

# Practice Exam 2 Answer Key

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12. You have a firm with the following production function:

$$q = kl$$

- a. **In the short run, the firm has 5 units of capital. Write out the short-run production function.**

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Plug  $k = 5$  into the production function:

$$q = 5l$$

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- b. **Write down the total product, marginal product, and average product of labor for 0, 1, 2, 3, 4, and 5 workers.**

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Make a table:

$l$	$q = 5l$	$MP_l = q_2 - q_1$	$AP_l = q/l$
0	0	—	—
1	5	5	5
2	10	5	5
3	15	5	5
4	20	5	5
5	25	5	5

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- c. **In the long run, the marginal products of labor and capital, respectively, are:**

$$MP_l = k$$

$$MP_k = l$$

**Suppose your firm needs to produce 1,000 units of output, the price of labor is \$20, and the price of capital is \$50. Find the optimal (i.e. cost-minimizing) combination of labor and capital that produces 1,000 units.**

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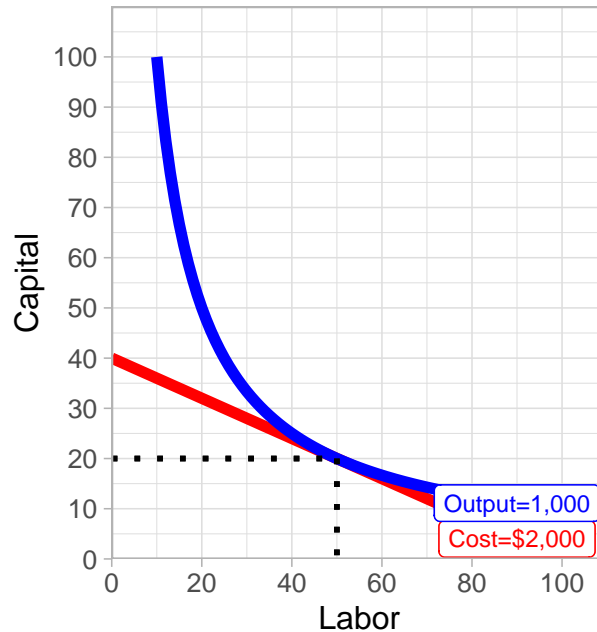
First, recognize that at the optimum, the slope of the isoquant curve (left) is equal to the slope of the isocost line:

$$\begin{aligned}\frac{MP_l}{MP_k} &= \frac{w}{r} \\ \frac{k}{l} &= \frac{10}{40} \\ \frac{k}{l} &= 0.4 \\ k &= 0.4l\end{aligned}$$

Now we take this and plug it into the production function, which has to produce 1,000 units.

$$\begin{aligned}
 q &= kl \\
 1000 &= (0.4l)l \\
 1000 &= 0.4l^2 \\
 2500 &= l^2 \\
 50 &= l
 \end{aligned}$$

If  $l^* = 50$ , then  $k^* = 0.4(50) = 20$ . Thus, the cost-minimizing combination of labor and capital that produces 1,000 units of output is 50 units of labor and 20 units of capital.



d. What is the total cost of producing with the optimal combination?

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$$\begin{aligned}
 wl + rk &= C \\
 20(50) + 50(20) &= C \\
 1000 + 1000 &= C \\
 2000 &= C
 \end{aligned}$$

The total cost of using 20 workers and 50 units of capital at current input prices is \$2,000.

e. Graph the isoquant, isocost line, and optimum on the graph above.

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It will help to find the isocost line equation to graph (by solving for k):

$$\begin{aligned}
 wl + rk &= C \\
 20l + 50k &= 2000 \\
 50k &= 2000 - 20l \\
 k &= 40 - 0.4l
 \end{aligned}$$

Alternatively, just calculate and graph the endpoints:

- If the firm only used capital, for \$2,000 it could use  $\frac{2000}{50} = 40$
- If the firm only used labor, for \$2,000 it could use  $\frac{2000}{20} = 100$

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f. **In the long run, does this production function exhibit constant, increasing, or decreasing returns to scale?**

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Suppose for example, we have 1 L and 1 K:

$$q = (1)(1) = 1$$

Now we double inputs to 2 L and 2 K:

$$q = (2)(2) = 4$$

Output has quadrupled, from 1 to 4 units, when we have doubled inputs from 1 to 2 K & L, so we have increasing returns to scale.

