Part 2 — Laplace transforms

```
clear
```

1. The Laplace transform

Find the Laplace transform of

$$f(t) = 0.0075 - 0.00034e^{-2.5t}cos(22t) + 0.087e^{-2.5t}sin(22t) - 0.0072e^{-8t}.$$

```
% make a symbol t
syms t
% the function f(t) represented by MATLAB
f_t = 0.0075 - 0.00034*exp(-2.5*t)*cos(22*t) + ...
    0.087*exp(-2.5*t)*sin(22*t) - 0.0072*exp(-8*t)
```

```
f_t =
```

$$\frac{87\sin(22\,t)\,\mathrm{e}^{-\frac{5\,t}{2}}}{1000} - \frac{17\cos(22\,t)\,\mathrm{e}^{-\frac{5\,t}{2}}}{50000} - \frac{9\,\mathrm{e}^{-8\,t}}{1250} + \frac{3}{400}$$

The Laplace transform of $f(t)$,

```
% make a symbol s
syms s
% this seems to be the best we can do although it just gives the sum of
% rational expressions instead of a rational expression of polynomials
% because you cannot convert symbol to transfer function
F = laplace(f_t, s)
```

```
F =
```

$$\frac{3}{400\,s} - \frac{9}{1250\,(s+8)} - \frac{17\left(s+\frac{5}{2}\right)}{50000\left(\left(s+\frac{5}{2}\right)^2+484\right)} + \frac{957}{500\left(\left(s+\frac{5}{2}\right)^2+484\right)}$$

```
clear
```

2. The inverse of the Laplace transform

Find the inverse Laplace transform of

```
% the transfer function to find the inverse Laplace of
F = zpk(-[3 5 7], [0 8 roots([1 10 100])'], 2)
```

```
F =

     2 (s+3) (s+5) (s+7)
  -------------------------
```

```
  s (s-8) (s^2 + 10s + 100)

Continuous-time zero/pole/gain model.
```

We can represent the transfer function in Matlab from its roots

```
% make a symbol s
syms s

% get zeros and poles
F_z = F.z{1};
F_p = F.p{1};
% zeroes and poles that are purely real
F_zr = F_z(imag(F_z) == 0);
F_pr = F_p(imag(F_p) == 0);
% zeroes and poles with negative impaginary part
F_zmi = F_z(imag(F_z) < 0);
F_pmi = F_p(imag(F_p) < 0);
% we don't need the positive imaginary parts because these are just the
% conjugates of the negative ones, and we can get them from the
% negative ones

% multiply the real zeroes/real poles (all linear polynomials)
F_linear = prod(s - F_zr)/prod(s - F_pr);
% multiply in the complex ones (all quadratic polynomials)
F_quadratic = prod(factorFromComplexRoot(F_zmi, s))/...
                    prod(factorFromComplexRoot(F_pmi, s));
% multiply linear, quadratic and gain
F_s = F_linear * F_quadratic * F.k(1)
```

F_s =

$$\frac{2\ (s+3)\ (s+5)\ (s+7)}{s\ ((s+5)^2+75)\ (s-8)}$$

Then, the inverse Laplace transform of $F(s)$,

```
% make a symbol t
syms t
f_t = ilaplace(F_s, t)
```

f_t =

$$\frac{2145\,e^{8t}}{976}+\frac{79\,e^{-5t}\left(\cos\left(5\sqrt{3}\,t\right)+9\sqrt{3}\,\sin\left(5\sqrt{3}\,t\right)\right)}{1220}-\frac{21}{80}$$

```
function factor = factorFromComplexRoot(complexRoot, s)
```

factorFromComplexRoot(complexRoot, s)

Returns the polynomial factor $P(s)$ that when fixed to 0, gives the given **complexRoot** for polynomial variable **s**.

Input Arguments

complexRoot : double = root for which to find the polynomial

Output Arguments

factor : the factor polynomial $P(s)$ that equals 0 at **s == complexRoot**

```
    factor = ((s - real(complexRoot)).^2 + imag(complexRoot).^2);
end % function factorFromComplexRoot(complexRoot, s)
```