Sensitivity to total capacity

- Estimating total capacity well turns out to be quite difficult
- Consider the sensitivity of voltage measurement to capacity:

$$S_{v_k}^{Q} = \frac{Q}{v_k}\frac{\mathrm{d}v_k}{\mathrm{d}Q} = \frac{Q}{v_k}\frac{\mathrm{d}}{\mathrm{d}Q}\left(\text{OCV}(z_k) + Mh_k - \sum_i R_i i_{R_i,k} - i_k R_0\right)$$

- Notice that
  1. Term in parenthesis is not directly a function of Q
  2. But, we are evaluating a total derivative, not a partial derivative
- So, there is still (some) hope that total capacity is observable through voltage measurements

Evaluating sensitivity through OCV

- Consider the first term in the sensitivity equation:

$$\frac{\mathrm{d}\text{OCV}(z_k)}{\mathrm{d}Q} = \frac{\partial \text{OCV}(z_k)}{\partial z_k}\frac{\mathrm{d}z_k}{\mathrm{d}Q}$$

- For most cells, OCV slope is very shallow, so $\partial \text{OCV}(z_k)/\partial z_k$ is very small. Further,

$$\begin{aligned}\frac{\mathrm{d}z_k}{\mathrm{d}Q} &= \frac{\mathrm{d}z_{k-1}}{\mathrm{d}Q} - \eta_{k-1} i_{k-1} \Delta t \frac{\mathrm{d}(1/Q)}{\mathrm{d}Q} \\ &= \frac{\mathrm{d}z_{k-1}}{\mathrm{d}Q} + \frac{\eta_{k-1} i_{k-1} \Delta t}{Q^2}\end{aligned}$$

- This total derivative can be calculated recursively

Considering recursive sensitivity through OCV

- From the previous slide, $\frac{\mathrm{d}z_k}{\mathrm{d}Q} = \frac{\mathrm{d}z_{k-1}}{\mathrm{d}Q} + \frac{\eta_{k-1} i_{k-1} \Delta t}{Q^2}$
  - Grows when i_k is in the same direction for a considerable amount of time and shrinks when i_k changes direction
  - For random i_k (*e.g.*, HEV) it is around the same order of magnitude as the second term, which can be computed from rms current
- The second term has Δt factor, which is often on the order of $1/3600$ or less
- Summary to date: Sensitivity of voltage to capacity through the OCV term is small, but can be maximized either by charging or discharging for long periods

Sensitivity through hysteresis

- Similarly, sensitivity of the voltage to capacity through hysteresis term is small (it is zero through the other terms)
- As a consequence, individual voltage measurements have very little information regarding capacity
- Must somehow combine many voltage measurements
- Simple ideas, like used to estimate $\widehat{R}_0$ will not work well
- So, we explore two basic approaches:
  - First, look at total-least-squares approaches, which are optimal
  - Next, look at KF-based approaches, which can work well (honors)

Summary

- An estimate of total capacity Q is a component of most SOH estimates
- The bad news is that Q is nearly unobservable: simple methods cannot estimate its value well
- The remainder of this course will explore reasons why one common approach to estimating Q is incorrect, and how to estimate Q correctly
  - Next week, we explore why ordinary-least-squares regression is the wrong approach, and why total-least-squares regression is correct
  - Then, we look at computationally efficient methods for computing the total-least-squares solution
  - Examples will demonstrate these claims