Simultaneous state and parameter estimation

- You have now learned how to use Kalman filters to perform state estimation and parameter estimation independently
  - How about both at the same time?
- There are two approaches to doing so: joint estimation and dual estimation
- These are discussed generically in this lesson, then more specifically in the remaining lessons this week

Generic joint estimation

- In joint estimation, state and parameter vectors are combined, KF simultaneously estimates this augmented state vector

$$\begin{aligned}\begin{bmatrix} x_k \\ \theta_k \end{bmatrix} &= \begin{bmatrix} f(x_{k-1}, u_{k-1}, \theta_{k-1}, w_{k-1}) \\ \theta_{k-1} + r_{k-1} \end{bmatrix} \\ y_k &= h(x_k, u_k, \theta_k, v_k).\end{aligned}$$

- To simplify notation, let $\mathbb{X}_k$ be the augmented state, $\mathbb{W}_k$ be the augmented noise, and $\mathcal{F}$ be the augmented state equations:

$$\begin{aligned}\mathbb{X}_k &= \mathcal{F}(\mathbb{X}_{k-1}, u_{k-1}, \mathbb{W}_{k-1}) \\ y_k &= h(\mathbb{X}_k, u_k, v_k).\end{aligned}$$

Evaluating generic joint estimation

- Advantage:
  - Quite straightforward to implement: with augmented model of system state dynamics and parameter dynamics defined, we simply apply a nonlinear KF method
- Disadvantages:
  - Large matrix operations due to the high dimensionality of the resulting augmented model
  - Potentially poor numeric conditioning due to the vastly different time scales of the states/parameters in augmented state vector

Generic dual estimation: state evolution

- In dual estimation, separate KFs are used for state and parameter estimation
- Computational complexity lower, matrices may be numerically better conditioned
- However, by decoupling state from parameters, any cross-correlations between changes are lost, leading to potentially poorer accuracy
- Model of state dynamics again explicitly includes parameters as vector θ_k

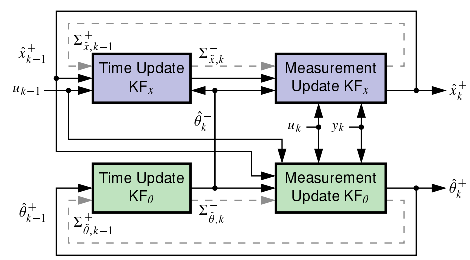
$$x_k = f(x_{k-1}, u_{k-1}, w_{k-1}, \theta_{k-1})$$
$$y_k = h(x_k, u_k, v_k, \theta_{k-1})$$

- Non-time-varying numeric values required by the model may be embedded within $f(\cdot)$ and $h(\cdot)$, and are not included in θ_k

Generic dual estimation: parameter evolution

- Also slightly revise model of parameter dynamics to include effect of state equation explicitly

$$\theta_k = \theta_{k-1} + r_{k-1}$$
$$d_k = h\big(f(x_{k-1}, u_{k-1}, \bar{w}_{k-1}, \theta_{k-1}), u_k, e_k, \theta_{k-1}\big)$$



- The dual filters can be viewed by drawing a block diagram
  - The interactions will be made clearer later
- The process essentially comprises two KFs running in parallel—one adapting state and one adapting parameters—with some information exchange

Summary

- Two approaches to adapting a model's states and parameters at same time
- Joint estimation augments both in large state vector, uses single standard nonlinear KF with this augmented system
- Dual estimation uses separate nonlinear KFs for state and parameter estimation
- Joint estimation easier conceptually but can be slower due to large matrix operations and possibly poorer matrix conditioning
- Dual estimation generally faster, but ignores any correlation between state and parameter estimation in its updates