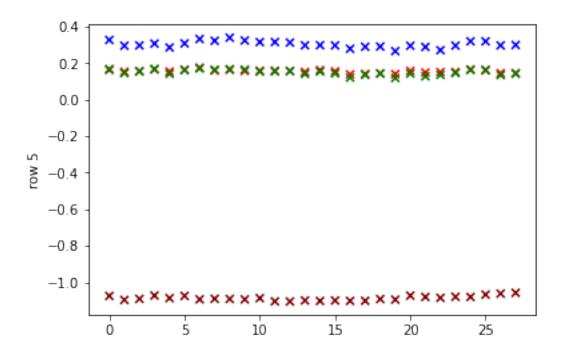


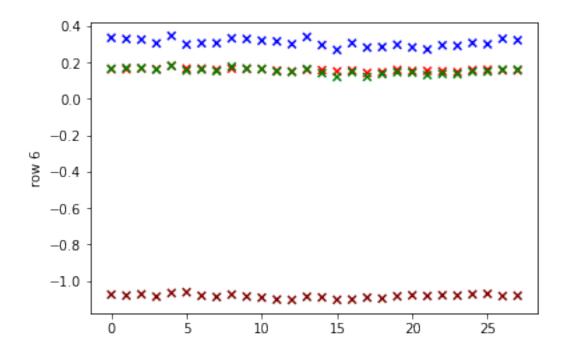


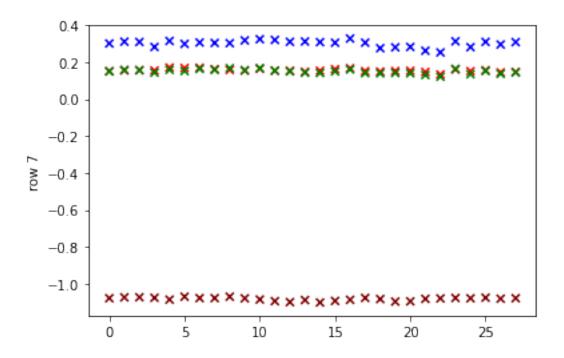


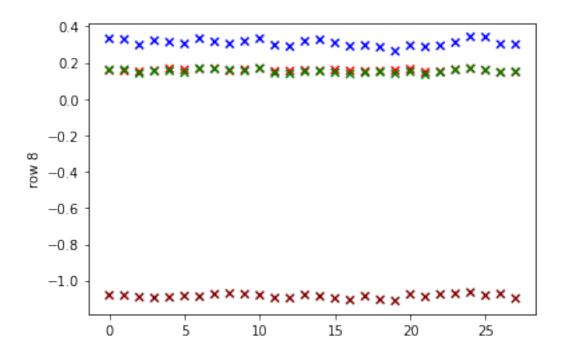


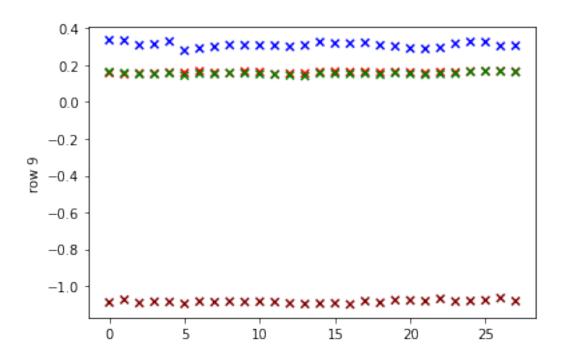


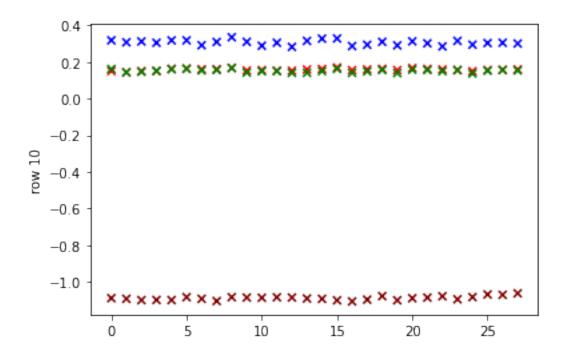


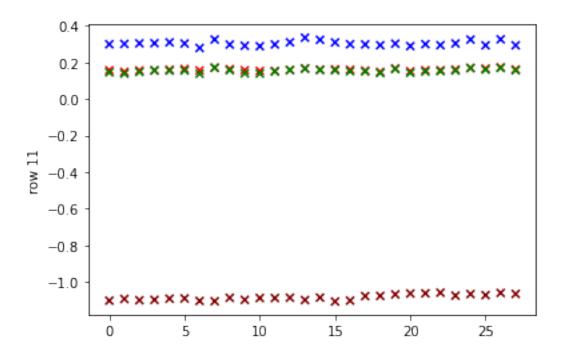


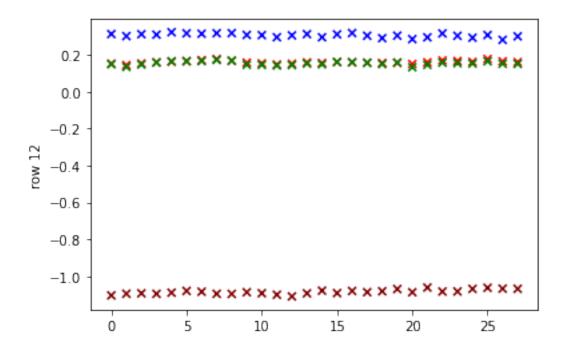


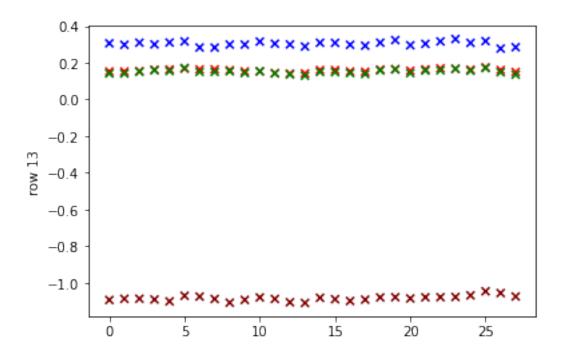


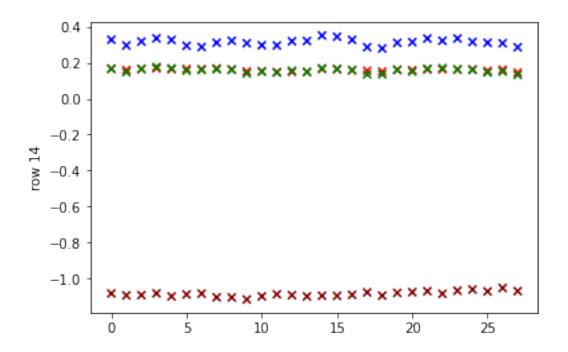


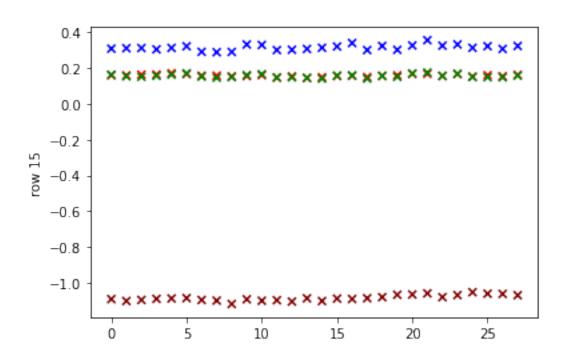


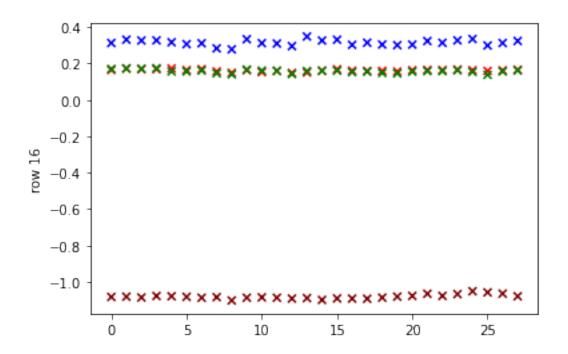


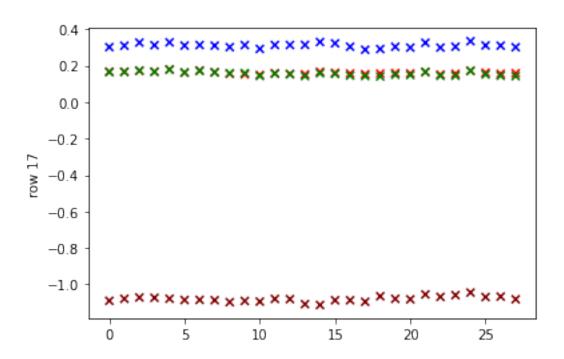


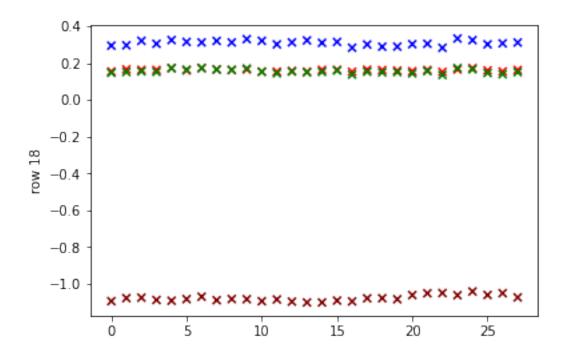


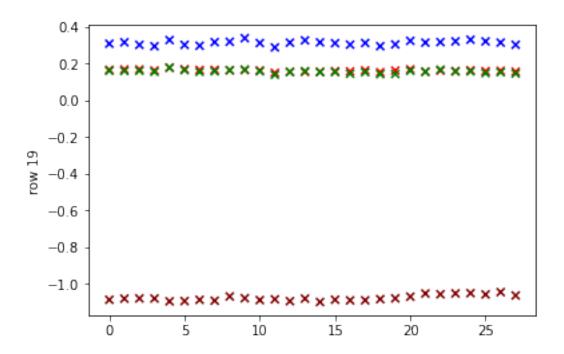


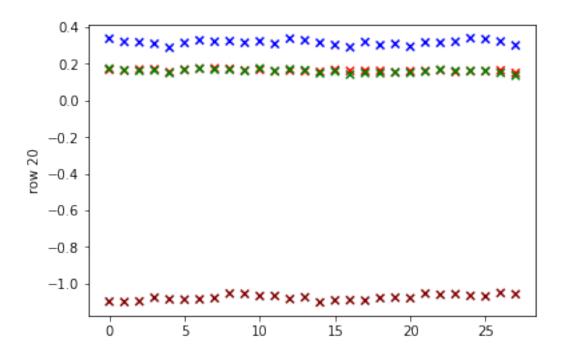


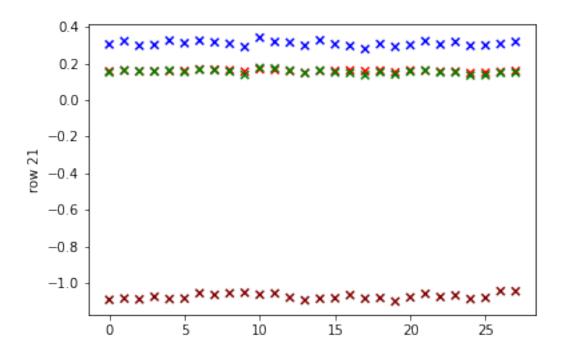


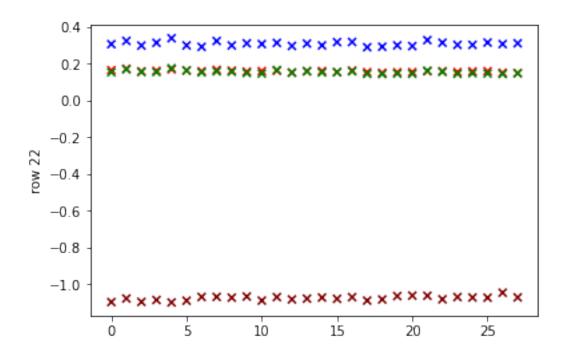


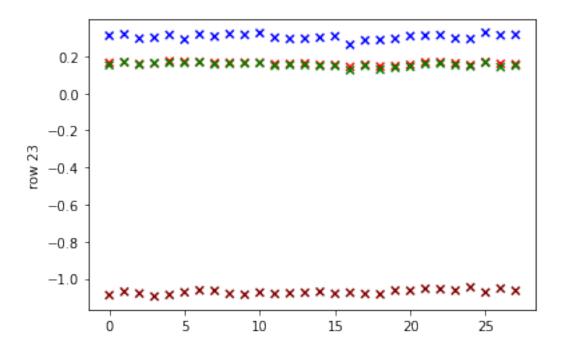


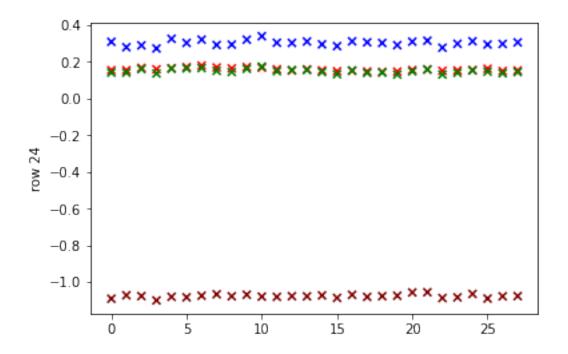


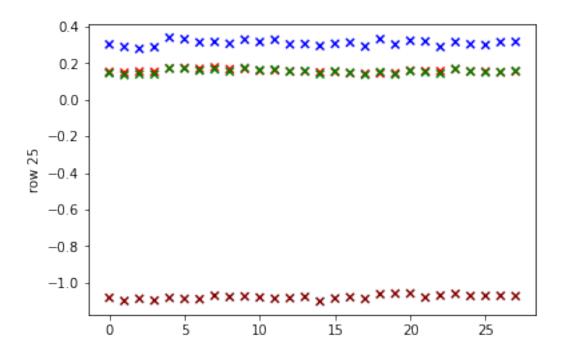


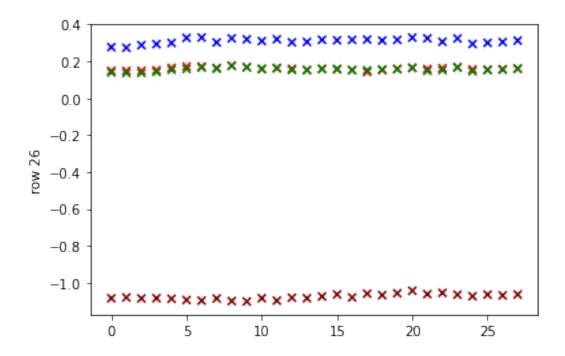


