Steps 1a–1b

- Generic Gaussian probabilistic inference solution can be divided into two main steps, each having three sub-steps
- **General step 1a**: State prediction time update
  - Each time step, compute an updated prediction of the present value of x_k, based on prior information and the system model

$$\hat{x}_k^- = \mathbb{E}\big[x_k \mid \mathbb{Y}_{k-1}\big] = \mathbb{E}\big[f(x_{k-1}, u_{k-1}, w_{k-1}) \mid \mathbb{Y}_{k-1}\big]$$

- **General step 1b**: Error covariance time update
  - Determine the predicted state-estimate error covariance matrix $\Sigma_{\tilde{x},k}^-$ based on prior information and the system model
  - We compute $\Sigma_{\tilde{x},k}^- = \mathbb{E}\big[(\tilde{x}_k^-)(\tilde{x}_k^-)^T\big]$, where $\tilde{x}_k^- = x_k - \hat{x}_k^-$

Step 1c

- **General step 1c**: Predict system output y_k
  - Predict the system's output using prior information

$$\hat{y}_k = \mathbb{E}\big[y_k \mid \mathbb{Y}_{k-1}\big] = \mathbb{E}\big[h(x_k, u_k, v_k) \mid \mathbb{Y}_{k-1}\big]$$

- Summarizing general step 1

Step 1a: State prediction time update

$$\hat{x}_k^- = \mathbb{E}\big[x_k \mid \mathbb{Y}_{k-1}\big] = \mathbb{E}\big[f(x_{k-1}, u_{k-1}, w_{k-1}) \mid \mathbb{Y}_{k-1}\big].$$

Step 1b: Error covariance time update

$$\Sigma_{\tilde{x},k}^- = \mathbb{E}\big[(\tilde{x}_k^-)(\tilde{x}_k^-)^T\big] = \mathbb{E}\big[(x_k - \hat{x}_k^-)(x_k - \hat{x}_k^-)^T\big].$$

Step 1c: Predict system output

$$\hat{y}_k = \mathbb{E}\big[y_k \mid \mathbb{Y}_{k-1}\big] = \mathbb{E}\big[h(x_k, u_k, v_k) \mid \mathbb{Y}_{k-1}\big].$$

Prediction

Step 2

- **General step 2a**: Estimator gain matrix L_k
  - Compute the estimator gain matrix $L_k = \Sigma_{\tilde{x}\tilde{y},k}^- \Sigma_{\tilde{y},k}^{-1}$
- **General step 2b**: State estimate measurement update
  - Compute the posterior state estimate by updating the prediction using the L_k and the innovation $y_k - \hat{y}_k$

$$\hat{x}_k^+ = \hat{x}_k^- + L_k(y_k - \hat{y}_k)$$

- **General step 2c**: Error covariance measurement update
  - Compute the posterior error covariance matrix

$$\begin{aligned}\Sigma_{\tilde{x},k}^+ &= \mathbb{E}\big[(\tilde{x}_k^+)(\tilde{x}_k^+)^T\big] \\ &= \Sigma_{\tilde{x},k}^- - L_k \Sigma_{\tilde{y},k} L_k^T\end{aligned}$$

Summarizing step 2

- Summarizing general step 2:

Step 2a: Estimator gain matrix

$$L_k = \Sigma^-_{\tilde{x}\tilde{y},k}\Sigma^{-1}_{\tilde{y},k}.$$

Step 2b: State estimate measurement update

$$\hat{x}^+_k = \hat{x}^-_k + L_k(y_k - \hat{y}_k).$$

Step 2c: Error covariance measurement update

$$\Sigma^+_{\tilde{x},k} = \Sigma^-_{\tilde{x},k} - L_k\Sigma_{\tilde{y},k}L_k^T.$$

Correction

- **KEY POINT:** Estimator outputs state estimate $\hat{x}^+_k$, error covariance estimate $\Sigma^+_{\tilde{x},k}$
  - That is, we have high confidence that the truth lies within $\hat{x}^+_k \pm 3\sqrt{\mathrm{diag}(\Sigma^+_{\tilde{x},k})}$
  - Estimator then waits until next sample interval, updates k, proceeds to step 1a

Summary

Step 1a: State prediction time update

$$\hat{x}^-_k = \mathbb{E}\big[x_k \mid \mathbb{Y}_{k-1}\big] = \mathbb{E}\big[f(x_{k-1}, u_{k-1}, w_{k-1}) \mid \mathbb{Y}_{k-1}\big].$$

Step 1b: Error covariance time update

$$\Sigma^-_{\tilde{x},k} = \mathbb{E}\big[(\tilde{x}^-_k)(\tilde{x}^-_k)^T\big] = \mathbb{E}\big[(x_k - \hat{x}^-_k)(x_k - \hat{x}^-_k)^T\big].$$

Step 1c: Predict system output

$$\hat{y}_k = \mathbb{E}\big[y_k \mid \mathbb{Y}_{k-1}\big] = \mathbb{E}\big[h(x_k, u_k, v_k) \mid \mathbb{Y}_{k-1}\big].$$

Prediction

Step 2a: Estimator gain matrix

$$L_k = \Sigma^-_{\tilde{x}\tilde{y},k}\Sigma^{-1}_{\tilde{y},k}.$$

Step 2b: State estimate measurement update

$$\hat{x}^+_k = \hat{x}^-_k + L_k(y_k - \hat{y}_k).$$

Step 2c: Error covariance measurement update

$$\Sigma^+_{\tilde{x},k} = \Sigma^-_{\tilde{x},k} - L_k\Sigma_{\tilde{y},k}L_k^T.$$

Correction