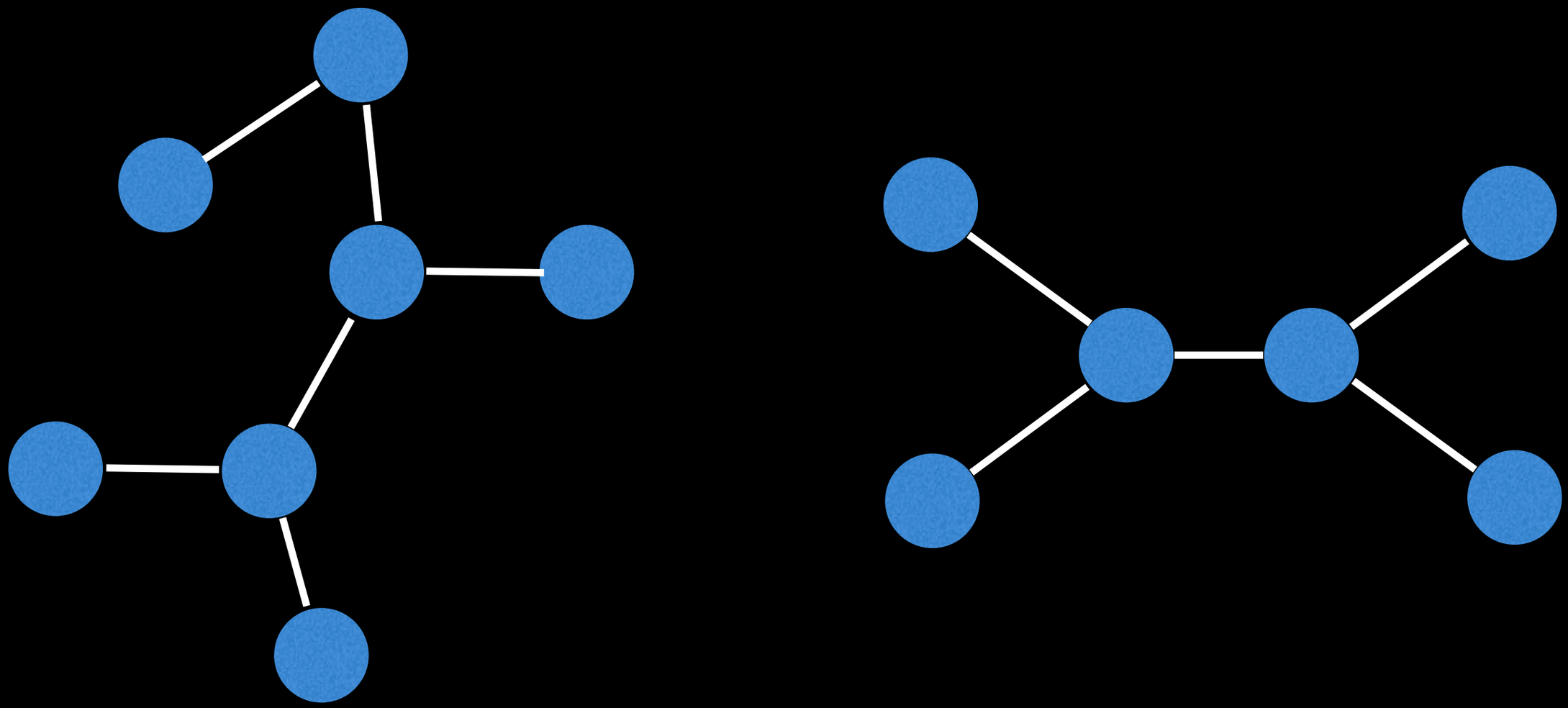
Center(s) of a tree

William Fiset

Center(s) of undirected tree

An interesting problem when you have an undirected tree is finding the tree's **center node(s)**. This could come in handy if we wanted to select a good node to root our tree 😉



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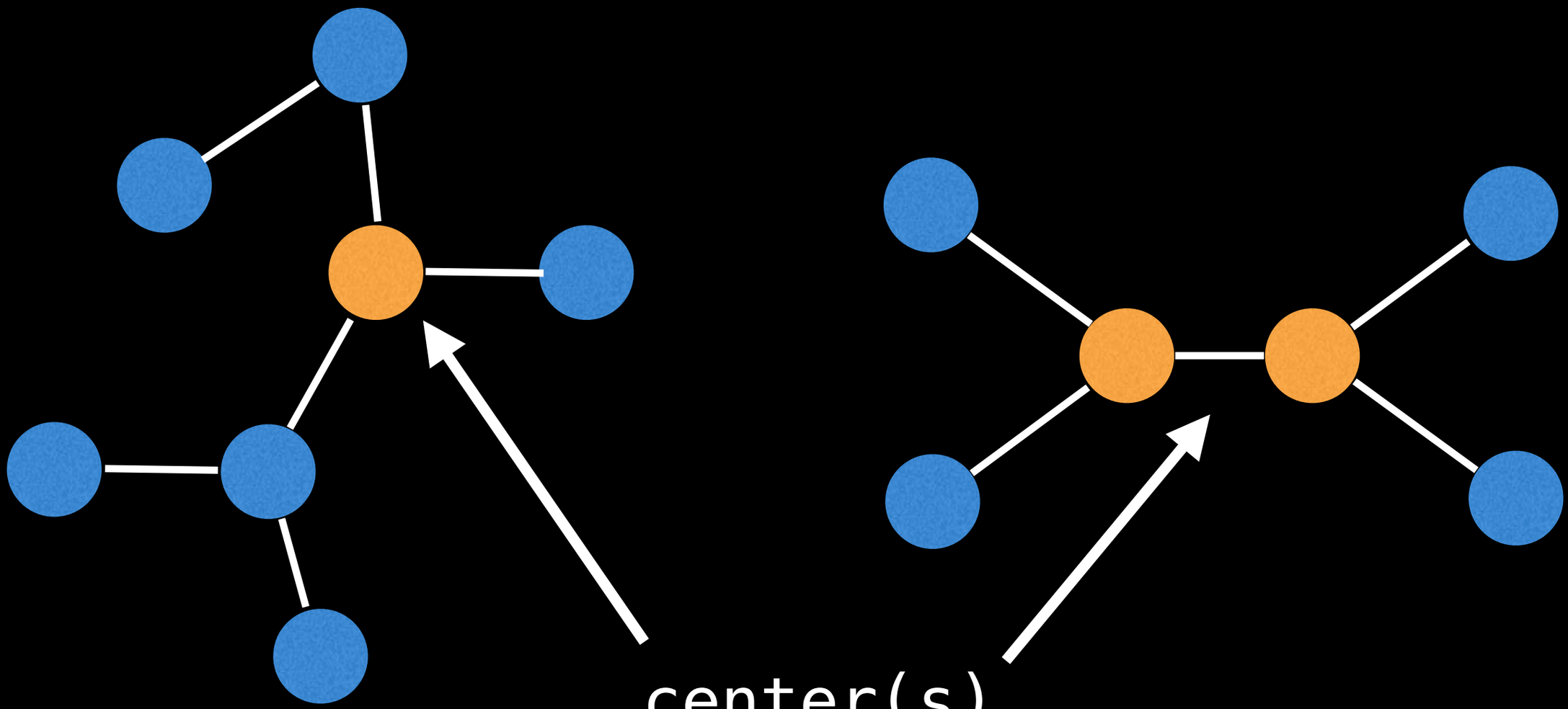

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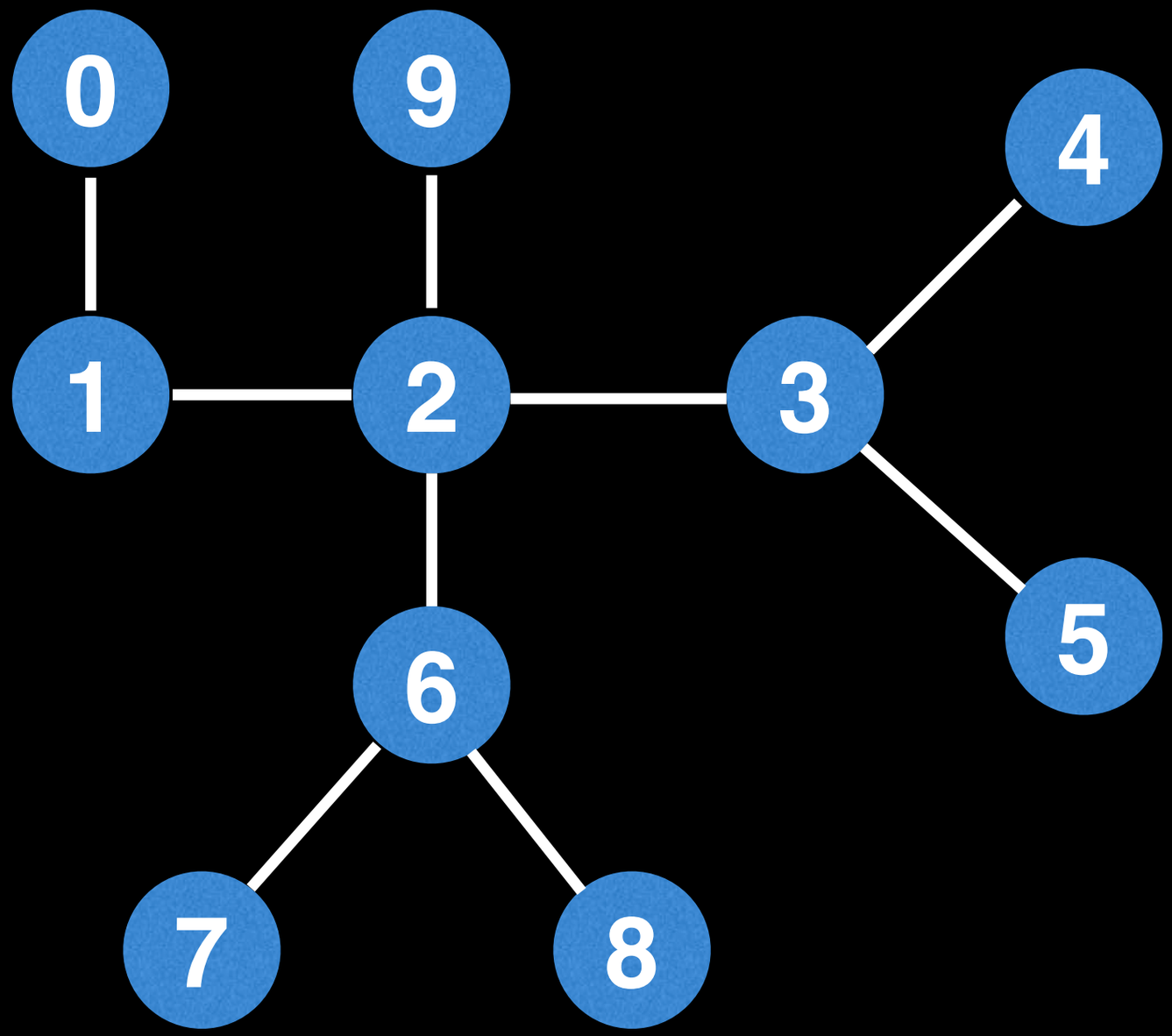

Notice that the center is always the middle vertex or middle two vertices in every longest path along the tree.

