ECE 4522 MATLAB Assignment 2

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***Abstract*—This assignment introduces various types of systems and provides hands on experience with the impulse response. It cements the concept of convolution using MATLAB including effects on a sound sample, and expands it with the concept of a restoring system to undo convolution. We will find the error plot and worst-case error to compare out actual result with the expected input and prove that a restoring system is possible.**

***Index Terms*—convolution, finite-duration impulse response, FIR, discrete-time system, echo system, restoration system, digital signal processing, cascading systems**

I. Objectives

The objectives of this report are to prove the concept of a restoring system and to provide a concrete example of an echo system on a sound sample.

After this lab, students will be able to identify various types of systems including echo systems, restoring systems, and cascading systems.

II. Methods

`./Ece4522/MatlabAssignment2/A.m` is a template included in the specification demonstrating a simple convolution of a sinusoidal wave running through a running average system and plots the input and output.

`./Ece4522/MatlabAssignment2/B1.m` has multiple parts. Part B.1, performs a similar convolution on a different wave. Part B.2 then performs a Restoration System which attempts to recover the original system. Part B.3 finds what the error (difference) in the worst-case was when the restoration was attempted. Finally, Part B.4 reads in a sound sample, applies a similar convolution to part B.1, and saves the result.

`./Ece4522/MatlabAssignment2/C.m` applies two systems to be cascaded to an impulse signal, which will later be compared to the same cascade found mathematically.

III. Results

A. Part B.1 FIR System

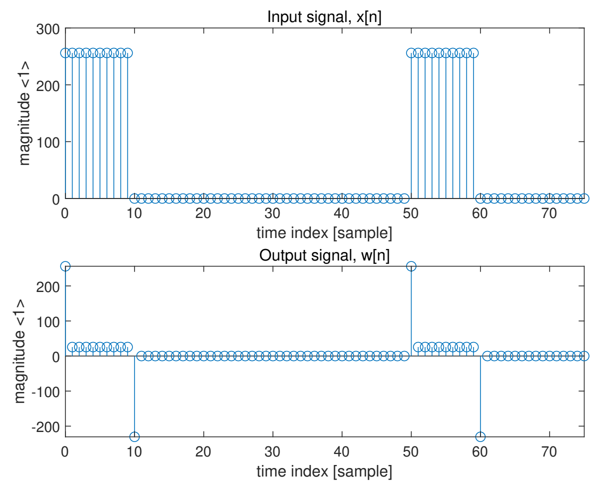

Fig. 1. Input signal $x(n)$ and output signal $w[n]$ of an FIR system.

B. Part B.2 Restoration System (b)

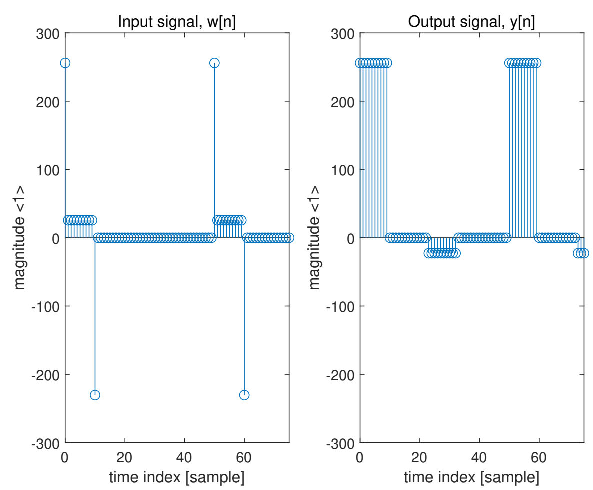

Fig. 2. Input signal $w(n)$ and output signal $y[n]$ of a restoration system. This system aims to restore the signal $x(n)$.

C. Part B.2 Restoration System (c)

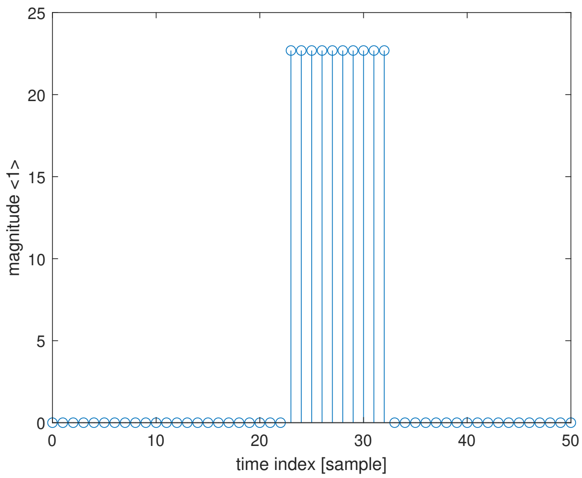

Fig. 3. Error (difference) between $y(n)$ found in part B.2(b) and the original $x(n)$.

IV. Insights

A. Part B.1 FIR System

The input in Fig. 1 represents 256 times the remainder of $n \in \mathbb{Z}[0..100]$ divided by 50. Whereas, the output is equivalent to the input at the current sample, subtracted by the input at the immediately previous sample attenuated to 0.9 times its amplitude.

