BUEC 311: Business Economics, Organization and Management
Strategic Behaviour
Game Theory and Business Strategy

Fall 2021

Outline

1. Oligopoly Games: static, repeated, dynamic
2. Auctions

Strategic Interaction

- Thus far: We have primarily focused on cases where firms are not interacting strategically.
    - But we often need to understand the potential decisions of rivals.
    - We need a toolbox for understanding strategic decision making.

- Game Theory: A set of tools used to analyze strategic decision making.
    - Idea: Model strategic interactions as a game in which players interact according to a set of rules.
        - Players decide strategies based on payoffs, the level of information, and their rationality.
        - Outcome of a game is a Nash Equilibrium; depends on information and rationality.
    - Game theory can be used to understand strategic behaviour by firms, outcomes in bargaining, and auctions.

Outline

1. Oligopoly Games: static, repeated, dynamic
2. Auctions

Oligopoly Games

- Ex. A duopoly game between American Airlines and United Airlines
  - Players and rules:
    - Two players: American and United, play a static game to decide how many passengers to fly per quarter. Each airline's objective is to maximize profit.
    - Rules: Firms announce output levels simultaneously, but cannot communicate otherwise (no side deals or coordination is allowed).
    - Complete information: Firms know all strategies and payoffs.
  - Strategies:
    - Each firm's strategy is to take one of two available actions: ether choose low output (48k passengers per quarter) or high output (64k passengers per quarter).
    - Both firms know all strategies and the corresponding payoffs for each firm.
    - We can summarize these strategies in a payoff matrix (or profit matrix).

Oligopoly Games

		American Airlines	
		$q_A = 64$	$q_A = 48$
United Airlines	$q_U = 64$	4.1 4.1	3.8 5.1
	$q_U = 48$	5.1 3.8	4.6 4.6

Note: Quantities are in thousands of passengers per quarter; (rounded) profits are in millions of dollars per quarter. The payoff to American Airlines is in the upper-right corner of each cell and the payoff to United Airlines is in the lower left.

Figure: The Payoffs for American and United

Dominant Strategies

- If one is available, a rational player always uses a dominant strategy.

Definition (Dominant Strategy)

A dominant strategy is a strategy that produces a higher payoff (profit) than any other strategy the player can use, no matter what its rivals do.

- In our airline duopoly example, high-output (64k) is the dominant strategy for both firms.
  - High output yields the highest profit *regardless of what the other firm is doing*.
  - Hence, the dominant strategy solution is $q_U = q_A = 64$.

Payoffs

- A dominant strategy solution does not necessarily lead to the best outcome for firms.
  - In our example, United and American choose strategies that do not maximize their joint or combined profit.
    - Each firm could earn $4.6 million if they both chose to produce a low level of output (48k).
  - Game between United and American is an example of a Prisoner's Dilemma.
    - All players have dominant strategies that lead to a profit that is inferior to what they could have achieved if they cooperated.
    - Individual incentives cause players to choose strategies that do not maximize joint profits.

Prisoner's Dilemma Example

- Suppose that United and American are now choosing whether or not to invest in new planes. Currently, each airline earns a profit of $25 billion using their old fleet of planes. If American upgrades to new planes and United does not, then American steals some of United's customers and increases its profits to $35 billion, while United's profits fall to $10 billion. Similarly, if United upgrades and American does not, United's profits increase to $35 billion and American's profits fall to $10 billion. If both airlines upgrade to new planes, then they each will earn $20 billion. What will each firm do? What will they earn in equilibrium?

Best Responses

- Many games do not have a dominant strategy solution. In this case, we can use the approach of <u>best response</u> to determine the outcome of a game.

Definition (Best Response)

A best response is the strategy that maximizes a players payoff (profit) given its beliefs about the strategies of its rivals.

- A dominant strategy is a strategy that is a best response to all possible strategies a rival might use.

- In the absence of a dominant strategy, each firm can determine its best response to <u>any possible</u> strategy chosen by its rivals.

Nash Equilibrium

- Best responses are the basis of a Nash Equilibrium.

Definition (Nash Equilibrium)

A Nash equilibrium is a set of strategies such that if, when all other players use these strategies, no player can obtain a higher profit by choosing a different strategy.

- In a Nash equilibrium, players are "best-responding" to each other.
  - This means the Nash equilibrium is self enforcing.
- Two steps to find the Nash Equilibrium:
  1. Determine each player's best response to any given strategy of the other player.
  2. Check whether pairs of strategies are best responses for both firms; these pairs are Nash equilibria.