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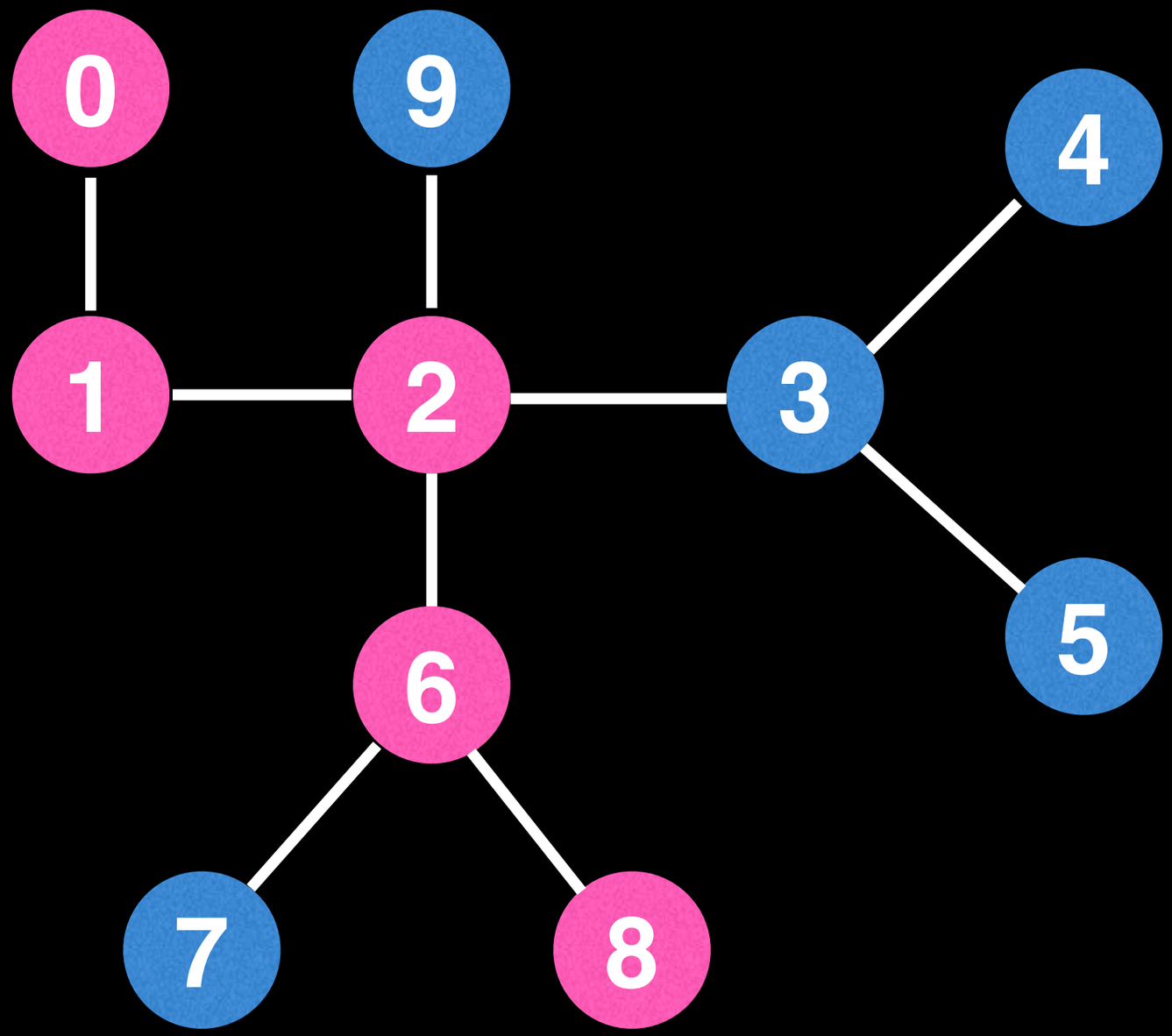

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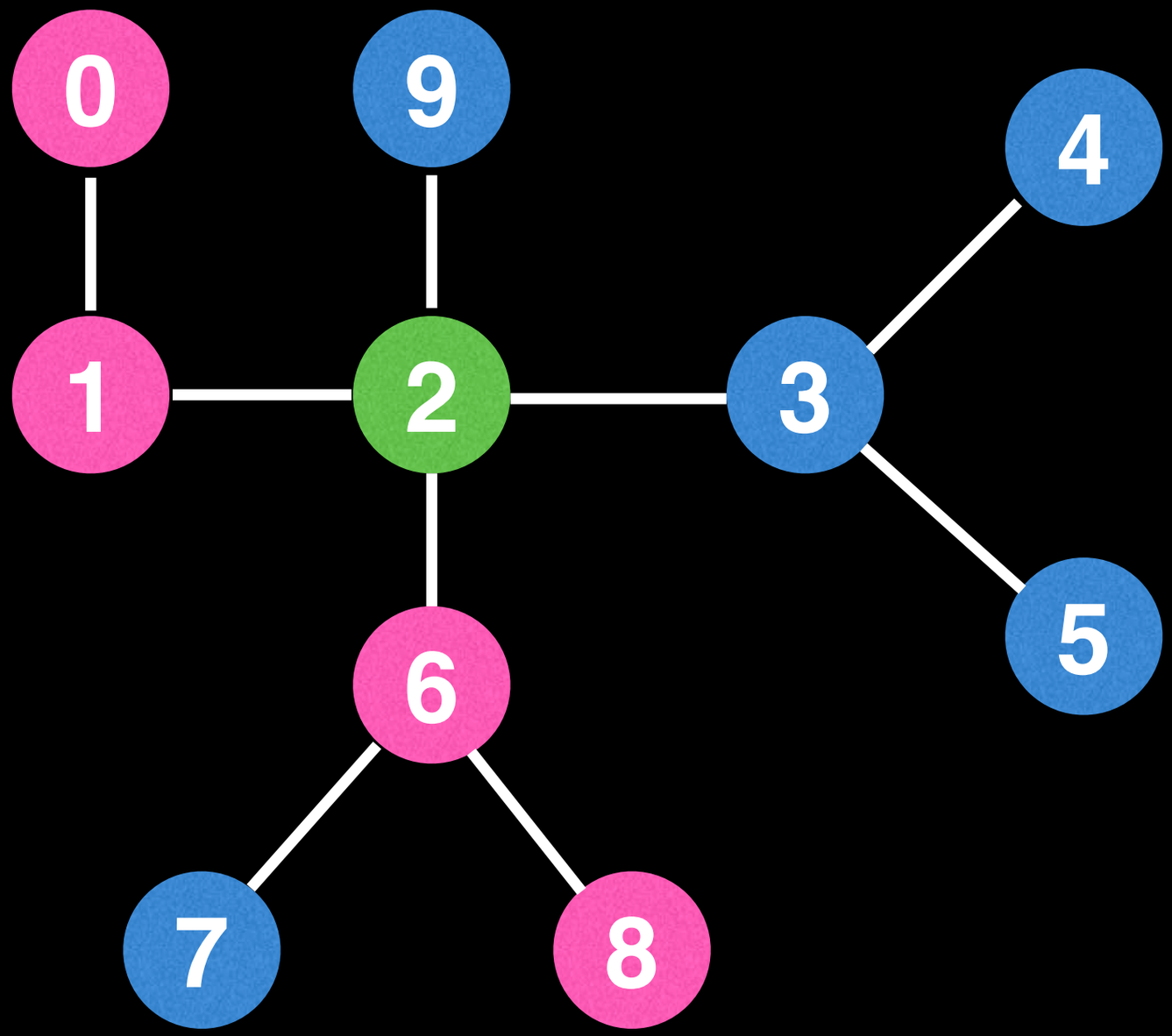

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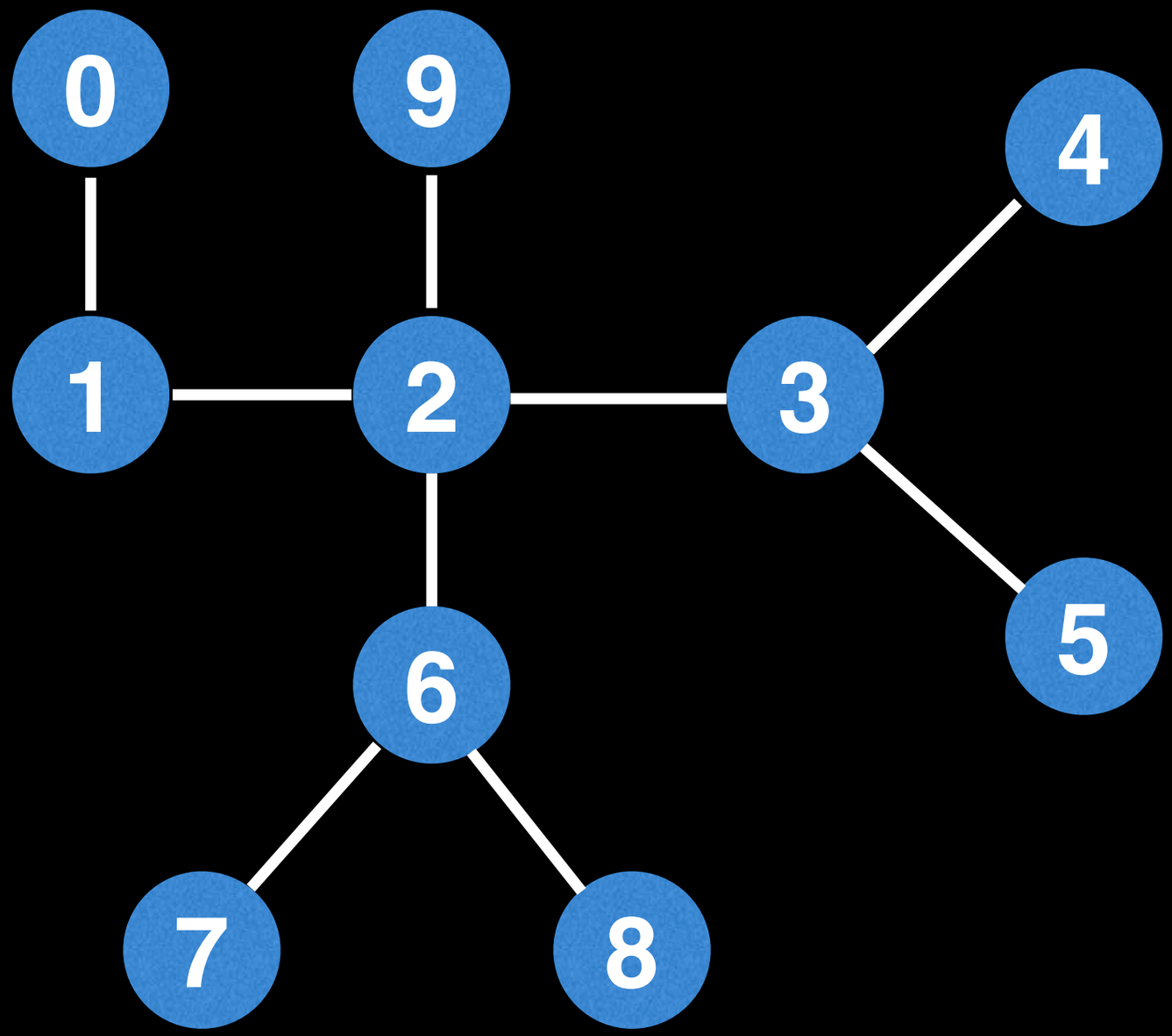