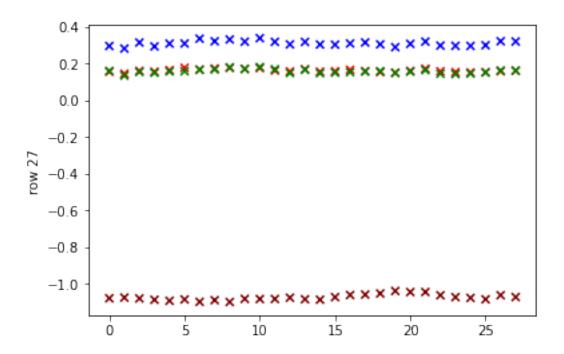












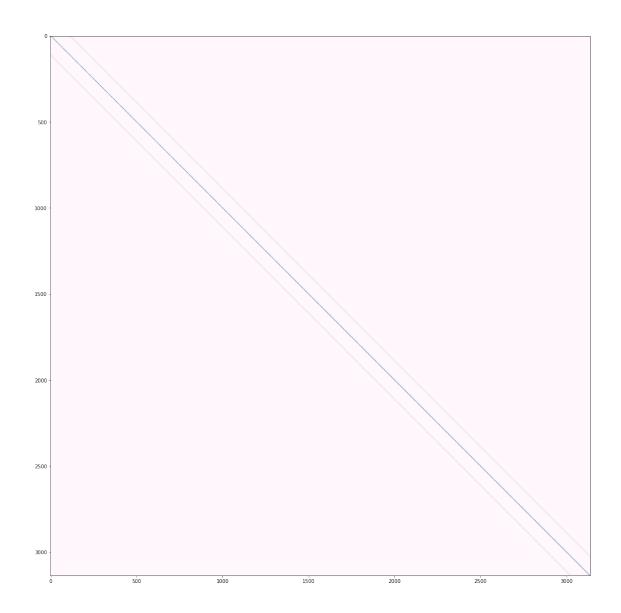
We can see from these scatter plots that: \* the red, green and blue channel are consistently approximately semetric ( $\in [-0.5, 0.5]$ ) (especially the red and green colors). Thus they are nonlong tail distributions. \* the near infrared (NIR) channel are consistently highly skewed negative ( $\in [-\infty, -1.0]$ ). Thus this is a long tail distribution.

Therefore, if there were missing values for us to replace, we would use the mean for the red, green and blue channels, and the median for the NIR channel.

Next, we see the correlations to find any features that we can remove.

```
[11]: # calculate the square correlations
X_cor2 = X_train.cov()**2
X_var = X_std**2
X_varprod = pd.DataFrame([(v0*v1) for v1 in X_var] for v0 in X_var)
# normalize X_cor2 using the product of the corresponding variances
X_cor2 = (X_cor2 / X_varprod)
```

```
[12]: # find strong correlations
is_strong_corr = (X_cor2 >= R2_TOLERANCE)
# graph results
plt.figure(figsize=(20, 20))
plt.imshow(is_strong_corr, cmap='PuBu')
plt.show()
```



```
[13]: # create a table of the strong correlations per feature

i_strong_corr_per_feat = pd.DataFrame(is_strong_corr[k].to_numpy().nonzero() for

→k in range(N_FEATS))

i_strong_corr_per_feat
```

```
      3132
      [3016, 3020, 3128, 3132, 3133, 3134]

      3133
      [3132, 3133, 3134]

      3134
      [3132, 3133, 3134]

      3135
      [3023, 3135]
```

[3136 rows x 1 columns]

Going by domain knowledge of the features, we know that they are all linearly independent.

There seem to be strong correlations at about 112 flattened pixels from each pixel in either direction of most pixels corresponding to  $112 \, \text{px} \times \frac{1 \, \text{row}}{28 \, \text{px}} = 4 \, \text{rows}$  up or down respectively. However, it was not consistent enough to rely on for reducing dimensionality.

