Recursive computation of weighted least squares

- WLS computes $\widehat{Q} = c_2/c_1$ based on $\{(x_i, y_i)\}_{i=1\ldots N}$, where

$$c_1 = \sum_{i=1}^{N} \frac{x_i^2}{\sigma_{y_i}^2} \quad \text{and} \quad c_2 = \sum_{i=1}^{N} \frac{x_i y_i}{\sigma_{y_i}^2}$$

- But, notice we can update summations recursively whenever new data pair available

$$c_{1,n} = \sum_{i=1}^{n} \frac{x_i^2}{\sigma_{y_i}^2} = c_{1,n-1} + \frac{x_n^2}{\sigma_{y_n}^2} \quad \text{and} \quad c_{2,n} = \sum_{i=1}^{n} \frac{x_i y_i}{\sigma_{y_i}^2} = c_{2,n-1} + \frac{x_n y_n}{\sigma_{y_n}^2}$$

- Then, we can write capacity estimate after n updates as $\widehat{Q}_n = c_{2,n}/c_{1,n}$
- Benefits of recursive approach:
  - Minimizes storage requirements
  - Evens out computational requirements when n gets large

Initializing recursion

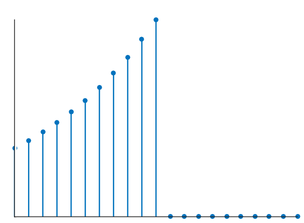
- But, recursion requires initial estimates of $c_{1,0}$ and $c_{2,0}$
- One approach is simply to set $c_{1,0} = c_{2,0} = 0$
- Alternately, can recognize that cell having nominal capacity Q_{nom} has that capacity over a state-of-charge range of 1.0
  - Therefore, we can initialize with a synthetic zeroth “measurement” where $x_0 = 1$ and $y_0 = Q_{\text{nom}}$
- The value for $\sigma_{y_0}^2$ can be set to the manufacturing variance of the nominal capacity
  - That is, $c_{1,0} = 1/\sigma_{y_0}^2$ and $c_{2,0} = Q_{\text{nom}}/\sigma_{y_0}^2$

Fading memory problem setup

- Can adapt method to allow fading memory of past data
- Start by modifying WLS cost function to place more emphasis on most recent measurements and less on measurements from distant past
- We define the fading-memory weighted-least-squares (FMWLS) cost function as

$$\chi^2_{\text{FMWLS}} = \sum_{i=1}^{N} \gamma^{N-i} \frac{(y_i - \widehat{Q} x_i)^2}{\sigma_{y_i}^2},$$

where the forgetting factor γ is in the range $0 \ll \gamma \leq 1$



Fading memory solution

- Using same method as last lesson, can find

$$\widehat{Q} = \sum_{i=1}^{N} \gamma^{N-i} \frac{x_i y_i}{\sigma_{y_i}^2} \Bigg/ \sum_{i=1}^{N} \gamma^{N-i} \frac{x_i^2}{\sigma_{y_i}^2}$$

- This solution may also easily be computed in a recursive manner
  - Compute $\widehat{Q}_n = \tilde{c}_{2,n}/\tilde{c}_{1,n}$ based on two running sums (initialized same as before)

$$\tilde{c}_{1,n} = \sum_{i=1}^{n} \gamma^{N-i} \frac{x_i^2}{\sigma_{y_i}^2} = \gamma \tilde{c}_{1,n-1} + \frac{x_n^2}{\sigma_{y_n}^2}$$

$$\tilde{c}_{2,n} = \sum_{i=1}^{n} \gamma^{N-i} \frac{x_i y_i}{\sigma_{y_i}^2} = \gamma \tilde{c}_{2,n-1} + \frac{x_n y_n}{\sigma_{y_n}^2}$$

Summary

- Have now derived WLS and FMWLS methods to estimate Q
- In summary, both WLS and FMWLS have nice properties:
  1. They give a closed-form solution for $\widehat{Q}$
  2. Only simple operations—multiplication, addition, and division—are required
  3. The solutions can very easily be computed in a recursive manner
  4. Fading memory can easily be added, allowing adaptation of $\widehat{Q}$ to adjust for true cell total capacity changes
- But, remember, that these solutions are biased
  - Will use as baseline against which to compare TLS-type solutions