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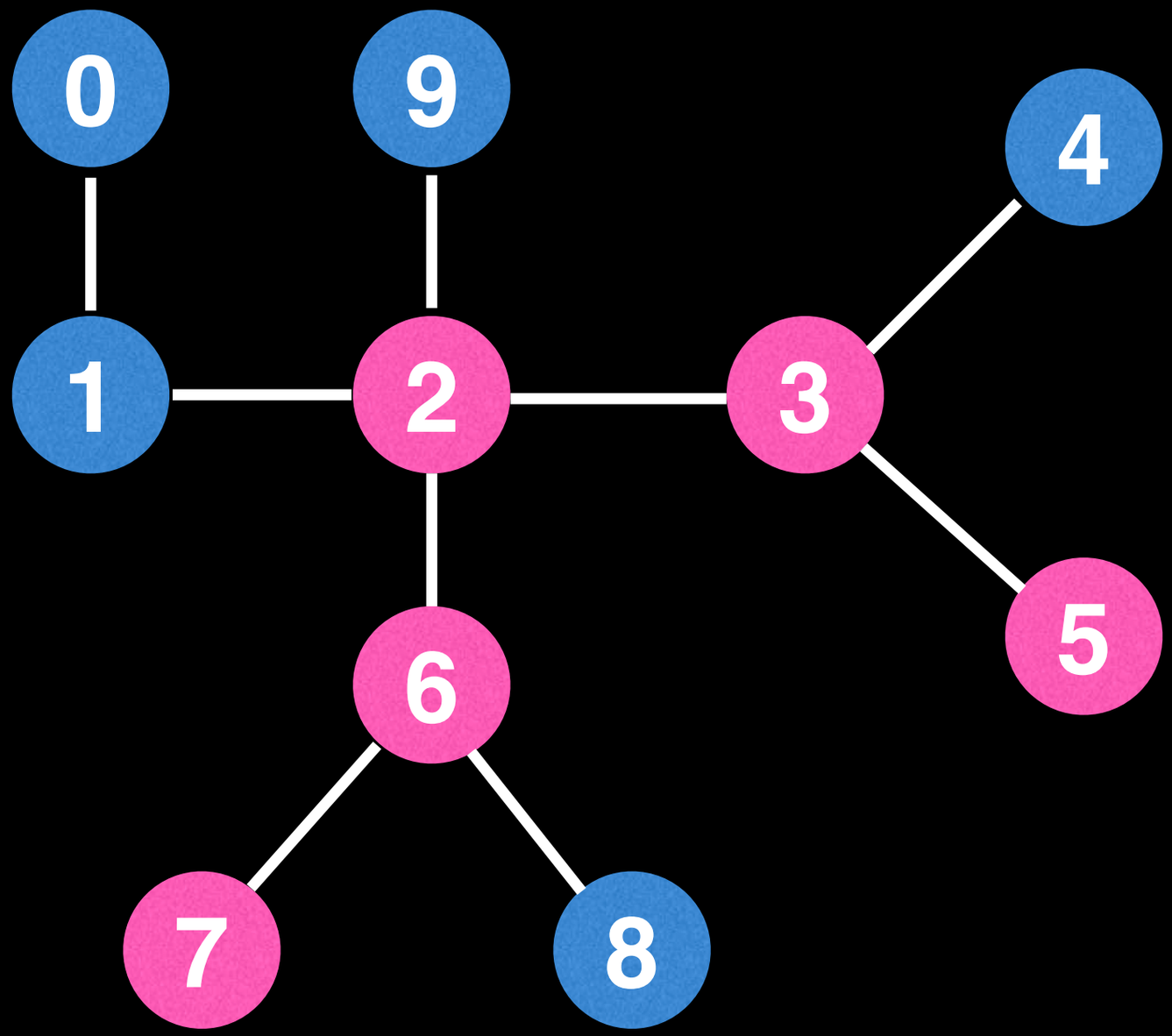

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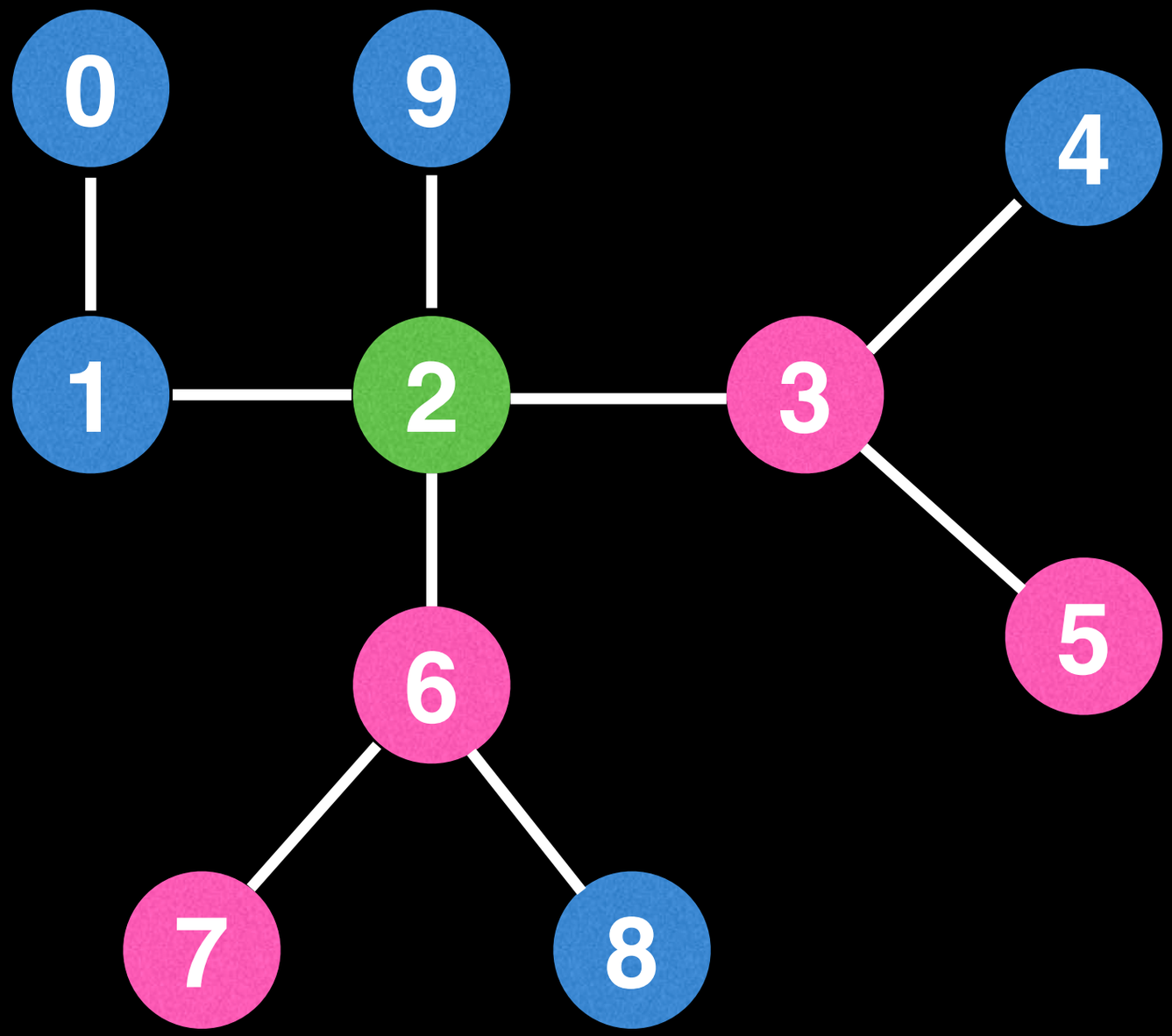

