BUEC 311: Business Economics, Organization and Management
Strategic Behaviour Part 2
Strategies Over Time and Dynamic Games

Fall 2020

Strategic Interaction Over Time

- Last topic: Strategic interaction in static setting.
    - But in practice, many interactions occur dynamically over time.

- Dynamic games: Games where players play the game over and over, and move either repeatedly or sequentially.

Outline

1. Repeated Games
2. Sequential Games
3. ~~Deterring Entry~~
4. ~~Cost and Innovation Strategies~~
5. ~~Disadvantages of Moving First~~
6. ~~Behavioural Game Theory~~

Outline

Repeated Games

- A repeated game is a game in which a static *constituent* game is repeated a finite and pre-specified number of times, or is repeated indefinitely.
- We still need to know:
  - Players
  - Rules
  - Information
  - Payoffs
- Key difference from a static game: How we think about actions and strategies.

Repeated Games

- In a repeated game:
  - An action is a single move that a player makes at a specified time, such as choosing an output level or a price.
  - A strategy is a battle plan that specifies the *full set* of actions that a player will make throughout the game.
    - It may involve actions that are conditional on prior actions of other players, or on new information available at a given time.

Repeated Games

- As an example, we will revisit game between American and United.
- Recall: The Nash equilibrium in the static game is both firms producing high (64k passengers) and making $4.1 million.

Repeated Games

		American Airlines	
		$q_A = 64$	$q_A = 48$
United Airlines	$q_U = 64$	4.1 / 4.1	3.8 / 5.1
	$q_U = 48$	5.1 / 3.8	4.6 / 4.6

* Nash equil.

? No

? No

?

not a static Nash equilibrium

Note: Quantities are in thousands of passengers per quarter; (rounded) profits are in millions of dollars per quarter.

Repeated Games

- Now assume that the same game gets repeated indefinitely.
  - Now firms must consider both current and future profits.
- With repetition, the outcome may be different than in the static game.
  - Depends on the strategies used by the firms.

Repeated Games

- Suppose, for example, that American adopts the following strategy:
  - It cheap-talks United that it will produce the collusive or cooperative quantity of 48k in the first period.
  - But its subsequent decisions depend on United:
    - If United produces 48k in period t, American will produce 48k in period $t+1$.
    - If United produces 64k in period t, American will produce 64k in period $t+1$.
- What is United's best response to this strategy?

Repeated Games

	United profits	
1	4.6	5.1
2	4.6	4.1
3	4.6	4.1
4	4.6	4.1
5	4.6	⋮
⋮	⋮	⋮
∞	4.6	4.1

← 0.5m in period 1

← -0.5m forever from period 2 onward

		American Airlines: $q_A = 64$	American Airlines: $\boxed{q_A = 48}$
United Airlines	$\boxed{q_U = 64}$	United 4.1, American 4.1	United 5.1, American 3.8
United Airlines	$q_U = 48$	United 3.8, American 5.1	United 4.6, American 4.6

Note: Quantities are in thousands of passengers per quarter; (rounded) profits are in millions of dollars per quarter.

remember: gains to deviation for United or for American in the static game

A	B	
4.6	5.1	4.1 + 1
* 4.6	4.1	4.1
4.6	4.1	⋮
4.6	4.1	⋮
4.6	⋮	⋮
⋮	⋮	4.1

$$NPV_{Perpetuity} \quad \frac{V}{r} \boxed{(1+r)}$$

$$NPV_A = \frac{4.6}{r}(1+r) = \boxed{96.6} @ 5\%$$

$$NPV_B = \frac{4.1}{r}(1+r) + 1 = 87.1 @ 5\%$$

Repeated Games

Choice is now 4.6m forever vs 5.1 once and then 4.1 forever

		American Airlines $q_A = 64$	**American Airlines** $q_A = 48$
United Airlines	$q_U = 64$	United: 4.1; American: 4.1	United: 5.1; American: 3.8
United Airlines	$q_U = 48$	United: 3.8; American: 5.1	United: 4.6; American: 4.6

Nash equil in repeated game

Note: Quantities are in thousands of passengers per quarter; (rounded) profits are in millions of dollars per quarter.

remember: gains to deviation for United or for American in the static game

Repeated Games

- American's strategy is an example of a trigger strategy.
    - Trigger strategy: Rival's defection from a collusive outcome *triggers* punishment.
- If United adopts the same trigger strategy, the Nash-equilibrium is the collusive outcome.
    - Neither firm has an incentive to deviate.
    - One period gains from doing so are not sufficient to offset all future losses.
- In reality, cooperation may not be sustainable because of regulation, bounded rationality, or if the firm cares little about future profits.

