

Fall 2020

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- Last topic: Strategic interaction in static setting.
 - But in practice, many interactions occur dynamically over time.
- Dynamic games: Games where players play the game over and over, and move either repeatedly or sequentially.

Outline

- 1 Repeated Games
- 2 Sequential Games
- 3 ~~Detering~~ Entry
- 4 ~~Cost and Innovation~~ Strategies
- 5 ~~Disadvantages~~ of Moving First
- 6 ~~Behavioural~~ Game Theory

1 Repeated Games

2 Sequential Games

3 Detering Entry

④ Cost and Innovation Strategies

5 Disadvantages of Moving First

6 Behavioural Game Theory

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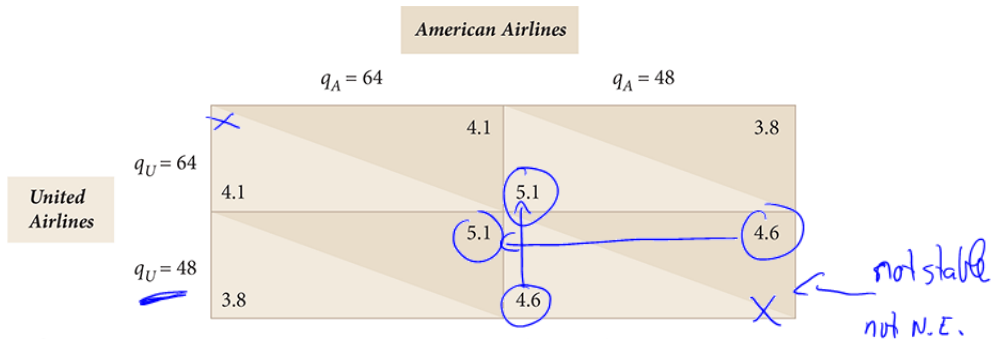
Repeated Games

- In a repeated game:
 - An action is a single move that a player makes at a specified time, such as choosing an output level or a price.
 - A strategy is a battle plan that specifies the *full set* of actions that a player will make throughout the game.
 - It may involve actions that are conditional on prior actions of other players, or on new information available at a given time.

Repeated Games

- As an example, we will revisit game between American and United.
- Recall: The Nash equilibrium in the static game is both firms producing high (64k passengers) and making \$4.1 million.

Repeated Games



Note: Quantities are in thousands of passengers per quarter; (rounded) profits are in millions of dollars per quarter.

Repeated Games

- Now assume that the same game gets repeated indefinitely.
 - Now firms must consider both current and future profits.
- With repetition, the outcome may be different than in the static game.
 - Depends on the strategies used by the firms.

Repeated Games

- Suppose, for example, that American adopts the following strategy:
 - It cheap-talks United that it will produce the collusive or cooperative quantity of 48k in the first period.
 - But its subsequent decisions depend on United:
 - ● If United produces 48k in period t , American will produce 48k in period $t + 1$.
 - If United produces 64k in period t , American will produce 64k in period $t + 1$.
- What is United's best response to this strategy?

Repeated Games

		American Airlines	
		$q_A = 64$	$q_A = 48$
United Airlines	$q_U = 64$	4.1	3.8
	$q_U = 48$	3.8	4.6

Handwritten annotations on the table:
 - Blue circles around the values 5.1 in the middle column (corresponding to $q_A = 48$).
 - Red circles around the values 5.1 and 4.6 in the middle row (corresponding to $q_U = 48$).
 - A red arrow points from the circled 4.6 to the circled 5.1.
 - A blue 'X' is drawn in the bottom-right cell (3.8, 4.6).
 - A blue arrow points from the circled 5.1 to the circled 4.6.

Note: Quantities are in thousands of passengers per quarter; (rounded) profits are in millions of dollars per quarter.

