

Homework 7: Stationary State of 2D Heat Equation

Part I – Finding Minima and Maxima by Hand

Use the Numpy package to fill a vector x with 1000 random numbers between 0 and 1. Then proceed without using any predefined functions or libraries and determine the value and the index of the smallest elements of this vector.

Part II – Heat Equation

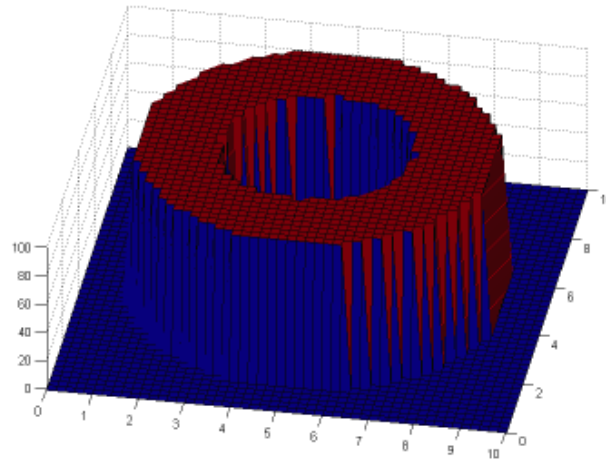


Figure 1: Initial temperature distribution that mimics the letter "O".

This homework set is very much based on the 2D calculation in this week's computer lab but we want to start from different initial conditions. The figure above shows the letter "O". Write some Python code that approximately represent *the second letter of your first name in capitalized form* and use it as initial condition for the 2D heat equation with the following boundary conditions:

(1) Always keep that temperature fixed at 0 at the four boundaries. Repeatedly apply the update formula (**) from the lab assignment to find the stationary state in 2D. *Write an .mp4 animation file* in reasonably good resolution and submit it along with your notebook.

(2) In your first simulation, all heat should have been drained from the system after sufficient time. Let us change the boundary conditions in order to prevent any heat from leaking out. The integral under the 2D curve represents the amount of thermal energy of the system that must be kept constant. To realize this in your simulation, you need to be extra careful at the boundaries and set

$$T_{\text{new}}[i,0] = (T[i,0] + T[i+1,0] + T[i-1,0] + T[i,1]) / 4.0$$

and in the corner specify

$$T_{\text{new}}[0,0] = (2.0 * T[0,0] + T[1,0] + T[0,1]) / 4.0$$

Please replicate these commands for the other three boundaries and corners.

Run your code again and submit a second .mp4 file. Please verify that the amount of thermal energy (sum of the temperatures on all points) remains constant throughout.