BUEC 311: Business Economics, Organization and Management
Producer Behaviour, Part 1

Fall 2020

Outline

1. Motivation: Employment decisions
2. The production function.
3. Production: short-run vs. long-run
4. Returns to scale.
5. Innovation

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Employment decisions.

- How do businesses decide how many workers to hire (or fire)?

The Production Process

- To understand how firms make input decisions, we first need a framework that helps us understand the *production process* of the firm.

Definition (Production process)

The production process (or *technology*) of the firm describes how firms transform *inputs* into *outputs*.

Inputs

human capital
firm-specific
job-specific
general

- Inputs are the resources used in production.
  - Capital (K): Land, buildings, machinery/equipment.
  - Labor (L): Skilled and unskilled (or less skilled) workers.
  - Materials (M): Natural resources, raw materials, processed products.

- Outputs are the result of the production process.
  - Physical product: Car/computer chip/gasoline.
  - Service: Haircut/consulting services/automobile tune-up.

Pollution

Tech/Knowledge.

Noise

The Production Function

- We can summarize a firm's production process using a *production function.*

Definition (Production Function)

The production function summarizes the maximum quantity of output that can be produced with different combinations of inputs, *given current knowledge about technology and organization*.

- Note: because it displays the maximum level of output that can be produced, the production function only reports efficient production processes.
    - What are we assuming about how firms use inputs?

The Production Function

- As an example, suppose a firm only uses capital and labor to produce output. Then its production function is given by:

y

$$q = f(L, K)$$

where q units of output are produced by combining L units of labor services and K units of capital via production function $f(-)$.

- What other inputs might we consider?

↳ environment /pollution /energy

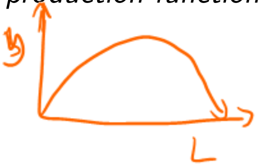
The Production Function

- Typically, we determine $f(-)$ by looking at data.

$y = f(K, L)$

$u(\text{skiing}, \text{other goods})$

- Empirical evidence suggests that the *Cobb-Douglas production function* fits the data well for many industries.



- The Cobb-Douglas production function is given by:

.7 .3

$$q = AL^{\alpha}K^{\beta}$$

where A, α and β are all positive constants.

The Production Function

- The production function contains useful information: it tells us how the output of a firm changes is it changes its inputs (K, L, and M).
- Often, what inputs a firm can change depends on the time horizon:
  - In the *short run*, at least one factor of production cannot be changed. As such, inputs can be *fixed* or *variable*.
  - In the *long run*, all factors of production can be changed, meaning all inputs are variable.

Production in the Short Run

- In the short run, at least one input is fixed.
  - Typically, fixed input is capital.
- Consider a production process with capital and labor as inputs. In the short run, total product (total output) is given by:

$$q = f(L, \bar{K})$$

- In this case, output can only change if the firm changes L.

Production in the Short Run

- Two key questions for management in the short run:
    1. How much does output change if we hire an additional worker?
    2. What happens to the average productivity of our workforce if we hire an additional worker?
- Aside: Why focus on the effects of hiring an additional worker?

Production in the Short Run

- To answer question 1, we need to know the *marginal product of labor*.

Definition (Marginal Product of Labor)

The marginal product of labor is the change in total output that results from employing an extra unit of labor, holding other factors (capital, materials) constant. That is:

$$MP_L = \frac{\Delta q}{\Delta L}$$

$\frac{\partial q}{\partial L}$

Production in the Short Run

- To answer question 2, we need to know the *average product of labor*.