Finally, the labels are one-hot encoded, representing

| terrain type | 1000 | 0100 | 0010 | 0001 |
|--------------|------|------|------|------|
| barren_land  | 1    | 0    | 0    | 0    |
| trees        | 0    | 1    | 0    | 0    |
| grassland    | 0    | 0    | 1    | 0    |
| none         | 0    | 0    | 0    | 1    |
|              |      |      |      |      |

Now to see how balanced the target data is, we use a bar plot.

```
---y_train----

0 1 2 3

0 0 1 0 0

1 1 0 0 0

2 0 0 0 1

3 1 0 0 0

4 1 0 0 0

...
```

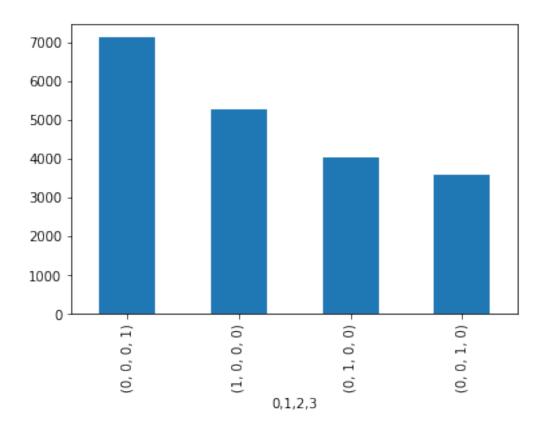
```
19995 0 0
           1 0
19996 1
         0
           0
              0
19997
           0
              0
19998
     0
        1
            0
              0
19999 0
         0
            1
              0
```

### [20000 rows x 4 columns]

===value counts=== 2 3 0 1 0 1 0 dtype: int64

rate of difference = 1 2 3

1 NaN
1 NaN
1 O NaN
1 O NaN
1 type: float64



We see that these total

| encoding | multiplicity |
|----------|--------------|
| 1000     | 5276         |
| 0100     | 4032         |
| 0010     | 3572         |
| 0001     | 7120         |
| Σ        | 20000.       |

Now consider that we have a vector  $\vec{v}$  of 4 Boolean integers. Well in our case,

$$\mathbf{1}_{4}^{\mathsf{T}}\mathbf{v}=1.$$

Thus for any order, it is the case that  $v_4 = 1 - (v_1 + v_2 + v_3)$ . Therefore, we can remove one of these label columns and have a linearly independent set of columns. I choose to remove 0001 because it is the last column.

```
[15]: # columns to keep for linear independence
y_lind_cols = y_train.columns[0:-1]
# remove the last column of the the labels
y_train_lind = y_train[y_lind_cols]
# confirm the removal
print(y_train_lind.shape)
(20000, 3)
```

#### 1.2 Split the data and normalize the features of examples according to training data

```
[16]: # read in the testing data (stored in a separate set of files)
X_test = pd.read_csv(X_TEST_FILENAME, header=None, index_col=None)
y_test = pd.read_csv(Y_TEST_FILENAME, header=None, index_col=None)

# remove the last label column for test data for linear independence too
y_test_lind = y_test[y_lind_cols]

# data shape constants
(N_TEST_XAMPS, N_TEST_FEATS) = X_test.shape
(_, N_TEST_LBLS) = y_test_lind.shape

# confirm the shapes
print("# test examples:\t{}".format(N_TEST_XAMPS))
print("# test features:\t{}".format(N_TEST_FEATS))
print("# test labels:\t{}".format(N_TEST_LBLS))
```

# test examples: 20000
# test features: 3136

#### # test labels: 3

Next we normalize. However, first we check if there are outliers (1.5 IQRs outside of the range  $[Q_1, Q_3]$ ) on  $\mathbf{x}$ 's data frame.

```
[17]: def find_outliers(X):
          r'''
           Finds the outliers in data frame X.
           @param X : pd.DataFrame = to search for outliers
           @return (inlier_min, inlier_max, outliers) = lower and upper
           bounds of outliers, and the outliers in X
           111
          # find the interquartile range
          q1, q3 = (X.quantile(q=q) for q in np.array([1, 3])*0.25)
          iqr = q3 - q1
          # calculate limits of the outlier
          inlier_min = (q1 - 1.5*iqr)
          inlier_max = (q3 + 1.5*iqr)
          # find any values out of range
          outliers = ((X < inlier_min) | (X > inlier_max))
          # return the result
          return (inlier_min, inlier_max, outliers)
      # def find_outliers(X)
```

```
[18]: # check if outliers
    (inlier_min, inlier_max, outliers) = find_outliers(X_train)
# print the limits
print(r'===outlier lower limit===')
print(inlier_min)
print()
print(r'===outlier upper limit===')
print(inlier_max)
print()
# print whether any outliers
print(r'===has outliers?===')
print(outliers.any())
```

```
===outlier lower limit===
0
         6.5
1
        25.5
2
        24.5
3
        72.5
4
         6.5
        . . .
3131
        72.5
3132
        9.0
3133
        28.0
3134
        23.0
3135
        72.5
```

```
Length: 3136, dtype: float64
===outlier upper limit===
0
        250.5
1
        221.5
2
        196.5
3
        252.5
        250.5
        . . .
3131
        252.5
        249.0
3132
        220.0
3133
3134
        199.0
3135
        252.5
Length: 3136, dtype: float64
===has outliers?===
0
        True
1
        True
2
        True
3
        True
        True
        . . .
3131
        True
3132
        True
3133
        True
3134
        True
3135
        True
Length: 3136, dtype: bool
```

Since we found at least 1 column with outliers, we will use standardization (*Z*-score normalization), rather than min-max normalization.

```
===standardized X_train mean statistics===
         3.136000e+03
count
         5.560037e-19
mean
         1.500497e-16
std
min
        -3.765876e-16
25%
        -1.129763e-16
50%
        -1.421085e-18
75%
         1.108447e-16
         3.765876e-16
max
dtype: float64
===standardized X_train standard devaition statistics===
         3.136000e+03
count
         1.000000e+00
mean
std
         9.023635e-17
         1.000000e+00
min
25%
         1.000000e+00
50%
         1.000000e+00
         1.000000e+00
75%
         1.000000e+00
max
dtype: float64
```

From the statistics of the mean and standard deviation, we verify that all features' means  $\bar{x} \approx 0$  and  $S_X \approx 1$  as expected.

