Derivatives for SOC state

- To implement EKF using ESC model, need $\hat{A}_k$, $\hat{B}_k$, $\hat{C}_k$, $\hat{D}_k$
- We first examine components of state equation to find $\hat{A}_k$, $\hat{B}_k$
- Model true cell current as $i_k + w_k$, but measure i_k only (w_k is current-sensor error)
- Assume $\eta_k = 1$, leverage adaptivity of EKF overcome this inaccuracy
- Then, state-of-charge equation can be written

$$z_{k+1} = z_k - \frac{\Delta t}{Q}(i_k + w_k)$$

- The two derivatives that we need for this term are (with Q having units [A s]):

$$\left.\frac{\partial z_{k+1}}{\partial z_k}\right|_{z_k=\hat{z}_k^+} = 1, \quad \text{and} \quad \left.\frac{\partial z_{k+1}}{\partial w_k}\right|_{w_k=\bar{w}} = -\frac{\Delta t}{Q}$$

Derivatives for R–C states

- If $\tau_j = \exp(-\Delta t/(R_j C_j))$, then resistor-currents state equation can be written as

$$i_{R,k+1} = \underbrace{\begin{bmatrix} \tau_1 & 0 & \cdots \\ 0 & \tau_2 & \\ \vdots & & \ddots \end{bmatrix}}_{A_{RC}} i_{R,k} + \underbrace{\begin{bmatrix} 1-\tau_1 \\ 1-\tau_2 \\ \vdots \end{bmatrix}}_{B_{RC}} (i_k + w_k)$$

- The two derivatives can be found to be

$$\left.\frac{\partial i_{R,k+1}}{\partial i_{R,k}}\right|_{i_{R,k}=\hat{i}_{R,k}^+} = A_{RC}, \quad \text{and} \quad \left.\frac{\partial i_{R,k+1}}{\partial w_k}\right|_{w_k=\bar{w}} = B_{RC}$$

Derivatives for dynamic hysteresis state (1)

- If $A_{H,k} = \exp\left(-\left|\frac{(i_k+w_k)\gamma\Delta t}{Q}\right|\right)$, then hysteresis state

$$h_{k+1} = A_{H,k}h_k + (A_{H,k} - 1)\,\mathrm{sgn}(i_k + w_k)$$

- Taking partial with respect to the state and evaluating at the setpoint (noting that $w_k = \bar{w}$ is a member of the setpoint),

$$\left.\frac{\partial h_{k+1}}{\partial h_k}\right|_{\substack{h_k=\hat{h}_k^+ \\ w_k=\bar{w}}} = \exp\left(-\left|\frac{(i_k + \bar{w}_k)\,\gamma\Delta t}{Q}\right|\right) = \bar{A}_{H,k}$$

Derivatives for dynamic hysteresis state (2)

- Problem: $\partial h_{k+1}/\partial w_k$ does not exist at $i_k + w_k = 0$
  - If we assume that $i_k + w_k > 0$,
$$\frac{\partial h_{k+1}}{\partial w_k} = -\left|\frac{\gamma \Delta t}{Q}\right| \exp\left(-\left|\frac{\gamma \Delta t}{Q}\right| |(i_k + w_k)|\right)(1 + h_k)$$
  - If we assume that $i_k + w_k < 0$,
$$\frac{\partial h_{k+1}}{\partial w_k} = -\left|\frac{\gamma \Delta t}{Q}\right| \exp\left(-\left|\frac{\gamma \Delta t}{Q}\right| |(i_k + w_k)|\right)(1 - h_k)$$
- Overall, evaluating at Taylor-series linearization setpoint,
$$\left.\frac{\partial h_{k+1}}{\partial w_k}\right|_{\substack{h_k = \hat{h}_k^+ \\ w_k = \bar{w}}} = -\left|\frac{\gamma \Delta t}{Q}\right| \bar{A}_{H,k}\left(1 + \operatorname{sgn}(i_k + \bar{w})\hat{h}_k^+\right)$$

Derivatives for instantaneous hysteresis state (2)

- The zero-state hysteresis equation is defined as
$$s_{k+1} = \begin{cases} \operatorname{sgn}(i_k + w_k), & |i_k + w_k| > 0, \\ s_k, & \text{else} \end{cases}$$
- If we consider $i_k + w_k = 0$ to be a zero-probability event, then
$$\frac{\partial s_{k+1}}{\partial s_k} = 0, \qquad \text{and} \qquad \frac{\partial s_{k+1}}{\partial w_k} = 0$$

Derivatives for output equation

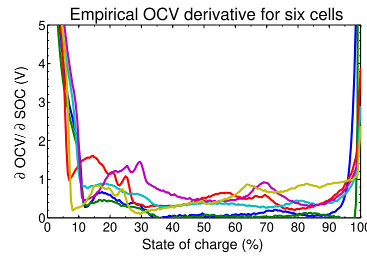
- We now look at the components that determine $\hat{C}_k$ and $\hat{D}_k$
- The ESC-model output equation is
$$y_k = \text{OCV}(z_k) + M_0 s_k + M h_k - \sum_j R_j i_{R_j,k} - R_0 i_k + v_k$$
  no longer considering i_k to have w_k noise added to it (this would add correlation between process noise and sensor noise).
- We have
$$\left.\frac{\partial y_k}{\partial s_k}\right| = M_0, \quad \left.\frac{\partial y_k}{\partial h_k}\right| = M, \quad \left.\frac{\partial y_k}{\partial i_{R_j,k}}\right| = -R_j, \quad \left.\frac{\partial y_k}{\partial v_k}\right| = 1$$

Derivative of OCV function

- Can use (evenly spaced) OCV data to approximate

$$\frac{\partial y_k}{\partial z_k}\bigg|_{z_k=\hat{z}_k^-} = \frac{\partial \mathrm{OCV}(z_k)}{\partial z_k}\bigg|_{z_k=\hat{z}_k^-}$$

- The figure shows empirical OCV derivative relationships for six different lithium-ion cells
- There is a little noise, which could be filtered (with a zero-phase filter!)
  - Filtering not really necessary




```
% Find dOCV/dz at SOC = z from {SOC,OCV} data
function dOCVz = dOCVfromSOC(SOC,OCV,z)
  dZ = SOC(2) - SOC(1); % Find spacing of SOC vector
  dUdZ = diff(OCV)/dZ;  % Scaled forward finite difference
  dOCV = ([dUdZ(1) dUdZ] + [dUdZ dUdZ(end)])/2; % Avg of fwd/bkwd diffs
  dOCVz = interp1(SOC,dOCV,z); % Could make more efficient than this...
```


Summary

- To implement EKF on ESC model, need noise assumptions
  - Assumed process noise is additive to electrical current
  - Assumed sensor noise is additive to voltage measurement
- Then, must find all partial derivates for $\hat{A}_k$, $\hat{B}_k$, $\hat{C}_k$, $\hat{D}_k$
  - This was a little complicated for the hysteresis state, since the update equation is not differentiable at all points
  - However, by a limiting argument, we found all matrices we need
- We are ready to implement EKF on ESC model!