Definition (Average Product of Labor)

The average product of labor is the amount of output produced per worker on average. That is:

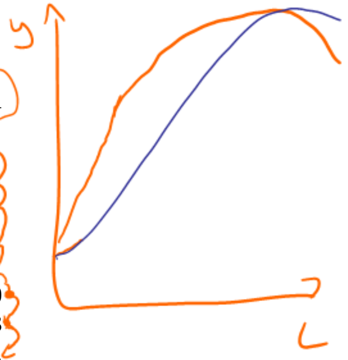
$$AP_L = \frac{q}{L}$$

Short Run Example: Selling Ice Cream

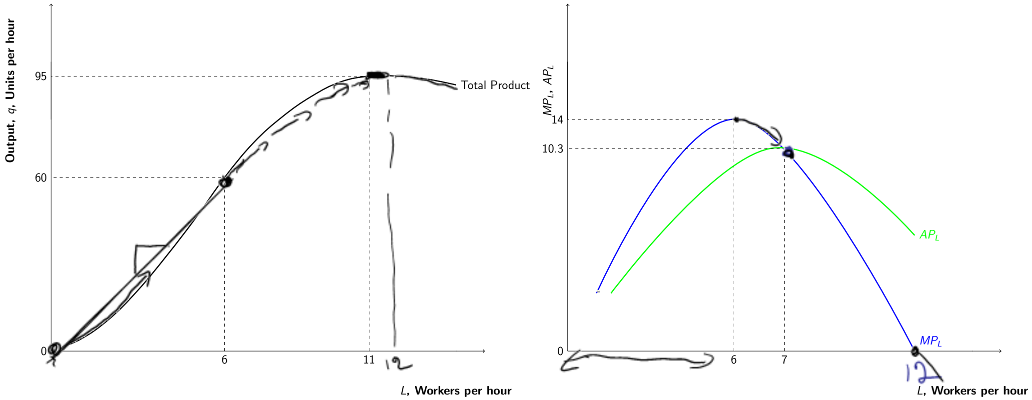
- As an example, consider Axel's ice cream shop.
- Axel's capital stock is fixed; he currently has 6 ice cream machines and is unable to add more due to space constraints.
- Axel is interested in understanding how changing his workforce affects output and productivity.

Short Run Example: Selling Ice Cream

Capital (K)	Labor (L)	Output (q)	MP_L	AP_L
6	0	0	-	-
6	1	4	4	4.0
6	2	11	7	5.5
6	3	21	10	7.0
6	4	33	12	8.3
6	5	46	13	9.2
6	6	60	14	10.0
6	7	72	12	10.3
6	8	81	9	10.1
6	9	88	7	9.8
6	10	93	5	9.3
6	11	95	2	8.6
6	12	95	0	7.9
6	13	94	-1	7.2
6	14	92	-2	6.6



Short Run Example: Selling Ice Cream



Production in the Short Run

- The total, average, and marginal product curves are all geometrically related.
- The AP_L and MP_L curves:
  - If the MP_L curve is above the AP_L curve, the AP_L curve must rise with extra workers.
  - If the MP_L curve is below the AP_L curve, the AP_L curve must fall with extra workers.
  - As a result, the AP_L curve peaks where $MP_L = AP_L$.
- The AP_L and MP_L curves can be derived from the total product curve. For L workers:
  - The AP_L is equal to the slope of a straight line from the origin to the total product curve with L workers.
  - The MP_L is the slope of the total product curve with L workers.

Production in the Short Run

- MP_L and AP_L are determined by the shape of the total product curve.
- Empirical evidence suggests the shape of the total product curve is determined by the *Law of Diminishing Marginal Returns*.

Definition (Law of Diminishing Marginal Returns)

If a firm keeps increasing the usage of an input, holding all other inputs and technology constant, then the corresponding increases in output will eventually become smaller (diminish).

- How fast diminishing returns kick in depends on the firms technology.

Production in the Long Run

- In the long run, all inputs are variable.
- This means that firms can choose to produce a single level of output in a variety of ways.
  - Firms can substitute inputs for each other while continuing to produce a given level of output $\bar{q}$.
- We can depict all of the efficient input combinations that produce a given level of output $\bar{q}$ using an *isoquant*.
- For a firm that uses capital and labor to produce, an isoquant is given by:

$\bar{q} = 20$

$$\bar{q} = f(L, K)$$

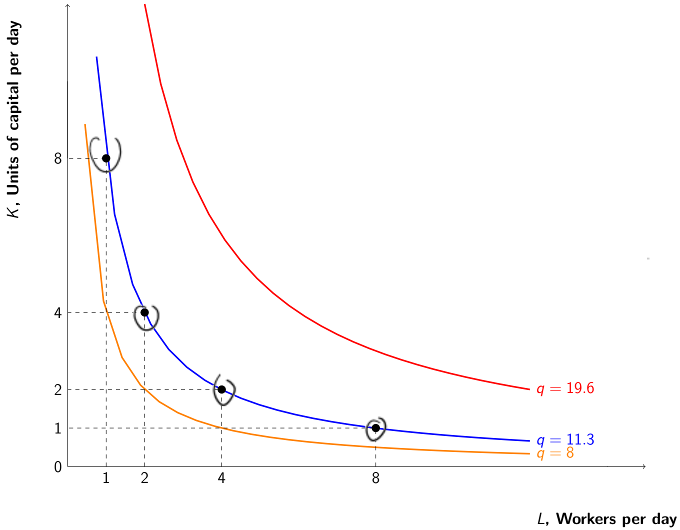
Production in the Long Run

- As an example, let's again consider the case of Axel who is planning on opening another ice cream shop.
- Now both capital and labor are variable; Axel can choose from many different possible combinations of inputs to produce the same level of output.

Production in the Long Run

				Labor (L)				
Capital (K)	1	2	3	4	5	6	7	8
1	4.0	5.7	6.9	8.0	8.9	9.8	10.6	11.3
2	5.7	8.0	9.8	11.3	12.6	13.9	15.0	16.0
3	6.9	9.8	12.0	13.9	15.5	17.0	18.3	19.6
4	8.0	11.3	13.9	16.0	17.9	19.6	21.2	22.6
5	8.9	12.6	15.5	17.9	20.0	21.9	23.7	25.3
6	9.8	13.9	17.0	19.6	21.9	24.0	25.9	27.7
7	10.6	15.0	18.3	21.2	23.7	25.9	28.0	29.9
8	11.3	16.0	19.6	22.6	25.3	27.7	29.9	32.0

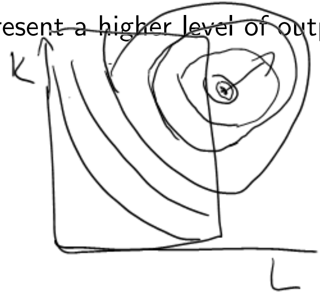
Production in the Long Run



Production in the Long Run

- Isoquants have three key properties:
  1. Isoquants farther from the origin represent a higher level of output.
  2. Isoquants do not cross.
  3. Isoquants slope downward.

positive marginal products.



Production in the Long Run

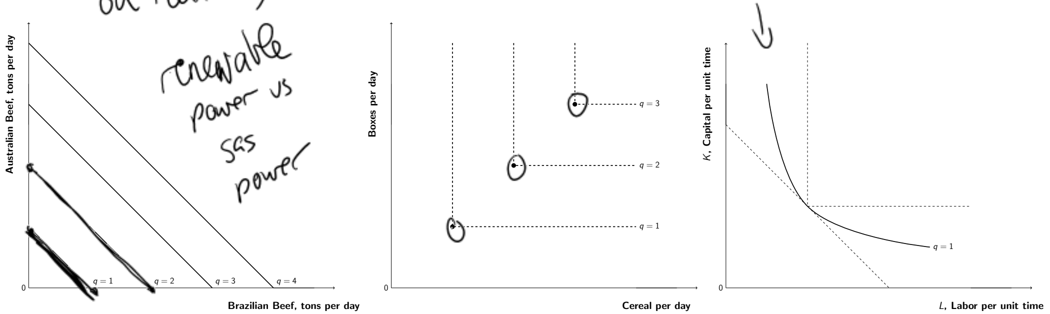
- The shape of an isoquant is informative for understanding how easily a firm can substitute between inputs to produce a given level of output.
    - If inputs are perfect substitutes, isoquants are straight lines.
    - If inputs can not be substituted at all and must be used in fixed proportions, isoquants are right angles.
    - If inputs can be substituted imperfectly for each other, isoquants are convex to the origin.

