## Finding transfer functions

- Single-particle model gives insight into physics-based models, but does not have enough resolution to control degradation
  - We need (at least) a 1-d model of the cell
- There are different approaches to creating reduced-order physics-based models
- Here, I introduce a “transfer-function” method, that produces very small models and can work very well
  1. First, assume linear behavior (so, linearize nonlinear PDEs using Taylor series)
  2. Then, take Laplace transforms of all PDEs
  3. Combine transforms to find transfer functions for all variables vs. applied current





## Example transfer function

- For example, transfer function for interphase potential difference uses

$$\nu^2(s) = \frac{L^2\left(\frac{a_s}{\sigma_{\text{eff}}}+\frac{a_s}{\kappa_{\text{eff}}}\right)}{R_{s,e} + \left[\frac{\partial U_{\text{ocp}}}{\partial c_{s,e}}\Big|_{c_{s,0}}\right]\frac{R_s}{FD_s}\left[\frac{1}{1-R_s\sqrt{s/D_s}\coth(R_s\sqrt{s/D_s})}\right]}$$

and is expressed in final form as

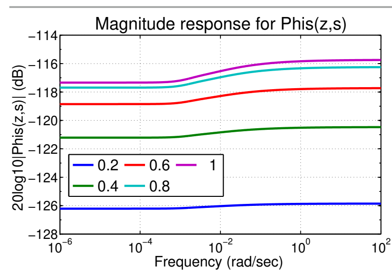
$$\frac{\widetilde{\Phi}_{s\text{-}e}^{\text{neg}}(z,s)}{I_{\text{app}}(s)} = \frac{L^{\text{neg}}\left[\sigma_{\text{eff}}^{\text{neg}}\cosh(\nu^{\text{neg}}(s)z)+\kappa_{\text{eff}}^{\text{neg}}\cosh(\nu^{\text{neg}}(s)(z-1))\right]}{A\sigma_{\text{eff}}^{\text{neg}}\kappa_{\text{eff}}^{\text{neg}}\nu^{\text{neg}}(s)\sinh(\nu^{\text{neg}}(s))}$$

- Yikes!

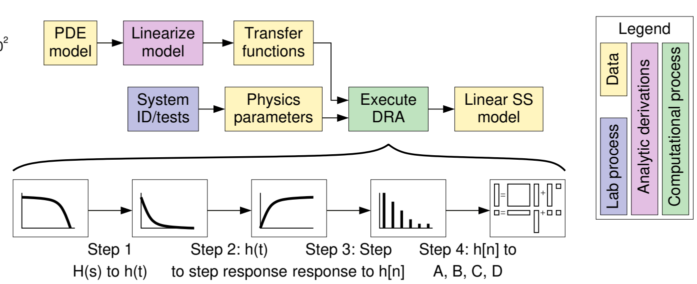




## Generate ROM with DRA



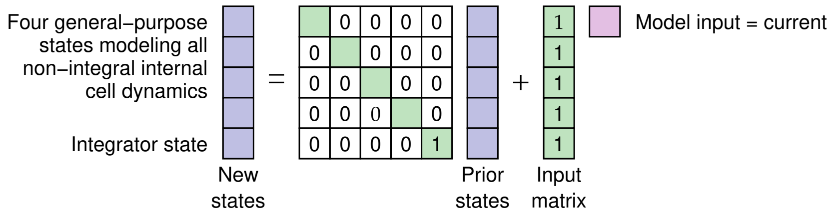

- Although transfer functions are mathematically complex, actual behaviors are generally simple
- Magnitude response shows this simplicity



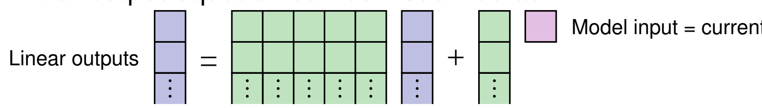

- We use “discrete-time realization algorithm” (DRA) to create reduced-order discrete-time state-space model

## Final model form

- For any linearization setpoint, model is in state-space form
- The state equation can be visualized as




- The linear output equation can be visualized as








## Cell voltage

- Cell voltage is a nonlinear combination of the linear outputs

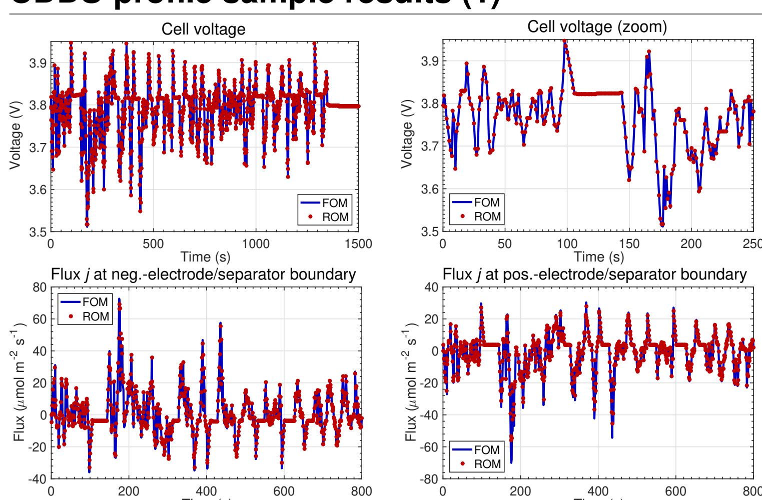
$$\begin{aligned} v(t) = F\left(R_{\text{film}}^{\text{pos}} j^{\text{pos}}(0,t) - R_{\text{film}}^{\text{neg}} j(0,t)\right) + \left[\tilde{\phi}_e(L^{\text{tot}},t)\right]_1 \\ + (\eta^{\text{pos}}(0,t) - \eta^{\text{neg}}(0,t)) + \left[\tilde{\phi}_e(L^{\text{tot}},t)\right]_2 \\ + \left(U_{\text{ocp}}^{\text{pos}}(c_{s,e}^{\text{pos}}(0,t)) - U_{\text{ocp}}^{\text{neg}}(c_{s,e}^{\text{neg}}(0,t))\right) \end{aligned}$$

