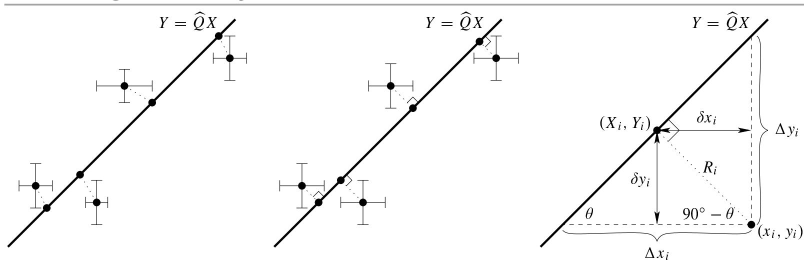
Desire a simple WTLS solution

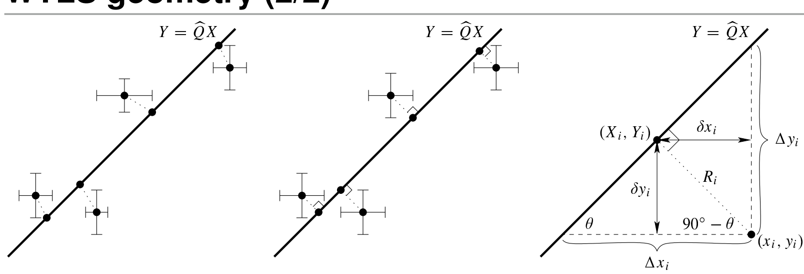
- Know that WLS solution is biased, but WTLS demands too much computation to use in practical BMS
- Have seen proportional-uncertainty version of WTLS—which we call TLS—that is feasible to implement: but, uncertainties are not necessarily proportional in practice
- We desire solution that approximates WTLS problem but allows $\sigma_{x_i}^2$ and $\sigma_{y_i}^2$ to be nonproportional, but which yields a recursive solution for feasible implementation in an embedded system
- In this lesson, you will learn an approach we can take, based on geometry of WTLS solution

WTLS geometry (1/2)



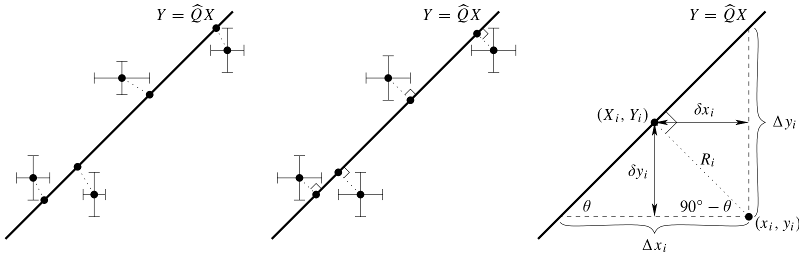
- Figure shows WTLS geometry, motivates approximate solution to be developed
- Left frame shows relationship between data point (x_i, y_i) and its optimized map (X_i, Y_i) on $Y_i = \widehat{Q}X_i$ when $\sigma_{x_i}^2$ and $\sigma_{y_i}^2$ are arbitrary
  - The error bars on each data point illustrate the uncertainties in each dimension, which are proportional to σ_{x_i} and σ_{y_i}

WTLS geometry (2/2)



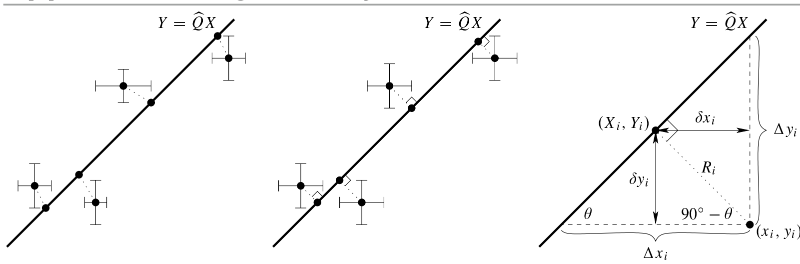
- We see that the distance between x_i and X_i is not necessarily equal to the distance between y_i and Y_i—depends on respective error bounds
- If quality of x_i is better (poorer) than quality of y_i, distance to its map X_i should be shorter (greater) than the distance from y_i to its map Y_i

TLS geometry



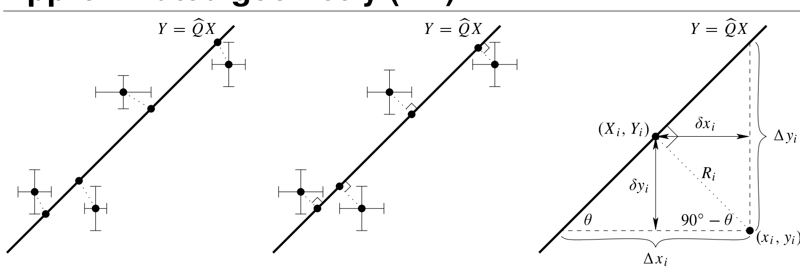
- Middle frame shows relationship between data point (x_i, y_i) and its optimized map (X_i, Y_i) on $Y_i = \widehat{Q}X_i$ when $\sigma_{x_i}^2$ and $\sigma_{y_i}^2$ are equal
- Distance between x_i and X_i is equal to distance between y_i and Y_i, and line joining data point (x_i, y_i) and (X_i, Y_i) is perpendicular to the line $Y_i = \widehat{Q}X_i$
  - If σ_{x_i} and σ_{y_i} are unequal but proportional, x- or y-axis may be scaled to yield transformed data points with equal variances, and hence same idea applies

Approximated geometry (1/2)



- Right frame illustrates definitions that will be used to derive an approximate weighted total least squares (AWTLS) solution
- Motivated by TLS, we enforce that line joining data point (x_i, y_i) and (X_i, Y_i) be perpendicular to line $Y_i = \widehat{Q}X_i$—will result in a recursive solution

Approximated geometry (2/2)



- However, as with the WTLS solution, we weight distance between x_i and X_i differently from distance between y_i and Y_i in optimization cost function
- This will give a better total capacity estimate than TLS when the uncertainties on x_i and y_i are not proportional

Summary

- WTLS solution maps (x_i, y_i) to (X_i, Y_i) on $Y = \widehat{Q}X$ with nonperpendicular line
  - Optimal but not practical to implement
- TLS maps with perpendicular line
  - Optimal only for proportional σ_{x_i} and σ_{y_i} but practical
- Will use observation of orthogonality to propose sub-optimal mapping (x_i, y_i) to (X_i, Y_i) that is perpendicular, but also weights uncertainties in σ_{x_i} and σ_{y_i}
- Will continue to use geometry from figure to right as we proceed beyond this point