Intro to Algorithms and Computation

Goals:

1. Solve Computational Problems
2. Prove correctness
3. Argue efficiency
4. Communication

Problems are inputs mapped to outputs.

We want to create algorithms for:

- General problems
    - Arbitrarily sized inputs

Algorithm

A function that takes inputs and maps it to a single correct output.

For birthday problem:

- Maintain record
- Interview students in some order
    - Check if birthday in record
        - If it is, return pair
    - Otherwise, add new student to record
- Return none

Induction

Inductive Hypothesis: if first k students contain a match, algorithm returns a match before interviewing student k+1.

Base Case: k = 0
True

Assume inductive hypothesis is true for k=k'

If k' contains a match, it is already returned by induction.
Else, if k' + 1 students contains match

Algorithm checks k' against all students

Efficiency
Don't measure time, instead count ops (fundamental operations)
Expect performance to depend on size of our input (n)

- O (upper bounds)
- Ω lower bounds (omega)
- Θ both (theta)

Running Time / Time Complexity
O(1) Constant Time
(O(log n)) Logarithmic Time
O(n) Linear Time
(O(n log n)) Linearithmic Time
$O(n^2)$ Quadratic Time
$2^{O(\log n)}$ = poly(n) Polynomial Time

Model of Computation (Wiki)
Word-RAM
Integer Arithmetic
Logical Operations
Bitwise Operations

constant time constant time constant time constant time constant time

Data Structures

[Next Lesson]