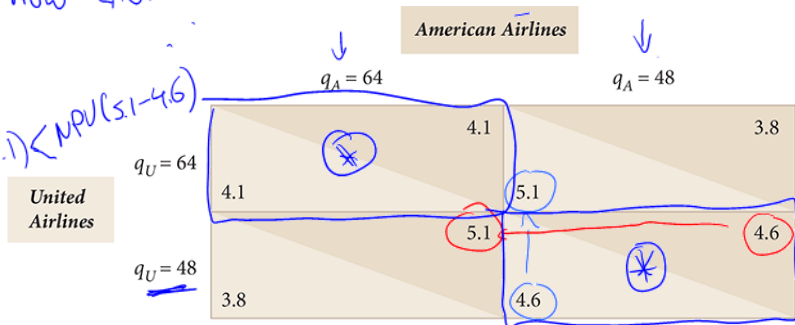
remember: gains to deviation for
 United or for American in the static
 game

Repeated Games

Choice is now 4.6 forever vs 5.1 once and then 4.1 forever

$$5.1 > 4.6$$

$$\text{If } NPV(4.6 - 4.1) < NPV(5.1 - 4.6)$$



American Payoffs

4.6	5.1
4.6	4.1
4.6	4.1
...	...
4.6	4.1

Note: Quantities are in thousands of passengers per quarter; (rounded) profits are in millions of dollars per quarter.

remember: gains to deviation for
 United or for American in the static
 game

$$NPV_{NEC} = \frac{4.6}{r} \times (1+r) - 92m \times (1+r) = 96.6$$

$$\frac{4.1}{r} \times (1+r) - 4.1 + 5.1 = 87.1m$$

4.1 $NPV_{4.1 \text{ forever}}$



4.1

$$NPV_P = \frac{P}{r} \quad \text{end of period payment}$$

$$\frac{P}{r} (1+r) \quad \text{beginning of period}$$

Repeated Games

- American's strategy is an example of a trigger strategy.
 - Trigger strategy: Rival's defection from a collusive outcome *triggers* punishment.
- If United adopts the same trigger strategy, the Nash-equilibrium is the collusive outcome.
 - Neither firm has an incentive to deviate.
 - One period gains from doing so are not sufficient to offset all future losses.
- In reality, cooperation may not be sustainable because of regulation, bounded rationality, or if the firm cares little about future profits.

Repeated Games

- Trigger strategy is just one possible option for American.
- They could instead adopt a tit-for-tat strategy.
 - Tit-for-tat: Cooperate in first round, then copy rival's action in each subsequent round.
- Tit-for-tat may induce cooperation if the payoff from deviating in any period is less than the loss from punishment in the subsequent period.
 - It depends on how firms discount the future.
- Cooperation is also more likely if the tit-for-tat strategy is modified to extend punishment for more than one period.
 - Extension of punishment needs to offset the one-time gains from not cooperating.

Repeated Games

- The equilibrium of the repeated game between American and United is an example of a collusive outcome.
- In most modern economies, explicit collusion is illegal.
 - However, antitrust and competition laws typically do not strictly prohibit choosing the cooperative (or cartel) quantity or price as long as no explicit agreement is reached.
 - Firms may be able to engage in implicit collusion or tacit collusion using trigger, tit-for-tat, or other similar strategies, as long as firms do not explicitly communicate with each other.
 - Tacit collusion lowers society's total surplus just as explicit collusion does.

Repeated Games

- Sustaining the cooperative outcome requires that players believe the game will repeat for ever.
- if there is a known end to the game, and players have complete foresight, the cooperation can be impossible to maintain.

Repeated Games

- To see this, suppose that American and United know that they will play the game a finite number of times (T).
- Suppose both firms use the trigger strategy that sustained collusion when the game was infinitely repeated.
- Now, the trigger strategy does not lead to a Nash Equilibrium.
- Why not?

Repeated Games

		American Airlines	
		$q_A = 64$	$q_A = 48$
United Airlines	$q_U = 64$	4.1	3.8
	$q_U = 48$	3.8	4.6

Handwritten notes in the top-left cell (4.1, 4.1):

- Q_4 (top left), Q_2 (top right)
- Q_3 (bottom left), Q_1 (bottom right)
- A blue arrow points from the bottom-right cell (4.6, 4.6) to the top-right cell (4.1, 3.8).

