

Let X_1, X_2, \dots, X_n be a random sample from the normal distribution with mean μ and variance $\sigma^2 = 2$.

$$H_0 : \mu \leq 3$$

$$H_1 : \mu > 3$$

Idea: Look at \bar{X} and reject H_0 in favor of H_1 if \bar{X} is “large”.

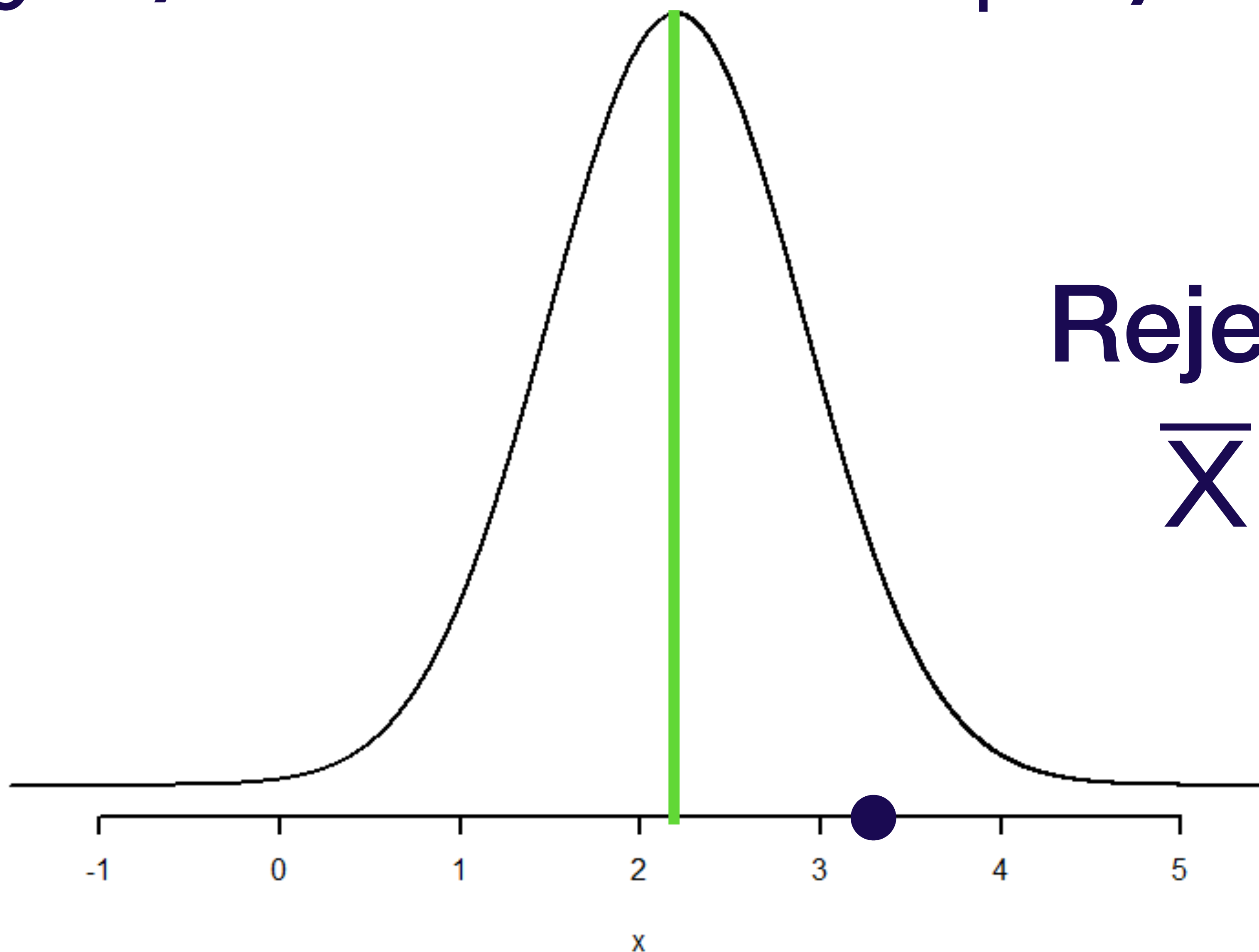
i.e. Look at \bar{X} and reject H_0 in favor of H_1 if $\bar{X} > c$ for some value c .