Production in the Long Run

crude oil
oil refinery

chemical processes

renewable
power vs
gas
power



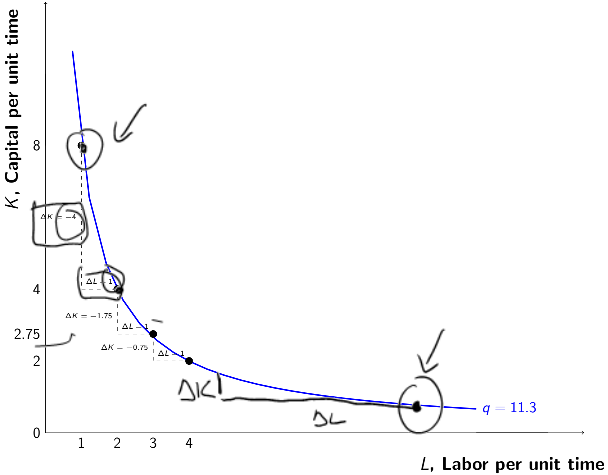
The Marginal Rate of Technical Substitution

- The slope of an isoquant is also informative.
  - The slope tells us how easy it is for a firm to replace one input with another, holding output constant.
  - The slope of the isoquant reflects the *marginal rate of technical substitution* (MRTS):

$$MRTS = \frac{\text{change in capital}}{\text{change in labor}} = \frac{\Delta K}{\Delta L}$$

  - The MRTS tells us how many units of one input can be replaced by one unit of another input, holding output levels constant.
  - This rate typically varies along an isoquant.

The Marginal Rate of Technical Substitution



The Marginal Rate of Technical Substitution

- We can also express the MRTS in terms of the marginal products of labor ($MP_L = \Delta q / \Delta L$) and capital ($MP_K = \Delta q / \Delta K$):

$$MRTS = -\frac{MP_L}{MP_K}$$

Marginal and Average Products and the MRTS

- Recall that the Cobb-Douglas production function is given by:

$$\boxed{q = AL^{\alpha}K^{\beta}}$$

$$\frac{\partial q}{\partial L} = \alpha A L^{(\alpha-1)} K^{\beta} = \alpha \frac{q}{L}$$

$$\frac{\partial q}{\partial K} = \beta A L^{\alpha} K^{\beta-1} = \frac{\beta q}{K}$$

- For this production function, the constants α and β determine the relationships between the average and marginal products of labor and capital.
  - $MP_L = \alpha q/L = \alpha AP_L$, so $\alpha = MP_L/AP_L$.
  - $MP_K = \beta q/K = \beta AP_K$, so $\beta = MP_K/AP_K$.

- Hence, for a Cobb-Douglas production function, the MRTS is given by:

$$\boxed{MRTS = -\frac{\alpha}{\beta}\frac{K}{L}}$$

$$y = 10L^{0.7}K^{0.3}$$

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Returns to Scale

- So far, we have thought about the effects of changing one input holding the other constant, and how we can change inputs holding output constant.
- An important related question: How does output change if a firm increases all of its inputs proportionately?
- The answer is important because it helps a firm determine its *scale* (size) in the long run.

Returns to Scale

- There are three possible outcomes if a firm increases all of its inputs proportionally.
  1. Constant Returns to Scale (CRS)
  2. Increasing Returns to Scale (IRS)
  3. Decreasing Returns to Scale (DRS)

Constant Returns to Scale: CRS

Definition (Constant Returns to Scale)

A production technology exhibits constant returns to scale if doubling inputs exactly doubles output. That is, if:

$$f(2L, 2K) = 2f(L, K) = 2q$$

- Implication: If a firm wants to double its output, it can build a second plant that uses the same amount of labor and equipment as the first plant.