Oligopoly Games

- As an example, consider a more complicated game between American and United.
  - Now both firms have 3 possible strategies:
    1. High output (96k passengers/quarter).
    2. Medium output (64k passengers/quarter).
    3. Low output (8k passengers/quarter).
  - Otherwise, the rules are the same as before:
    - Static simultaneous move game.
    - Perfect information.

Oligopoly Games

		American Airlines		
		$q_A = 96$	$q_A = 64$	$q_A = 48$
United Airlines	$q_U = 96$	0, 0	3.1, 2.0	4.6, 2.3
	$q_U = 64$	2.0, 3.1	4.1, 4.1	5.1, 3.8
	$q_U = 48$	2.3, 4.6	3.8, 5.1	4.6, 4.6

Note: Quantities are in thousands of passengers per quarter; (rounded) profits are in millions of dollars per quarter.

Oligopoly Games

- Determine equilibrium via two step method:
  1. Determine best responses for United:
     - If United chooses $q_U = 96$, American's best response is $q_A = 48$.
     - If United chooses $q_U = 64$, American's best response is $q_A = 64$.
     - If United chooses $q_U = 48$, American's best response is $q_A = 64$.

     And for American:
     - If American chooses $q_A = 96$, United best response is $q_U = 48$.
     - If American chooses $q_A = 64$, United best response is $q_U = 64$.
     - If American chooses $q_A = 48$, United best response is $q_U = 64$.
  2. Determine the Nash Equilibrium
     - The Nash equilibrium is $q_A = q_U = 64$.
     - This outcome is a Nash equilibrium because neither firm wants to deviate from its strategy *given what the other firm is doing.*
     - Note: The Nash Equilibrium does not maximize joint profits.

Oligopoly Games

- In general, whether or not the Nash equilibrium maximizes the combined payoff to players (i.e. profits for firms) depends on the payoff matrix.
- As an example, consider a static game where firms decide to 'advertise' or 'not advertise'.
- The effects of advertising depend on whether advertising brings new customers into the market.

Oligopoly Games

(a) Advertising Only Takes Customers from Rivals

		Firm 1	
		Do Not Advertise	*Advertise*
Firm 2	*Do Not Advertise*	2 2	3 0
	Advertise	0 3	1 1

(b) Advertising Attracts New Customers to the Market

		Firm 1	
		Do Not Advertise	*Advertise*
Firm 2	*Do Not Advertise*	2 2	4 3
	Advertise	3 4	5 5

Oligopoly Games

- Example highlights a phenomenon often observed in practice:
  - In oligopolistic markets, the effect of firm advertising depends on whether it helps (increases the size of the overall market) or hurts (steals customers) rivals.
- In some industries, advertising primarily steals customers from rivals.
  - E.g. market for cola; market for erectile dysfunction drugs.
- In other industries, advertising by any firm increases the size of the market.
  - E.g. market for beer; market for cigarettes.
- It is possible to observe market size and business stealing effects simultaneously.
  - E.g. Fast food; CPUs.

Repeated Games

- A repeated game is a game in which a static *constituent* game is repeated a finite and pre-specified number of times, or is repeated indefinitely.

- We still need to know:
  - Players
  - Rules
  - Information
  - Payoffs

- Key difference from a static game: How we think about actions and strategies.

Repeated Games

- In a repeated game:
    - An action is a single move that a player makes at a specified time, such as choosing an output level or a price.
    - A strategy is a battle plan that specifies the *full set* of actions that a player will make throughout the game.
        - It may involve actions that are conditional on prior actions of other players, or on new information available at a given time.

Repeated Games

- As an example, we will revisit game between American and United.
- Recall: The Nash equilibrium in the static game is both firms producing high (64k passengers) and making $4.1 million.

Repeated Games

		American Airlines	
		$q_A = 64$	$q_A = 48$
United Airlines	$q_U = 64$	4.1, 4.1	5.1, 3.8
	$q_U = 48$	3.8, 5.1	4.6, 4.6

Note: Quantities are in thousands of passengers per quarter; (rounded) profits are in millions of dollars per quarter.

Repeated Games

- Now assume that the same game gets repeated indefinitely.
  - Now firms must consider both current and future profits.
- With repetition, the outcome may be different than in the static game.
  - Depends on the strategies used by the firms.