Let X_1, X_2, \dots, X_n be a random sample from the normal distribution with mean μ and variance $\sigma^2 = 2$.

$$H_0 : \mu \leq 3$$

$$H_1 : \mu > 3$$

Reject H_0 if
 $\bar{X} > c.$



Errors in Hypothesis Testing

Your Decision

“accept”^{*} H_0 reject H_0

H_0 true



H_0 false



****fail to reject****

Errors in Hypothesis Testing

Your Decision

“accept”^{*} H_0 reject H_0

H_0 true



Type I error

H_0 false

Type II error



****fail to reject****

Errors in Hypothesis Testing

- Type I or Type II error
which is worse?
- This totally depends on how you set up your hypotheses and what is at stake.
- The null hypothesis is assumed to be true and the alternate hypothesis is what you are out to show.

Example:

You are a potato chip manufacturer and you want to ensure that the mean amount in 15 ounce bags is at least 15 ounces.

$$H_0 : \mu \leq 15$$

$$H_1 : \mu > 15$$

Example:

You are an angry consumer group and you want to show that the chip company is cheating its customers.

$$H_0 : \mu \geq 15$$

$$H_1 : \mu < 15$$

Example:

You are a potato chip manufacturer and you want to ensure that the mean amount in 15 ounce bags is at least 15 ounces.

$$H_0 : \mu \leq 15 \quad H_1 : \mu > 15$$

Type I error:

The true mean is ≤ 15 but you concluded it was > 15 . You are going to save some money because you won't be adding chips but you are risking a lawsuit!

Example:

You are a potato chip manufacturer and you want to ensure that the mean amount in 15 ounce bags is at least 15 ounces.

$$H_0 : \mu \leq 15 \quad H_1 : \mu > 15$$

Type II error:

The true mean is > 15 but you concluded it was ≤ 15 . You are going to be spending money increasing the amount of chips when you didn't have to.

Errors in Hypothesis Testing

Your Decision

fail to reject H_0 reject H_0

H_0 true



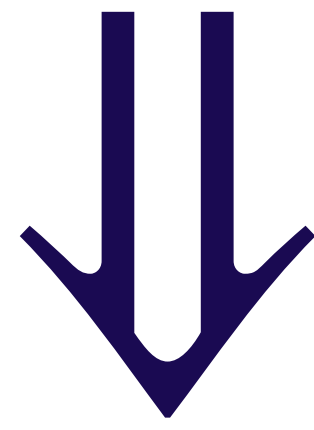
Type I error

H_0 false

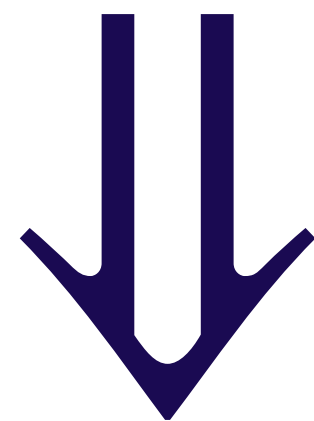
Type II error



random sample



sample mean \bar{X}

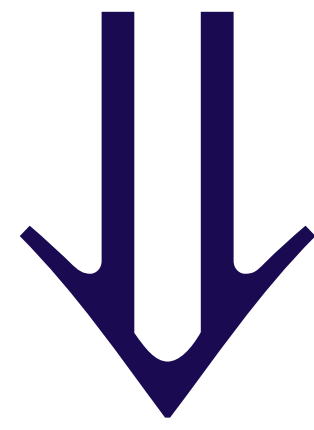


decision
(potentially with error)

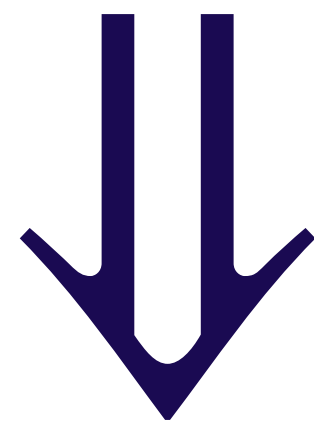
Question:

What is the probability that we will make a Type I or Type II error?

random sample



sample mean \bar{X}



decision
(potentially with error)

Related Question:

Can we control this error?