Even simpler, since this is a Cobb-Douglas function, adding the exponents

$$q = l^1 k^1$$

leads to  $1+1=2$ , which is greater than 1, so it is again, increasing returns to scale.

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13. Suppose you are a restaurant operating in the very competitive D.C. brunch market. You have a cost structure as follows, where  $q$  is hundreds of meals served per day.

$$C(q) = 2q^2 + 4q + 18$$

$$MC(q) = 4q + 4$$

- a. Write the equations for (i) fixed costs, (ii) variable costs, (iii) average fixed costs, (iv), average variable costs, and (v) average (total) costs.

Fixed costs are where costs don't change with output, so any term(s) where there is no variable  $q$  in them:  
 18 We could also see that if  $q = 0$ :

$$C(q) = 2q^2 + 4q + 18$$

$$C(0) = 2(0)^2 + 4(0) + 18$$

$$C(0) = 18$$

$$f = 18$$

Variable costs change with output, so any term(s) with a variable  $q$  in them:

$$VC(q) = 2q^2 + 4q$$

$$AFC(q) = \frac{FC}{q}$$

$$= \frac{18}{q}$$

$$AVC(q) = \frac{VC(q)}{q}$$

$$= \frac{2q^2 + 4q}{q}$$

$$= 2q + 4$$

$$AC(q) = \frac{C(q)}{q}$$

$$= \frac{2q^2 + 4q + 18}{q}$$

$$= 2q + 4 + \frac{18}{q}$$

Alternatively, we know that:

$$AC(q) = AVC(q) + AFC(q)$$

$$= (2q + 4) + \left(\frac{18}{q}\right)$$

$$= 2q + 4 + \frac{18}{q}$$

- b. The market price is currently \$12. Calculate the profit-maximizing quantity of output.

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In a perfectly competitive market, optimal quantity is where  $p = MR(q) = MC(q)$ .

$$\begin{aligned}p &= MC(q) \\12 &= 4q + 4 \\8 &= 4q2 &= q^*\end{aligned}$$

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c. **At the profit-maximizing quantity, calculate the average cost.**

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We know the average cost function from (a), just plug in the quantity we just found, 2.

$$\begin{aligned}AC(q) &= 2q + 4 + \frac{18}{q} \\AC(2) &= 2(2) + 4 + \frac{18}{(2)} \\AC(2) &= 4 + 4 + 9 \\AC(2) &= 17\end{aligned}$$

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d. **At the profit-maximizing price and quantity, calculate the total profit. Should this firm stay or exit the market in the long run?**

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$$\begin{aligned}\pi &= (p - AC(q))q \\ \pi &= (12 - 17)2 \\ \pi &= (-5)2 \\ \pi &= -10\end{aligned}$$

Since the firm is earning losses, it will want to *exit* in the *long run*.

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e. **Should this firm produce or shut down in the short run?**

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Find the shut-down price, the minimum of  $AVC$ , where  $AVC = MC$

$$\begin{aligned}AVC(q) &= MC(q) \\2q + 4 &= 4q + 4 \\q &= 0\end{aligned}$$

Plug in 0 into either  $MC(q)$  or  $AVC(q)$  to get a price of \$4. This is the shut down price. Since the current market price (12) is higher than the shut down price, the firm should *not* shut down in the short run, even though it is earning losses!

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f. **What price would the firm need to charge in order to break even?**

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Find the break-even price, the minimum of AC, where  $AC = MC$

$$\begin{aligned}AC(q) &= MC(q) \\2q + 4 + \frac{18}{q} &= 4q + 4 \\2q + \frac{18}{q} &= 4q \\\frac{18}{q} &= 2q \\18 &= 2q^2 \\9 &= q^2 \\3 &= q\end{aligned}$$

This is the quantity where average cost is minimized. Find what this cost is by plugging in 3 into either  $MC$  or  $AC$ :

$$\begin{aligned}MC(q) &= 4q + 4 \\MC(3) &= 4(3) + 4 \\MC(3) &= 16\end{aligned}$$

At a price of \$16, the firm breaks even ( $\pi = 0$ ).

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g. **In the long run, what must the equilibrium market price be for this industry, and why?**

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For a competitive market to be in long run equilibrium, all firms must earn an economic profit of \$0. The price must be equal to average cost (the break even price) for this to happen, which again is at \$16.

Visualize this problem below:

