MAS_1_0

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No Group

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0.0.1 1.1.1

There is no pure strategy being dominated by another pure strategy. However, if we look at player 1, we can see that a mixed strategy between U and M can dominate D. If we consider a probability vector P, we can easily see that, for example $\hat{P} \leftarrow [\frac{1}{3}, \frac{2}{3}, 0]$ will yield

$$EU_1(\{\hat{P}, S_1\}, s_2) > EU_1(D, s_2) \; \forall s_2 \in S_2$$

which essentially represents a dominant strategy.

0.0.2 1.1.2

Modeling the game as a sequential one with discrete time steps t where each player i iteratively chooses his best choice $BR_i(t, BR_j(t-1))$ will either result in a loop, indicating a mixed NE, or it will converge at a pure NE cell. In the case of this game, the game will always converge to (M, C), regardless of the initial cell, because (M, C) represents the cell at which no player has an incentive to deviate.

0.0.3 1.1.3

In this specific game, the NE maximimizes the welfare $W = U_1 + U_2$, derived from n-player economic games with $W(u_1, ..., u_n) = \sum_i^n u_i$. Solution properties (i.e. binary constraint mappings) such as max W are not induced by NEs. For example, the prisoners dilemma will result in a non "social"-optimal NE.

0.0.4 1.2.1

A rational strategy would be to iteratively eliminate dominated strategies.

Let $S = \{0, 1, 2, ..., 100\}$

Let S^* be the set of viable strategies.

Now $S^* \leftarrow S$

Now, if we assume every player is rational, we can immediately see that everything above 66.667 would not be played since it just cannot possibly be true. However, after the size of the viable set of strategies has been reduced, the exact same logic occurs (resulting from rationality). Therefore we see the viability threshold decreasing from $(100 \times \frac{2}{3}) \times \frac{2}{3}) \times \frac{2}{3}...$ ultimatively ending up at $S^* = \{1\}$. We can write this down in a sequential manner using discrete time steps t.

$$S^*_{t+1} \leftarrow \{x \in S^*_t \ : x <= \max S^*_t \times \frac{2}{3}\}$$

0.0.5 1.2.2

The above answer implies rationality accross all participants, which is a ridicilous assumption (Although a human can be rational in regards to a *specific, well-defined preference set*, humans, in general can never be rational - The likelihood of 50 humans being rational is zero). I would simulate the experiment computationally in the following way:

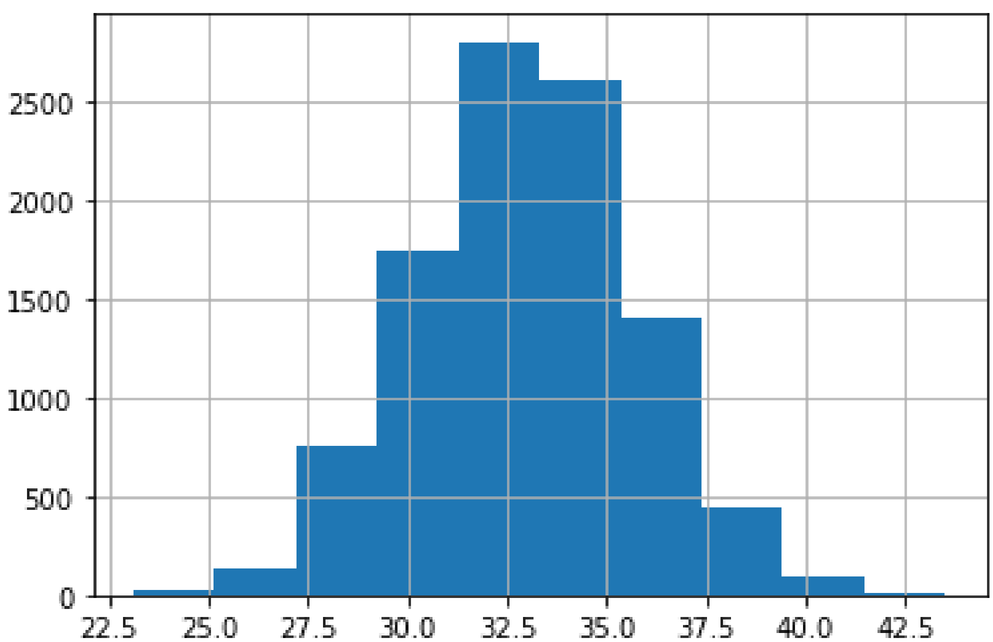
1.1 Draw $n = 50$ integers from $Unif(S)$

1.2 Calculate Mean*2/3

2. Repeat 10000 times and get distribution

[76]:
```
import numpy as np, pandas as pd
pd.Series([np.mean([int(np.random.uniform(0,100)) for _ in range(50)])*(2/3)␣
 ↪for __ in range(10_000)]).hist()
```

[76]: <AxesSubplot:>



Answer : Choose the mean which is equivalent to 2/3 * 50 = 33.333.

0.0.6 1.3.1

BR_i can be derived by maximizing the profit function π_i.

$$\pi_i = q_i p(q_1, q_2) - q_i c_i$$

F.O.C :

$$\frac{d\pi_i}{dq_i} = 0$$

yields

$$q_i^* = BR_i(q_{j \neq i}) = \frac{\alpha}{2\beta} - \frac{q_j}{2} - \frac{c_i}{2\beta}$$

Without providing a figure I can show that this is essentially the same process like in exc 1.1. Each player reacts accross his best response function (which is a line) $BR_i(t, BR_j(t-1))$ until they arrive at the crossing of both lines, the nash equilibrium q_i^{NE}.

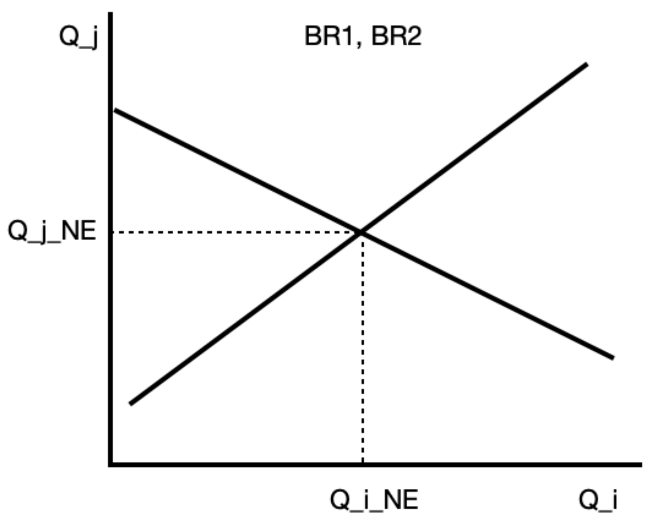
Solving

$$q_i(q_j^*) == q_i^*(q_j)$$

for q_i yields $q_i^{NE} = \frac{\alpha + c_j - 2c_i}{3\beta}$, the point at which the process will end and noone deviates (NE).

[77]:
```
from IPython.display import Image
Image(filename='/Users/leonarddariusvorbeck/Desktop/Bildschirmfoto 2020-11-06␣
↪um 15.50.21.png', width=600)
```

[77]:



0.0.7 1.4.1

For $a = 0.1$ and $b = 0.8$, we can define the set of dominated strategies $S' = \{c : c <= a \wedge c >= b\}$, which wont be played since any point $c \in [a, b]$ will result in a higher market share. He is in fact indifferent between any point in the interval because any point will yield a market share of 35%.

0.0.8 1.4.2

In the case where b is variable, the best response of c becomes a function of b. First, we define the general market share function

In the following I assume that $c = b$ is not a legal location.

$$M(c, b) = 1 - c + \frac{c-b}{2} \ \ if \ \ (c > b) \ \ else \ \ \frac{c-a}{2} + \frac{b-c}{2}$$

Unfortunately I cannot derive an analytical solution (I was also unable to get a meaningful equilibrium), I therefore solve it computationally. By doing that I was able to derive the following rule in order to $\max_c M(a, b, c)$

$$BR_c(b) \leftarrow b - \delta if \ \ \lim_{\delta \to 0} M(a = 0.1, b, c = b - \delta) > M(a = 0.1, b, c = b + \delta)$$

$$BR_c(b) \leftarrow b + \delta if \ \ \lim_{\delta \to 0} M(a = 0.1, b, c = b - \delta) < M(a = 0.1, b, c = b + \delta)$$

For $a = 0.1$ this will yield indifference-threshold of $b \approx 0.7$

0.0.9 1.4.3

Given 1.4.2 was correct, he should obviously choose 0.7, because he can essentially guarentee himself a market share $>= \ 0.3$. This is true because this forces the last player to locate himself $c \in [0.1, 0.7]$.

0.0.10 1.4.4

Choosing an initial point $a = \frac{2}{3}$ or $a = \frac{1}{3}$ will both ensure a market share of $M_a >= \frac{1}{3}$. The next player will then also choose the remaining spot at $\frac{1}{3}$ or $\frac{2}{3}$ because he knows that there will come another player after him. Note that player 2 cannot ``out-delta'' ($b \leftarrow a + \delta$) player 1 because he knows that player 3 would then also do this : $c \leftarrow a + \delta + \delta$ meaning if b doesnt make c indifferent he will lose market share.