Homework IV

2020년 2월 28일 금요일 오후 12:28

recursion tree or master's method

T(n) is constant for $n \le 2$

Q: Solving by Recursion-Tree

a) $T(n) = 36T(n/6) + 2n$

$a = 36$

$b = 6$

$n^{\log_b a} = n^{\log_6 36} = n^2$

$f(n) = 2n = O(n^{2-\varepsilon})$ $\quad \varepsilon = 1 > 0$

$\therefore T(n) = \Theta(n^2)$

b) $T(n) = 5T(n/3) + 17n^{1.2}$

$a = 5$

$b = 3$

$n^{\log_b a} = n^{\log_3 5} = n^{1.465}$

$f(n) = 17n^{1.2} = O(n^{1.465-\varepsilon})$ $\quad \varepsilon = 0.265 > 0$

$\therefore T(n) = \Theta(n^{1.465})$

c) $T(n) = 12T(n/2) + n^2 \lg n$

$a = 12$

$b = 2$

$n^{\log_b a} = n^{\log_2 12} = n^{3.585} >$ $\quad T(n) = \Theta(n^{\log_2 12})$

$f(n) = n^2 \lg n$

d) $T(n) = 3T(n/5) + T(n/2) + 2^n$

$T(n/2) \approx T(n/5)$

↳ $T(n) = 4T(n/5) + 2^n$

$a = 4$

$b = 5$

$n^{\log_5 4} = n^{0.86} <$

$f(n) = 2^n$

check regularity condition ✓

$a \cdot f(n/b) \le c \cdot f(n)$ for $c < 1$?

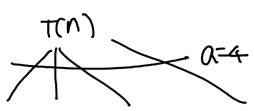
$4 \cdot f(n/5) \le c \cdot f(n)$ for $c < 1$?

$4 \cdot 2^{n/5} \le c \cdot 2^n$ for $c < 1$?

assume $c = 1/2$

$2^{n/5} \le \frac{1}{8} \cdot 2^n$ for all $n > 0$

$\therefore T(n) \in \Theta(2^n)$



e) **Bonus**

$T(n) = T(2n/5) + T(3n/5) + \Theta(n)$

$T(2n/5) \approx T(3n/5)$

$T(n) = 2T(2n/5) + \Theta(n)$

$a = 2$

$b = \frac{5}{2}$

$\log_{5/2} 2 = \log_{2.5} 2$ $\quad n^{\log_{2.5} 2} <$

$f(n) = \Theta(n)$

↳ check regularity con.

$2 \cancel{c} \frac{2n}{5} \le k \cdot \cancel{c} n$ $\quad k < 1$

$\frac{4}{5} n \le k n$ holds for $k < 1$

$\therefore T(n) = \Theta(n)$