Repeated Games

- Suppose, for example, that American adopts the following strategy:
    - It cheap-talks United that it will produce the collusive or cooperative quantity of 48k in the first period.
    - But its subsequent decisions depend on United:
        - If United produces 48k in period t, American will produce 48k in period $t+1$.
        - If United produces 64k in period t, American will produce 64k in period $t+1$.

- What is United's best response to this strategy?

Repeated Games

		American Airlines	
		$q_A = 64$	$q_A = 48$
United Airlines	$q_U = 64$	4.1 4.1	3.8 5.1
	$q_U = 48$	5.1 3.8	4.6 4.6

Note: Quantities are in thousands of passengers per quarter; (rounded) profits are in millions of dollars per quarter.

Repeated Games

- American's strategy is an example of a trigger strategy.
  - Trigger strategy: Rival's defection from a collusive outcome *triggers* punishment.
- If United adopts the same trigger strategy, the Nash-equilibrium is the collusive outcome.
  - Neither firm has an incentive to deviate.
  - One period gains from doing so are not sufficient to offset all future losses.
- In reality, cooperation may not be sustainable because of regulation, bounded rationality, or if the firm cares little about future profits.

Repeated Games

- Trigger strategy is just one possible option for American.
- They could instead adopt a tit-for-tat strategy.
    - Tit-for-tat: Cooperate in first round, then copy rival's action in each subsequent round.
- Tit-for-tat may induce cooperation if the payoff from deviating in any period is less than the loss from punishment in the subsequent period.
    - It depends on how firms discount the future.
- Cooperation is also more likely if the tit-for-tat strategy is modified to extend punishment for more than one period.
    - Extension of punishment needs to offset the one-time gains from not cooperating.

Repeated Games

- The equilibrium of the repeated game between American and United is an example of a collusive outcome.
- In most modern economies, explicit collusion is illegal.
  - However, antitrust and competition laws typically do not strictly prohibit choosing the cooperative (or cartel) quantity or price as long as no explicit agreement is reached.
  - Firms may be able to engage in implicit collusion or tacit collusion using trigger, tit-for-tat, or other similar strategies, as long as firms do not explicitly communicate with each other.
    - Tacit collusion lowers society's total surplus just as explicit collusion does.

Repeated Games

- Sustaining the cooperative outcome requires that players believe the game will repeat for ever.
- if there is a known end to the game, and players have complete foresight, the cooperation can be impossible to maintain.

Repeated Games

- To see this, suppose that American and United know that they will play the game a finite number of times (T).
- Suppose both firms use the trigger strategy that sustained collusion when the game was infinitely repeated.
- Now, the trigger strategy does not lead to a Nash Equilibrium.
- Why not?

Repeated Games

		American Airlines	
		$q_A = 64$	$q_A = 48$
United Airlines	$q_U = 64$	4.1 4.1	3.8 5.1
	$q_U = 48$	5.1 3.8	4.6 4.6

Note: Quantities are in thousands of passengers per quarter; (rounded) profits are in millions of dollars per quarter.

Repeated Games

- When the game is repeated a finite number of times, the only Nash Equilibrium is for both firms to produce a high level of output in all periods.
  - There is no cooperation again.

Sequential Games

- So far, we've maximized strategic interactions where players make simultaneous decisions one or many times times.
- But in many interactions, players alternate moves.
- We can model this type of strategic interaction as a sequential game.

Stackelberg Oligopoly

- As an example, we will again revisit the interaction between American and United, but we will now assume that the firms move sequentially in two stages:
  - First, American (the leader) chooses its output level.
  - Second, United (the follower) chooses its output level.
- This is an example of a Stackelberg oligopoly.
  - Stackelberg oligopoly involves one leader and one or more followers.

