Existence of Eulerian Paths and Circuits

William Fiset

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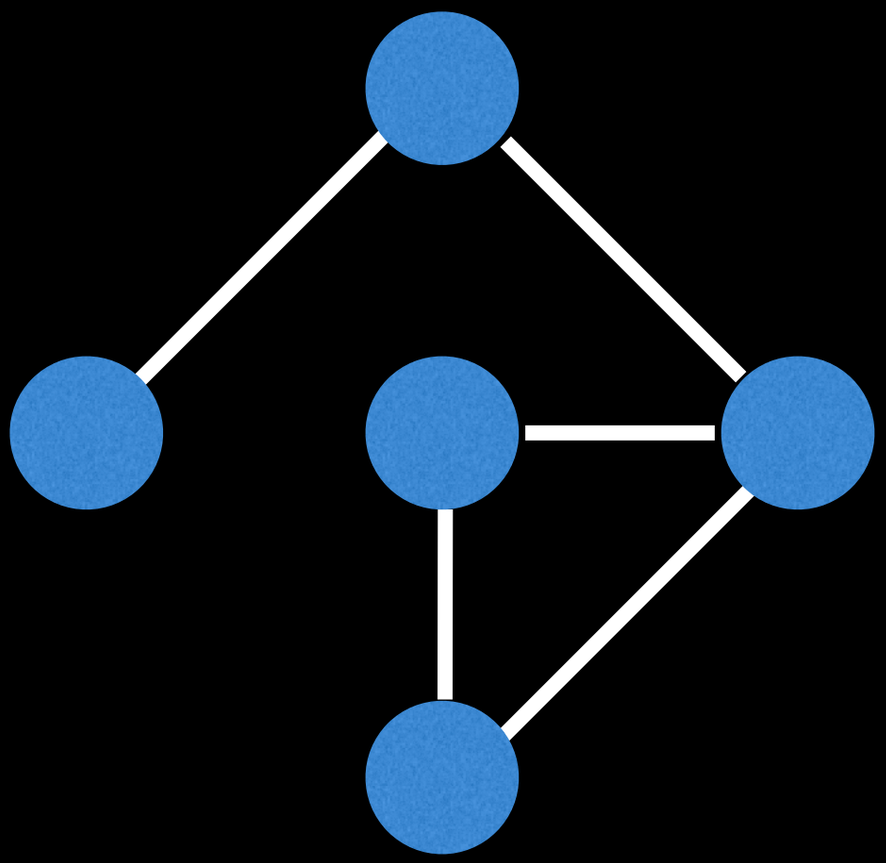

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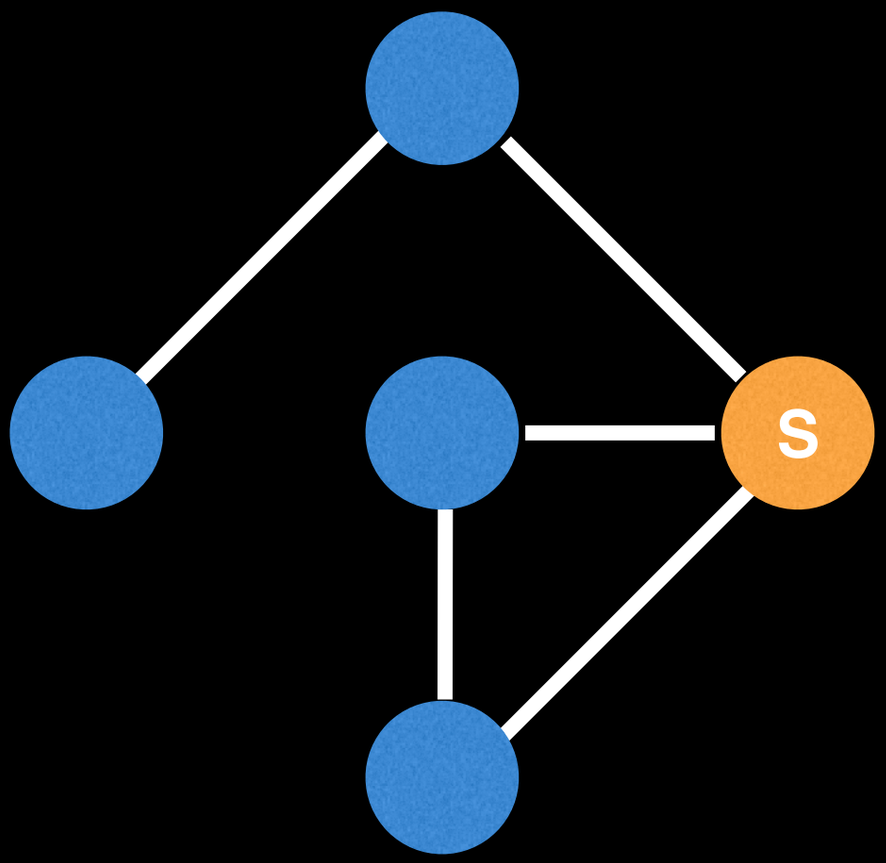

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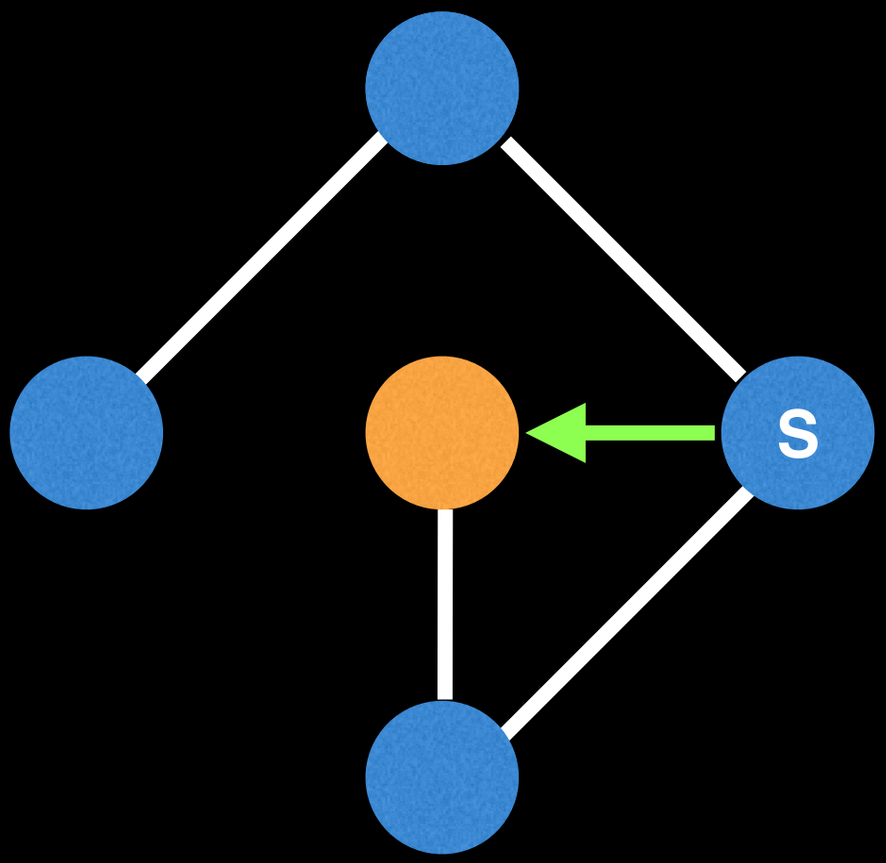

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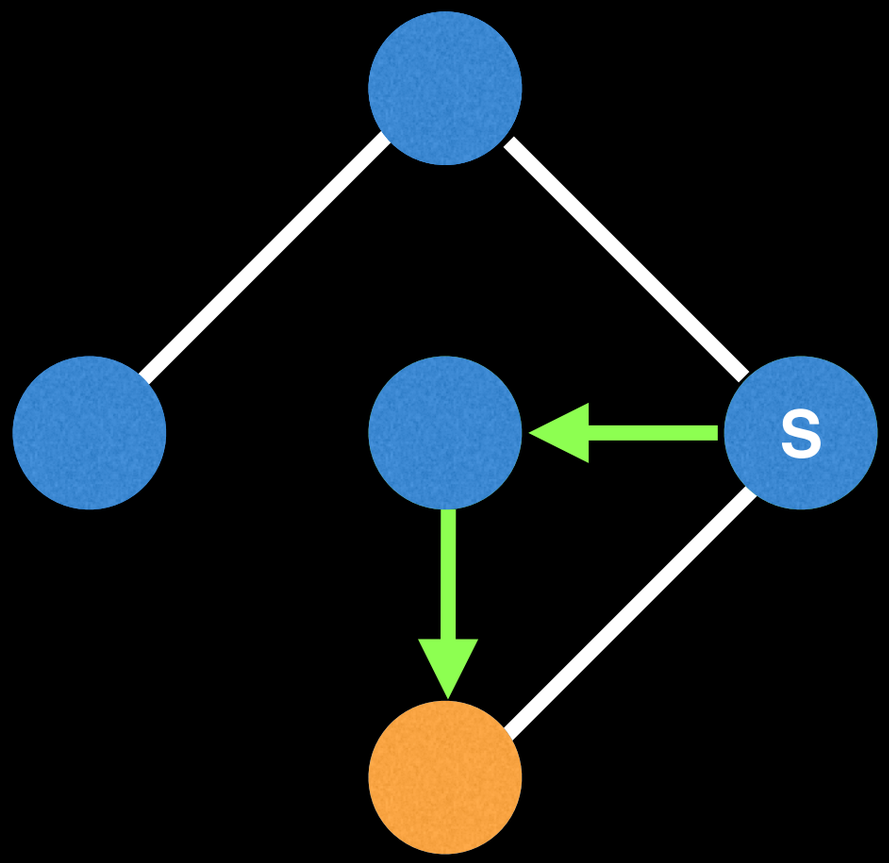

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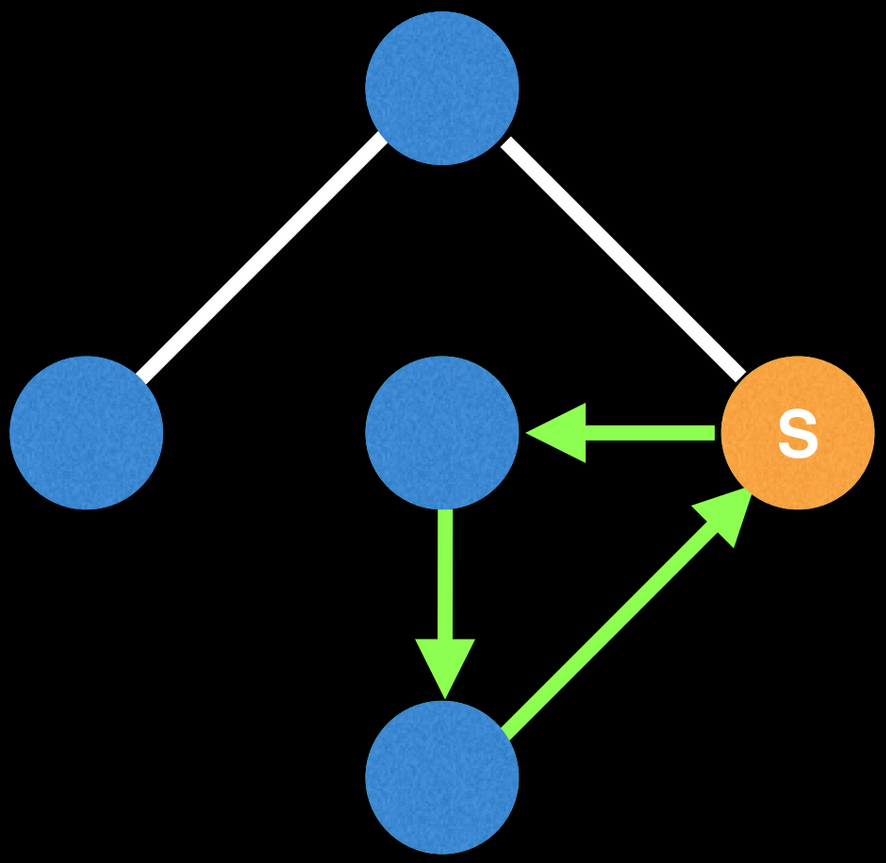

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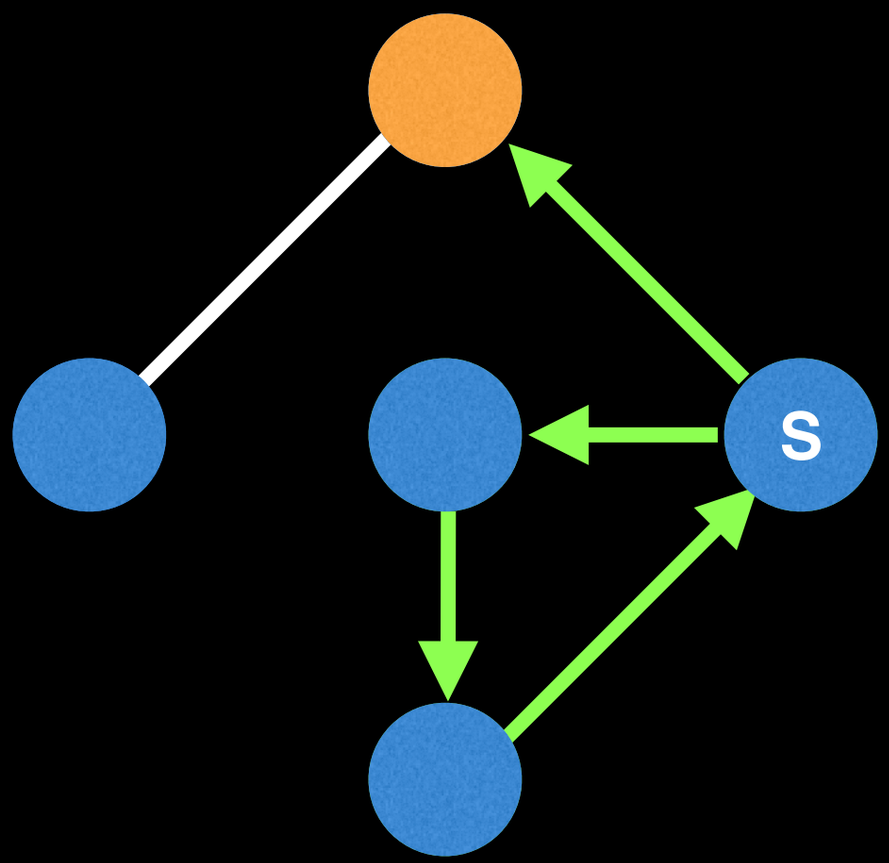

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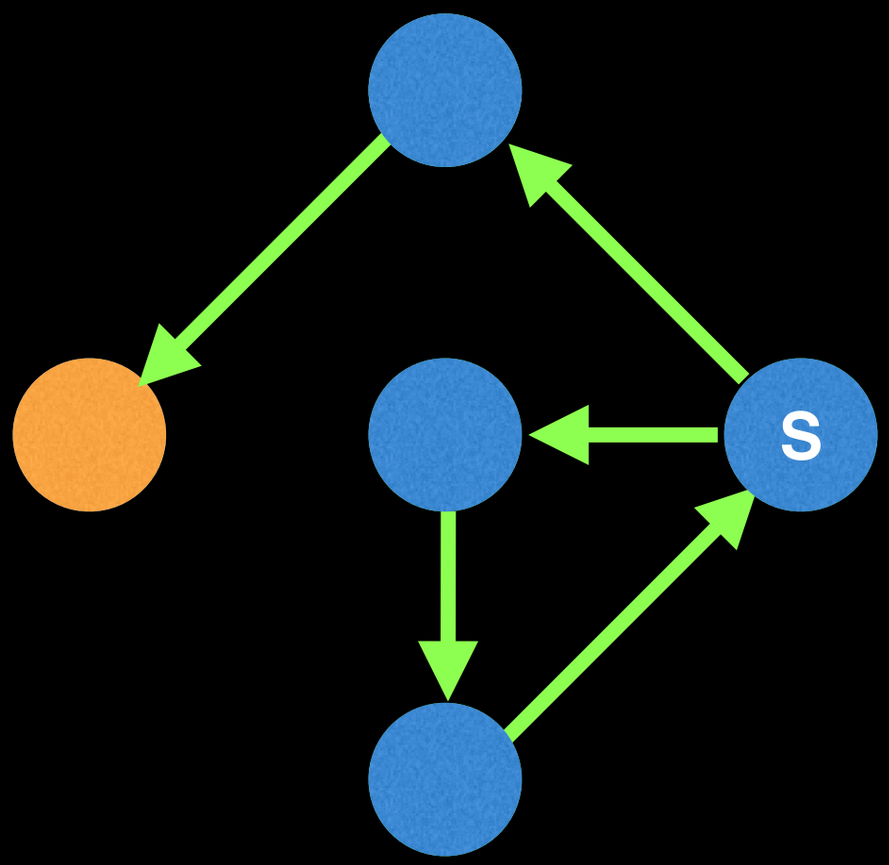

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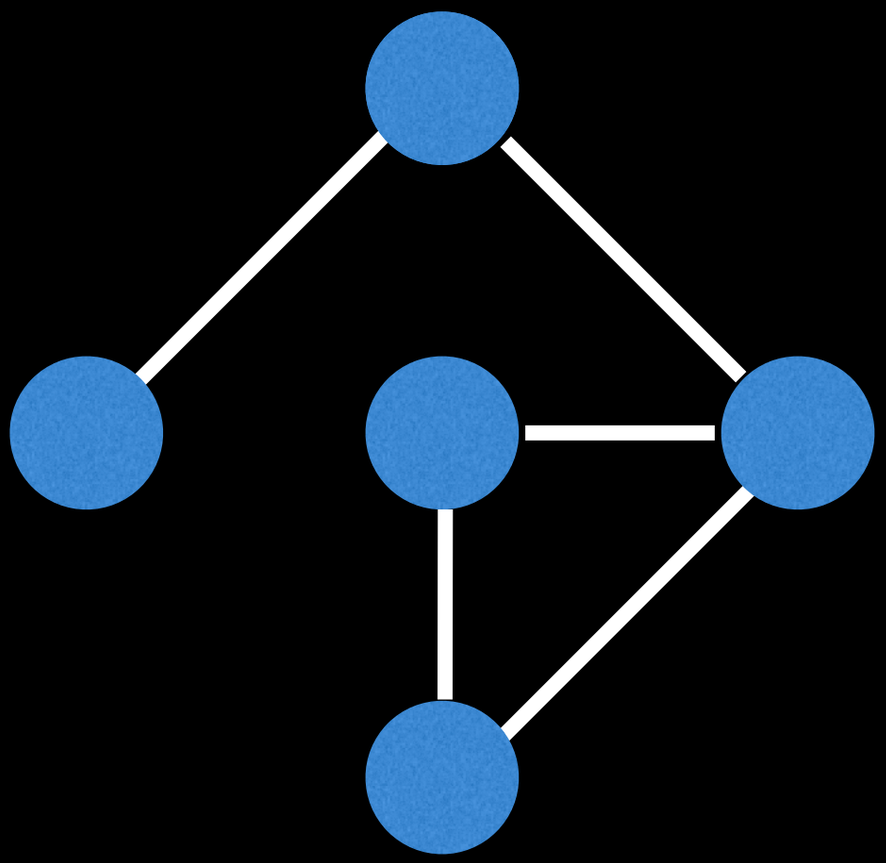

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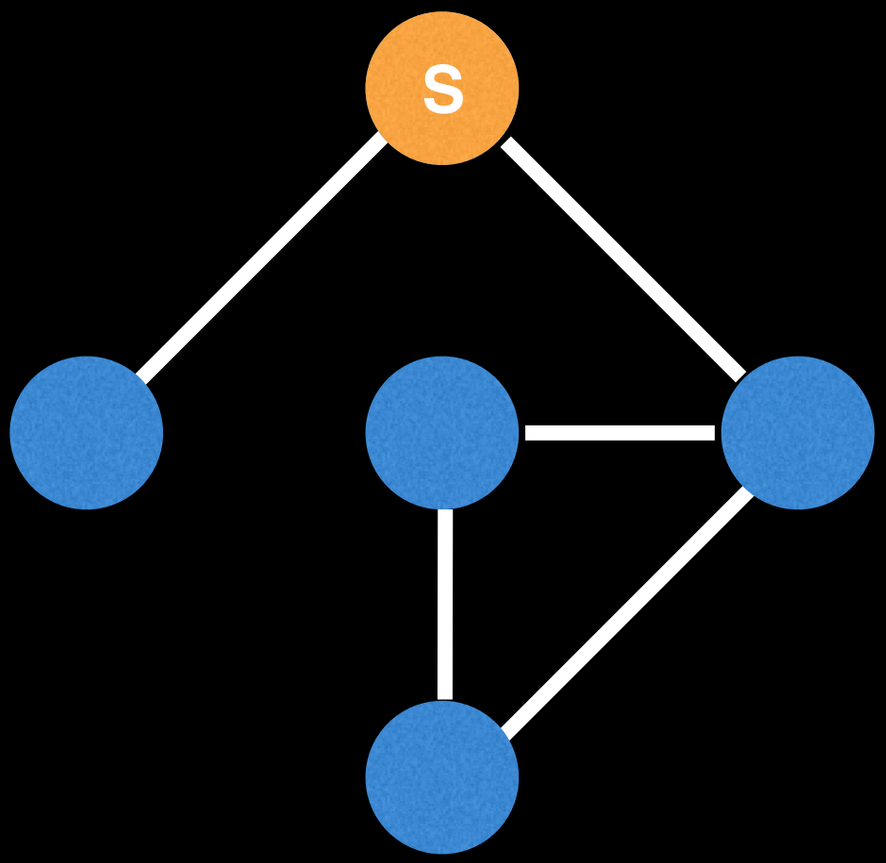

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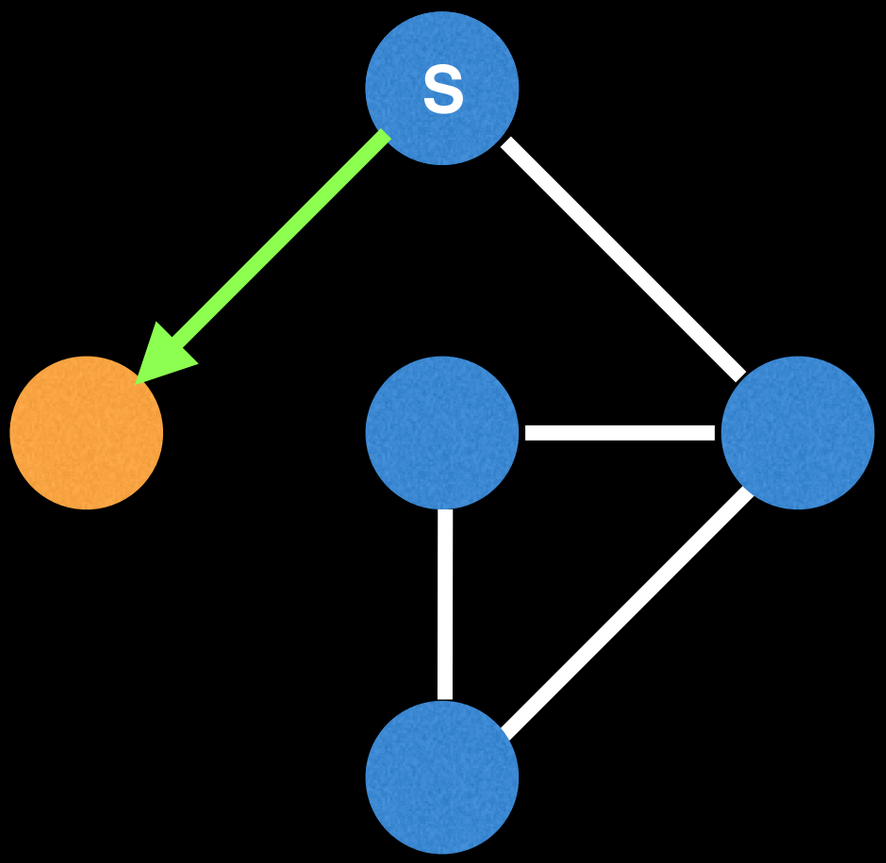

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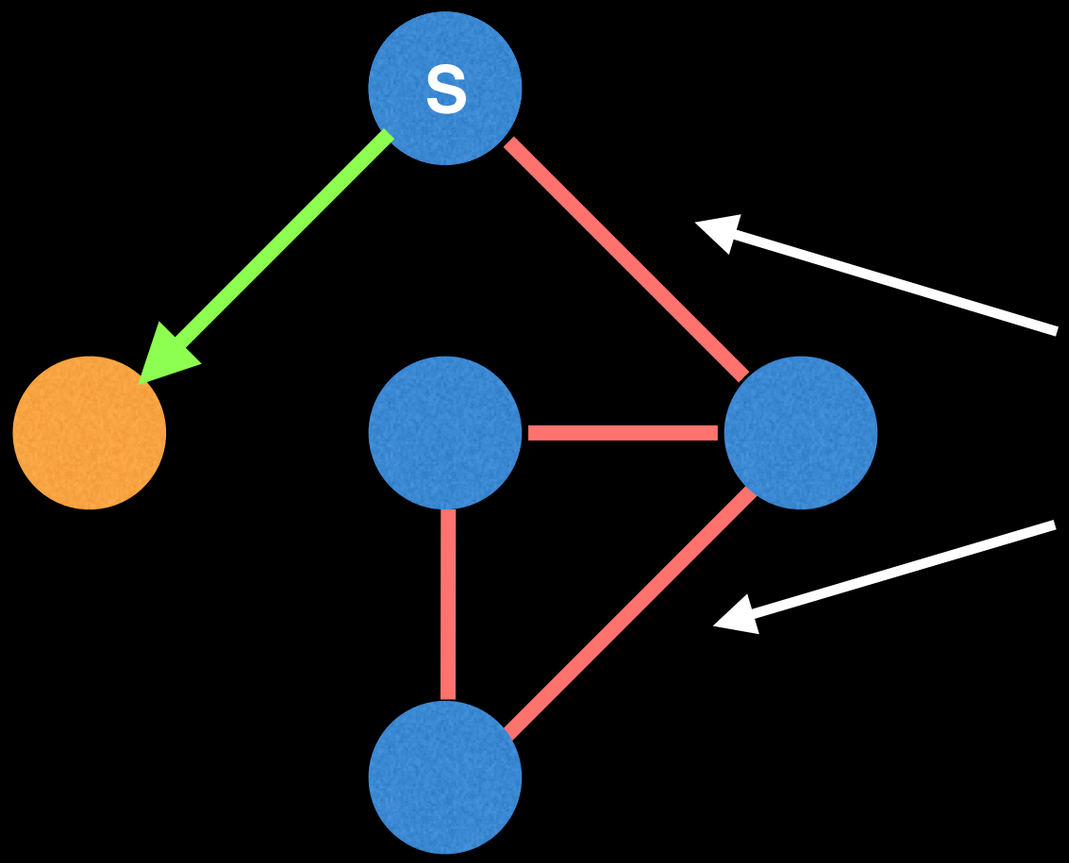

Choosing the wrong starting node can lead to having unreachable edges.

What is an Eulerian circuit?

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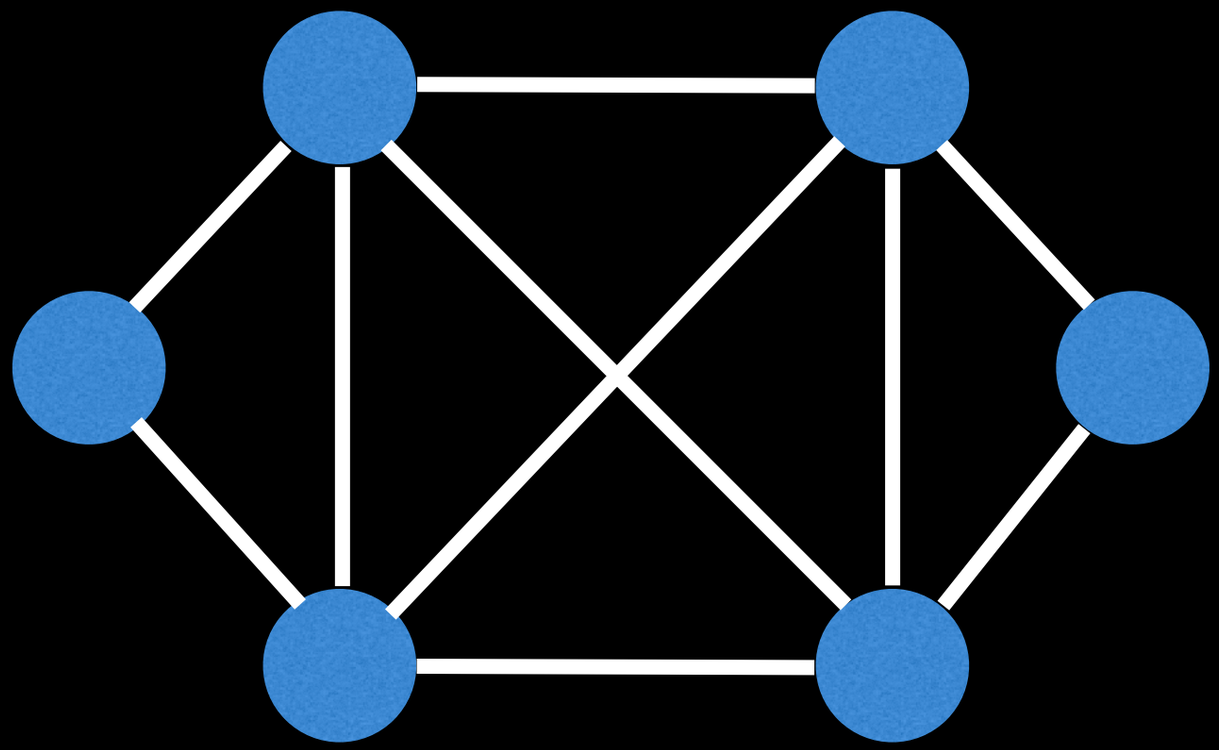

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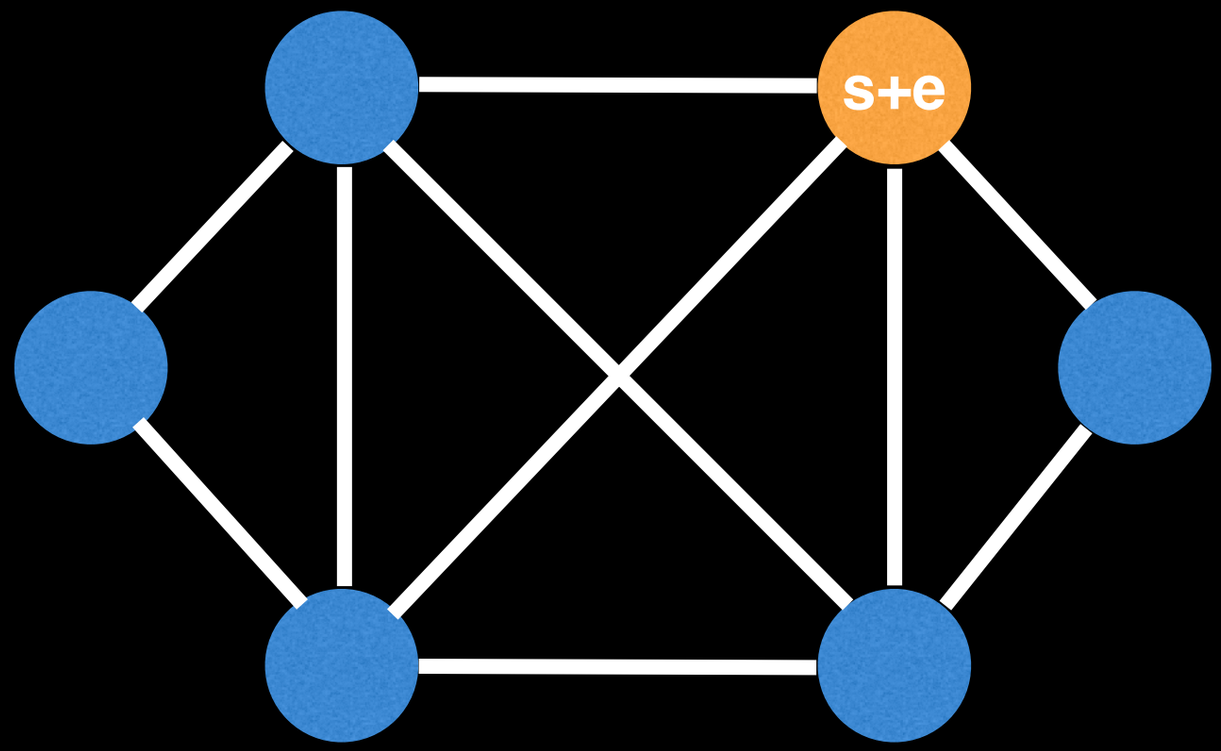

If you know the graph contains an Eulerian cycle then you can start anywhere.

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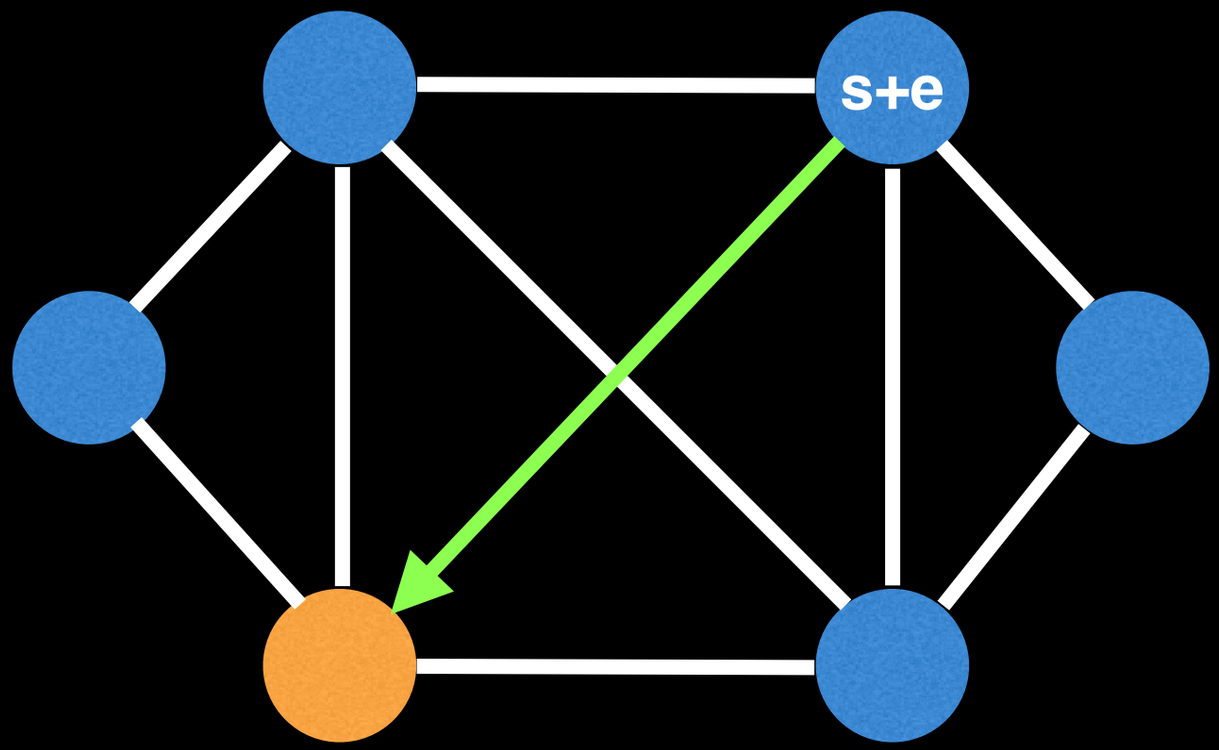

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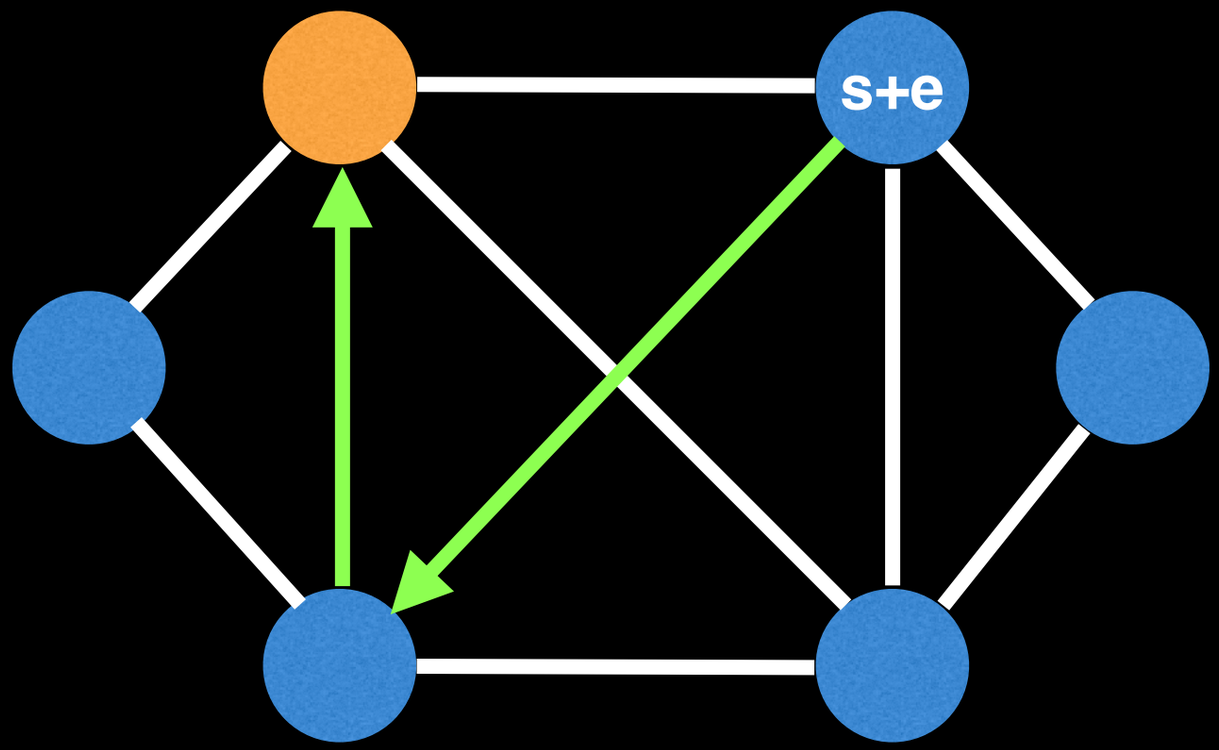

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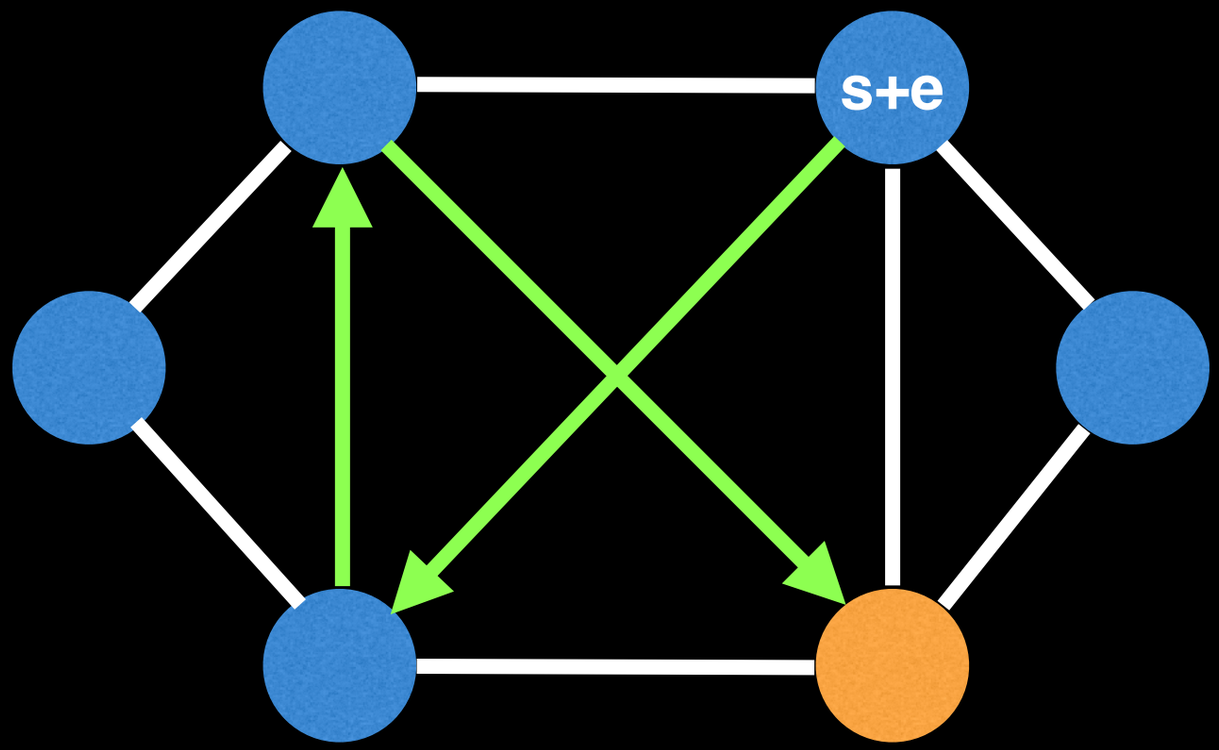

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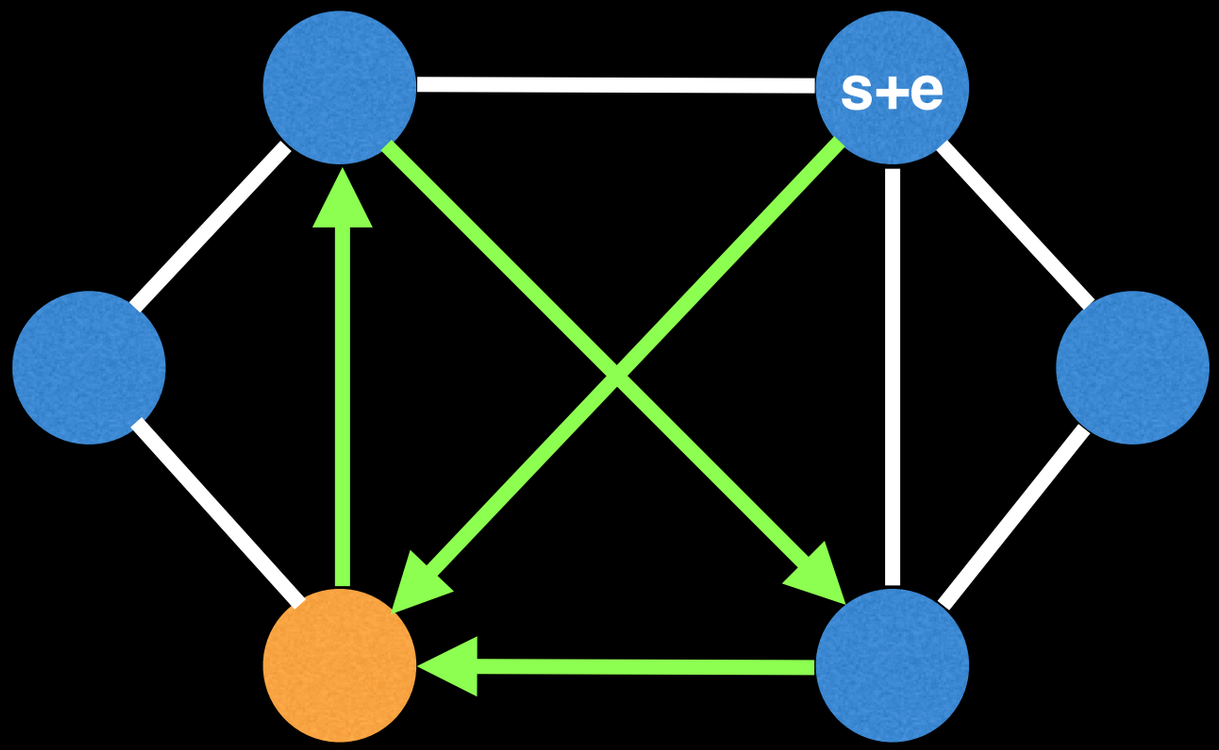

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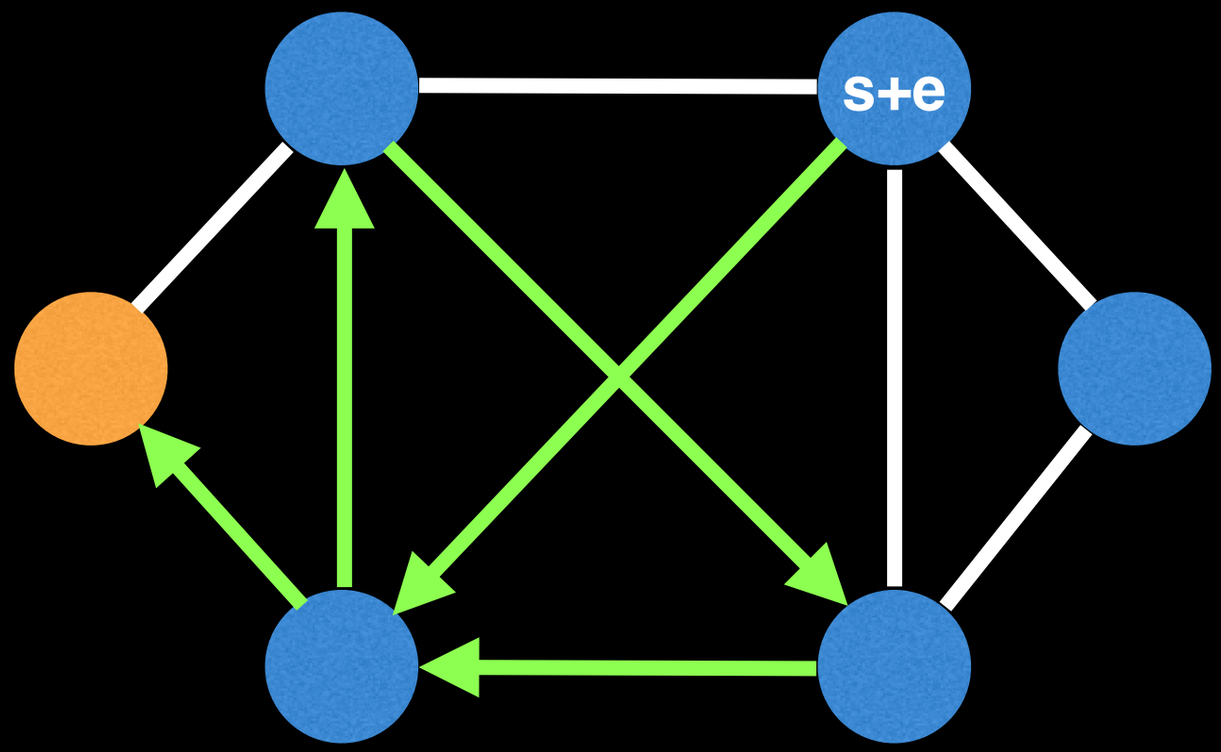

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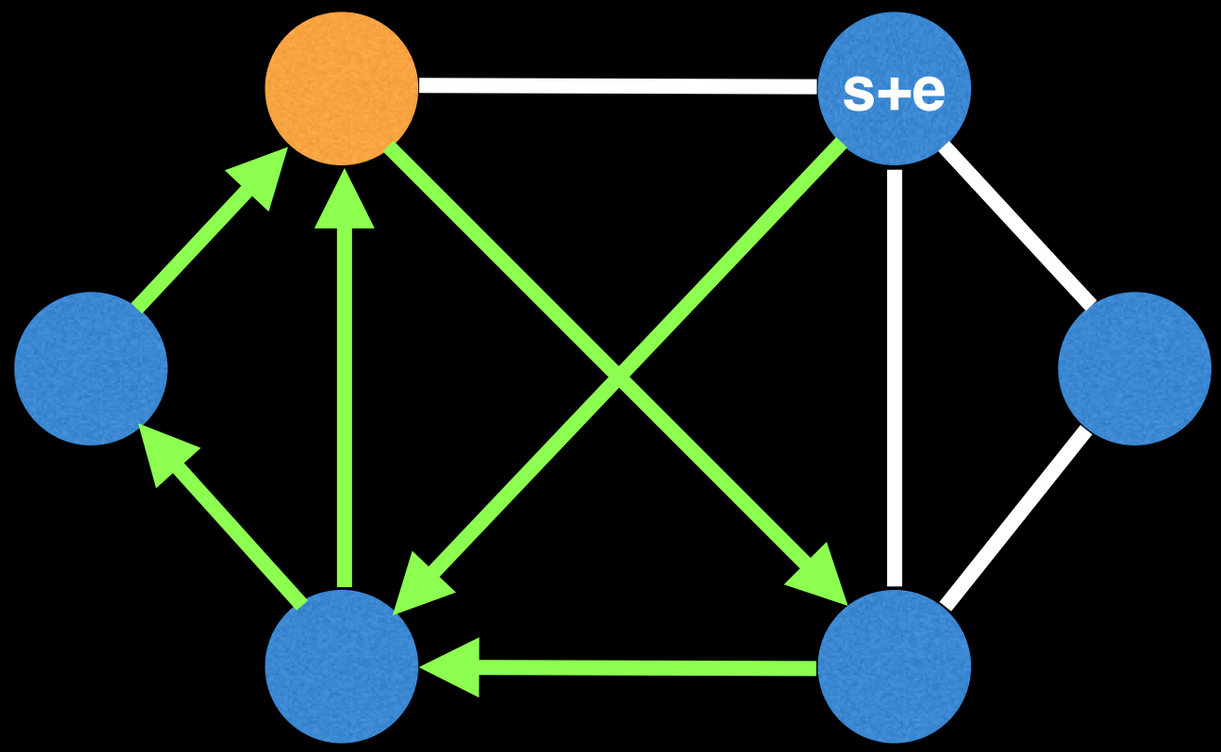

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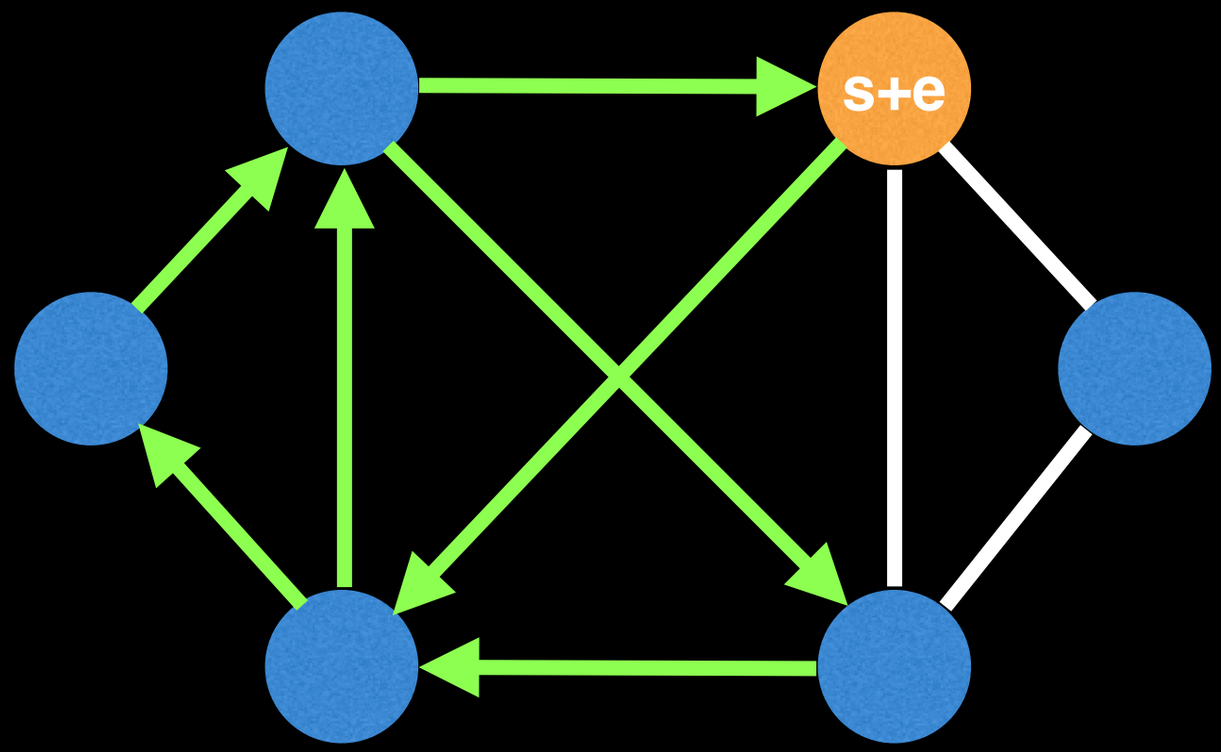

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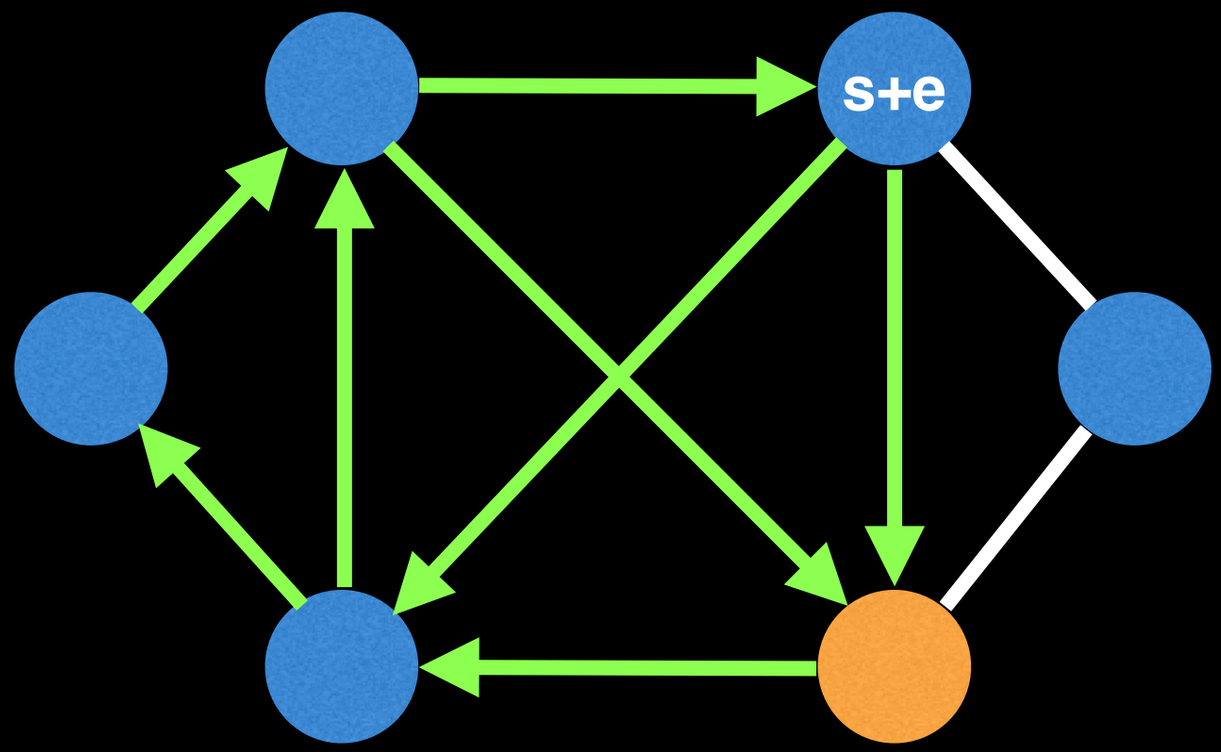

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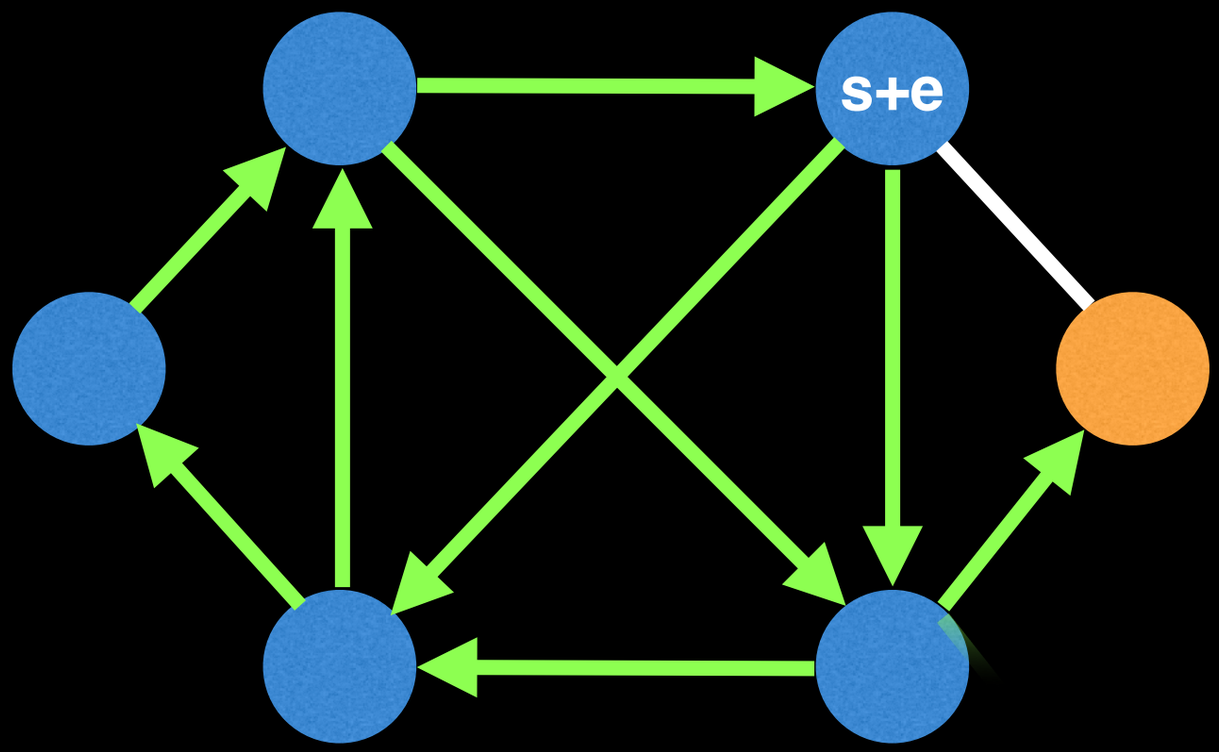

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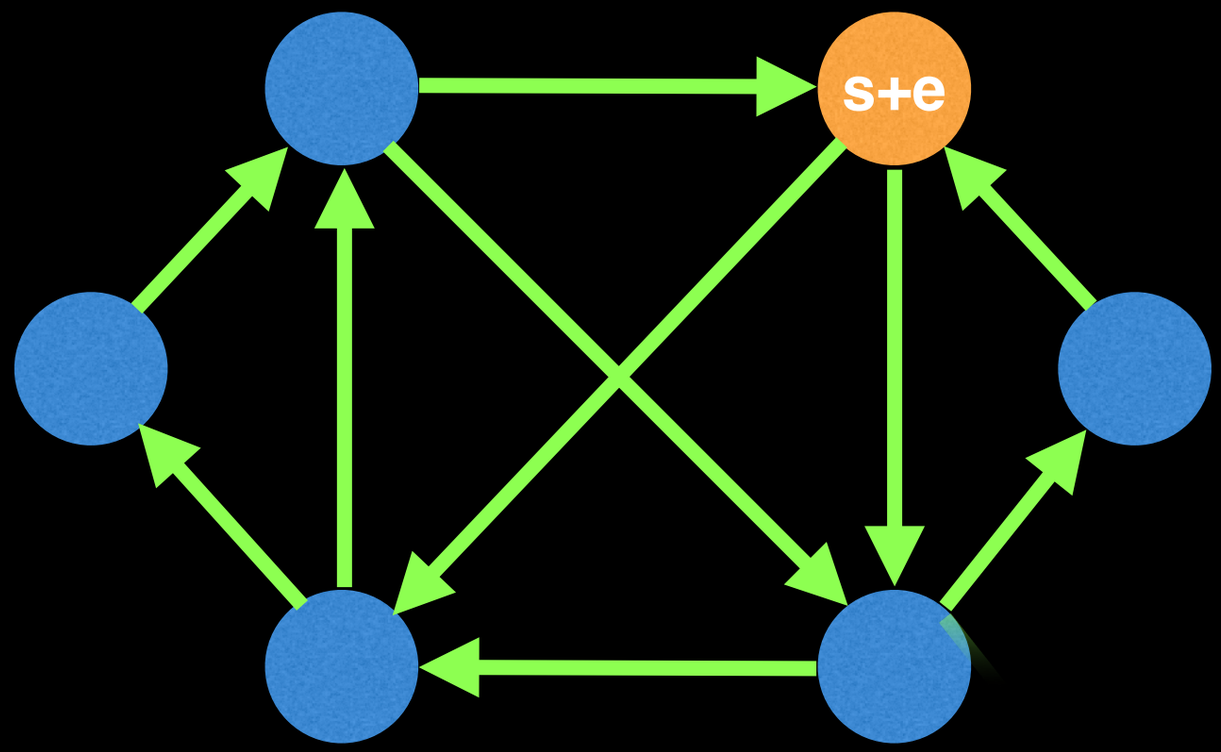

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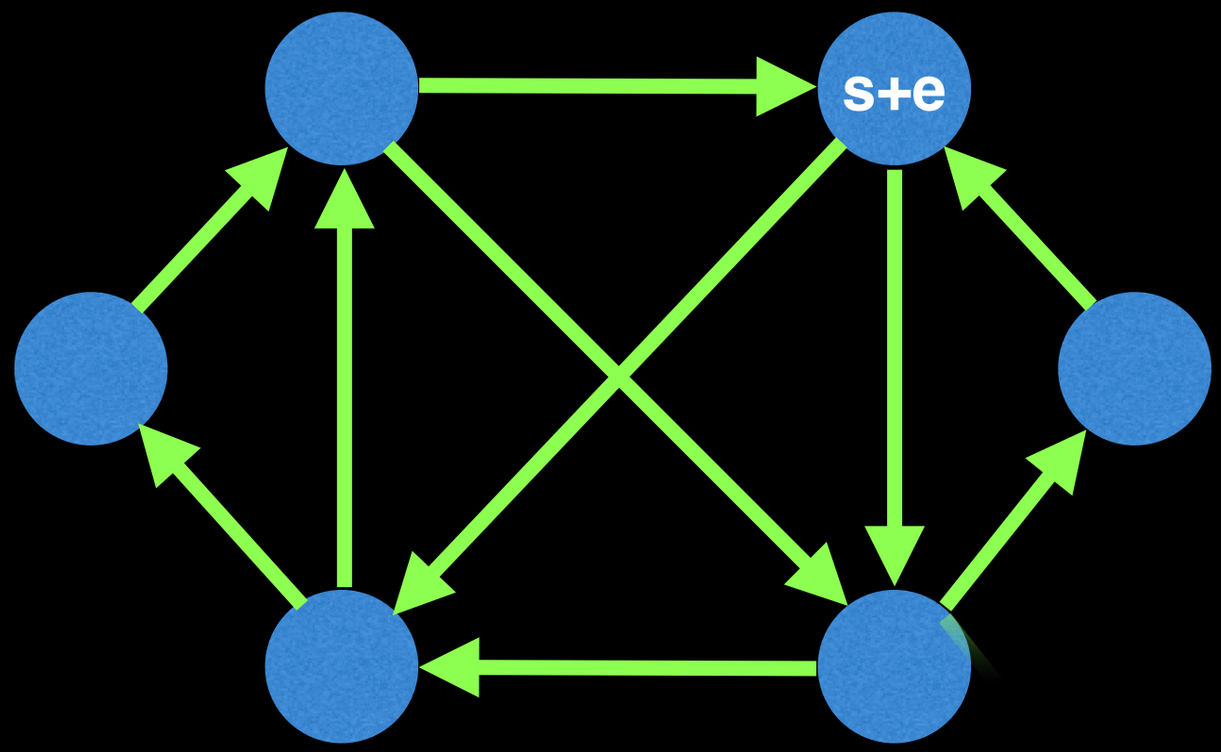

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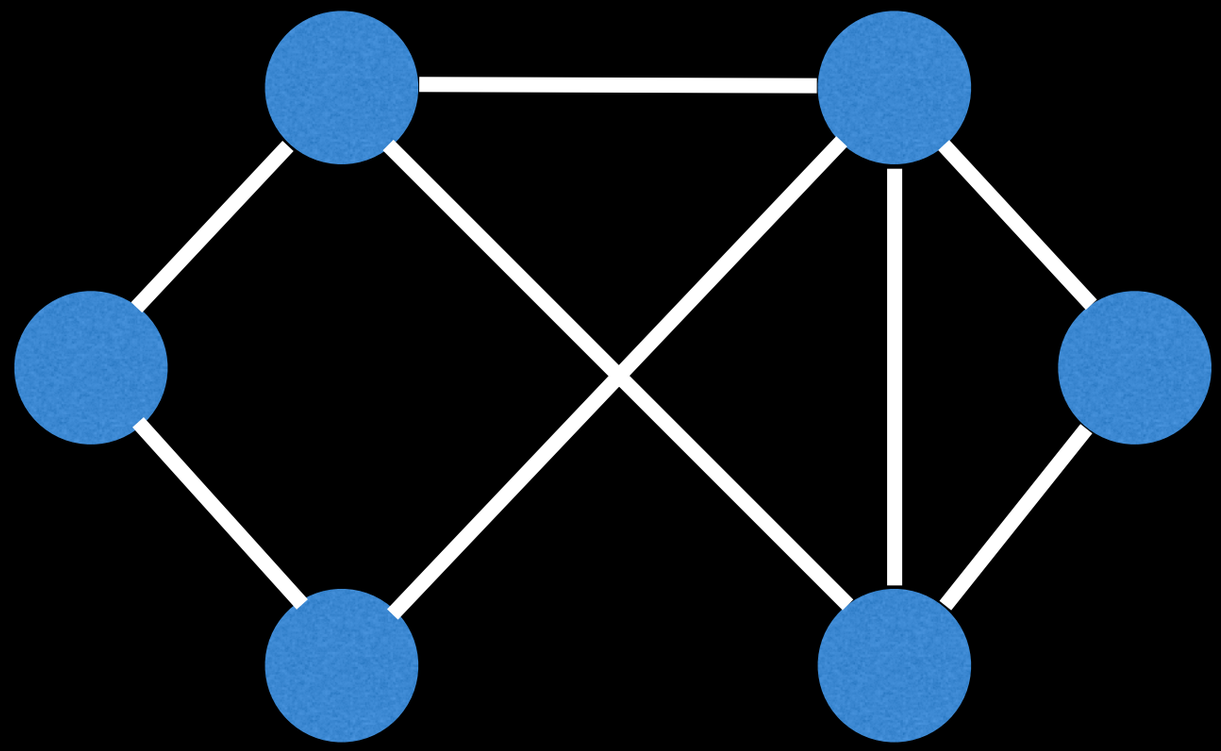

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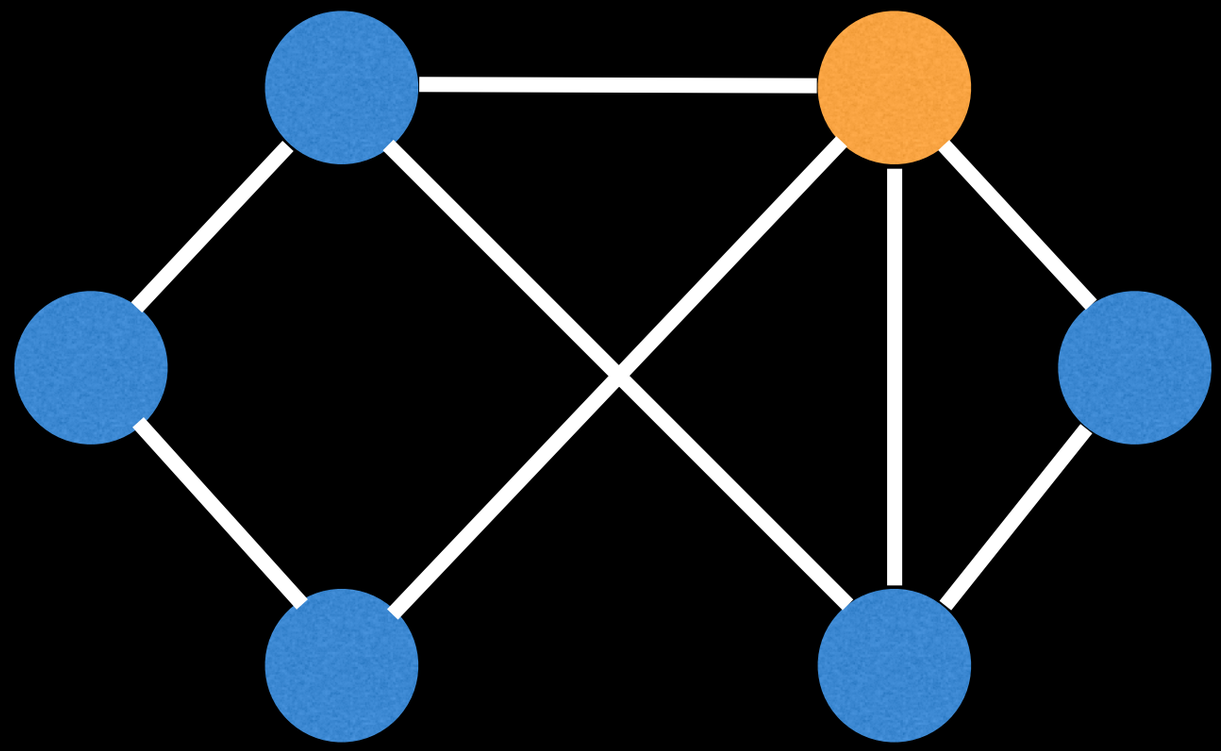

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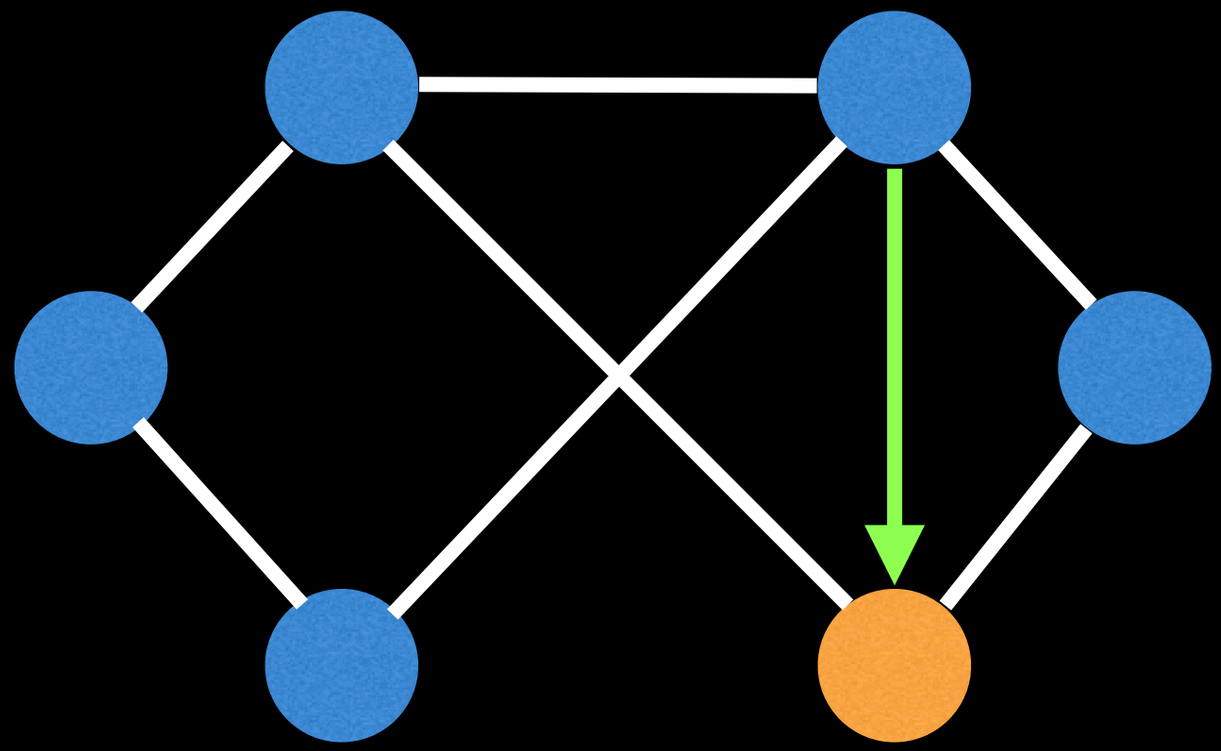

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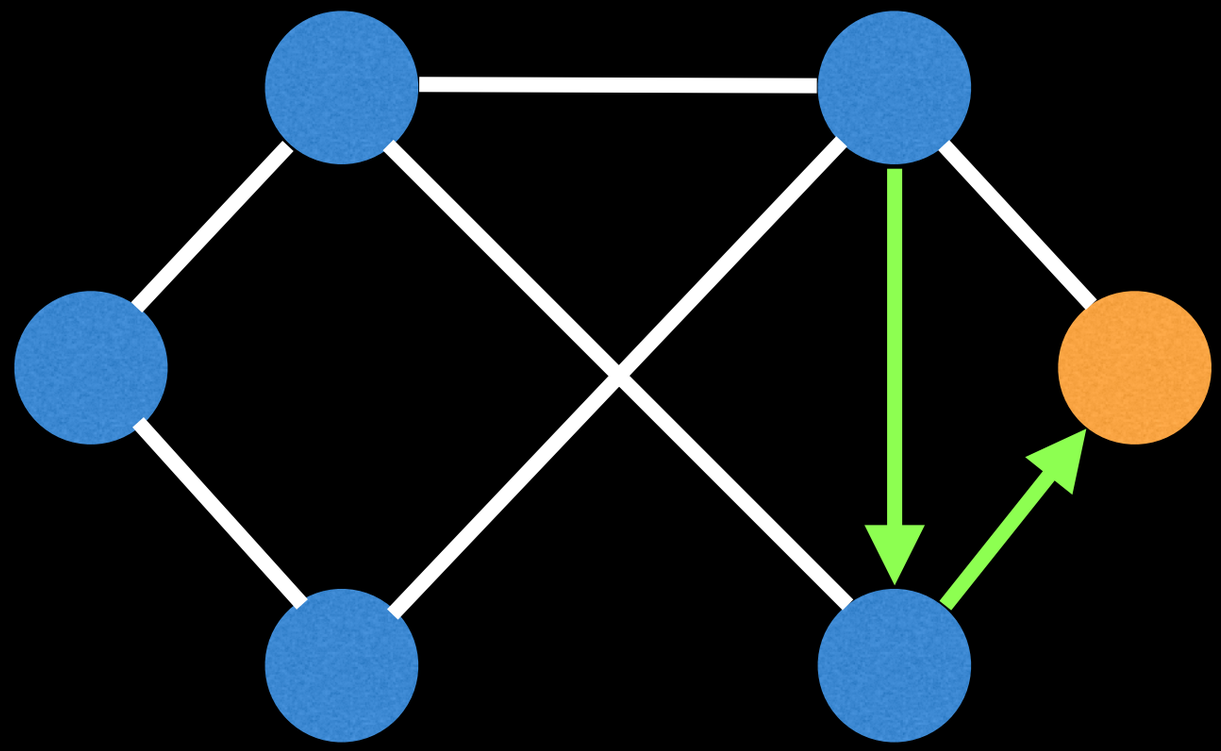

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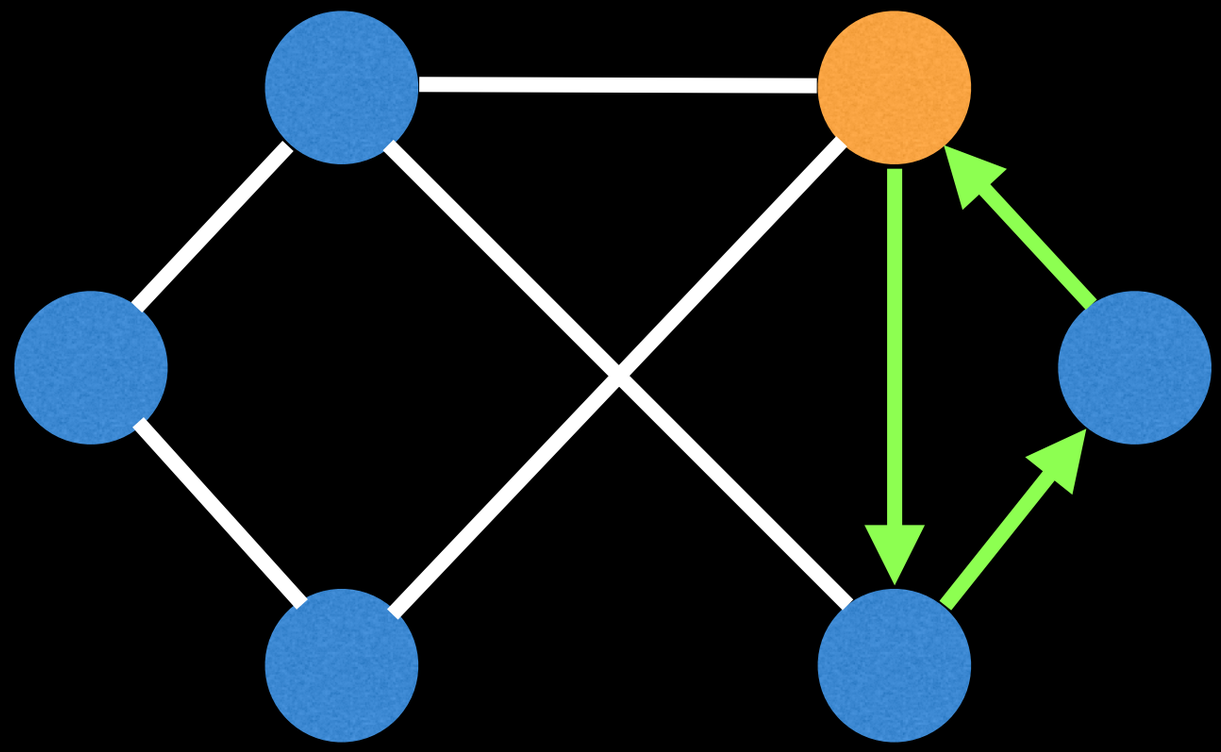

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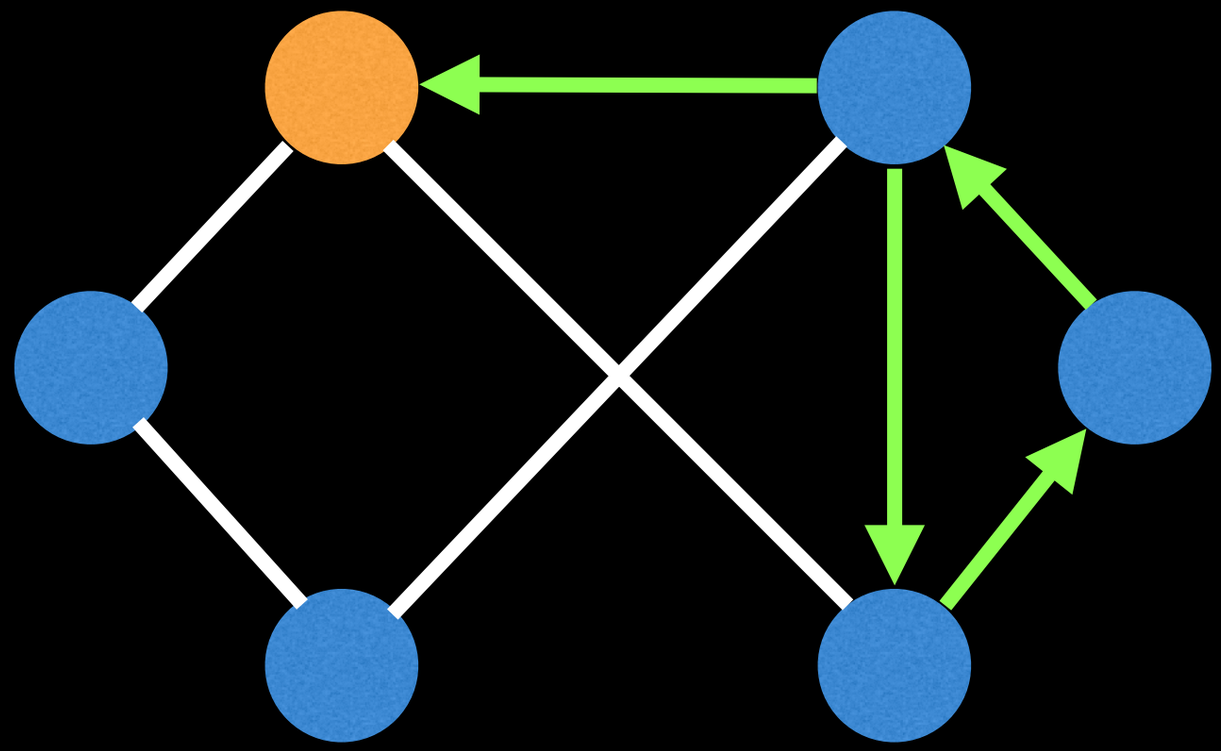

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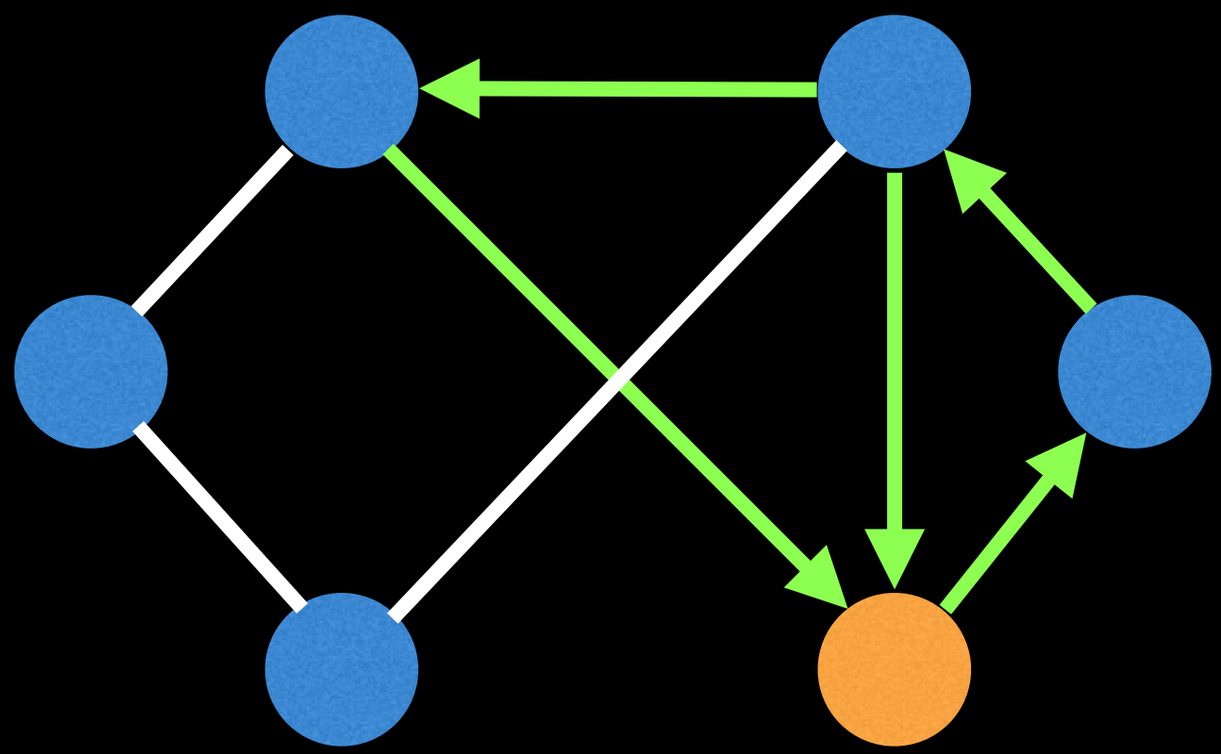

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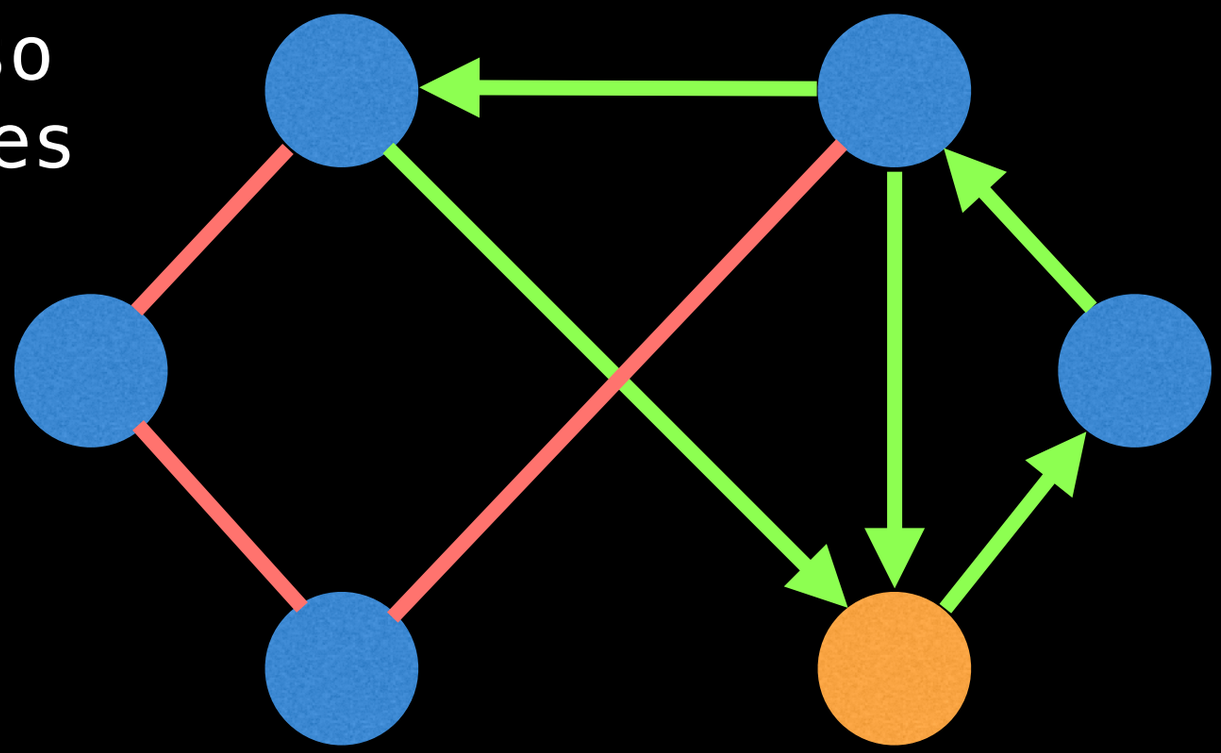
There are also unvisited edges remaining



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What conditions are required for a valid Eulerian Path/Circuit?

That depends on what kind of graph you're dealing with. Altogether there are four flavors of the Euler path/circuit problem we care about:

	Eulerian Circuit	**Eulerian Path**
Undirected Graph	Every vertex has an even degree.	Either every vertex has even degree or exactly two vertices have odd degree.
Directed Graph	Every vertex has equal indegree and outdegree	At most one vertex has (outdegree) - (indegree) = 1 and at most one vertex has (indegree) - (outdegree) = 1 and all other vertices have equal in and out degrees.

Node Degrees

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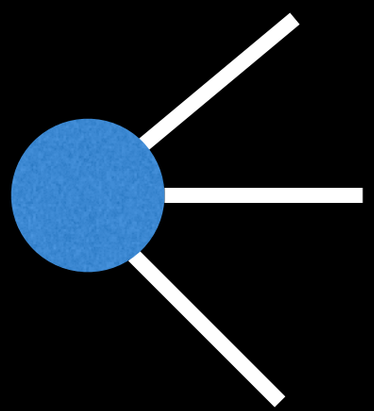
Undirected graph



Node degree = 3

The degree of a node is how many edges are attached to it.

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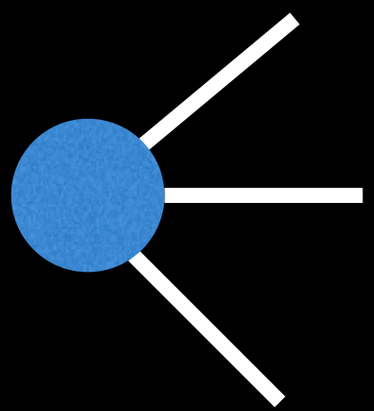

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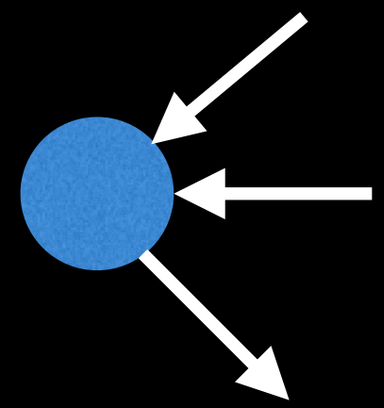

In degree = 2
Out degree = 1

The indegree is the number of incoming edges and outdegree is number of outgoing edges.

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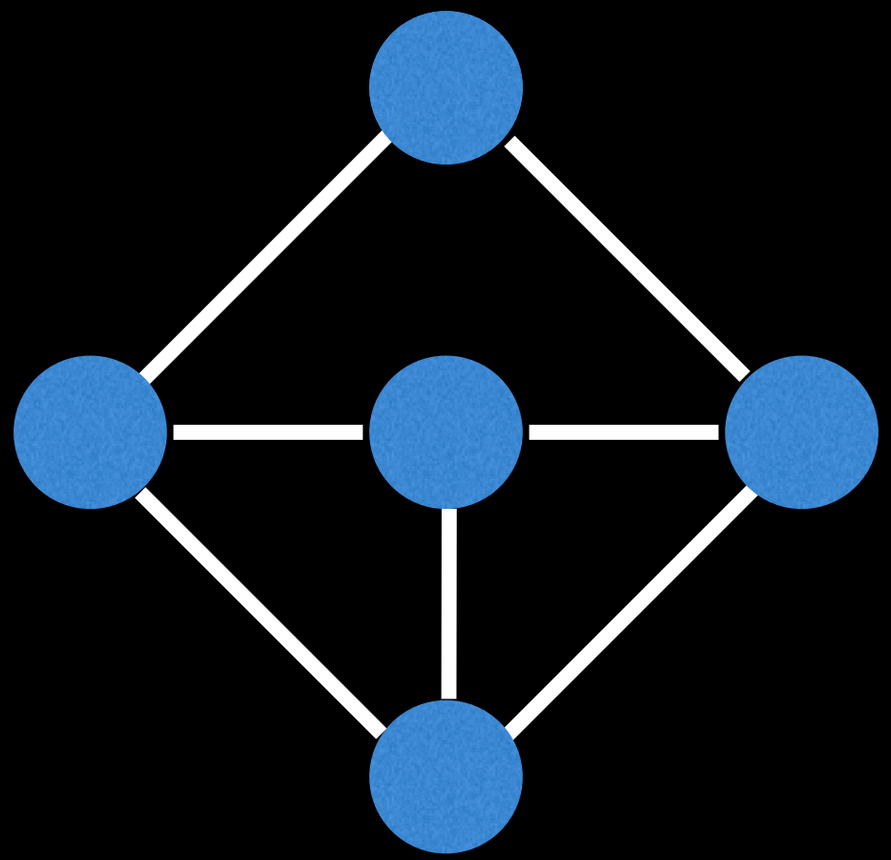

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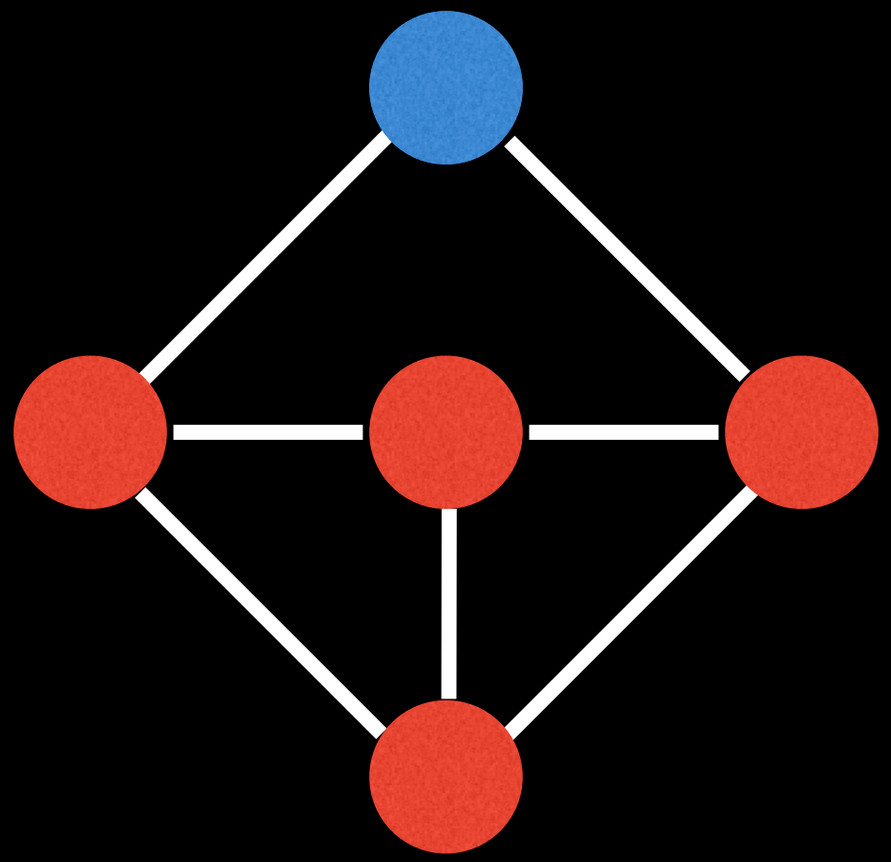

No Eulerian path or circuit.
There are too many nodes that have an odd degree.

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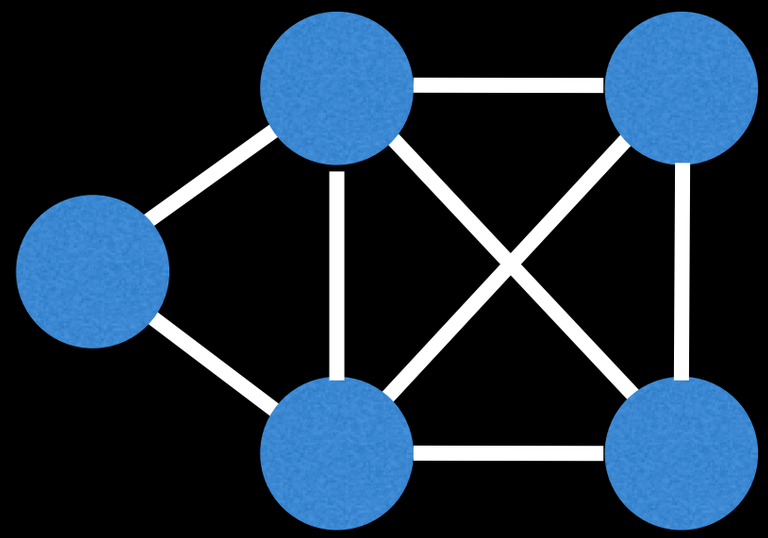

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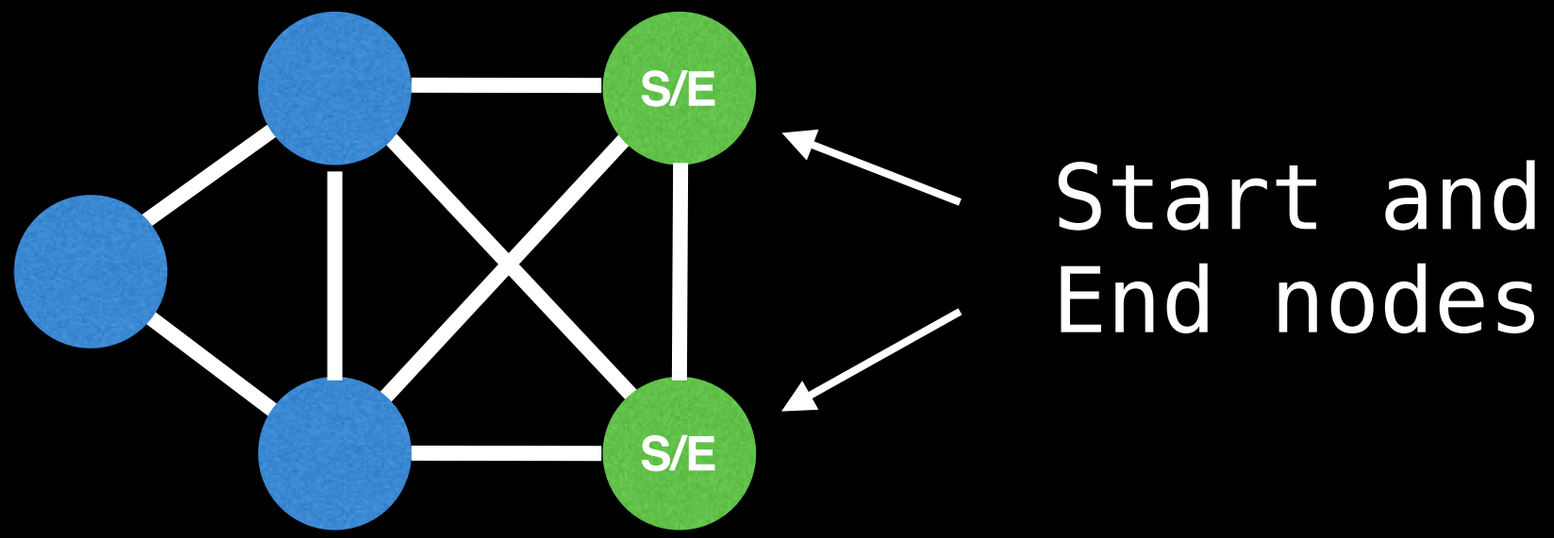
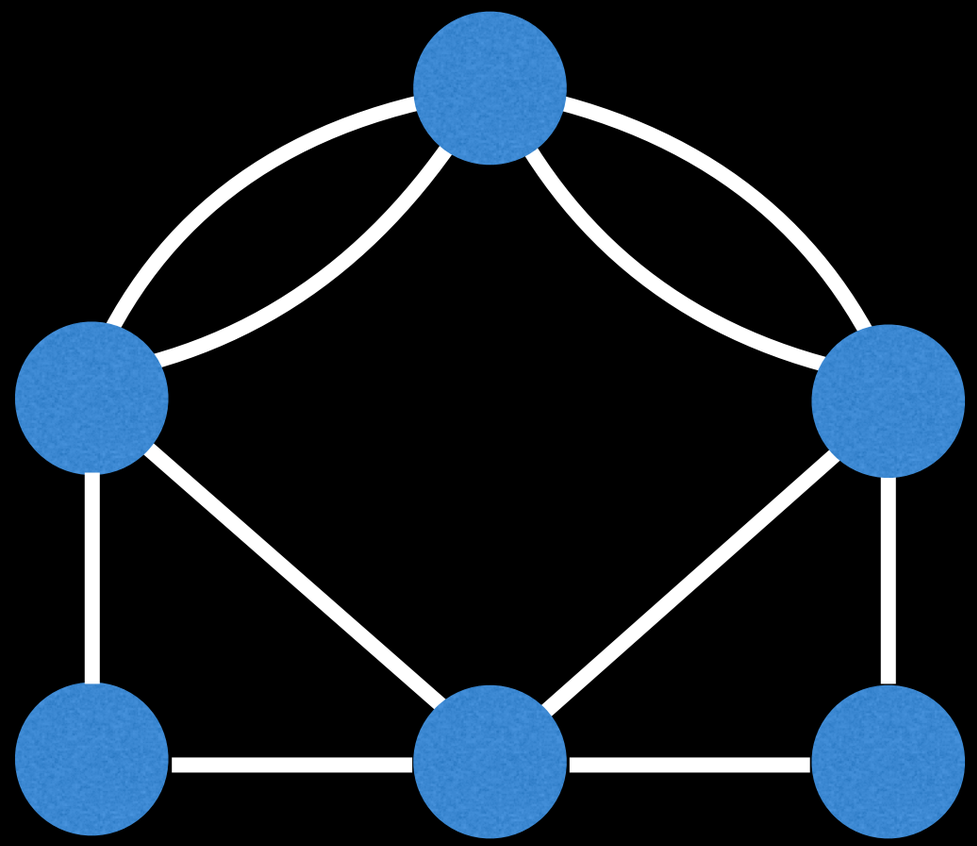
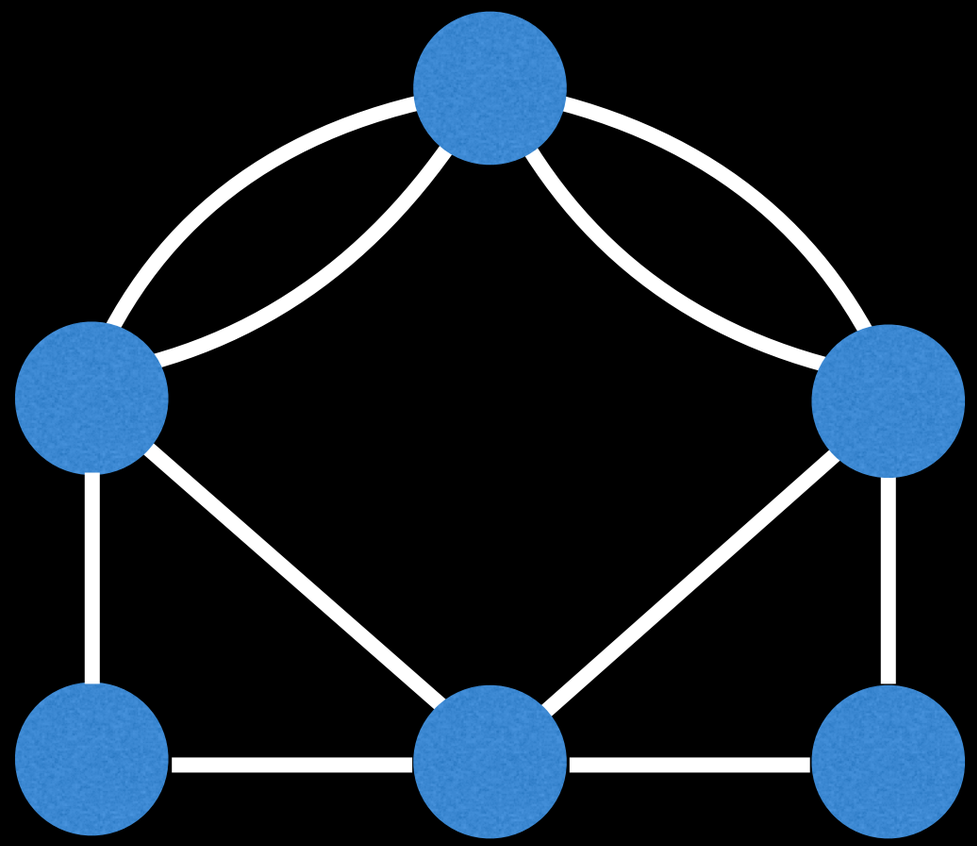


Only Eulerian path.

Does this undirected graph have an Eulerian path/circuit?

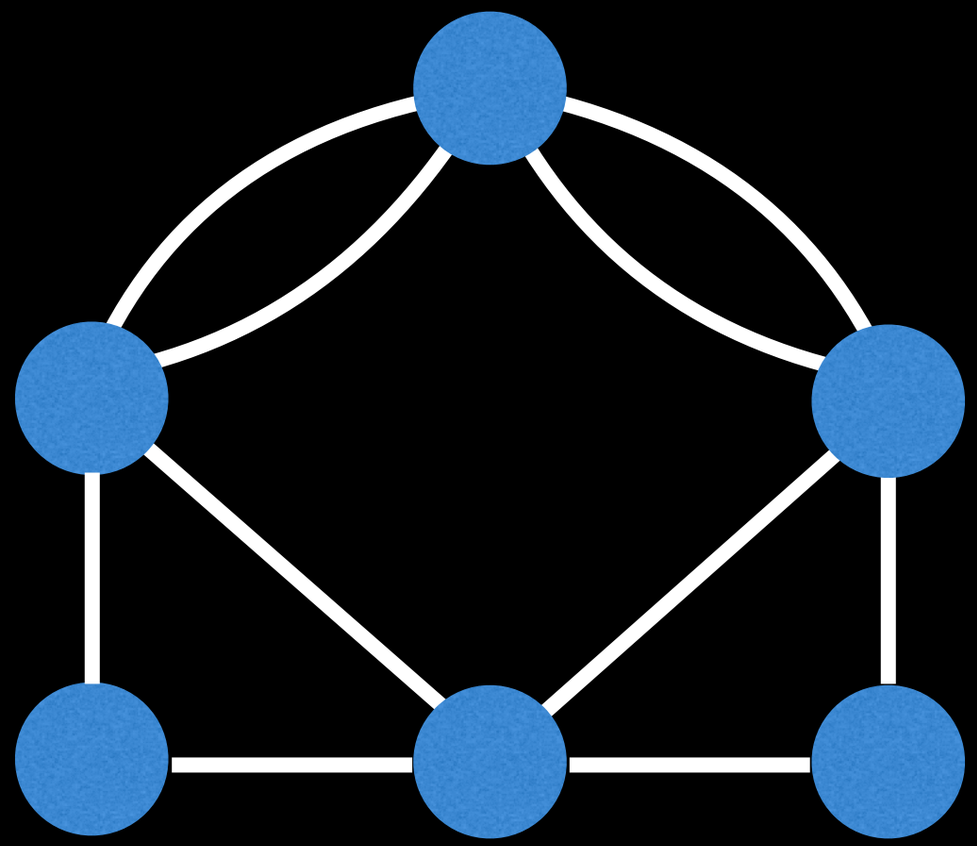


Does this undirected graph have an Eulerian path/circuit?



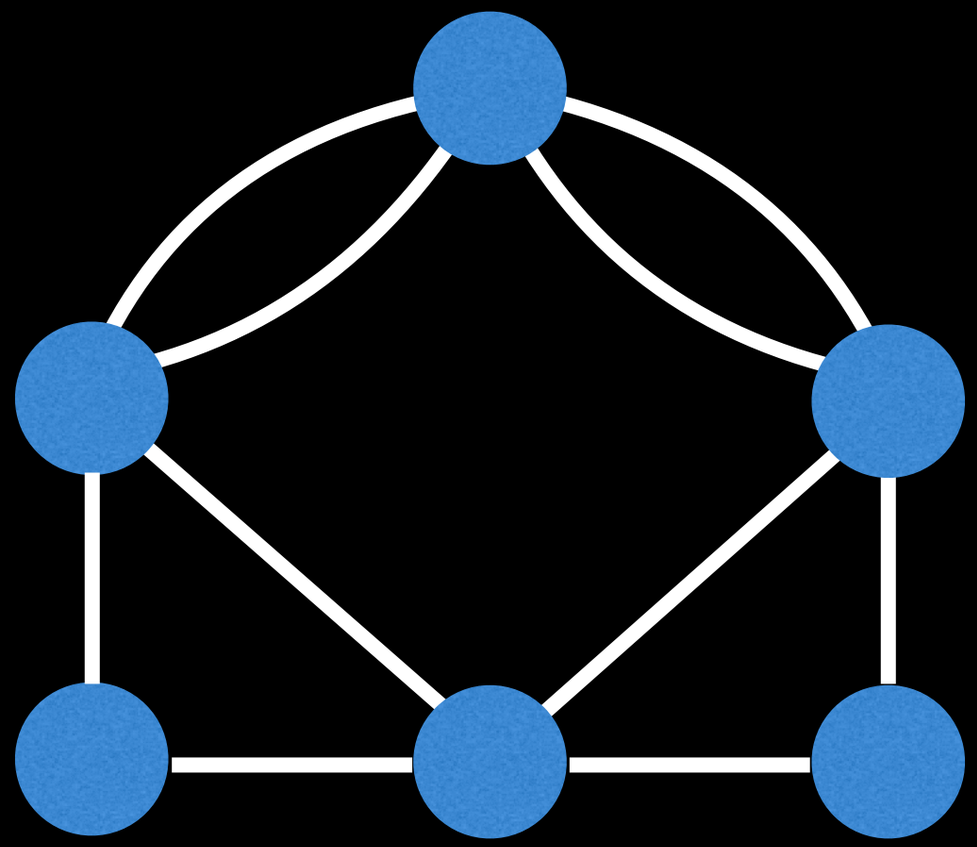
Yes! It has both an Eulerian path and circuit.

Does this undirected graph have an Eulerian path/circuit?



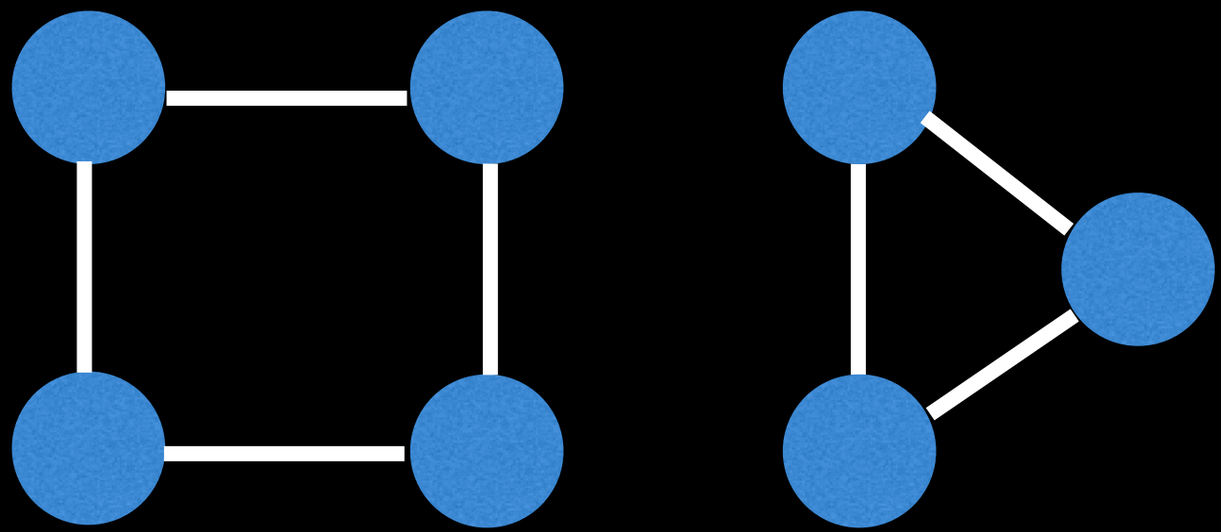
True or false: if a graph has an Eulerian circuit, it also has an Eulerian path.

Does this undirected graph have an Eulerian path/circuit?

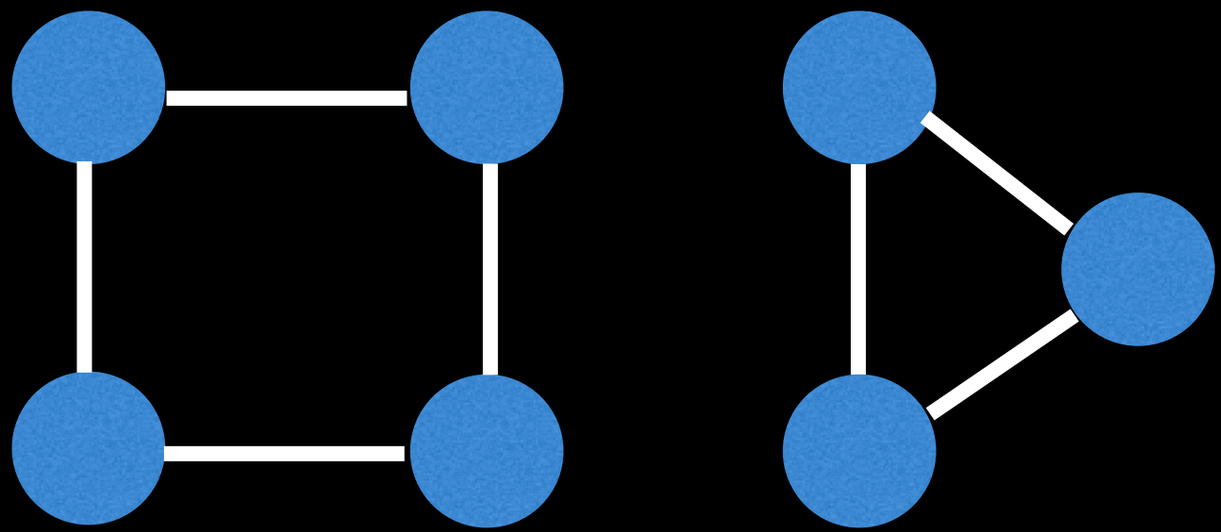


True or false: if a graph has an Eulerian circuit, it also has an Eulerian path.

Does this undirected graph have an Eulerian path/circuit?

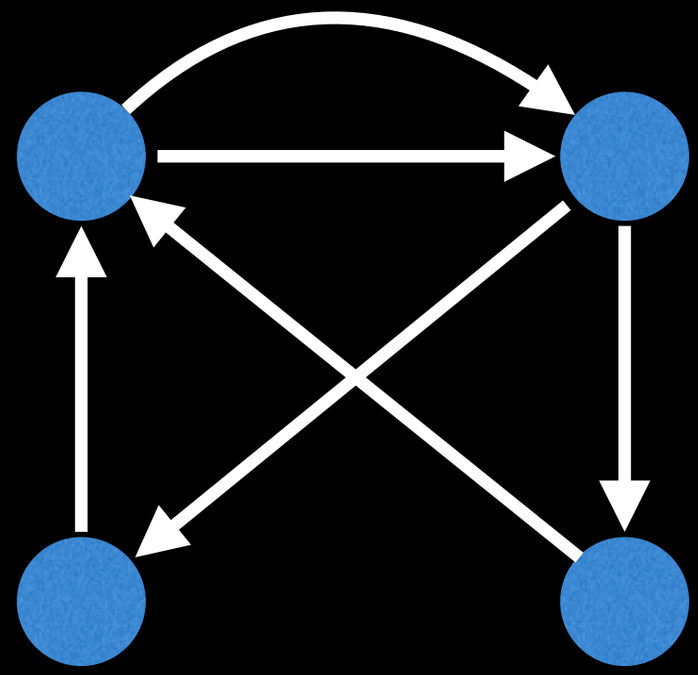


Does this undirected graph have an Eulerian path/circuit?

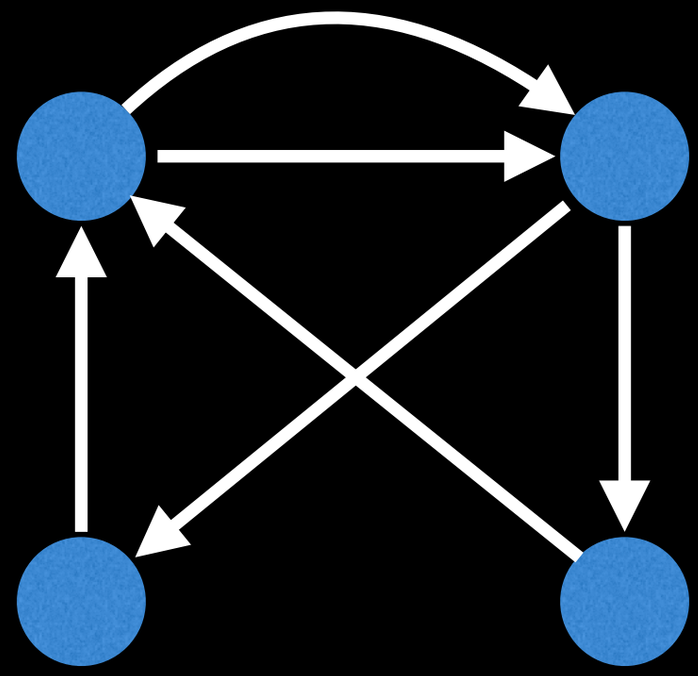


There are no Eulerian paths/circuits. An additional requirement when finding paths/circuits is that all vertices with nonzero degree need to belong to a single connected component.

Does this directed graph have an Eulerian path/circuit?

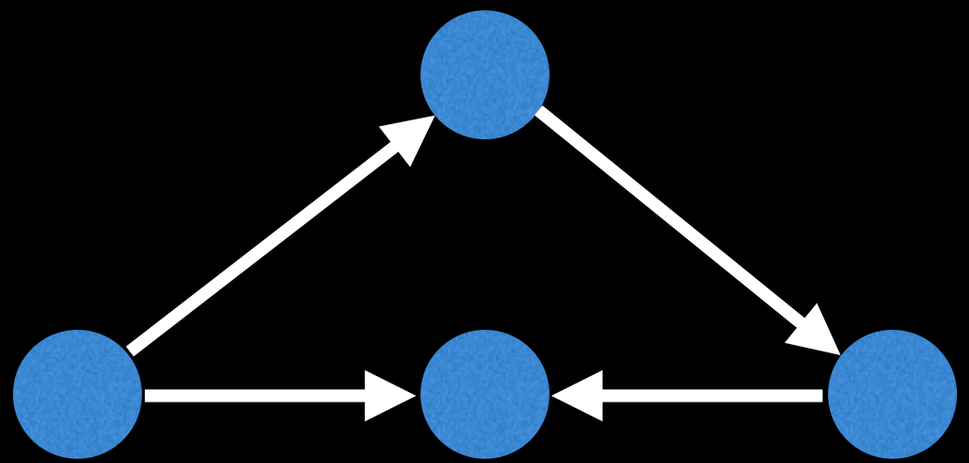


Does this directed graph have an Eulerian path/circuit?

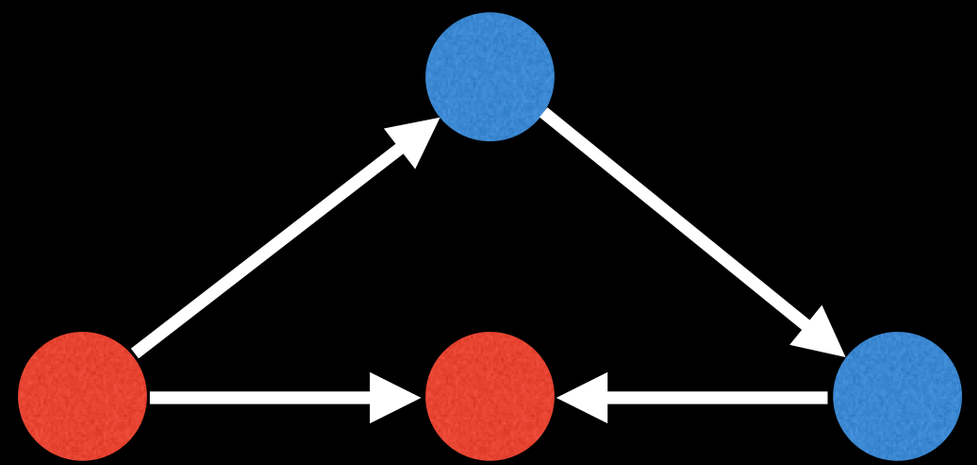


Yes, it has both an Eulerian path and an Eulerian circuit because all in/out degrees are equal.

Does this directed graph have an Eulerian path/circuit?

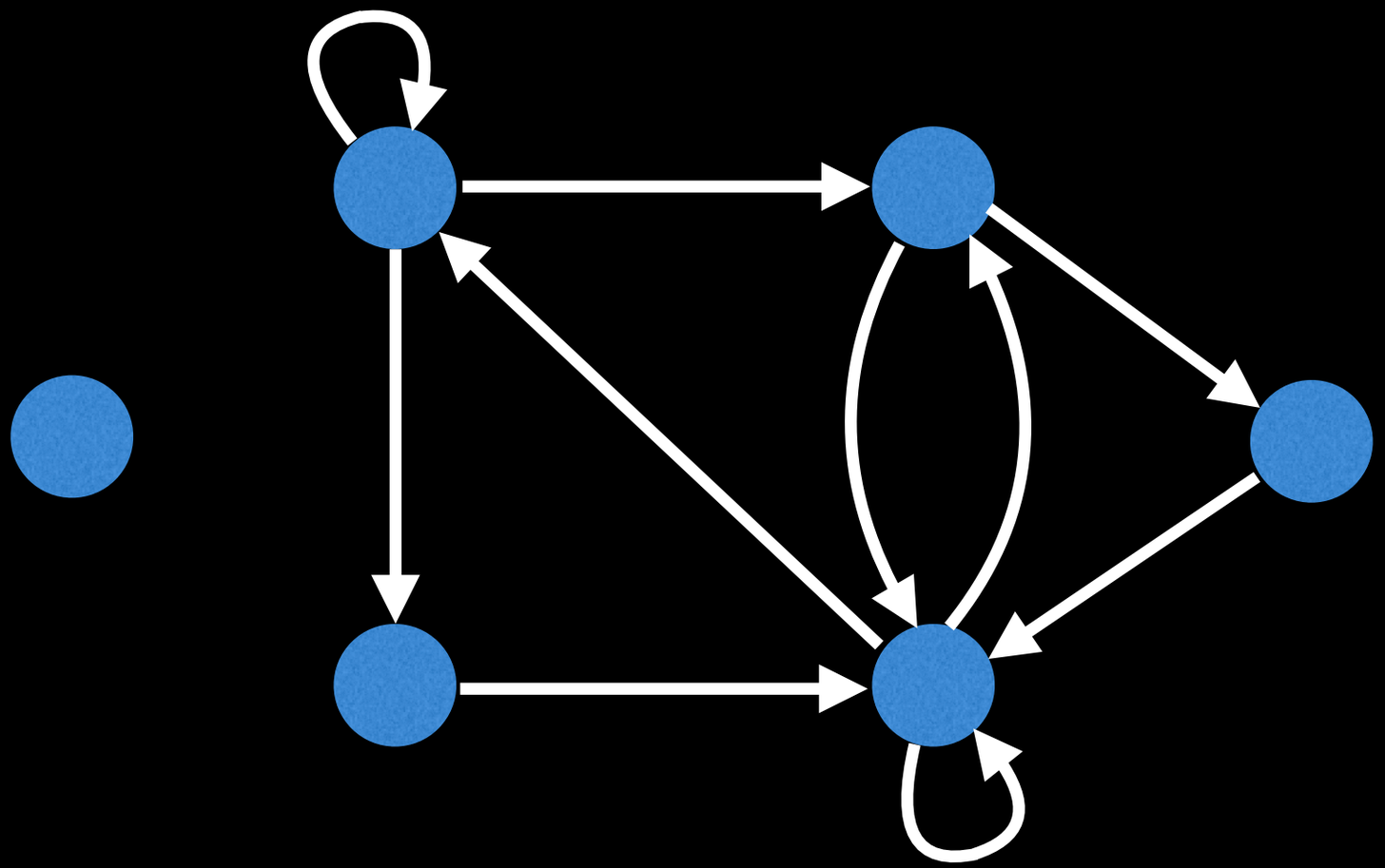


Does this directed graph have an Eulerian path/circuit?

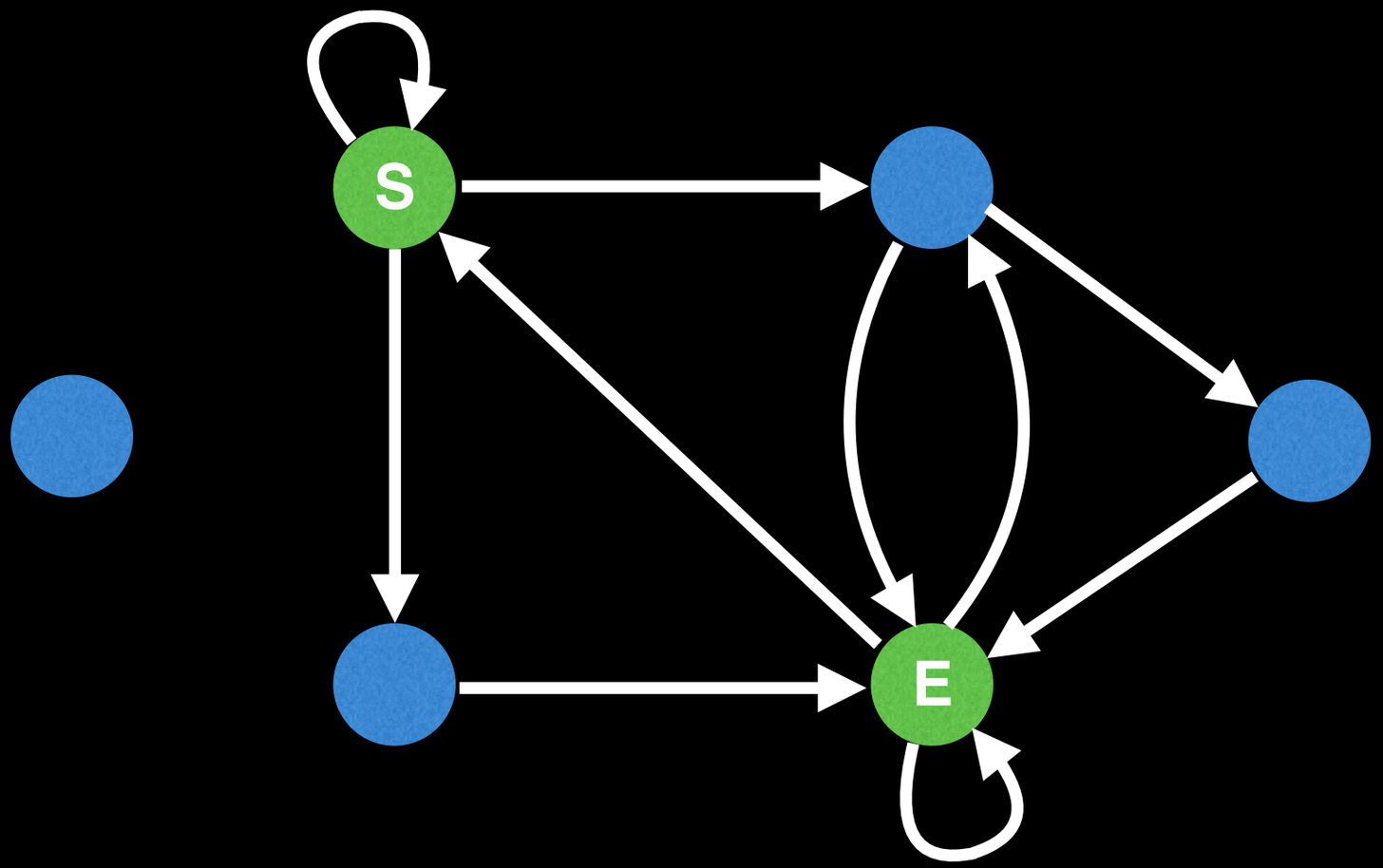


No path or circuit. The red nodes have either too many in coming or outgoing edges.

Does this directed graph have an Eulerian path/circuit?

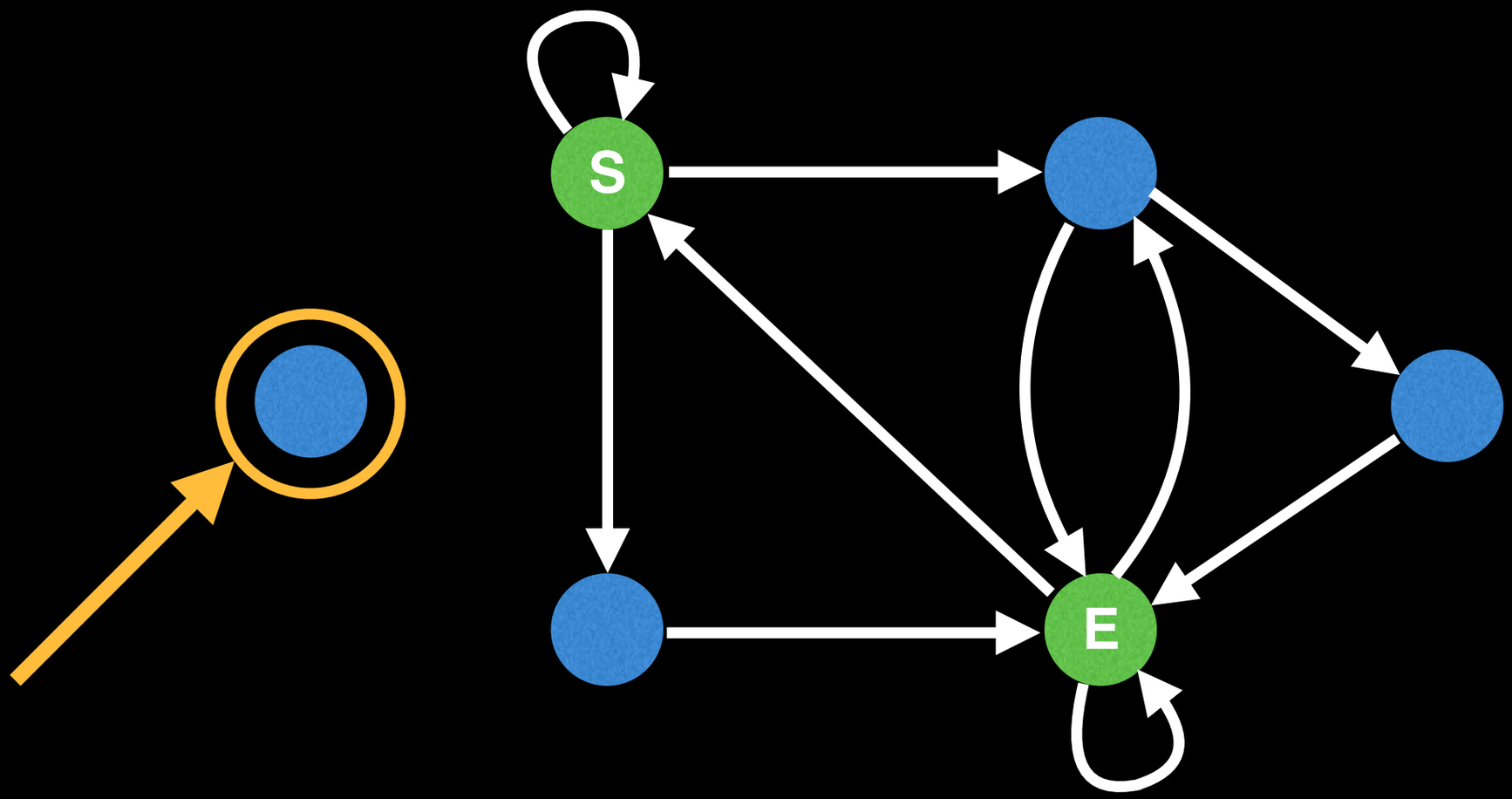


Does this directed graph have an Eulerian path/circuit?



This graph has an Eulerian path, but no Eulerian circuit. It also has a unique start/end node for the path.

Does this directed graph have an Eulerian path/circuit?



Note that the singleton node has no incoming/outgoing edges, so it doesn’t impact whether or not we have an Eulerian path.