MAS_3

November 21, 2020

Leonard Vorbeck

No Group

Student Number : 2709813

[4]:
```
import pandas as pd
import numpy as np
```

0.0.1 3.1

Action space for both players in g_{-1} is given by

$$\{S, C\} \times \{S, C\} = \{SS, SC, CS, CC\}.$$

Matrix Form

[7]:
```
A = ["SS", "SC", "CS", "CC"]
pd.DataFrame([[(1,1),(1,1),(1,1),(1,1)],
               [(1,1),(1,1),(1,1),(1,1)],
              [(0,3),(0,3),(2,2),(2,2)],
              [(0,3),(0,3),(1,2),(3,3)]], columns=A, index=A)
```

[7]:

	SS	SC	CS	CC
SS	(1, 1)	(1, 1)	(1, 1)	(1, 1)
SC	(1, 1)	(1, 1)	(1, 1)	(1, 1)
CS	(0, 3)	(0, 3)	(2, 2)	(2, 2)
CC	(0, 3)	(0, 3)	(1, 2)	(3, 3)

The pure NEs are $\{(SS, SS), (SS, SC), (SC, SS), (SC, SC), (CC, CC)\}$.

There are three non-trivial subgames g_3, g_2, g_1 and the whole game g_{-1}

g_1

[17]:
```
pd.DataFrame({"S":[(1,2)], "C":[(3,3)]}, index=["Joint-Payoff"])
```

[17]:

	S	C
Joint-Payoff	(1, 2)	(3, 3)

g_2

[18]:
```
pd.DataFrame({"S":[(2,2),(1,2)], "C":[(2,2), (3,3)]}, index=["S", "C"])
```

[18]:

	S	C
S	(2, 2)	(2, 2)
C	(1, 2)	(3, 3)

g_3 For g_3 we have action space $\{S, C\} \times \{S, C\}$ for player 2 again.

[19]:
```
pd.DataFrame({"SS":[(0,3),(0,3)], "SC":[(0,3),(0,3)],
              "CS":[(2,2), (1,2)], "CC":[(2,2), (3,3)]}, index=["S", "C"])
```

[19]:

	SS	SC	CS	CC
S	(0, 3)	(0, 3)	(2, 2)	(2, 2)
C	(0, 3)	(0, 3)	(1, 2)	(3, 3)

$(3, 3) \rightarrow (CC, CC)$ is a subgame perfect NE, as it is present in every sub-game.

Using Backwards-Induction the result is obvious immediately :

$$\text{A} : U_2(C) > U_2(S) \implies g_1 \leftarrow (3,3)\text{B} : \text{A} \implies g_2 \leftarrow (3,3)\text{C} : \text{B} \not\Longrightarrow g_3 \leftarrow (3,3)$$

g_3 cannot be replaced with $(3, 3)$ as player 2 is indifferent between $(0, 3)$, $(3, 3)$. However,if $P(S) > \frac{2}{3} \implies EU_1C > EU_1S$.

Note that if there wouldnt be this occurence of indifference, e.g havin $(0, 2)$ at this terminal, then we could replace the whole game with the sgpne ($g_{-1} \leftarrow (3, 3)$ would then be true.)

0.0.2 3.2

From here on boss is player 1, employee is player 2.

There are 4 non trivial subgames g_A (left bottom), g_B (left top), g_C (right bottom), g_D (right top).

On the left side we see

$$g_A \leftarrow (0,1) \implies g_B \leftarrow (0,1) \implies U_1(N) = (0,1)$$

On the right side we see

$$g_C \leftarrow (-1,-1) \implies g_D \leftarrow (1,0) \implies U_1(W) = (1,0)$$

Player 1 will play W, and Player 2 will play Q. Backwards-Induction proposes $(W, Q) = (1, 0)$, essentially replacing the whole game with $(1, 0)$.

