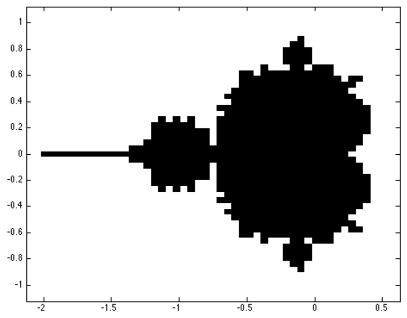
Computer Lab Assignment 5

Mandelbrot Set

This lab is focused on the Mandelbrot set that was discussed in the lecture. The goal is to produce this image in much higher resolution and to understand the algorithm for its generation:



(1) The Mandelbrot set emerges from the super simple iteration formula $z_{n+1} = z_n^2 + C$ where C is a constant and one starts from $z_0=0$. Let us assume all numbers are real for the moment. For what values of C does this series diverge? To find out, please write a simple loop the computes z_n up to $n_{max}=30$. Please define a **function** to do so and place it in a separate cell. Here is an example:

```
[1]: def Mandelbrot(z,C):
         zNew = z**2 + C
         return zNew
```

Print all z_n. If $|z_n|$ exceeds $z_{limit}=50.0$ for any n, let us assume the series diverges. For the following C values, determine whether the series diverges or not. *(We studied this question during lecture.)*

C	Diverges Yes/No
–4	
–2.001	
–2	
–1.999	
–1	
0	
+0.5	
+1	
+2	

(2) Now we want to convey in the information in this table in graphical form. Generate a dense array of C values from –4 to +2 and write a second Python function f(C) that returns 1 or 0 depending on whether the series z_n diverging or not. The generate a plot f(C) versus C. *(You may not need to write a function f(C) as long as the plot is correct. Also this will be a somewhat boring graph but it is just a precursor to what is to come.)*

(3) Now make z and C complex variables and repeat step (1). Most likely you will only need to change a single line in your code where you define C switch to `C=complex(-0.4,+0.2)`. The complex `abs(z)` function is defined to return $\sqrt{re^2 + im^2}$ and can be conveniently used to compare with z_{limit}.

C	Diverges Yes/No
$-0.4 + 0.2i$	
$0.4 + 0.6i$	
-1.7	
$-1.7 + 0.001i$	
-1.778	
$-1.778 + 0.001i$	
-1.25	
$-1.25 + 0.04i$	
$-0.125 - 0.9i$	
$-0.125 - 0.85i$	

(If you are not as confused as Mandelbrot was, when he first noticed the peculiar results, then you may have done something wrong ;=)

(4) Finally, we want to calculate the real Mandelbrot set where we change the real and imaginary parts of C=x+y×i using a fine grid of N points in x and y directions ranging from x= –2.0 to +0.5 and y= –1.25 and +1.25. You may start with N=201. For every point C, start the iteration over z_n from z_0=0. Define a N×N matrix named A. Fill its elements with 1 if the series has diverged within the first n_{max}=50 step. Otherwise set the matrix element to 0. Once every matrix element has been filled, use `imshow` to display it.

(5) Now we want to make the colors a bit more appealing. In cases, where the series z_n does not diverge, fill the matrix with n_{max} instead of 0. In cases where it diverges at some iteration $n<n_{\text{max}}$, fill the matrix with n. Does that look better?

(6) Zoom into the interval $-0.8<x<-0.7$ and $0.05<y<0.15$ and run the code again. Now increase the number of iterations n_{max} first to 75 and later to 100. What change do you see in the resulting image?

(7) Now zoom in further step by step until the size of the x and y intervals are only 0.001 or less. Adjust n_{max} as needed. Get lost in the neverworld of fractals. Save the image file and the parameters for the most beautiful image you obtained!

(8) Finally, run your code with only N=201 grid points and try these magic commands. They will display the contents of the 2D NumPy array `A`. Copy of your results into matrix `A`. (In case you choose to store your results as doubly-linked list, convert them with `A=np.matrix(list)`.)

```
%matplotlib osx
# On PCs, try %matplotlib qt
# On a Mac, could also try %matplotlib tk
# Please restart your Jupyter kernel after every failed attempt!!!
plt.rcParams['figure.figsize'] = [5, 5]
from matplotlib import cm

min = A.min()
A -= min
max = A.max()
while (True):
    A = (A + 2) % max;
    plt.imshow(A,cmap='gist_ncar', interpolation='nearest',animated=True)
    plt.draw()
    plt.pause(0.2)
```

and see what happens. (If an empty window appears please try changing the time argument in the pause command to something longer like 1.0)