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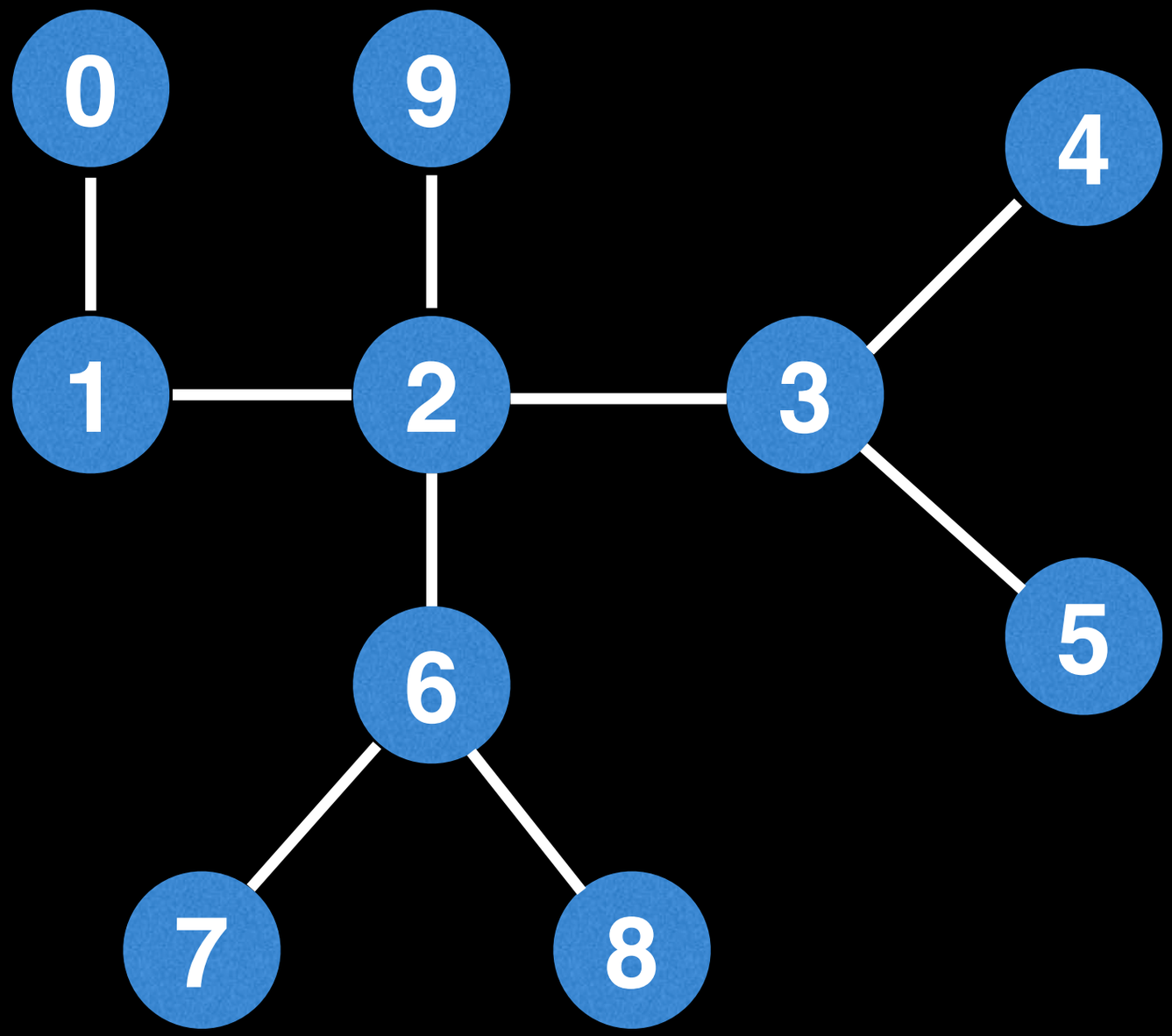

Another approach to find the center is to iteratively pick off each leaf node layer like we were peeling an onion.

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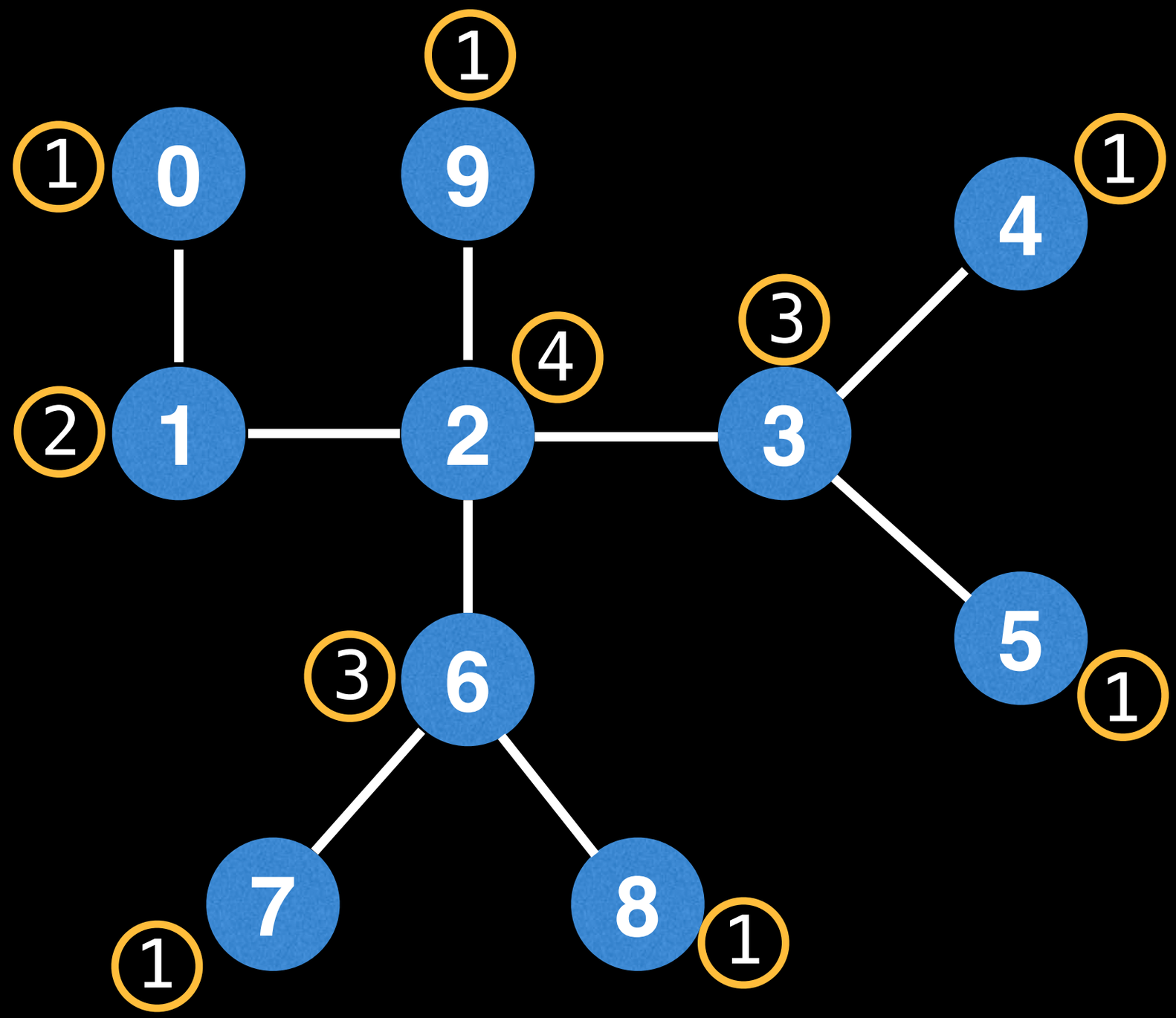

The orange circles represent the **degree** of each node. Observe that each leaf node will have a degree of 1.

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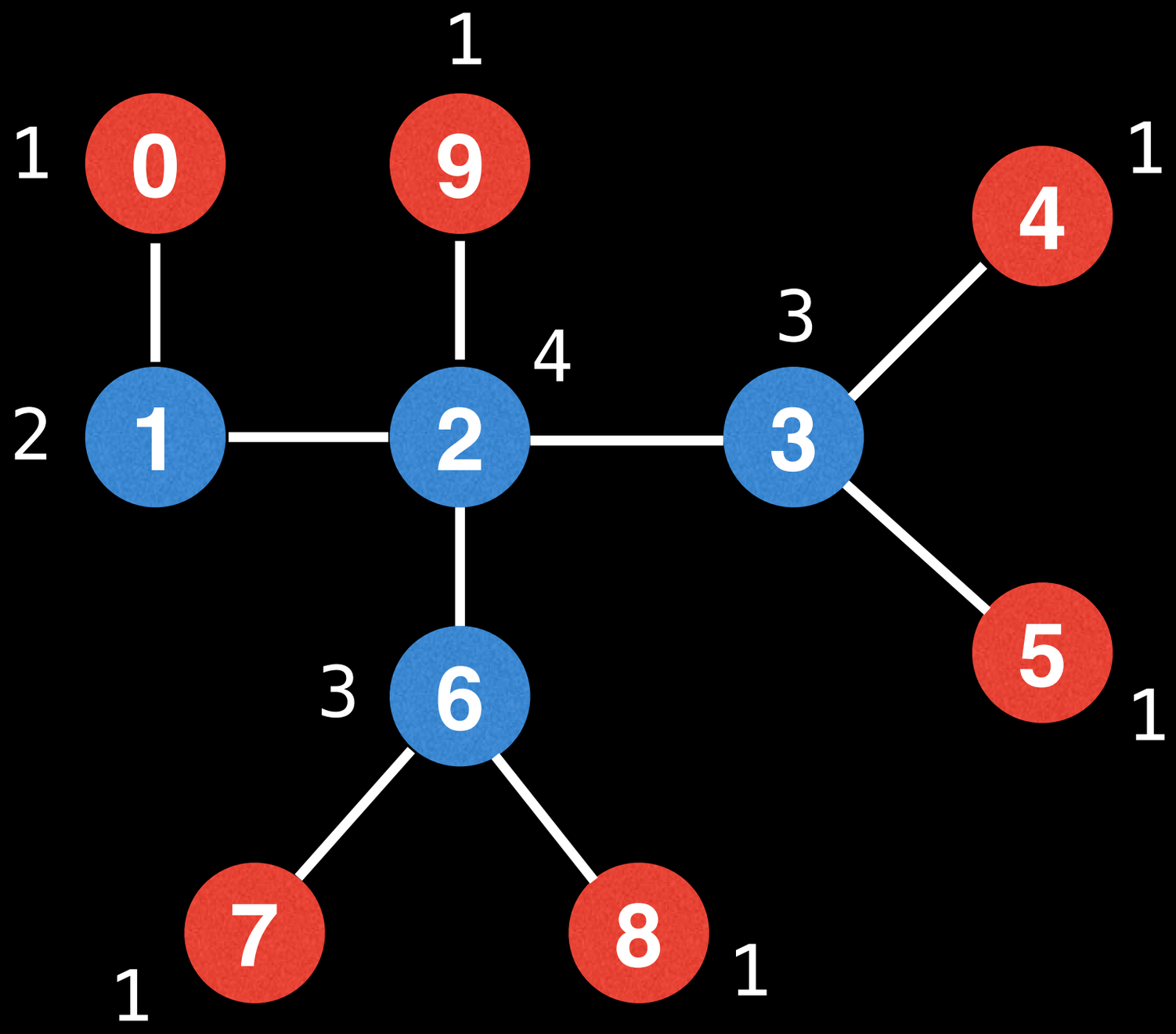

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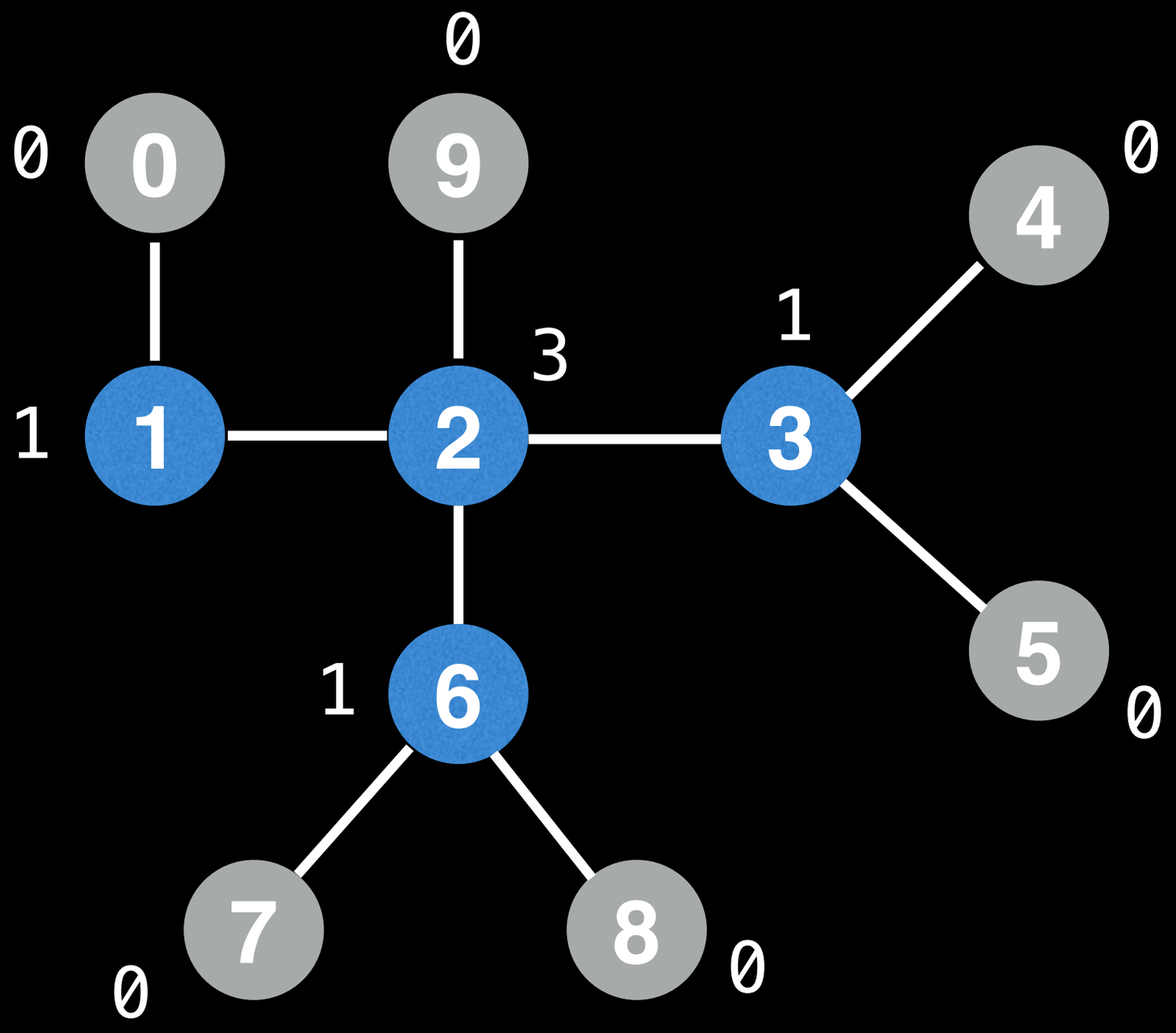

As we prune nodes also reduce the node degree values.

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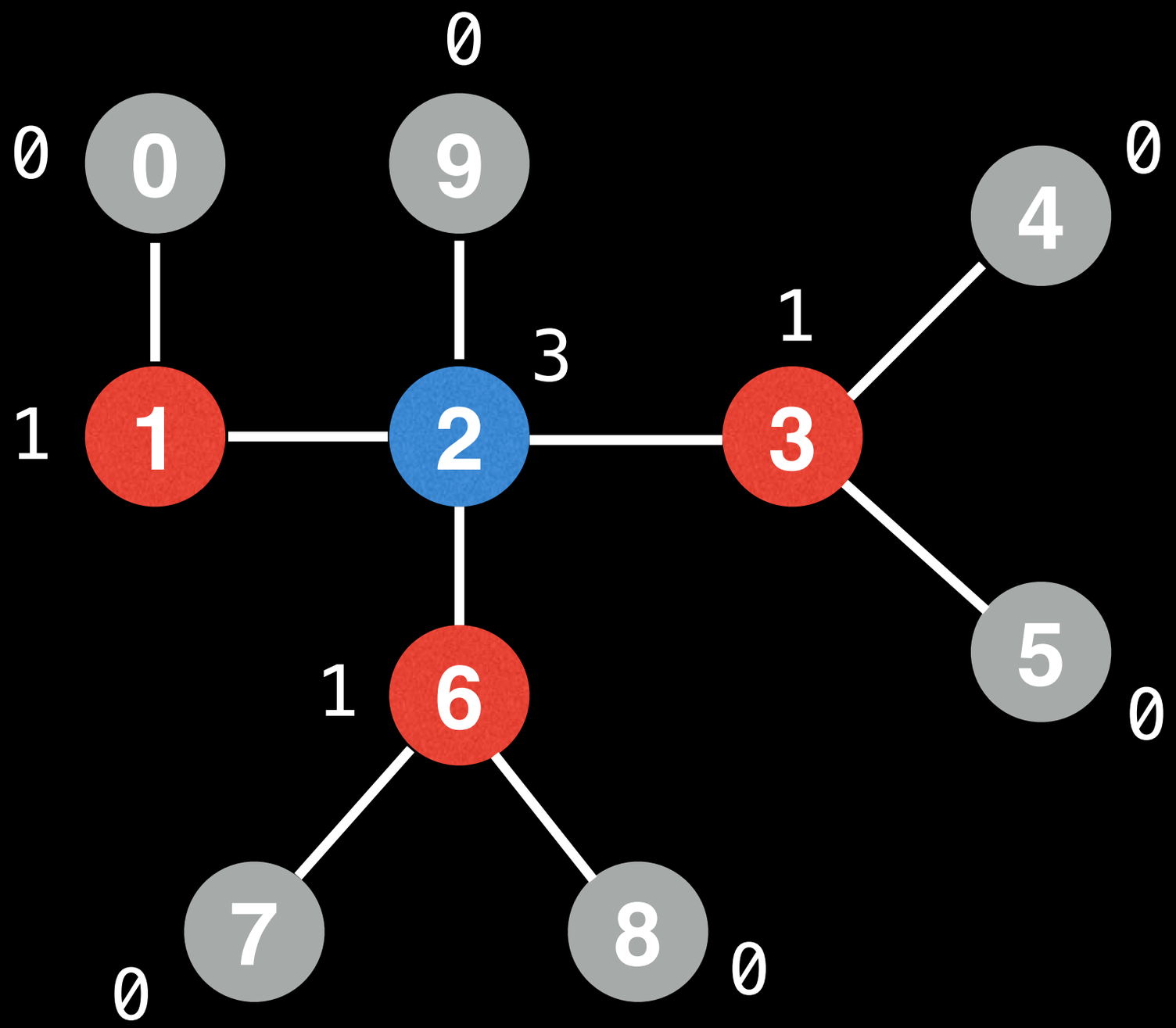

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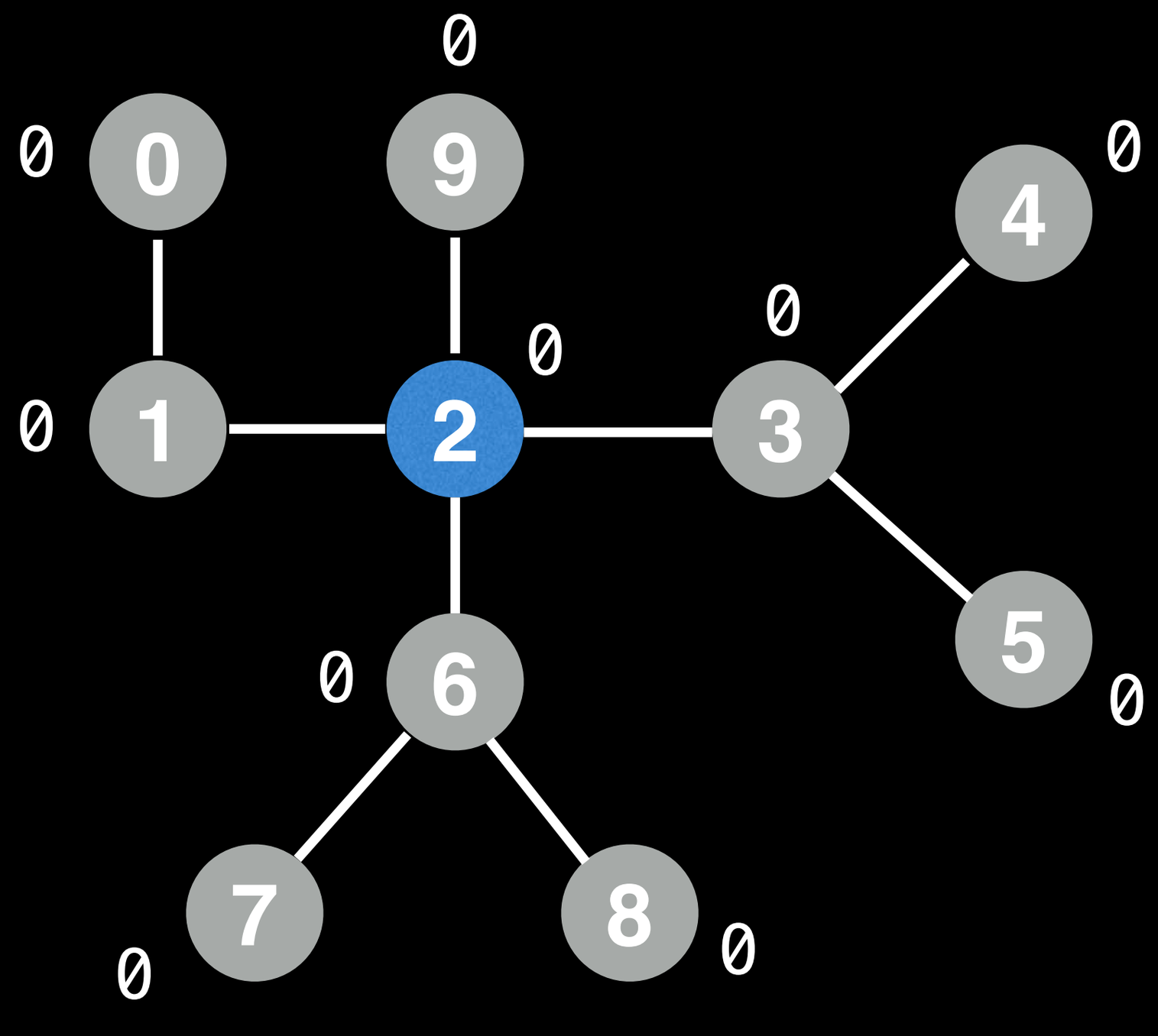