Repeated Games

- Trigger strategy is just one possible option for American.
- They could instead adopt a tit-for-tat strategy.
  - Tit-for-tat: Cooperate in first round, then copy rival's action in each subsequent round.
- Tit-for-tat may induce cooperation if the payoff from deviating in any period is less than the loss from punishment in the subsequent period.
  - It depends on how firms discount the future.
- Cooperation is also more likely if the tit-for-tat strategy is modified to extend punishment for more than one period.
  - Extension of punishment needs to offset the one-time gains from not cooperating.

Repeated Games

- The equilibrium of the repeated game between American and United is an example of a collusive outcome.
- In most modern economies, explicit collusion is illegal.
    - However, antitrust and competition laws typically do not strictly prohibit choosing the cooperative (or cartel) quantity or price as long as no explicit agreement is reached.
    - Firms may be able to engage in implicit collusion or tacit collusion using trigger, tit-for-tat, or other similar strategies, as long as firms do not explicitly communicate with each other.
        - Tacit collusion lowers society's total surplus just as explicit collusion does.

Repeated Games

- Sustaining the cooperative outcome requires that players believe the game will repeat for ever.

- if there is a known end to the game, and players have complete foresight, the cooperation can be impossible to maintain.

Repeated Games

- To see this, suppose that American and United know that they will play the game a finite number of times (T).
- Suppose both firms use the trigger strategy that sustained collusion when the game was infinitely repeated.
- Now, the trigger strategy does not lead to a Nash Equilibrium.
- Why not?

Repeated Games

		American Airlines	
		$q_A = 64$	$q_A = 48$
United Airlines	$q_U = 64$	4.1, 4.1	5.1, 3.8
	$q_U = 48$	3.8, 5.1	4.6, 4.6

Note: Quantities are in thousands of passengers per quarter; (rounded) profits are in millions of dollars per quarter.

Q
1 64,64
2 64,64
3 64,64
4 64,64

5 No QS, no commitment

Repeated Games

- When the game is repeated a finite number of times, the only Nash Equilibrium is for both firms to produce a high level of output in all periods.
  - There is no cooperation again.

Outline

1. Repeated Games
2. Sequential Games timing matters.
3. Deterring Entry
4. Cost and Innovation Strategies
5. Disadvantages of Moving First
6. Behavioural Game Theory

Sequential Games

- So far, we've maximized strategic interactions where players make simultaneous decisions.

- But in many interactions, players alternate moves.

- We can model this type of strategic interaction as a sequential game.

Stackelberg Oligopoly

- As an example, we will again revisit the interaction between American and United, but we will now assume that the firms move sequentially in two stages:
  - First, American (the <u>leader</u>) chooses its output level.
  - Second, United (the <u>follower</u>) chooses its output level.

- This is an example of a <u>Stackelberg</u> oligopoly.
  - Stackelberg oligopoly involves one leader and one or more followers.

Stackelberg Oligopoly

		American Airlines		
		$q_A = 96$	$q_A = 64$	$q_A = 48$
United Airlines	$q_U = 96$	0, 0	3.1, 2.0	4.6, 2.3
	$q_U = 64$	2.0, 3.1	4.1, 4.1	5.1, 3.8
	$q_U = 48$	2.3, 4.6	3.8, 5.1	4.6, 4.6

Note: Quantities are in thousands of passengers per quarter; (rounded) profits are in millions of dollars per quarter.

