EKF step 2a: Estimator gain matrix (1)

- Recall that the KF gain vector is $L_k = \Sigma^{-}_{\tilde{x}\tilde{y},k}\Sigma^{-1}_{\tilde{y},k}$
- Output prediction error may be approximated

$$\tilde{y}_k = y_k - \hat{y}_k = h(x_k, u_k, v_k) - h(\hat{x}^{-}_k, u_k, \bar{v}_k)$$

using Taylor-series expansion on first term

$$y_k \approx h(\hat{x}^{-}_k, u_k, \bar{v}_k) + \underbrace{\left.\frac{\mathrm{d}h(x_k, u_k, v_k)}{\mathrm{d}x_k}\right|_{x_k=\hat{x}^{-}_k}}_{\text{Defined as } \hat{C}_k}(x_k - \hat{x}^{-}_k) + \underbrace{\left.\frac{\mathrm{d}h(x_k, u_k, v_k)}{\mathrm{d}v_k}\right|_{v_k=\bar{v}_k}}_{\text{Defined as } \hat{D}_k}(v_k - \bar{v}_k)$$

- Note, much like we saw in Step 1b,

$$\frac{\mathrm{d}h(x_k, u_k, v_k)}{\mathrm{d}x_k} = \frac{\partial h(x_k, u_k, v_k)}{\partial x_k} \quad \text{and} \quad \frac{\mathrm{d}h(x_k, u_k, v_k)}{\mathrm{d}v_k} = \frac{\partial h(x_k, u_k, v_k)}{\partial v_k}$$

EKF step 2a: Estimator gain matrix (2)

- From these results, we can compute

$$\begin{aligned}\Sigma_{\tilde{y},k} &\approx \hat{C}_k\Sigma^{-}_{\tilde{x},k}\hat{C}^T_k + \hat{D}_k\Sigma_{\tilde{v}}\hat{D}^T_k,\\ \Sigma^{-}_{\tilde{x}\tilde{y},k} &\approx \mathbb{E}[(\tilde{x}^{-}_k)(\hat{C}_k\tilde{x}^{-}_k + \hat{D}_k\tilde{v}_k)^T]\\ &= \Sigma^{-}_{\tilde{x},k}\hat{C}^T_k\end{aligned}$$

- These terms may be combined to get the Kalman gain

$$L_k = \Sigma^{-}_{\tilde{x},k}\hat{C}^T_k\left[\hat{C}_k\Sigma^{-}_{\tilde{x},k}\hat{C}^T_k + \hat{D}_k\Sigma_{\tilde{v}}\hat{D}^T_k\right]^{-1}$$

EKF step 2b/c: State/covariance meas. update

EKF step 2b: State estimate measurement update

- Computes posterior state estimate by updating prediction using estimator gain and innovation $y_k - \hat{y}_k$

$$\hat{x}^{+}_k = \hat{x}^{-}_k + L_k(y_k - \hat{y}_k)$$

EKF step 2c: Error covariance measurement update

- Finally, the updated covariance is computed as

$$\Sigma^{+}_{\tilde{x},k} = \Sigma^{-}_{\tilde{x},k} - L_k\Sigma_{\tilde{y},k}L^T_k$$

Summary

- EKF makes two fundamental assumptions to generalize KF
  1. Assume $\mathbb{E}[\text{fn}(x)] \approx \text{fn}(\mathbb{E}[x])$, which is not true in general
  2. Assume Taylor-series expansion to linearize system equations for covariances
- Applying these to generic-probabilistic-inference solution yields final three EKF steps
- EKF algorithm is now derived!