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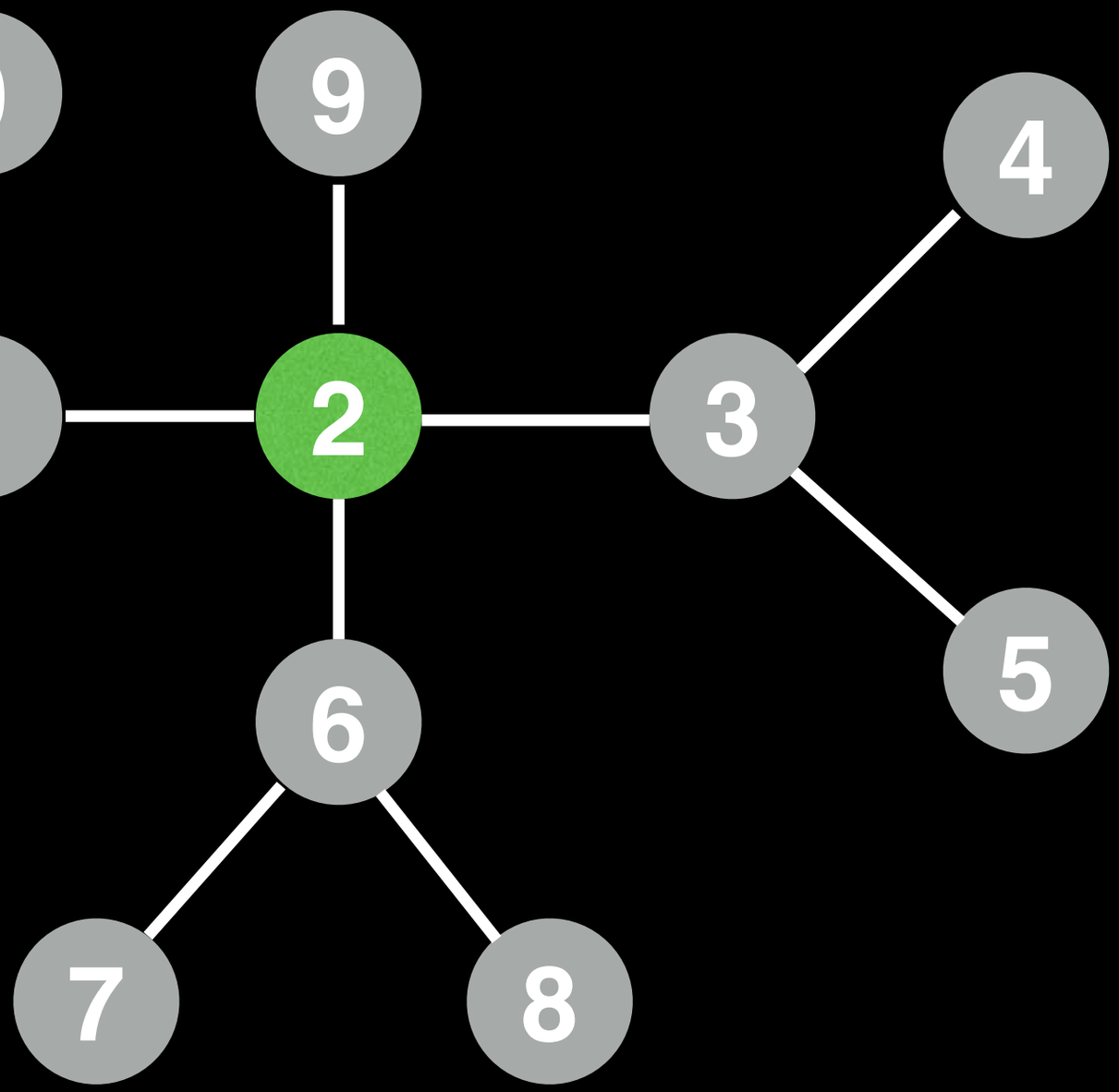

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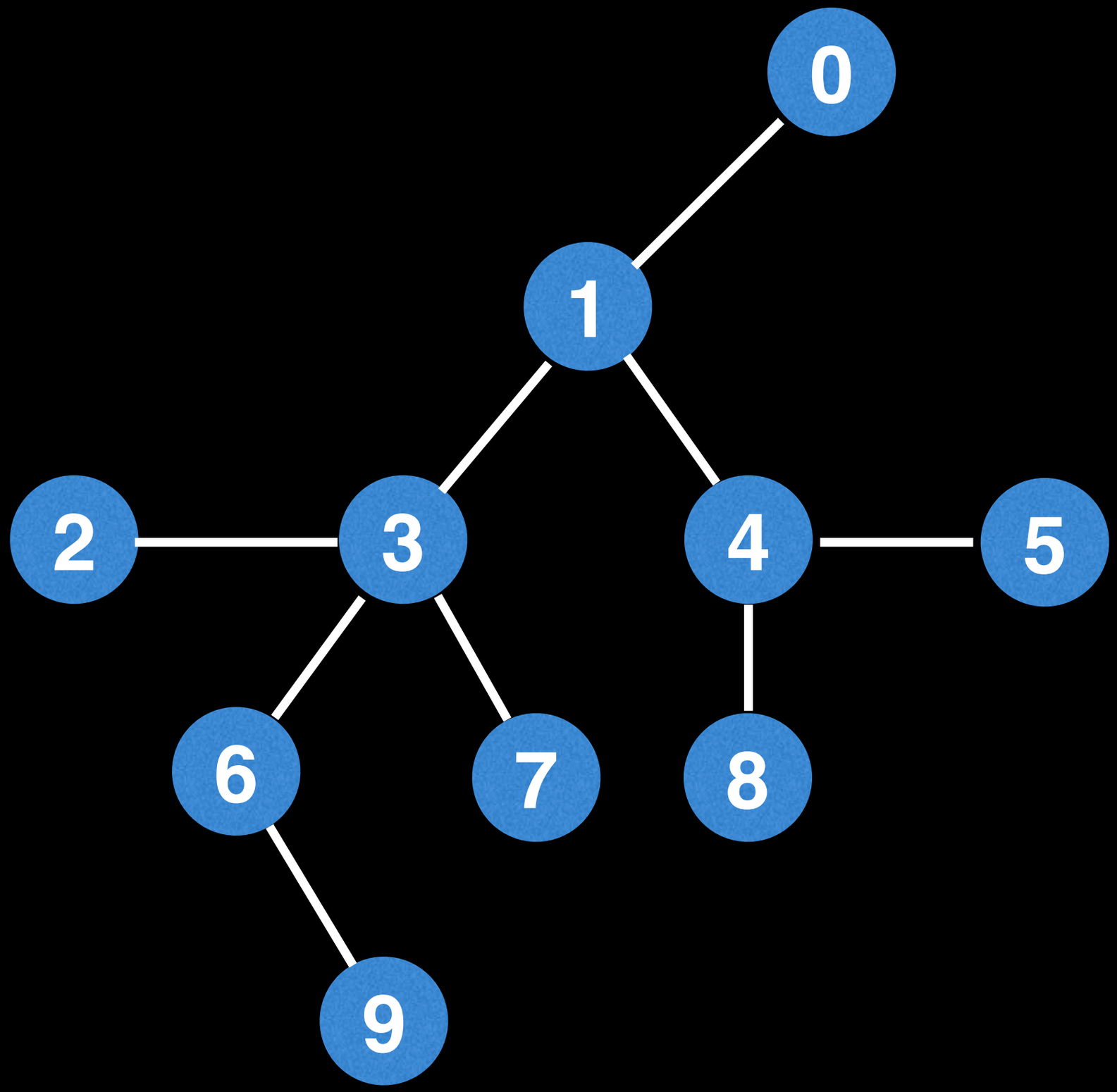

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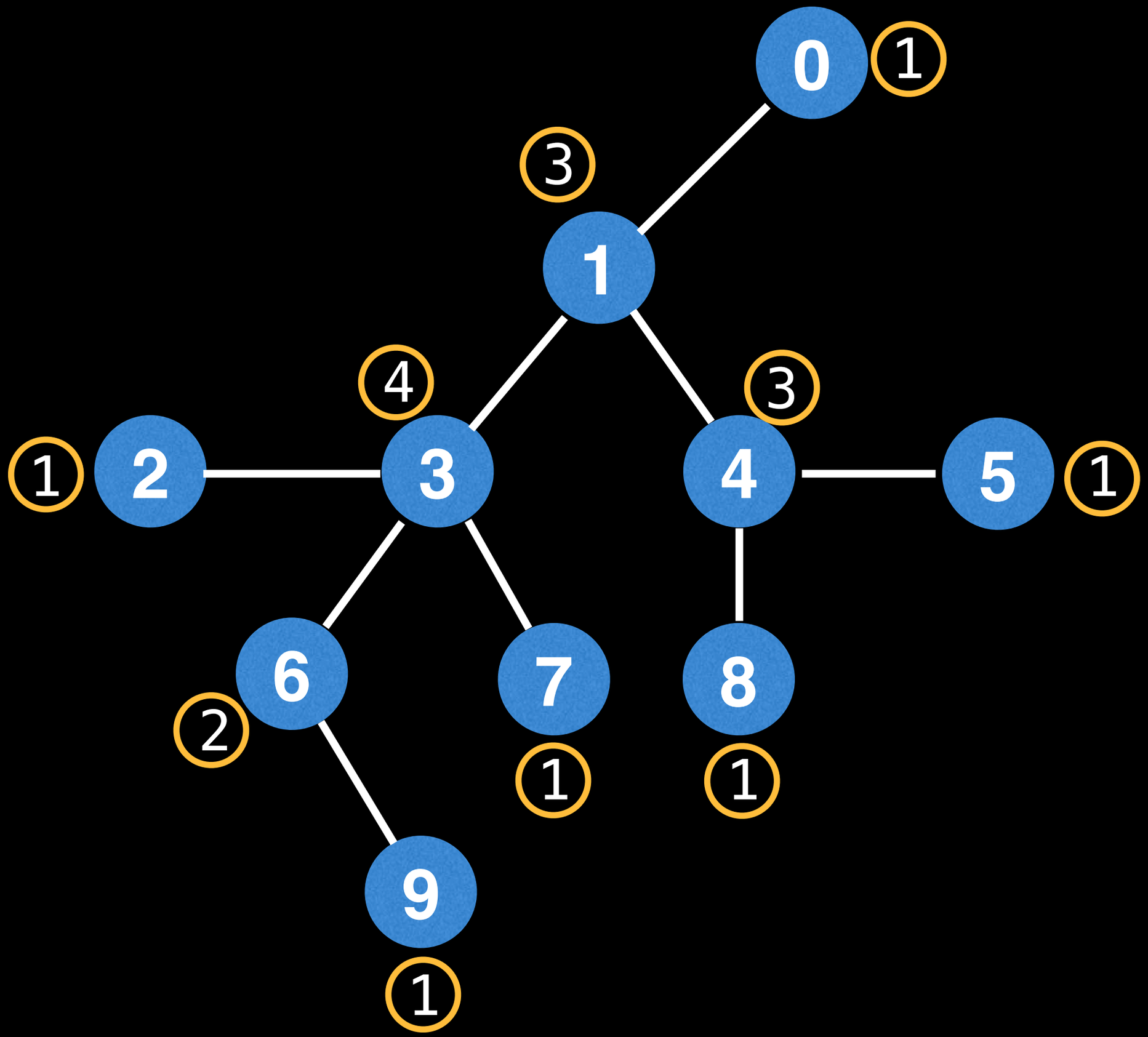