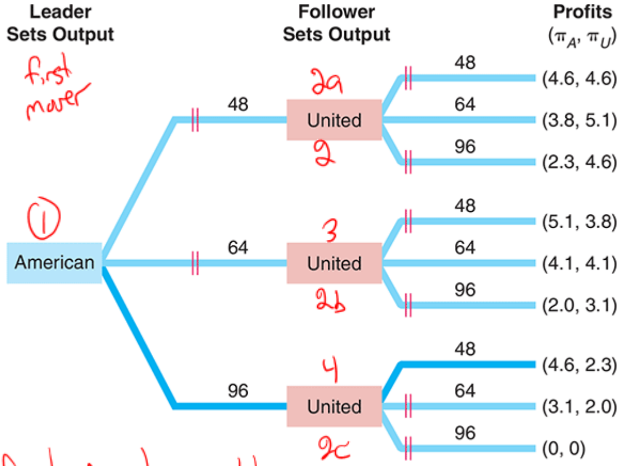
Figure: Payoffs in the Stackelberg game

Decision trees

- Key issue with the payoff matrix:
  - It does not show the sequential nature of the game.
- We can better illustrate the game using an extensive form diagram.
  - Also known as a game tree, or a decision tree.
  - The extensive form is a branched diagram that shows the players, the sequence of moves, the actions players can take at each move, the information that each player has about previous moves, and the payoff function over all possible strategy combinations.

