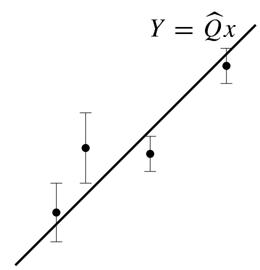
Linear structure of capacity equation

- Recall that the sensitivity of the cell voltage to cell capacity is very low, so noise tends to bias results
- Consider the SOC equation, $z[k_2] = z[k_1] - \dfrac{\Delta t}{Q} \displaystyle\sum_{k=k_1}^{k_2-1} \eta[k]i[k]$
- We can rearrange its terms to get: $\underbrace{-\Delta t \displaystyle\sum_{k=k_1}^{k_2-1} \eta[k]i[k]}_{y} = Q\,\underbrace{(z[k_2] - z[k_1])}_{x}$,

  where the obvious linear structure of $y = Qx$ becomes apparent
- Using a regression technique, for example, one may compute estimates of Q
  - One needs only to find values for "x" and "y"

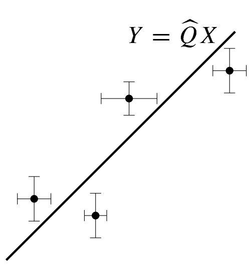
Problem with ordinary least squares

- The problem with using standard (least squares) linear regression techniques is that both summed current y and difference between state-of-charge values x have sensor noise or estimation noise associated with them
- The least squares linear regression problem is a solution to the equation $(y - \Delta y) = Qx$; that is, there is noise assumed on the measurements y, but not on the independent variable x



A better way to think about the linear relationship

- Total-capacity-estimation problem is implicitly of form $(y - \Delta y) = Q(x - \Delta x)$ since both the integrated current and SOC estimates have noise
- That is, because estimates of SOC are generally imperfect, there will be noise on the x variable, and using standard least squares linear regression results in a biased estimate of battery cell total capacity
  - Note that KF-based methods are (recursive) least squares: they will tend to be biased by noise



What to do?

- Usual approach to counteract this problem is to try to ensure that SOC estimates are as accurate as possible, then use standard least-squares estimation anyway
- For example, we might put constraints on how the capacity is estimated
  - Could force cell current to be zero before test begins and after the test ends (so that cell is in an equilibrium state and SOC estimates are as accurate as possible)
  - Eliminates to a large extent (but not completely) the error in the x variable, and makes the regression reasonably accurate
- Still does not correctly handle the residual noise in x: minimizes the noise, but never totally eliminates it
- Solution: use “total least squares” instead of “(ordinary) least squares” estimation

4.2.1: What's wrong with using ordinary least squares to estimate total capacity?

Summary

- We can write an equation that involves measurable quantities and is linear in Q
- Tempting to use (ordinary least squares) regression techniques to estimate Q!
- However, there are errors/noises in both the “x” and “y” terms in this regression
  - Ordinary-least-squares solutions will be biased by these noises
  - Instead, total-least-squares solutions should be used
- That is what we study next