Stackelberg Oligopoly

		American Airlines		
		$q_A = 96$	$q_A = 64$	$q_A = 48$
United Airlines	$q_U = 96$	0, 0	3.1, 2.0	4.6, 2.3
	$q_U = 64$	2.0, 3.1	4.1, 4.1	5.1, 3.8
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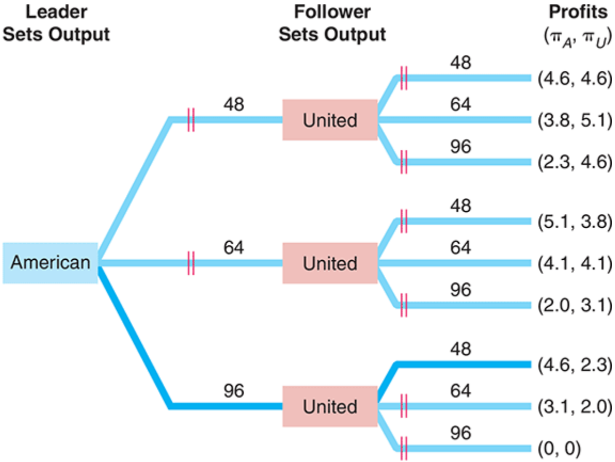
Note: Quantities are in thousands of passengers per quarter; (rounded) profits are in millions of dollars per quarter.

Figure: Payoffs in the Stackelberg game

Decision trees

- Key issue with the payoff matrix:
  - It does not show the sequential nature of the game.
- We can better illustrate the game using an extensive form diagram.
  - Also known as a game tree, or a decision tree.
  - The extensive form is a branched diagram that shows the players, the sequence of moves, the actions players can take at each move, the information that each player has about previous moves, and the payoff function over all possible strategy combinations.

Stackelberg Game Tree



Subgames

- The sequential game depicted in the extensive form has four subgames.

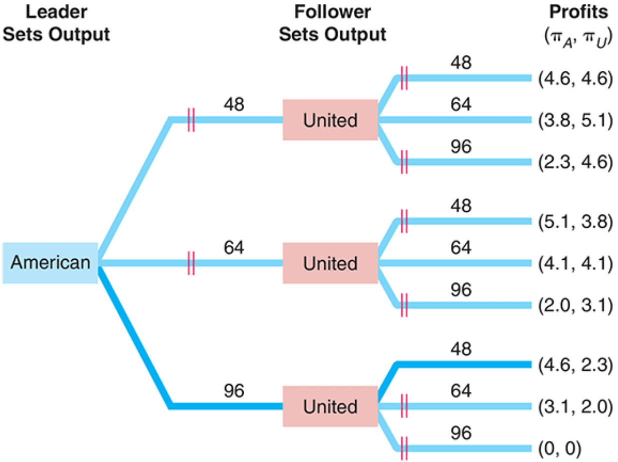
Definition (Subgame)

A subgame consists of all of the actions (and the corresponding payoffs) that a player can take at a given stage in the game, *given the actions that have already been taken*.

Subgame Perfection

- To predict the outcome of the sequential game, we need to know the set of strategies that form a Nash equilibrium in each subgame.
    - These strategies yield the subgame-perfect Nash Equilibrium.
- We can solve for the subgame-perfect Nash Equilibrium through backward induction.
    - First, we determine the best response by the last player to move, then we determine the best response for the player who makes the next-to-last move, and so on, until we reach the first move of the game.

Subgame Perfection



Subgame Perfection

- In the game between American and United, American first determines what United (the follower), will do in the second stage of the game in each of the three subgames.
  - This is the q_U with the highest profit at each node.
- American then determines the best action in the first stage given the choices that United will make *conditional on its actions* in the second stage.
  - This amounts to choosing the q_A with the highest profit.
- Thus, American chooses $q_A = 96$ in the first stage, and United chooses $q_U = 48$ in the second stage.
  - This is a subgame perfect Nash equilibrium; neither firm wants to change its strategy given what the other player is doing.

Simultaneous vs. Sequential Games

- It is worth noting that if this game was played simultaneously, the Nash equilibrium would be $q_A = q_U = 64$.

- Simultaneous and sequential games have different solutions because of credible threats and first mover advantage.
  - For a firm's strategy to be a <u>credible threat</u>, rivals must believe that the firm's strategy is rational (that is, it works in the firm's best interest).
  - In the simultaneous move game between United and American, United will not believe a threat by American to produce 96. However, in the sequential game, commitment to produce 96 is credible because American makes the first move.

Other Examples

- Entry Deterrence
- Sports - a golf or tennis match
- Limit pricing
- Innovation and R&D
- Bargaining

Outline

Auctions

Definition (Auction)

A sale in which a good or service is sold to the highest bidder.

- Game theory can be used to understand behaviour in auctions.
  - An auction is a game in which players (called <u>bidders</u>) devise bidding strategies without knowing the payoff functions of other players.
  - Bidders need to know the rules of the game:
    - The number of units being sold.
    - The format of bidding.
    - The value that potential bidders place on the good.