#### 1.3 Training the model

Now let's attempt the linear model.

```
[20]: linear = LinearRegression()
linear.fit(X_train, y_train_lind)
```

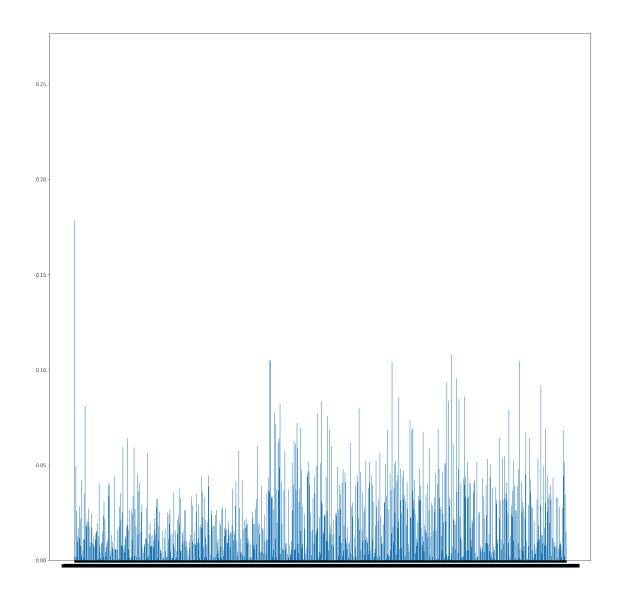
[20]: LinearRegression()

Let's inspect the weights (with bias first) to see which features are as important in determining the model.

```
[21]: print("bias:\t{}".format(linear.intercept_))
print("weights:\t{}".format(linear.coef_))
print("# weights:\t{}".format(linear.coef_.shape))

bias: [0.2638 0.2016 0.1786]
weights: [[ 0.00349978  0.03350413 -0.02830229 ... -0.00076971
-0.01991985
-0.01042235]
[-0.03050444  0.00311614  0.0097701 ...  0.040247  0.00532699
-0.00593741]
[-0.0461447  0.02486673  0.00224196 ... -0.0174027  0.01254749
```

```
-0.00033398]]
     # weights:
                     (3, 3136)
[22]: # get the learned weights
      W = np.insert(linear.coef_, 0, linear.intercept_)
      # flatten weights to a vector
      weights = W.flatten()
      # get the weight magnitude
      w_mag = abs(W).flatten()
      w_mag
[22]: array([0.2638 , 0.2016
                                 , 0.1786 , ..., 0.0174027 , 0.01254749,
             0.00033398])
[23]: # plot the bar graph
      # the bar graph from the data frame would be too compact, so we use plt.bar.
      # 3 significant figures in scientific notation should show all
      # (3 + 3196*3) = 9411 independent features.
      plt.figure(figsize=(20, 20))
      plt.bar(["{:+.3e}".format(weight) for weight in weights], w_mag)
      plt.show()
```



Find the measures of error agains the training data.

```
[24]: # check the model fit to training data
y_train_pred = linear.predict(X_train)

# mean errors
mae_train = mean_absolute_error(y_train_pred, y_train_lind)
mse_train = mean_squared_error(y_train_pred, y_train_lind)
rmse_train = np.sqrt(mse_train)

print()
print(r'===fit to the training set===')
print('MAE\t', mae_train)
print('MSE\t', mse_train)
```

```
print('RMSE\t', rmse_train)
```

## 1.4 Evaluating the model

Find the measures of error agains the testing data.

```
[25]: # check the model fit to testing data
    # use a data frame wrapper for y_test_pred
    y_test_pred = pd.DataFrame(linear.predict(X_test))

# mean errors
mae_test = mean_absolute_error(y_test_pred, y_test_lind)
mse_test = mean_squared_error(y_test_pred, y_test_lind)
rmse_test = np.sqrt(mse_test)

print()
print(r'===fit to the testing set===')
print('MAE\t', mae_test)
print('MAE\t', mse_test)
print('RMSE\t', rmse_test)
```

```
===fit to the testing set===
MAE 0.27122718739782964
MSE 0.12413254391711742
RMSE 0.35232448668396216
```

Let us analyze visually. First, let's reconstruct the linear dependent column of the labels.

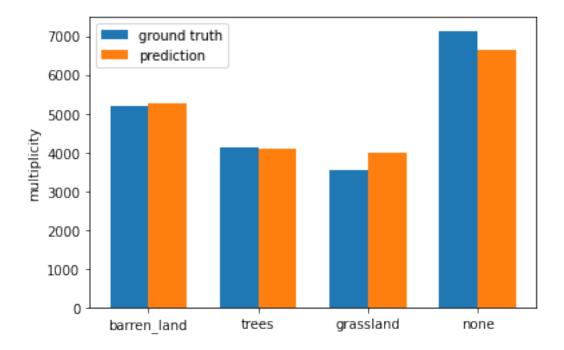
```
[26]: # add the dependent column to the prediction
# v4 = 1 - (v1 + v2 + v3)
y_test_pred_dep = pd.concat((y_test_pred, (1 - y_test_pred.sum(axis=1))),
→axis=1, ignore_index=True)
```

```
[27]: labels = [r'barren_land', r'trees', r'grassland', r'none']
x = np.arange(len(labels)) # the label locations
width = 0.35 # the width of the bars

# print label counts
print(r"=== y value counts===")
print(y_test.sum(axis=0))
print()
print(r"=== y' value row sums===")
```

```
0 5194
1 4117
2 3554
3 7135
dtype: int64

=== y' value row sums===
0 5268.622657
1 4080.350228
2 4010.181841
3 6640.845274
dtype: float64
```



By visual inspection, the model produces a close fit. However, let us normalize the mean errors.

First, let's check for overfit.

```
[29]: print("MAE test:train ratio:\t{}".format(mae_test/mae_train))
print("RMSE test:train ratio:\t{}".format(rmse_test/rmse_train))
```

MAE train:test ratio: 0.8367269764498956 RMSE train:test ratio: 0.8430650654615408

We find that the test-train error ratios *MAE* and *RMSE* are each greater than approximated 1. This means that the testing errors are strictly greater than the training errors. Thus, we have a case of overfit. We will later solve that through ridge regularization.