- So, to compute cell voltage, a minimum of these transfer functions at these locations must be processed
- ROMs on the order of five states (similar computational complexity to 4 R–C equivalent circuit) often work very well



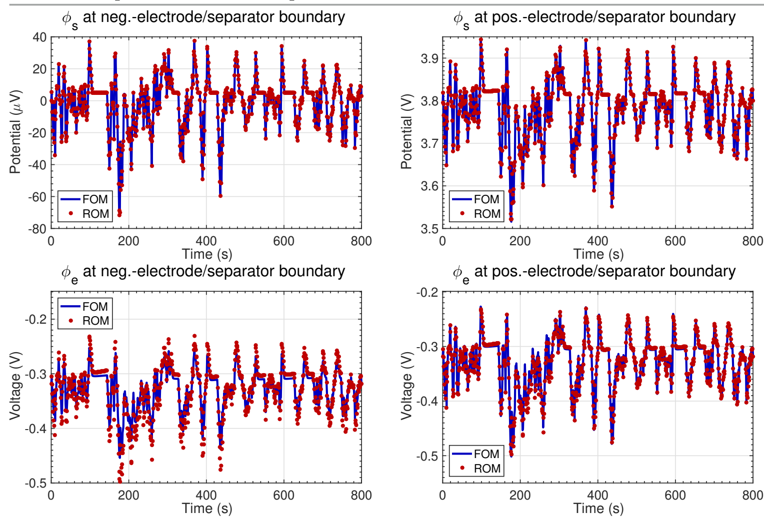


## UDDS-profile sample results (1)




- First, voltage (< 1 mV RMS error), lithium interphase flux

## UDDS-profile sample results (2)

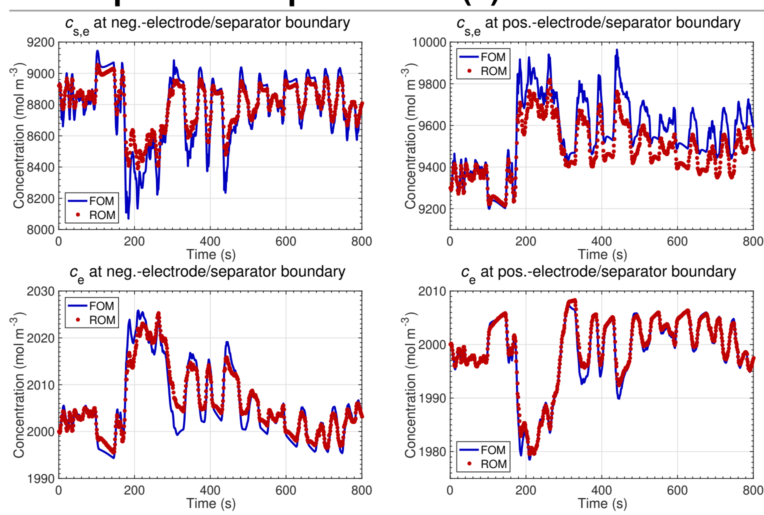



- Next, we see solid and electrolyte phase potential





## UDDS-profile sample results (3)




- Finally, solid-surface and electrolyte concentration of Li





## Summary

- Need at least a 1-d cell model to predict/mitigate degradation
- One approach is via transfer functions
  1. First, assume linear behavior (so, linearize nonlinear PDEs using Taylor series)
  2. Then, take Laplace transforms of all PDEs
  3. Combine transforms to find transfer functions for all variables vs. applied current
- Then, execute discrete-time realization algorithm (DRA)
  - Transfer function ▷ continuous-time impulse response ▷ continuous-time step response ▷ discrete-time unit pulse response ▷ state-space model (via Ho–Kalman algorithm)
- Final discrete-time reduced-order state-space model having around 5 states produces highly accurate predictions of all internal cell variables of interest

## For further study

- Early work describing creation of transfer functions and the DRA can be found in chapters 5–6 of
  - Plett, Gregory L., *Battery Management Systems, Volume 1: Battery Modeling*
  - See also `http://mocha-java.uccs.edu/BMS1/index.html`.
- There are detailed lecture notes and lecture videos on these topics from which you can learn at `http://mocha-java.uccs.edu/ECE5710/index.html` (Topics 5, 6).
- This is an area of active research... keep a look out for further papers refining both the transfer functions and realization algorithms!