Increasing Returns to Scale: IRS

Definition (Increasing Returns to Scale)

A production technology exhibits increasing returns to scale if doubling inputs more than doubles output. That is, if:

$$f(2L, 2K) > 2f(L, K) = 2q$$

- Implication: If a firm wants to increase output, it may better off building a single larger plant or expanding an existing facility instead of building two small plants.

$$f(2L, 2K) = 3f(L, K)$$

Decreasing Returns to Scale: DRS

Definition (Decreasing Returns to Scale)

A production technology exhibits decreasing returns to scale if doubling inputs less than doubles output. That is, if:

$$f(2L, 2K) < 2f(L, K) = 2q$$

- Implication: If a firm wants to increase output, it may be better off building two small plants rather than expanding an existing facility.

4. Returns to Scale: A Cobb-Douglas Example

- Which of the following Cobb-Douglas production functions exhibit CRS? DRS? IRS?
  1. $q = 5L^{0.25}K^{0.25}$
  2. $q = 5L^{1.25}K^{0.25}$
  3. $q = 5L^{0.25}K^{0.75}$

K = 2 L = 2
K = 4 L = 4

K = 200 L = 200
K = 400 L = 400

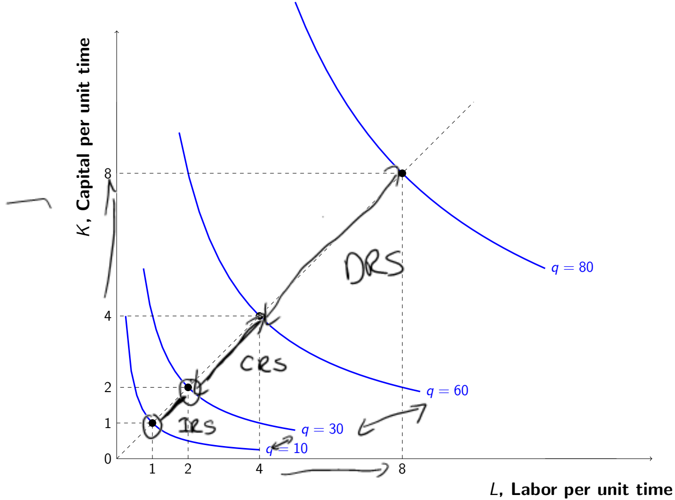
Varying Returns to Scale

- Many production functions exhibit *varying returns to scale*.

Definition (Varying Returns to Scale)

A production function exhibits varying returns to scale if it exhibits increasing returns to scale for low input levels, constant returns to scale for moderate input levels, and decreasing returns for large input levels.

Varying Returns to Scale



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Innovation

- Recall: A production function represents a method for turning inputs into outputs.

- The underlying production process that the production function captures depends on the available technologies and the manner in which production is organized.

- Changes in these technologies or in the organization of production will also affect output.

Technological Progress

- Changes in technology used in production are referred to as *process innovations* or *technological progress*.

Definition (Process Innovation)

A process innovation is a new idea, device or method that allows more output to be produced with the same level of inputs.

Technological Progress

- Technological progress can be *neutral*, or *non-neutral*.
  - It is neutral if more output is produced using the same ratio of inputs, i.e an increase in total factor productivity $y = \underline{A} L^{\alpha} K^{\beta}$
  - It is non-neutral if it is *capital saving* or *labor saving*.
    - Capital saving technological progress: produce same level of output using less capital.
    - Labor saving technological progress: produce same level of output using less labor.

Organizational Innovation

- Changes in the organization of production are referred to as *organizational innovation*.

Definition (Organizational Innovation)

An organizational innovation is a new way of organizing a firm that allows more output to be produced with a given level of inputs.

- Example: Henry Ford
    - Introduced interchangeable parts.
    - Introduced a conveyor belt and assembly line into his operations.

Firm Theory Part 1: Key Takeaways

1. The production function tells us how firms are able to turn inputs into output.
2. In the short run, firms alter production decisions by changing the use of variable inputs.
3. In the long run, firms alter production decisions by changing the use of all inputs.
4. In the long run, the effects of changing input levels depends on returns to scale.
5. The quantity of output obtained from a level of inputs can be increased by technological progress and through organizational innovation.