Models for nonlinear Kalman filters

- KF assumes a cell model of the form

$$\begin{aligned} x_k &= A_{k-1}x_{k-1} + B_{k-1}u_{k-1} + w_{k-1} \\ y_k &= C_k x_k + D_k u_k + v_k \end{aligned}$$

- But, cell models are nonlinear, so the standard KF recursion doesn't apply directly
- We now generalize to the nonlinear case, with system dynamics described as

$$\begin{aligned} x_k &= f(x_{k-1}, u_{k-1}, w_{k-1}) \\ y_k &= h(x_k, u_k, v_k) \end{aligned}$$

- Functions $f(\cdot)$ and $h(\cdot)$ may be time-varying, but we generally omit the time dependency from the notation for ease of understanding

Kinds of nonlinear Kalman filters

- Three basic KF generalizations for nonlinear systems
  - Extended Kalman filter (EKF): Analytic linearization of model at each point in time: problematic, but still popular
  - Sigma-point (Unscented) Kalman filter (SPKF/UKF): Statistical/empirical linearization of model at each point in time: can be much better than EKF at same computational complexity
  - Particle filters: Most precise, but often thousands of times more computations required than either EKF/SPKF
- This week, we study EKF; next week SPKF
- Particle filters are beyond the scope of this course

Prediction steps of nonlinear Kalman filters

- EKF and SPKF both follow same set of general steps as KF
- Recall general step 1

Step 1a: State prediction time update

$$\hat{x}_k^- = \mathbb{E}\big[x_k \mid \mathbb{Y}_{k-1}\big] = \mathbb{E}\big[f(x_{k-1}, u_{k-1}, w_{k-1}) \mid \mathbb{Y}_{k-1}\big].$$

Step 1b: Error covariance time update

$$\Sigma_{\tilde{x},k}^- = \mathbb{E}\big[(\tilde{x}_k^-)(\tilde{x}_k^-)^T\big] = \mathbb{E}\big[(x_k - \hat{x}_k^-)(x_k - \hat{x}_k^-)^T\big].$$

Step 1c: Predict system output

$$\hat{y}_k = \mathbb{E}\big[y_k \mid \mathbb{Y}_{k-1}\big] = \mathbb{E}\big[h(x_k, u_k, v_k) \mid \mathbb{Y}_{k-1}\big].$$

Prediction

- EKF and SPKF simply have different expressions from KF for evaluating the expectation operations (because of different system model)

Update steps of nonlinear Kalman filters

- EKF and SPKF both follow same set of general steps as KF
- Recall general step 2

Step 2a: Estimator gain matrix
$$L_k = \Sigma^-_{\tilde{x}\tilde{y},k}\Sigma^{-1}_{\tilde{y},k}.$$
Step 2b: State estimate measurement update
$$\hat{x}^+_k = \hat{x}^-_k + L_k(y_k - \hat{y}_k).$$
Step 2c: Error covariance measurement update
$$\Sigma^+_{\tilde{x},k} = \Sigma^-_{\tilde{x},k} - L_k\Sigma_{\tilde{y},k}L^T_k.$$

Correction

- Again, EKF and SPKF simply have different expressions from KF for evaluating the expectation operations implied in Step 2a (because of different system model)

Summary

- Battery cells are nonlinear systems, therefore need nonlinear models and nonlinear Kalman filters
- We will study two different approaches to generalizing KF equations for nonlinear systems, one leading to EKF and the other to SPKF/UKF[1]
- EKF and SPKF (and variants) still obey predict/correct sequence of six steps
- So, derivation will involve evaluating those six steps under different sets of assumptions

[1] cf. `http://www.ieeeghn.org/wiki/index.php/First-Hand:The_Unscented_Transform` re. name "unscented"