Next, let's compare the *MAE* and *RMSE* of the test data with the mean absolute deviation around the median and standard deviation respectively of the training data.

```
[103]: # find the norms
L1_y_train = (abs(y_train_lind).dot(np.array([1, 1, 1])))
L2_y_train = (((y_train_lind**2).dot(np.array([1, 1, 1])))**(1.0/2.0))

# for MAE, we find the ratio to the mean absolute deviation around the median of_____
the L1-norm

q2 = L1_y_train.quantile(q=0.5)
mad = np.linalg.norm((L1_y_train - q2), ord=1)/N_XAMPS
mae_iqr_test = mae_test/mad
```

```
# for RMSE, we find the ratio by the standard deviation of the L2-norm
std = L2_y_train.std()
rmse_std_test = rmse_test/std

# display the results
print("MAD:\t{}".format(mad))
print("SD:\t{}".format(std))
print()

print("MAE test:MAD train:\t{}".format(mae_iqr_test))
print("RMSE test:SD train:\t{}".format(rmse_std_test))
```

MAD: 0.356

SD: 0.4788271752659708

MAE test:MAD train: 0.7618741219040158 RMSE test:SD train: 0.7358072074507027

Now since the ratios are both < 1, this means that the mean absolute error is much less than one interquartile range, and the root mean square error is less than one standard deviation.

This means that the model has a strong predictive power. However, because both are close to 1, it is a moderately strong predictive power.

### 1.5 Ridge regularization to reduce overfitting

Finally, let's apply ridge regularization to see if we can reduce overfitting.

Let's establish the baseline for the linear regression.

Here, I attempt to calculate accuracy, recall, precision and  $F_1$  score.

```
precision = n_tp/(n_tp + n_fp)
f1 = mean((recall**-1, precision**-1))**-1

print('threshold for positive is >= ', q2_train)
print()
print('accuracy \t', accuracy)
print('recall \t', recall)
print('precision\t', precision)
print('f1 score \t', f1)
```

threshold for positive is >= [0. 0. 0.]

accuracy 0.3844

recall 0.32501155802126674 precision 0.30288668677294267 f1 score 0.3135593220338983

Here, we use a tolerance of 0.5 for a positive label. Then we use macro-averaging because the labels are independent.

accuracy 0.58665 recall 0.4493992231017458 precision 0.6065020921872838

f1 score 0.5036355376125148

Next, let's compare the accuracy, recall, precision and F1 scores for the range  $[-10 \, dB..11 \, dB]$ .

```
[90]: lambda_db = np.array(range(-10, 11)) # different regularization hyper parameters
→in dB
lambda_lin = 10**(lambda_db/20) # convert to linear scale

# lists to plot
recalls = [ recall ]
precisions = [ precision ]
```

```
# list for maximum ('best') F1 score
f1\_scores = [f1]
for i_lambda, lambda_val in enumerate(lambda_lin):
    # test the model fit to testing data
    ridge = Ridge(alpha=lambda_val)
    ridge.fit(X_train, y_train_lind)
    y_test_pred = ridge.predict(X_test)
    # calculate the scores through macro-averaging
    accuracy = accuracy_score(y_test_lind, y_test_pred >= LABEL_TOLERANCE)
    recall = recall_score(y_test_lind, y_test_pred >= LABEL_TOLERANCE,__
 →average='macro')
    precision = precision_score(y_test_lind, y_test_pred >= LABEL_TOLERANCE,__
 →average='macro')
    f1 = f1_score(y_test_lind, y_test_pred >= LABEL_TOLERANCE, average='macro')
    # append to the lists
    recalls.append(recall)
    precisions.append(precision)
    f1_scores.append(f1)
    # display result for this lambda
    print(fr'==regularization parameters = {lambda_db[i_lambda]} dB = __
 →{lambda_val}===')
    print('accuracy \t', accuracy)
    print('recall \t', recall)
    print('precision\t', precision)
    print('f1 score \t', f1)
    print()
# next lambda_val
===regularization parameters = -10 dB = 0.31622776601683794===
accuracy
                0.58715
                0.44922446696412854
recall
               0.6075849506159723
precision
f1 score
               0.5037411510816644
===regularization parameters = -9 dB = 0.35481338923357547===
                0.5872
accuracy
recall
                0.4492886435781704
precision
                0.6077762406861051
                0.5038066425016782
f1 score
===regularization parameters = -8 dB = 0.3981071705534972===
                 0.5873
accuracy
recall
                 0.4494337852962524
```

0.607844995207374 precision 0.5039229368765102 f1 score ===regularization parameters = -7 dB = 0.44668359215096315=== 0.58735 accuracy 0.4493528201922122 recall precision 0.6079392009526936 f1 score 0.5038767122931914 ===regularization parameters = -6 dB = 0.5011872336272722=== 0.58735 accuracy recall 0.449271855088172 0.6079398509472012 precision 0.5038157451076203 f1 score ===regularization parameters = -5 dB = 0.5623413251903491=== accuracy 0.58755 recall 0.4490842730210176 precision 0.6081144087457719 f1 score 0.5036484574642933 ==regularization parameters = -4 dB = 0.6309573444801932=== accuracy 0.5876 recall 0.4490842730210176 precision 0.6083680618451495 0.503697137969315 f1 score ===regularization parameters = -3 dB = 0.7079457843841379=== accuracy 0.58735 recall 0.4486370071517444 precision 0.6078782887164116 f1 score 0.5031616356837981 ===regularization parameters = -2 dB = 0.7943282347242815=== 0.58745 accuracy recall 0.44885028804432964 precision 0.6078626211036924 f1 score 0.5032986730138832 ===regularization parameters = -1 dB = 0.8912509381337456=== 0.58755 accuracy 0.4486755319067122 recall 0.60803368099758 precision 0.5031846728113596 f1 score ===regularization parameters = 0 dB = 1.0=== accuracy 0.58775

