



## The pack “bar” filter, using ESC model

- Pack-average “bar” filter SPKF estimates following quantities:
  - Pack-average state-of-charge, pack-average diffusion current(s), the pack-average hysteresis voltage
- To implement this SPKF, need a state-space model of pack-average states and how those pack-average states produce a measurable output
  - This lesson will derive pack-average state equation
  - Current-sensor bias state included in model as  $i_k^b = i_{k-1}^b + n_{k-1}^b$  where  $n_k^b$  is fictitious noise source that allows SPKF to adapt bias estimate



## Example “bar”-filter state equation development

- For example, starting with a single-cell SOC equation

$$z_k^{(i)} = z_{k-1}^{(i)} - i_{k-1} \Delta t / Q^{(i)}$$

$$\frac{1}{N_s} \sum_{i=1}^{N_s} z_k^{(i)} = \frac{1}{N_s} \sum_{i=1}^{N_s} z_{k-1}^{(i)} - \frac{i_{k-1} \Delta t}{N_s} \sum_{i=1}^{N_s} \frac{1}{Q^{(i)}} = \frac{1}{N_s} \sum_{i=1}^{N_s} z_{k-1}^{(i)} - \frac{i_{k-1} \Delta t}{N_s} \sum_{i=1}^{N_s} Q_{inv}^{(i)}$$

$$\bar{z}_k = \bar{z}_{k-1} - i_{k-1} \Delta t \bar{Q}_{inv}$$

- Note the new concept of “inverse capacity” to make the equations simpler.
  - If we’re estimating all cells’ capacities, then have time-varying quantity  $\bar{Q}_{inv,k-1}$
- If we also consider the current-bias state,  $\bar{z}_k = \bar{z}_{k-1} - (i_{k-1} - i_{k-1}^b) \Delta t \bar{Q}_{inv,k-1}$



## “Bar”-filter model state equations

- Dynamics of all pack-average states, parameters:

$$\bar{z}_k = \bar{z}_{k-1} - (i_{k-1} - i_{k-1}^b) \Delta t \bar{Q}_{inv,k-1}$$

$$\bar{i}_{R_j,k} = A_{RC} \bar{i}_{R_j,k} + B_{RC} (i_{k-1} - i_{k-1}^b)$$

$$A_{h,k} = \exp \left( - \left| (i_{k-1} - i_{k-1}^b) \gamma \Delta t \bar{Q}_{inv,k-1} \right| \right)$$

$$\bar{h}_k = A_{h,k} \bar{h}_{k-1} + (1 - A_{h,k}) \operatorname{sgn}(i_{k-1} - i_{k-1}^b)$$

$$\bar{R}_{0,k} = \bar{R}_{0,k-1} + n_{k-1}^{\bar{R}_0}$$

$$\bar{Q}_{inv,k} = \bar{Q}_{inv,k-1} + n_{k-1}^{\bar{Q}_{inv}}$$

$$i_k^b = i_{k-1}^b + n_{k-1}^b,$$

where  $n_k^{\bar{R}_0}$ ,  $n_k^{\bar{Q}_{inv}}$  are fictitious noise sources that allow SPKF to adapt parameters



## “Bar”-filter model output equation

- Pack bar-filter SPKF uses this model of pack-average states and the measurement equation

$$\bar{y}_k = \text{OCV}(\bar{z}_k) + M\bar{h}_k - \sum_j R_j \bar{i}_{R_j,k} - \bar{R}_{0,k}(i_k - i_k^b) + v_k,$$

where  $v_k$  models sensor noise



## Summary

- xKF to implement pack-average “bar” filter must have state and measurement equations
- You have learned how to develop these averaged equations, using the SOC equation as an example
- You have also seen one method for parameter adaptation using xKF (there will be more of this in the next course)
- The next step is to see how to implement the “delta” filters