Note: Quantities are in thousands of passengers per quarter; (rounded) profits are in millions of dollars per quarter.

Repeated Games

- When the game is repeated a finite number of times, the only Nash Equilibrium is for both firms to produce a high level of output in all periods.
 - There is no cooperation again.

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- So far, we've maximized strategic interactions where players make simultaneous decisions.
- But in many interactions, players alternate moves.
- We can model this type of strategic interaction as a sequential game.

Stackelberg Oligopoly

Oligopoly.

- As an example, we will again revisit the interaction between American and United, but we will now assume that the firms move sequentially in two stages:
 - First, American (the leader) chooses its output level.
 - Second, United (the follower) chooses its output level.
- This is an example of a Stackelberg oligopoly.
 - Stackelberg oligopoly involves one leader and one or more followers.

Stackelberg Oligopoly

		American Airlines		
		$q_A = 96$	$q_A = 64$	$q_A = 48$
United Airlines	$q_U = 96$	0 0	2.0 3.1	2.3 4.6
	$q_U = 64$	3.1 2.0	4.1 4.1	3.8 5.1
	$q_U = 48$	4.6 2.3	5.1 3.8	4.6 4.6

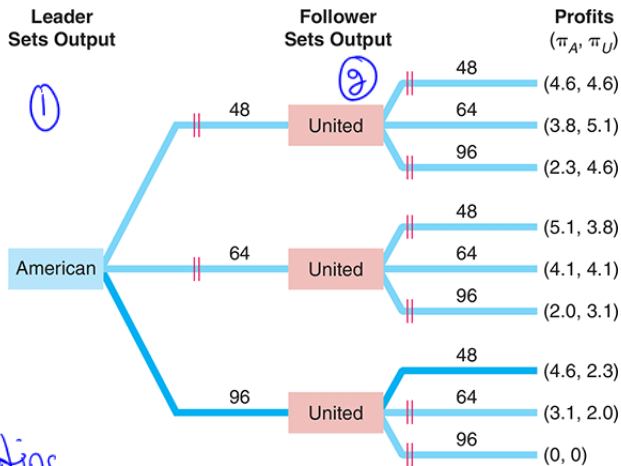
Note: Quantities are in thousands of passengers per quarter; (rounded) profits are in millions of dollars per quarter.

Figure: Payoffs in the Stackelberg game

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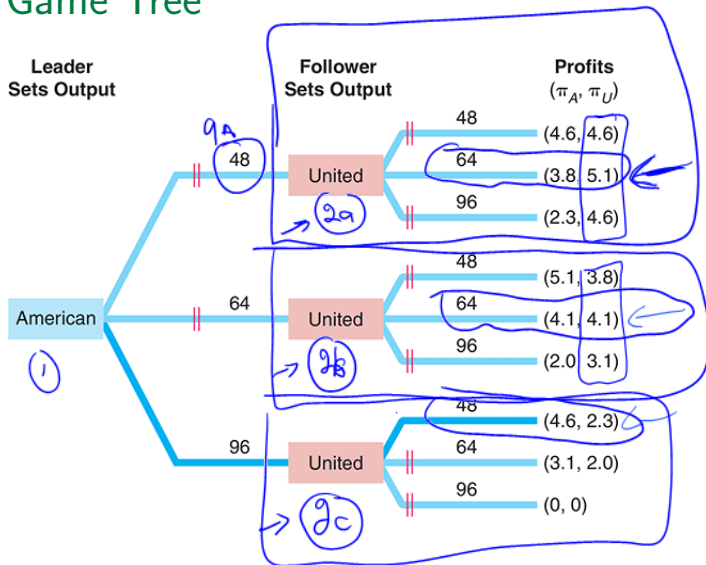
Stackelberg Game Tree