0.44873970852075407

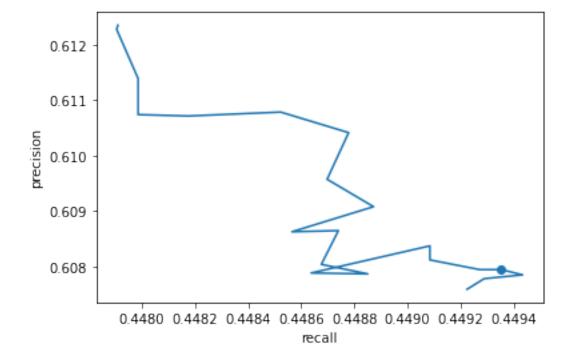
recall

0.6086427593859304 precision 0.5033432141352383 f1 score ===regularization parameters = 1 dB = 1.1220184543019633=== 0.58775 accuracy recall 0.4485649523831367 precision 0.6086225636833118 f1 score 0.5032103547844932 ===regularization parameters = 2 dB = 1.2589254117941673=== 0.588 accuracy 0.448872024309299 recall 0.6090748139440184 precision 0.5035466488795411 f1 score ===regularization parameters = 3 dB = 1.4125375446227544=== accuracy 0.5881 recall 0.44869726817168165 precision 0.6095694906461742 f1 score 0.5035110328600362 ===regularization parameters = 4 dB = 1.5848931924611136=== accuracy 0.5885 recall 0.4487782332757218 precision 0.6104109099997941 0.5037766968830625 f1 score ===regularization parameters = 5 dB = 1.7782794100389228=== accuracy 0.5885 recall 0.44852251203406435 precision 0.6107817976750681 0.5036020810947259 f1 score ===regularization parameters = 6 dB = 1.9952623149688795=== 0.58845 accuracy recall 0.44817299975882957 precision 0.6107110409285051 f1 score 0.5032615372821362 ===regularization parameters = 7 dB = 2.2387211385683394=== 0.58855 accuracy recall 0.44798541769167527 precision 0.6107363934997151 0.5030861896774851 f1 score ===regularization parameters = 8 dB = 2.51188643150958=== accuracy 0.58875 recall 0.44798541769167527

```
precision
                 0.6113871721572975
f1 score
                 0.5032827113710475
===regularization parameters = 9 dB = 2.8183829312644537===
                 0.5891
accuracy
recall
                 0.44790445258763506
precision
                 0.6122752617389601
f1 score
                 0.5034322630651811
===regularization parameters = 10 dB = 3.1622776601683795===
                 0.58925
accuracy
recall
                 0.4479094003626063
                 0.6123493532783794
precision
f1 score
                 0.5033598847989803
```

We can compare recall and precision by plotting them as follows where the disc represents the model with no regularization.

```
[91]: plt.plot(recalls[1:], precisions[1:])
  plt.scatter(recalls[0], precisions[0])
  plt.xlabel('recall')
  plt.ylabel('precision')
  plt.show()
```



Finally, the maximum F1 score gives us.

Best model is 'regularization = -8 dB = 0.3981071705534972' with F1 score = 0.5039229368765102.

