

Legendre 多项式为 $[-1,1]$ 上的正交多项式, 满足递推公式

$$\begin{cases} (n+1)P_{n+1}(x) = (2n+1)xP_n(x) - nP_{n-1}(x), & n=1,2,\dots \\ P_0(x)=1, & P_1(x)=x. \end{cases}$$

用 Newton 法求解 $P_{30}(x)=0$ 的根 x_1, x_2, \dots, x_{30} 。要求:

- (1) 给出算法思想和流程;
- (2) 编程实现(附可运行程序), 计算结果与下表对比。

-0.996893484074650	-0.983668123279747	-0.960021864968308
-0.926200047429274	-0.882560535792053	-0.829565762382768
-0.767777432104826	-0.697850494793316	-0.620526182989243
-0.536624148142020	-0.447033769538089	-0.352704725530878
-0.254636926167890	-0.153869913608584	-0.0514718425553177
0.0514718425553177	0.153869913608584	0.254636926167890
0.352704725530878	0.447033769538089	0.536624148142020
0.620526182989243	0.697850494793316	0.767777432104826
0.829565762382768	0.882560535792053	0.926200047429274
0.960021864968308	0.983668123279747	0.996893484074650