Stackelberg Game Tree

Second mover

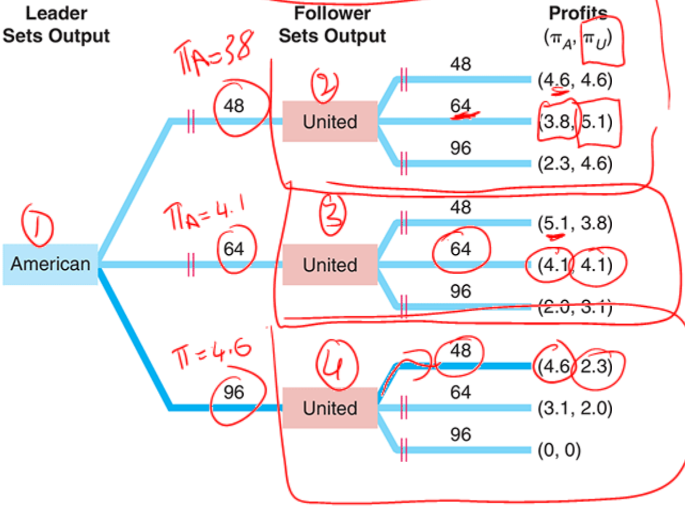


stage games

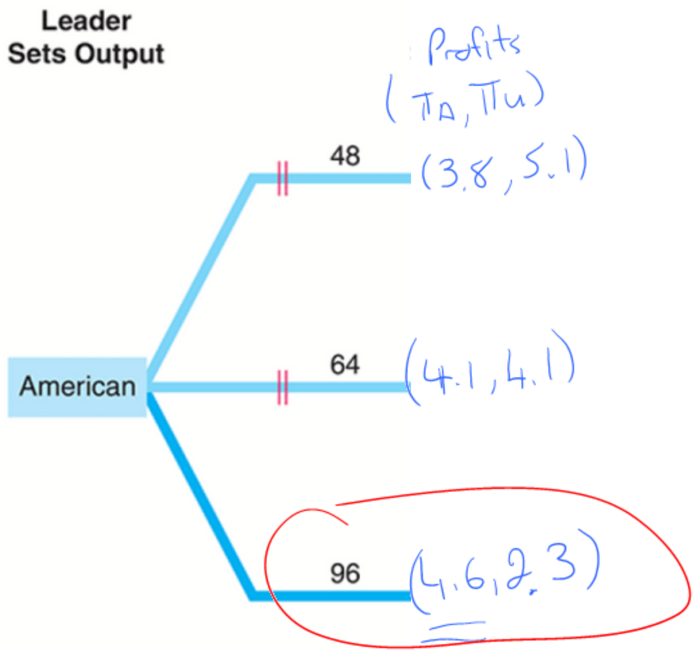
subgame perfect Nash equilibrium

Stackelberg Game Tree

backward induction
recursion

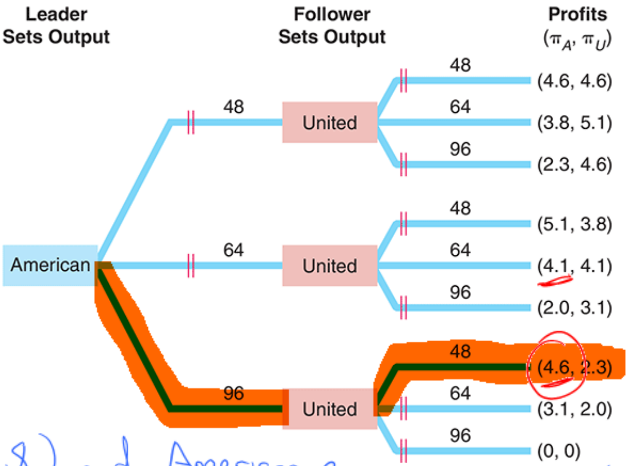


subgame



Leader
Sets Output
Profits
(π_A, π_U)
American
48
(3.8, 5.1)
64
(4.1, 4.1)
96
(4.6, 2.3)

Stackelberg Game Tree



SPNE is (96, 48) and American earns 4.6m to United's 2.3m

Subgames

- The sequential game depicted in the extensive form has four subgames.

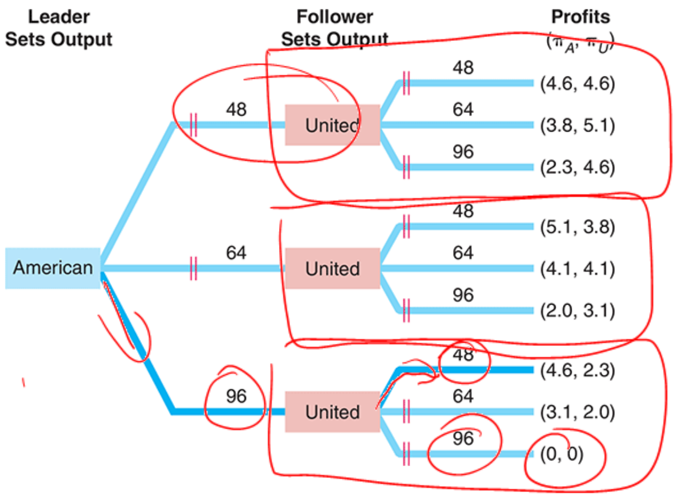
Definition (Subgame)

A subgame consists of all of the actions (and the corresponding payoffs) that a player can take at a given stage in the game, *given the actions that have already been taken*.

Subgame Perfection

- To predict the outcome of the sequential game, we need to know the set of strategies that form a Nash equilibrium in each subgame.
    - These strategies yield the <u>subgame-perfect Nash Equilibrium</u>.
- We can solve for the subgame-perfect Nash Equilibrium through backward induction.
    - First, we determine the best response by the last player to move, then we determine the best response for the player who makes the next-to-last move, and so on, until we reach the first move of the game.

Subgame Perfection



Subgame Perfection

- In the game between American and United, American first determines what United (the follower), will do in the second stage of the game in each of the three subgames.
    - This is the q_U with the highest profit at each node.
- American then determines the best action in the first stage given the choices that United will make *conditional on its actions* in the second stage.
    - This amounts to choosing the q_A with the highest profit.
- Thus, American chooses $q_A = 96$ in the first stage, and United chooses $q_U = 48$ in the second stage.
    - This is a subgame perfect Nash equilibrium; neither firm wants to change its strategy given what the other player is doing.

Simultaneous vs. Sequential Games

- It is worth noting that if this game was played simultaneously, the Nash equilibrium would be $q_A = q_U = 64$.

- Simultaneous and sequential games have different solutions because of credible threats and first mover advantage.
  - For a firm's strategy to be a credible threat, rivals must believe that the firm's strategy is rational (that is, it works in the firm's best interest).
  - In the simultaneous move game between United and American, United will not believe a threat by American to produce 96. However, in the sequential game, commitment to produce 96 is credible because American makes the first move.

Other Examples

- Entry Deterrence
- Sports - a golf or tennis match
- Limit pricing
- Innovation and R&D
- Bargaining