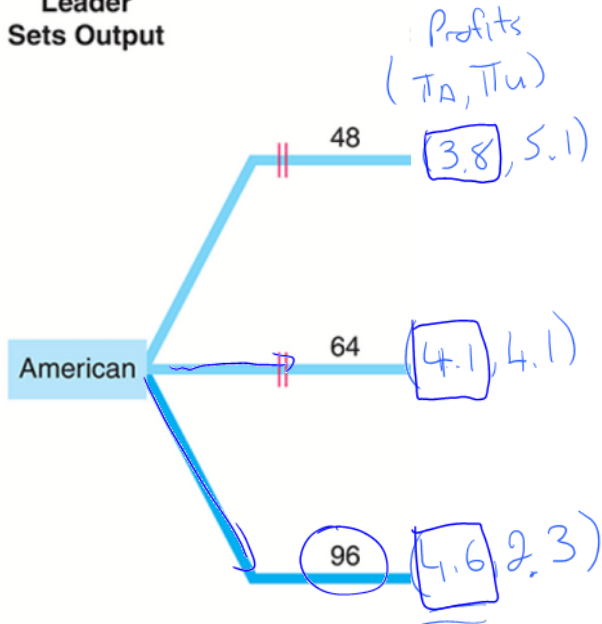
stage game

Recursion

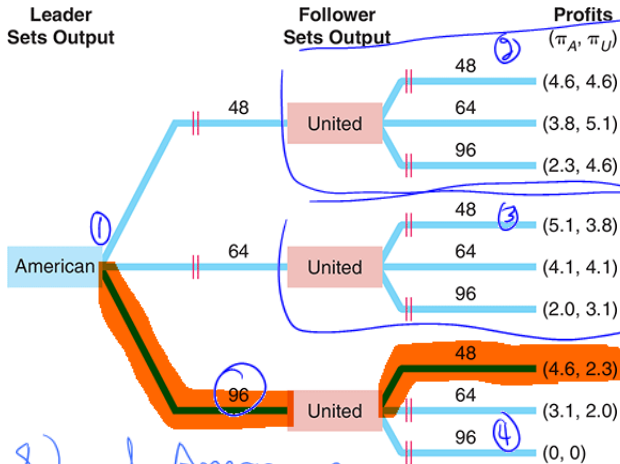
Stackelberg Game Tree



Leader
Sets Output



Stackelberg Game Tree



static
Nash
equilibrium

Subgame
perfect
Nash equilibrium

SPNE is (96, 48) and American earns 4.6m to United's 2.3m

Subgames

- The sequential game depicted in the extensive form has four subgames.

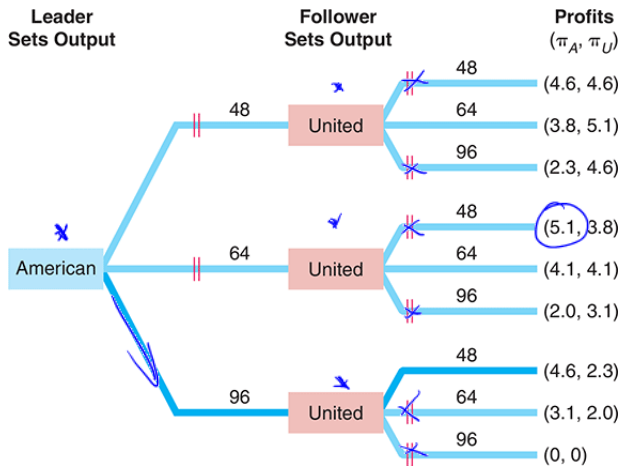
Definition (Subgame)

A subgame consists of all of the actions (and the corresponding payoffs) that a player can take at a given stage in the game, *given the actions that have already been taken*.

Subgame Perfection

- To predict the outcome of the sequential game, we need to know the set of strategies that form a Nash equilibrium in each subgame.
 - These strategies yield the subgame-perfect Nash Equilibrium.
- We can solve for the subgame-perfect Nash Equilibrium through backward induction. (recursion)
 - First, we determine the best response by the last player to move, then we determine the best response for the player who makes the next-to-last move, and so on, until we reach the first move of the game.

Subgame Perfection



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- Entry Deterrence
- Sports - a golf or tennis match
- Limit pricing
- Innovation and R&D
- Bargaining