# Part III Sampling script

```
r'''
 Canonical: https://github.com/lduran2/CIS3715_DataScience_2022/blob/main/final/
 \hookrightarrow sample_dataset.py
 Samples a set of X, y from examples of dataset files.
          : Leomar Durán <a href="https://github.com/lduran2">https://github.com/lduran2</a>
          : CIS 3715/Principles of Data Science
 For
 CHANGELOG:
    v1.2.7 - 2022-04-19t01:37Q
        `mainarg` for each arg, delete `df` after use
    v1.2.6 - 2022-04-12t17:01Q
        `sampleCsvFile` returns tuple of lists
    v1.2.5 - 2022-04-12t16:10Q
        lists->tuples where possible
    v1.2.4 - 2022-04-11t01:30Q
        fixed df headers written
    v1.2.3 - 2022-04-10t23:45Q
        stacks->iterators, implemented associated (X,y)
    v1.2.2 - 2022-04-10t22:16Q
        fixed df index written, IDed feat/tgt files
    v1.2.1 - 2022-04-10t21:35Q
        modularized `sample_dataset.py`
    v1.2.0 - 2022-04-10t17:22Q
        sampling is done
    v1.1.0 - 2022-04-10t14:56Q
        counting row of each file
    v1.0.0 - 2022-04-07t07:34Q
        gets the options for sampling
import sys
                                     # gets command line arguments
from getopt import getopt
                                     # to handle command line options
import csv
                                    # for list read
                                     # for sampling
import random
import pandas as pd
                                    # for dataframe
                                   # for handling paths
from pathlib import Path
from cqs_iter import CqsIter # for iterator
```

```
# constants
K_EXAMPLES = 20000
                                   # default to sampling 20,000
OPTIONS = r'n:'
                                   # short commandline option names
LONG_OPTIONS = (r'nsamples=', )
                                  # long commandline option names
SEED = 42
                                    # seed for sampling
def main(K_EXAMPLES=K_EXAMPLES):
    # apply the seed
   random.seed(SEED)
    # get the options from commandline
    option_to_values, argv = getopt(sys.argv[1:], OPTIONS, LONG_OPTIONS)
    # loop through options (key, value) pair
    for kopt, opt_value in option_to_values:
        # store if K_EXAMPLES key
        if (kopt in (r'-n', r'--nsamples')):
            K_EXAMPLES = int(opt_value)
        # end if (kopt in (r'-n', r'--nsamples'))
    # next kopt
    # report number of samples
    print(r"===sampling to {} examples===".format(K_EXAMPLES))
    # loop through filenames in argu
    for X_filename in argv:
        # sample X, y represented by that filename
        mainarg(X_filename, K_EXAMPLES)
    # next X_filename
# end def main(K_EXAMPLES=K_EXAMPLES)
def mainarg(X_inname, K_EXAMPLES=K_EXAMPLES):
   r'''
     Samples a set of X, y given a dataset's filename.
     Qparam X_inname : str = filename '*/X_*' representing a dataset
        X, y
     Oparam K_EXAMPLES : int = # indices to sample
   print()
    # set up file names
    # input paths X, y
    X_inpath = Path(X_inname)
    y_inpath = X_inpath.parent / r''.join((r'y', X_inpath.name[1:]))
    # output paths X, y
    samp_label = (r'_samp', str(K_EXAMPLES), X_inpath.suffix)
    X_outpath = X_inpath.parent / r''.join((X_inpath.stem, *samp_label))
    y_outpath = X_inpath.parent / r''.join((y_inpath.stem, *samp_label))
    # report reading/writing
```

```
print(r'===reading from===')
    print(X_inpath)
    print(y_inpath)
   print(r'===writing to===')
    print(X_outpath)
    print(y_outpath)
    # perform conversion
    with open(X_inpath) as X_csvfile, open(y_inpath) as y_csvfile:
        # sample the csvfiles
        # y_csvfile 1st because it will be used to find
            dimensionality and is smaller
        csvfiles = (y_csvfile, X_csvfile)
        outlists = tuple([] for k in range(len(csvfiles)))
        sampleCsvFile(outlists, csvfiles, K_EXAMPLES)
        # loop through dataframes
        for (outlist, outpath) in zip(outlists, (y_outpath, X_outpath)):
            # report the writing
            print(r"writing to {}".format(outpath))
            # convert to dataframe
            df = pd.DataFrame(outlist)
            # write to each csv file
            df.to_csv(outpath, header=False, index=False)
            # delete the reference to dataframe
            del df
        # next (outlist, outpath)
    # end with X_csvfile, y_csvfile
# end def mainarq(X_inname, K_EXAMPLES)
def sampleCsvFile(dests, infiles, K_EXAMPLES):
   r'''
     Samples K_EXAMPLES from infiles into destination lists,
     representing dataframes.
     Oparam dests : tuplelike<listlike> = K-sampled examples from
        infiles to be output
     Oparam infiles : file = source files in CSV format
     @param K_EXAMPLES : int = # indices to sample
     Oreturn the tuple of K-sampled lists, dest.
    # create readers for rows as lists
    csvins = tuple(csv.reader(infile) for infile in infiles)
    # get the dimensionality (from 1st file)
    (N_EXAMPLES, _) = shape_2d(csvins[0])
    # rewind the file
    infiles[0].seek(0)
    # report number of examples
    print(r"===sampling from {} examples===".format(N_EXAMPLES))
    # sample the indices
    i_samples = sampleIndexes(N_EXAMPLES, K_EXAMPLES)
```

```
# for each csv input file and destination listlike
    for i_csvin, (csvin, dest) in enumerate(zip(csvins, dests)):
        # report file being copied into memory
        print(r"{}. copying from {}".format(i_csvin, infiles[i_csvin]))
        # iterator on samples
        iit = CqsIter(iter(i_samples))
        # copy the csv file
        idxdCpy(dest, csvin, iit)
    # next csvin
    # return the dataframe
    return dests
# end def sampleCsvFile(dests, infiles, K_EXAMPLES)
def shape_2d(tuple2d):
   r'''
     Finds the shape of bidimensional tuple-like representing a
     dataframe.
     @param tuple2d : tuplelike<tuplelike> = to traverse
     Oreturn (N_EXAMPLES, N_FEATURES) = # of examples, # features of
        the dataframe
    # loop through the rows
    for irow, row in enumerate(tuple2d):
        pass
    # next row
    # store the dimensionality of the file
    (N_EXAMPLES, N_FEATURES) = (irow, len(row))
    # return the dimensionality
    return (N_EXAMPLES, N_FEATURES)
# end def shape_2d(tuple2d)
def sampleIndexes(N, K):
   r'''
     Creates a list of the K sampled indices [0..N[.
     Oparam N : int = # elements in original iterable
     Oparam K : int = # indices to sample
     Oreturn list of sampled indices.
    # indices to be sampled
    iis = range(N)
    # sample them
    i_samples = random.sample(iis, K)
    # sort them
    i_sorted = sorted(i_samples)
    # return sorted indexes
   return i_sorted
# end def sampleIndexes(N, K)
```

```
def idxdCpy(dest, src, iit):
   r'''
     Copies elements whose index matches those from the index iterator
     in order from src in dest.
     Oparam dest : list<T> = destination list
     Oparam src : iterable = source iterable
     @param iit : CqsIter<int> = the index iterator
     Oreturn the list copy, dest.
    for idx, el in enumerate(src):
        # return if no more samples
        if (not(iit)):
            return dest
        # end if (not(iit))
        consumeIdxd(dest.append, idx, el, iit)
    # next el
    return dest
# end def idxdCpy(dest, src, iit)
def consumeIdxd(consume, idx, el, iit):
   r'''
     Calls the consume callback on the given element if its index is
     next in the index iterator.
     Oparam consume : function<? super T> = to consume elements
     Oparam idx : int = index of current element
     Oparam\ el\ :\ T\ =\ element\ to\ consume
     @param iit : CqsIter<int> = the index iterator
    # consume only if element is next indexed
    if (idx != iit.get()):
        return
    # end if (idx != iit.get())
    # add this element
    consume(el)
    # pop the sample found
    iit.next()
# end def consumeIdxd(consume, idx, el, iit)
if __name__ == "__main__":
   main()
# if __name__ == "__main__"
```

## Part IV

## Command-query separated iterator

```
r'''
 Canonical: https://github.com/lduran2/CIS3715_DataScience_2022/blob/main/final/
 \rightarrow cqs_iter.py
 A command-query separated version of iterator.
           : Leomar Durán <a href="https://github.com/lduran2">https://github.com/lduran2</a>
           : CIS 3715/Principles of Data Science
 For
 Osee https://wiki.kluv.in/a/CommandQuerySeparation (wiki.c2.com)
 CHANGELOG:
    v1.0.1 - 2022-04-11t00:41Q
         `isValid`->`__bool__`, implemented `__iter__`
    v1.0.1 - 2022-04-10t23:26Q
        handling no more elements in iterator
    v1.0.0 - 2022-04-10t22:59Q
        initial implementation
class CqsIter:
    # Represents no more elements in an iterator.
    __default = object()
    def __init__(self, it):
        r'''
         Adapts the iterator `it`.
         @param it : iter = the backing iterator
         111
        self._it = it
        CqsIter.__next(self)
    # end def __init__(self, it)
    def next(self):
        r'''
         Moves the iterator to the next element.
        CqsIter.__next(self)
    # def next(self)
    def __bool__(self):
        r'''
         Returns true if the iterator is valid, i.e., if there is a current \Box
 \hookrightarrow element.
         Oreturn true if the iterator is valid; false otherwise.
        return (self._curr is not CqsIter.__default)
```

```
# def isValid(self)
   def get(self, default=__default):
       r / / /
         Returns the current element whereto the iterator points.
         Oparam default : object = to return when no more elements
         Oreturn the current element whereto the iterator points.
        # if no more elements
        if (self._curr is CqsIter.__default):
            # and no default, then raise exception
            if (default is CqsIter.__default):
                raise StopIteration
            else:
                # otherwise, return the given default
                return default
            # end if (default is CqsIter.__default) //
       return self._curr
    # end def get(self, default=CqsIter.__default)
   def __iter__(self):
       r'''
        Returns the backing iterator.
         Oreturn the backing iterator.
       return self._it
   def __next(self):
       r'''
         Private function implements #next.
         Osee #next
        self._curr = next(self._it, CqsIter.__default)
    # end def __next(self)
# end class CqsIter
```