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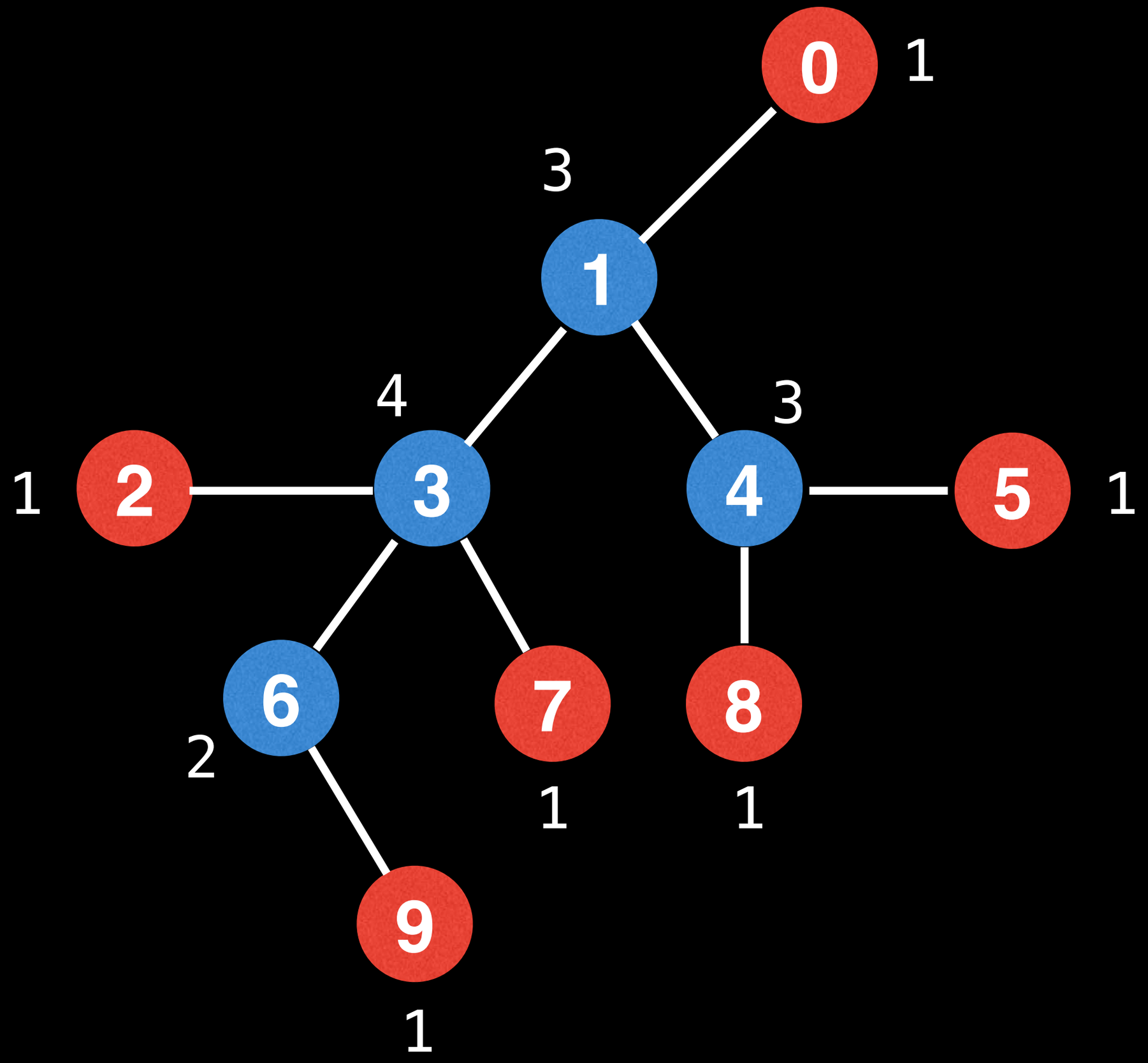

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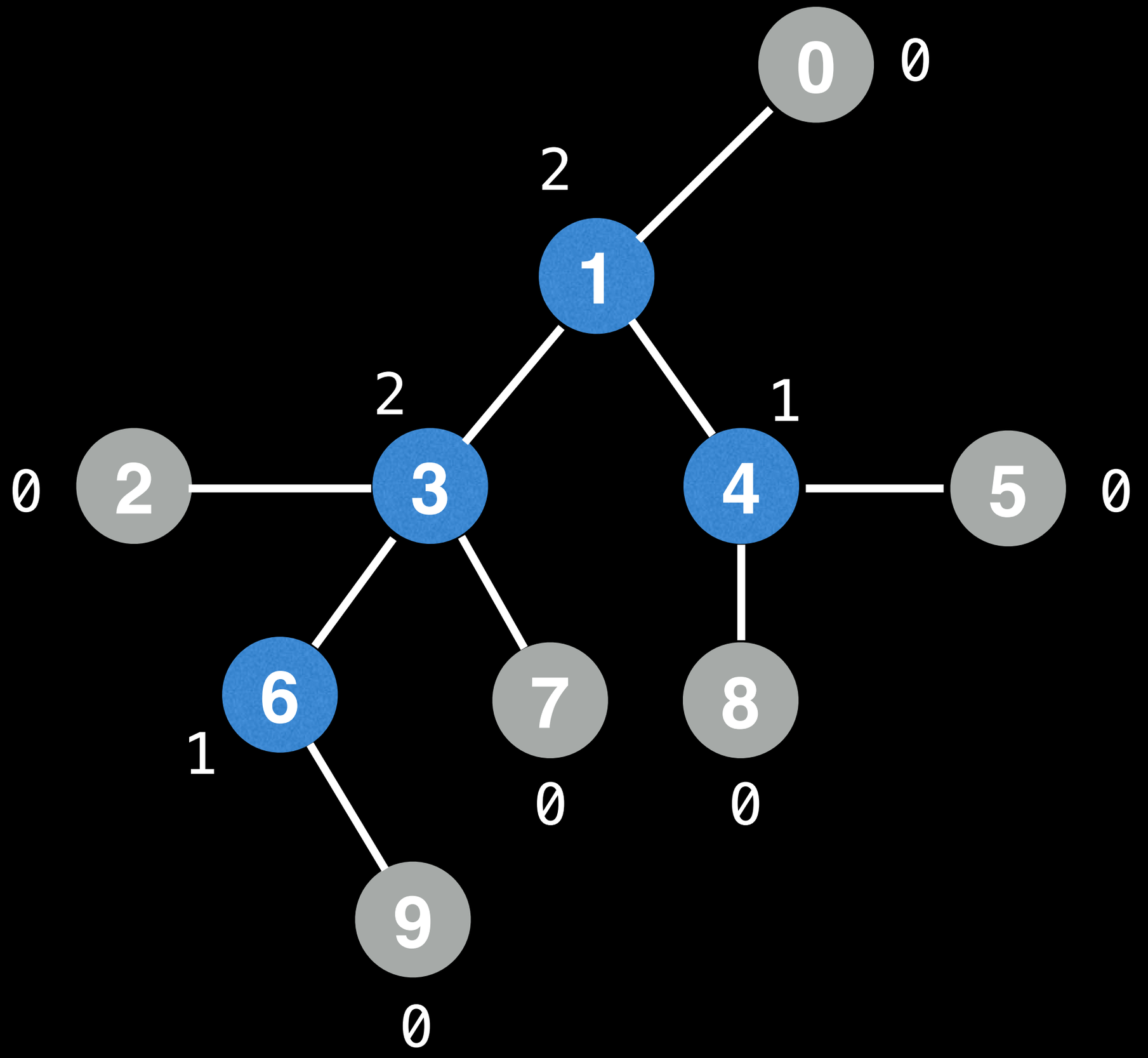

