Λ	Birth rate
μ	Death rate
ψ	Vaccination rate
τ_I, τ_J	Infection rates
γ_I, γ_J	Recovery rates
δ	Relative susceptibility of vaccinated individuals to infection with strain I
χ	Proportion of strain I infections who recover to compartment S[1]

Table 1: Parameter definitions for our two-strain SIS model with vaccination and demography.

Consider an SIS-type disease with two strains, call them I and J. Assume also that we have a vaccine for this disease which is fully effective against strain J and partially effective against strain I. If we include demography, the dynamics are as follows:

$$S' = \Lambda - \mu S - \tau_I SI - \tau_J SJ + \chi\gamma_I I + \gamma_J J - \psi S \tag{1}$$
$$V' = \psi S - \mu V - \delta\tau_I VI + (1-\chi)\gamma_I I \tag{2}$$
$$I' = \tau_I SI + \delta\tau_I VI - \mu I - \gamma_I I \tag{3}$$
$$J' = \tau_J SJ - \mu J - \gamma_J J \tag{4}$$

Parameters are defined in Table 1.

The disease-free equilibrium can be computed in the usual way, giving $S_0 = \Lambda/(\mu+\psi)$ and $V_0 = \psi S_0/\mu$. Note that the equilibrium population size is always $\Lambda/\mu =: N_*$ so, as we would expect, $S_0 + V_0 = N_*$. A local stability analysis gives the following two $\mathcal{R}_0$-type values: The values in the book assume $\Lambda = \mu$. Since I'm trying to avoid this assumption so I can see what role is played by the population size, I need to re-derive these quantities. Tldr: The following quantities are wrong and need to be re-derived.

$$\mathcal{R}_0^I = \frac{\tau_I(\mu+\delta\psi)}{(\mu+\gamma_I)(\mu+\psi)}$$
$$\mathcal{R}_0^J = \frac{\tau_I(\mu+\delta\psi)}{(\mu+\gamma_J)(\mu+\psi)}$$

We are actually more interested in the endemic equilibria. To this end, we now assume that $I, J \neq 0$. This assumption allows us to cancel I and J in equations 3 and 4. More to come.

[1] The χ parameter can be thought of as the combined effect of what proportion of the population is vaccinated and what fraction of strain I infections produce antibodies which are functionally equivalent to the vaccine.