Since the impulse response $h[n] := [1 \quad -0.9] \approx [1 \quad -0.9 \quad 0 \quad 0 \quad \cdots \quad 0]$, we find that

$$\begin{aligned} & y[n] \\ := & \sum_{k=0}^{M} h[k]x[n-k] \\ = & 1x[n-0] - 0.9x[n-1] + 0x(n-2) + \cdots + 0x[n-M] \\ = & x[n] - 0.9x[n-1] \end{aligned}$$

is the effect of the system impulse response on the input signal.

To find the length of this output signal, let's define $\nu(\vec{v})$ as the length of the vector for any row vector $\vec{v}$. Then $\nu(w[\cdot]) = \nu(x[\cdot]) + \nu(h[\cdot]) - 1$. In this case, $\nu(w[\cdot]) = \nu[0..100] + \nu[1 \quad -0.9] - 1 = 101 + 2 - 1 = 102$.

B. B.3 Worst-Case Error

The error plot in Fig. 3 shows the absolute difference between the original (and expected) signal $x[n]$ and the signal $y[n]$ that was received from the restoration system. It shows that when $n \in [0, 23) \cup (32, 50]$, the error is 0, but in 22.6891 when $n \in [23, 32]$. The worst-case error shows the maximum absolute error, which is in fact 22.6891. Compared to the amplitude, this is a rate of $\frac{22.6891}{256} \approx 0.0886$, which is still very significant.

If the worst-case error had been of a much lower magnitude, at about 7.3×10^{-15}, such as the value at $y[22]$ then the difference with $x[n]$ would not be noticeable at all in its plot.

C. Part B.4 An Echo System

$y_1[n] = x_1[n] + rx_1[n-P]$ represents an echo system because it adds onto $x_1[n]$, a repetition delayed by P samples at a rate of r the original's strength as defined by an echo system.

To have a $90\,\%$ strength echo delayed by 0.2 seconds, $r := 90\,\% = 0.9$. Now

$$x_a(t) = x_a(nT) = x_1[n].$$

Then $t - 0.2\,\mathrm{s} = (n-P)T = nT - PT$. So $0.2\,\mathrm{s} = PT = P/8000\,\mathrm{Hz}$. Thus $P := (0.2\,\mathrm{s})(8000\,\mathrm{Hz}) = 1600\,\mathrm{samples}$.

So a $r := 0.9$ and $P := 1600\,\mathrm{samples}$ will satisfy the requirements for this system.

Now, the impulse response of the echo system

$$h[n] = \delta[n] + r\delta[n-P] := \begin{cases} 1 & \text{if } n = 0; \\ 0.9 & \text{if } n = 1600; \\ 0 & \text{otherwise.} \end{cases}$$

and the length $\nu(h[\cdot]) = P + 1 = 1600 + 1 = 1601$.

D. Part C. Cascading Two Systems

Since FIR System-1 impulse response $h_1[n] := [1 \quad -q] \approx [1 \quad -q \quad 0 \quad 0 \quad \cdots \quad 0]$, then the overall impulse response

$$\begin{aligned} & h[n] \\ := & \sum_{\ell=0}^{M} r^{\ell} h_1[n-\ell] \\ = & \sum_{\ell=0}^{M} h_1[\ell] r^{(n-\ell)} && \text{by commutativity.} \\ = & 1r^{(n-0)} - qr^{(n-1)} \\ = & r^n - qr^{(n-1)}. \end{aligned}$$

Well $h[0] = 0.9^0 = 1$ and $h[m] = 0.9^m - 0.9(0.9^{(m-1)}) = 0.9^m - 0.9^m = 0$ for all $m \neq 0$.

Thus the overall impulse response

$$h[n] = [1], \tag{1}$$

the impulse signal.

Now for an impulse signal to give $y[n] = \sum_{k=0}^{M} h[k]x[n-k] = x[n]$, note that it's the case that

$$x[n] = 1x[n-0] + 0x[n-1] + 0x[n-2] + \cdots + 0x[n-M].$$

So

$$1x[n-0] + 0x[n-1] \cdots + 0x[n-M] = \sum_{k=0}^{M} h[k]x[n-k].$$

So $h[n] = [1] = (h_1 * h_2)[n] \iff: P(h_1, h_2)$.

As found in the MATLAB script mathematically resulting in equation (1), it is the case that the FIR System-1 represented by $h_1[\cdot]$ and the FIR System-2 represented by $h_2[\cdot]$ do meet the condition expressed by $P(h_1, h_2)$.

V. CONCLUSION

We have proven that it is mathematically possible for two systems $h_1[\cdot], h_2[\cdot]$ to create cascade with the latter forming a restorative system as long as they satisfy $P(h_1, h_2) :\Leftrightarrow (h_1 * h_2)[n] = [1]$.

We have also identified moving average and echo impulse responses, and demonstrated cascading systems.