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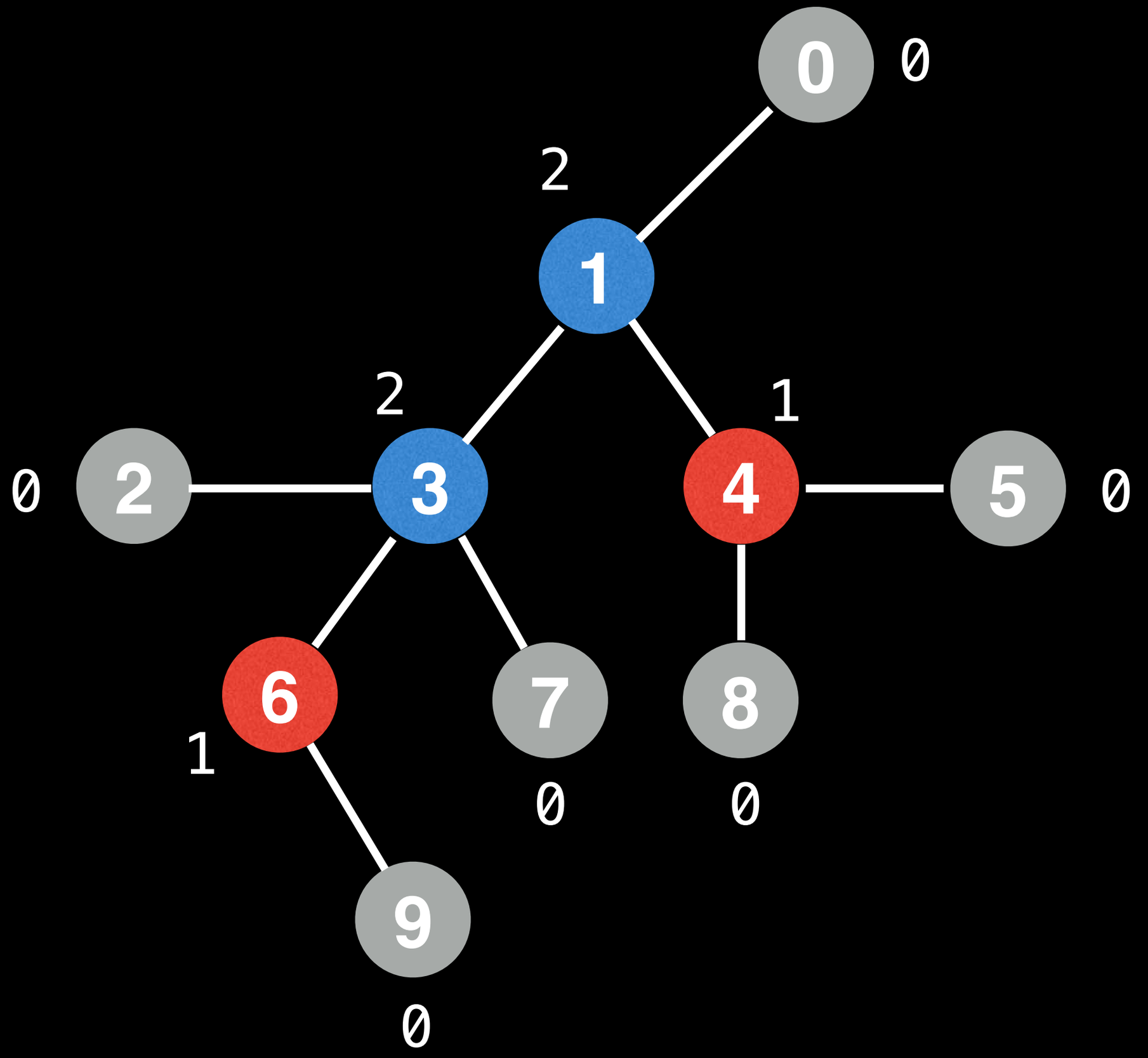

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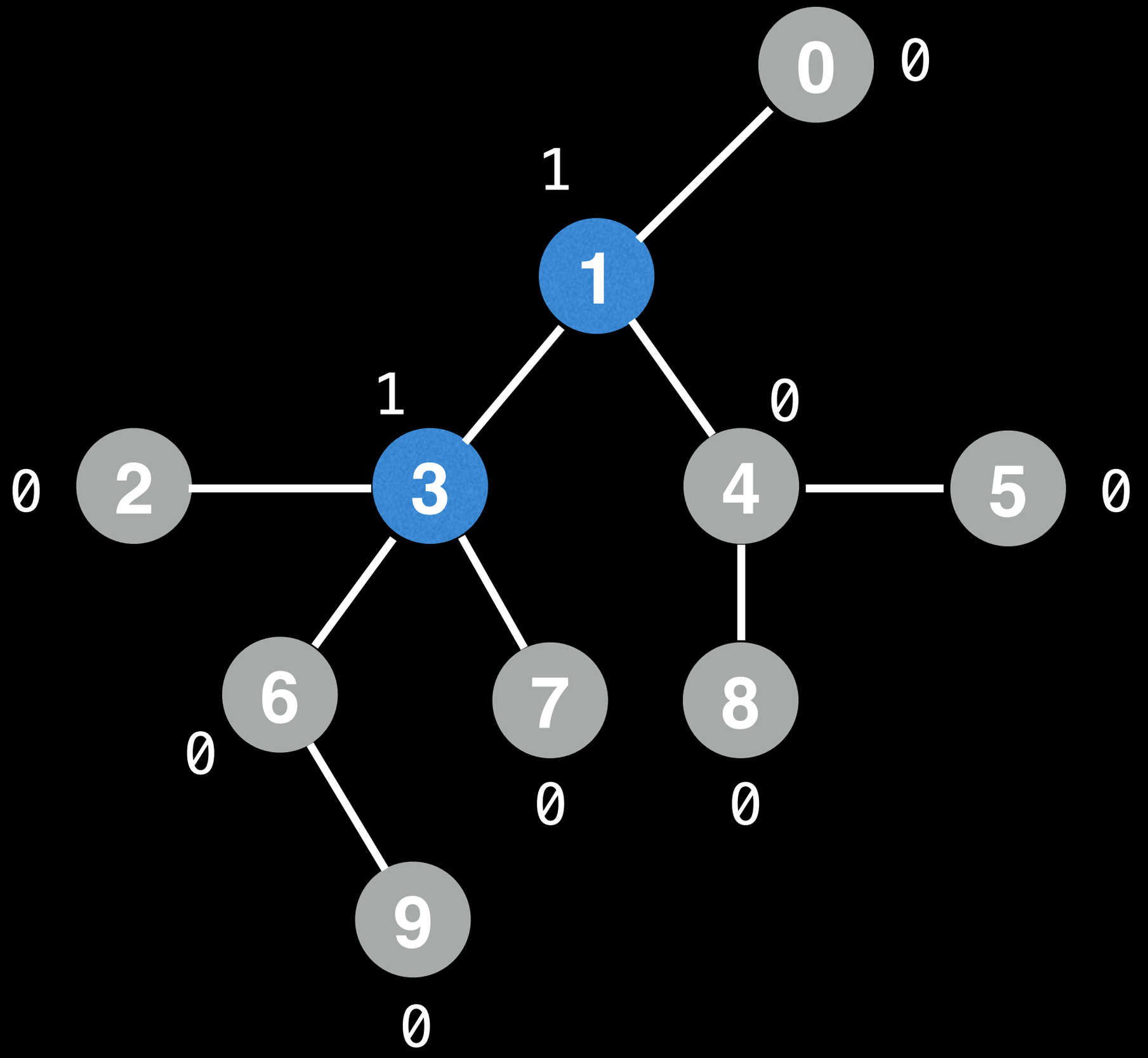

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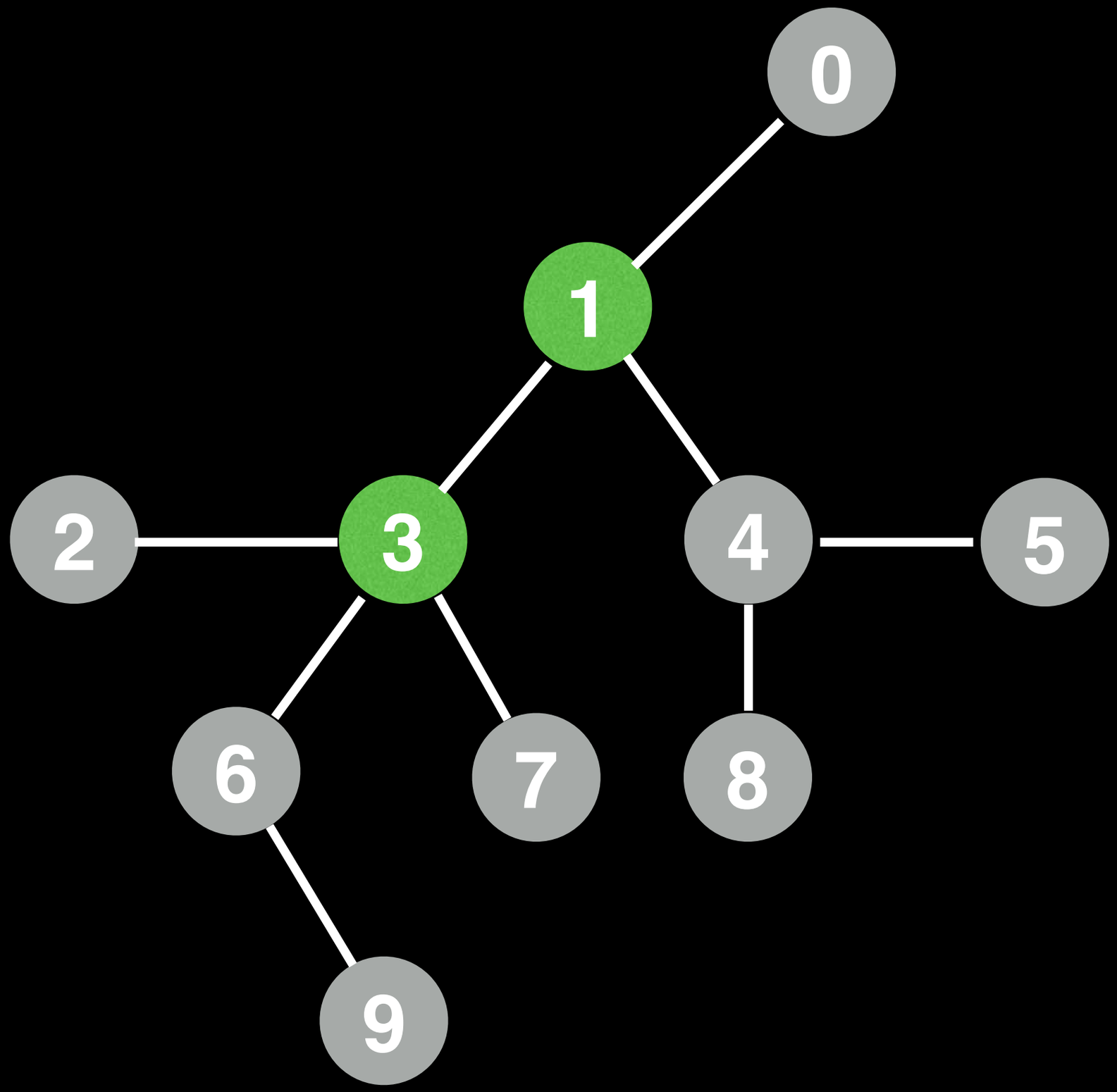

Some trees have two centers

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# g = tree represented as an undirected graph
function treeCenters(g):
  n = g.numberOfNodes()
  degree = [0, 0, …, 0] # size n
  leaves = []
  for (i = 0; i < n; i++):
    degree[i] = g[i].size()
    if degree[i] == 0 or degree[i] == 1:
      leaves.add(i)
      degree[i] = 0
  count = leaves.size()
  while count < n:
    new_leaves = []
    for (node : leaves):
      for (neighbor : g[node]):
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