Auctions

- Auctions are frequently used in practice:
  - Government auctions:
    - Government procurement, auctions for electricity and transport markets, auctions to concede portions of the airwaves for radio stations, mobile phones and wireless internet access; auctions for oil and gas leases.
  - Market transactions:
    - Goods commonly sold at auction are natural resource such as timber and drilling rights for oil, as well as houses, cars, agricultural products, horses, antiques and art. And of course, goods online in sites like eBay.

Elements of Auctions

- Number of units: auctions can be used to sell one or many units of a good.
- Format of bidding:
  - English auction: Ascending-bid auction process where the good is sold to the last bidder for the highest bid. Commonly used to sell art/antiques.
  - Dutch auction: Descending-bid auction process where the seller reduces the price until someone accepts it and buys at that price. Often used in government procurement.
  - Sealed-bid auction: Bidders submit bids simultaneously without seeing anyone else's bid and highest bidder wins. In a 1st price sealed-bid auction, the winner pays its own, highest bid. In a 2nd price sealed-bid auction, the winner pays the amount bid by the 2nd highest bidder.
- Value:
  - Private value: Individual bidders know how much the good is worth to them, but not how much other bidders value it.
  - Common value: The good has the same value to everyone, but no bidder knows exactly what that value is.

Second Price Sealed Bid Auctions

- Rules:
  - Each bidder has a different private value for a single indivisible good.
  - Bidders simultaneously submit sealed bids without knowledge of other bids.
- Design of auction means that amount that you bid affects whether you win, but it does not affect how much you pay if you win (which is equal to the second-highest bid).
- Best strategy: Bid your highest value.
  - This strategy weakly dominates all others.
  - Ex: Suppose that you value a folk art carving at \$100. If you bid \$100 and win, your $CS = 100 -$ 2nd price. If you bid less than \$100, you risk not winning. If you bid more than \$100, you risk ending up with negative CS.
  - Thus, bidding \$100 leaves you *at least as well off* as bidding any other value.

English Auctions

- Rules:
  - Each bidder has a different private value for a single indivisible good.
  - Ascending-bid auction process where the good is sold to the last bidder for the highest bid.
- Design of auction means that amount you bid affects whether you win and how much you pay.
- Best strategy: Raise the current highest bid as long as that value is less than the value you place on the good.
  - Ex: Again suppose that you value a folk art carving at \$100. If you bid an amount b and win, $CS = 100 - b$. CS is positive or zero for $b \leq 100$, but negative if $b > 100$. So it is best to raise bids up to \$100 and stop there.
  - If all participants bid up to their value, the winner will pay slightly more than the value of the second-highest bidder. Thus, the outcome of an English auction is essentially the same is a in a sealed-bid, second-price auction.

Other Auctions

- Two other common private value auctions:
  - Dutch Auction: Descending-bid auction where the seller reduces the price until someone accepts the offered price and buys at that price.
  - First-Price Sealed-Bid Auction: Bidders submit bids simultaneously without seeing other bids. Highest bidder wins and pays amount of bid.
- In both cases, the amount that you bid affects whether you win and pay.
- The best strategy in both auctions is to bid an amount that is equal to, or slightly greater than what you expect will be the second-highest bid, given that your value is the highest.
  - Bidders shade bids to less than their value to balance the effect of decreasing the probability of winning and increasing CS. Bid depends on beliefs about strategies of rivals.

Auctions

- Key point: Expected outcome is the same across private value auctions.
  - Winner is the person with the highest value, and the winner pays roughly the second-highest value.
- Is there any reason that a seller still might choose one format over an alternative?

Auctions

- Key feature of common-value auctions: the Winner's Curse.
  - Winner's bid exceeds the value of item up for bid; winner pays too much.
  - Occurs due to uncertainty about the true value of the good.
    - E.g. Timber land auctions/auctions for oil and gas leases.
- Best strategy to avoid Winner's Curse: Shade/reduce bids to below estimates of value.
  - The amount of reduction depends on number of other bidders; more bidders $\implies$ more likely winning bid is an overestimate.
- While Winner's Curse is a well known phenomenon, there is strong empirical evidence it continues to happen in practice (e.g in the corporate acquisition market).
  - One possible explanation: Bounded rationality.

Takeaways

1. Insights from game theory can be used to improve outcomes when making strategic decisions.
2. Sequential games add steps of information that may matter to the eventual outcome
3. Strategic interactions may or may not mean maximized profits