Player 1 :

$$A_1 \leftarrow \{N, W\} \times \{I, Fi\} \times \{F, Ft\}$$
$$= \{(N,I,F), (N,I,Ft), (N,Fi,F), (N,Fi,Ft), (W,I,F), (W,I,Ft), (W,Fi,F), (W,Fi,Ft)\}$$

For player 2 we have

$$A_2 \leftarrow \{H, S\} \times \{Q, T\} = \{(H, Q), (H, T), (S, Q), (S, T)\}$$

With Matrix

[13]:

```
pd.DataFrame({"(H,Q)":[(1,0) for _ in range(8)],
              "(H,T)":[(1,0),(1,0),(1,0),(1,0),(-2,1),(-1,-1),(-2,1),(-1,-1)],
              "(S,Q)":[(0,1),(0,1),(-1,-1),(-1,-1),(1,0),(1,0),(1,0),(1,0)],
              "(S,T)":
↪[(0,1),(0,1),(-1,-1),(-1,-1),(-2,1),(-1,-1),(-2,1),(-1,-1)]},
             index=["(N,I,F)", "(N,I,Ft)", "(N,Fi,F)", "(N,Fi,Ft)",
                    "(W,I,F)", "(W,I,Ft)", "(W,Fi,F)", "(W,Fi,Ft)"])
```

[13]:

	(H,Q)	(H,T)	(S,Q)	(S,T)
(N,I,F)	(1, 0)	(1, 0)	(0, 1)	(0, 1)
(N,I,Ft)	(1, 0)	(1, 0)	(0, 1)	(0, 1)
(N,Fi,F)	(1, 0)	(1, 0)	(-1, -1)	(-1, -1)
(N,Fi,Ft)	(1, 0)	(1, 0)	(-1, -1)	(-1, -1)
(W,I,F)	(1, 0)	(-2, 1)	(1, 0)	(-2, 1)
(W,I,Ft)	(1, 0)	(-1, -1)	(1, 0)	(-1, -1)
(W,Fi,F)	(1, 0)	(-2, 1)	(1, 0)	(-2, 1)
(W,Fi,Ft)	(1, 0)	(-1, -1)	(1, 0)	(-1, -1)

I can identify the following pure NEs :

$$X = [\{(N, I, F), (S, T)\}, \{(N, I, Ft), (S, T)\}, \{(N, Fi, F), (H, Q)\}, \{(N, Fi, F), (H, T)\}, \{(N, Fi, Ft), (H, Q)\}, \{(N,$$

from which only

$$X^* = [\{(W, I, Ft), (H, Q)\}, \{(W, I, Ft), (S, Q)\}, \{(W, Fi, Ft), (H, Q)\}, \{(W, Fi, Ft), (S, Q)\}]$$

induce the SGPNE.

Actually

$$X^* = \{\ \{(a_{11}, a_{12}, a_{13}), (a_{21}, a_{22})\} \mid a_{11} = W \wedge a_{22} = Q\}$$

By definition of the cartesian product, there will be trivial joint actions, but we essentially end up at the same SGPNE like in BI.

0.0.3 3.3

When player 2 goes into the market, he uses his best response function.

His utility function is

$$u_2(q_1, q_2) = P(q_1, q_2)q_2 - cq_2$$

with F.O.C :

$$\frac{du_2}{dq_2} = 0$$

which yields

$$q_2^*(q_1) = \frac{\alpha}{2\beta} - \frac{q_1}{2} - \frac{c}{2\beta}$$

Player 1 knows that and incorporates that into his maximization problem.

$$u_1(q_1, q_2^*) = P(q_1, q_2^*)q_1 - cq_1$$

s.t. F.O.C is satisfied by

$$\frac{du_1(q_1, q_2^*)}{dq_1} = 0$$

which yields a constant

$$q_1^* = \frac{\alpha - c}{2\beta}$$

Player 2's best response for this therefore is

$$q_2^*(q_1^*) = \frac{\alpha - c}{4\beta}$$

s.t.

$$\{q_1^*, q_2^*\} \leftarrow \{\frac{\alpha - c}{2\beta}, \frac{\alpha - c}{4\beta}\}$$

We see that $q_1^* > q_2^*$ and subsequently, there is a first mover advantage. In the cornout game we have $q_1^* = q_2^*$. We can conclude that player 1 has a larger quantity and profit in comparison to the simultanous game. The fraction that is lost for q_2 is the gain of q_1, meaning, player 2 has less than player 1 and also less than he would have in the simultanous game.