Problems with EKF

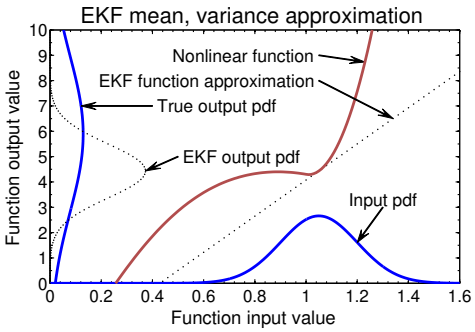
- Last week, we studied the EKF for battery state estimation
- EKF is best known and most used nonlinear Kalman filter
- However, it has serious flaws that can be remedied fairly easily

ISSUES: How input mean and covariance are propagated through static nonlinear function to create output mean and covariance estimates

- When computing mean estimates in Steps 1a and 1c, assumes $\mathbb{E}[\text{fn}(x)] \approx \text{fn}(\mathbb{E}[x])$
  - This is not true in general, and not necessarily even close to true (depending on "how nonlinear" the function $\text{fn}(\cdot)$ is)
- When finding covariances in Steps 1b and 2a, linearizes Taylor-series expansions
  - Nonlinear terms are dropped, resulting in a loss of accuracy

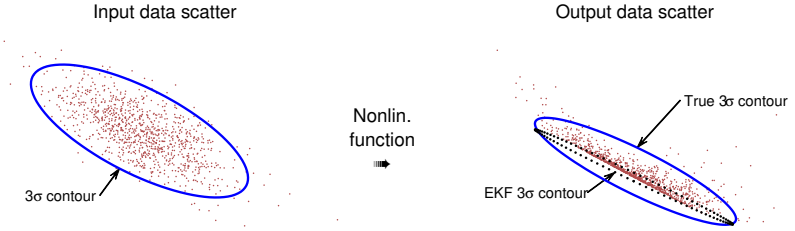
Illustrating problems in 1-d

- A simple 1-d example illustrates these two effects
- Input RV pdf is shown on horizontal axis, with mean 1.05
- Inputs passed through red nonlinear function to produce outputs
- Straight dotted line is linearized approximation used by EKF to find output mean and covariance
- EKF-approximated pdf is compared to a Gaussian pdf having same mean and variance of the true data on the vertical axis
- Notice that EKF approach is not producing an accurate estimate of either



Illustrating problems in 2-d

- Left: cloud of random Gaussian function input points; right: transformed set of output points for 2-d example
- Actual 3σ confidence interval is compared to EKF-estimated confidence interval
- "Sigma-point" approach can improve mean, covar propagation through nonlinear equations



Summary

- EKF is best-known nonlinear state estimator
- But, has some limitations due to assumptions made when approximating expectations
  - We know that $\mathbb{E}[\text{fn}(x)] \neq \text{fn}(\mathbb{E}[x])$ in general
  - We know that truncating a Taylor series loses accuracy
- This week, we study "sigma point" approach to approximating expectations, results in sigma-point Kalman filter (SPKF/UKF)
- SPKF generally outperforms EKF for nonlinear problems and has same computational complexity as EKF