

Suppose that  $X_1, X_2, \dots, X_n$  is a random sample from a distribution with pdf  $f(x; \theta)$ .

Let  $\Theta$  be the parameter space. Assume that the parameter is one-dimensional.

Consider testing

$$H_0 : \theta = \theta_0 \quad \text{vs} \quad H_1 : \theta \neq \theta_0$$

Let  $\lambda(\vec{X}) = L(\theta_0)/L(\hat{\theta})$  be the GLR for this test.

If the parameter does not define the support for the pdf, for example as in the  $\text{unif}(0, \theta)$ , we have...

## Wilks' Theorem

Under the assumption that  $H_0$  is true

$$-2 \ln \lambda(\vec{X}) \xrightarrow{d} \chi^2(1)$$

Note that

$$\alpha = P(\lambda(\vec{X}) \leq c; \theta_0)$$

$$= P(\ln \lambda(\vec{X}) \leq c_1; \theta_0)$$

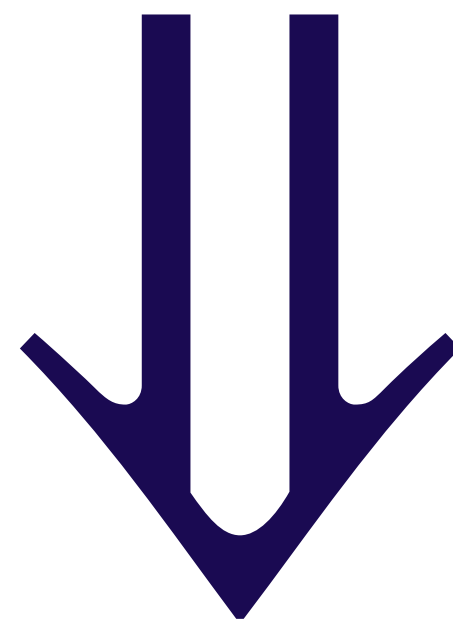
$$= P(\underbrace{-2 \ln \lambda(\vec{X})}_{\text{approximately } \chi^2(1)} \geq c_2; \theta_0)$$

approximately  $\chi^2(1)$   
for large sample size n

$$\approx P(W \geq c_2; \theta_0)$$

where  $W \sim \chi^2(1)$

$$\alpha = P(W \geq c_2; \theta_0)$$



$$c_2 = \chi^2_{\alpha, 1}$$

# “The Money Slide”

Suppose that  $X_1, X_2, \dots, X_n$  is a random sample from a distribution with pdf  $f(x; \theta)$ .

Suppose that  $\theta$  is not involved in the support of  $f$ .

Suppose that  $n$  is “large”.

An approximate GLRT of size  $\alpha$  for testing

$$H_0 : \theta = \theta_0 \quad \text{vs} \quad H_1 : \theta \neq \theta_0$$

is to reject  $H_0$  if  $-2 \ln \lambda(\vec{X}) > \chi_{\alpha, 1}^2$ .

## Example:

Suppose that  $X_1, X_2, \dots, X_n$  is a random sample from the continuous Pareto distribution with pdf

Find an approximate large sample GLRT of size  $\alpha = 0.05$  for

$$H_0 : \gamma = 1.8 \quad \text{vs} \quad H_1 : \gamma \neq 1.8$$

A likelihood is

$$L(\gamma) = \frac{\gamma^n}{\left[ \prod_{i=1}^n (1 + x_i) \right]^{\gamma+1}}$$

- The MLE for  $\gamma$  is

$$\hat{\gamma} = \frac{n}{\sum_{i=1}^n \ln(1 + X_i)}$$

- The restricted MLE for  $\gamma$  is

$$\hat{\gamma}_0 = 1.8$$

Compute  $\lambda(\vec{X}) = L(\gamma_0)/L(\vec{X})$ .

Compute  $-2 \ln \lambda(\vec{X})$ .

Reject  $H_0 : \gamma = 1.8$  if

$$-2 \ln \lambda(\vec{X}) > \chi_{\alpha,1}^2 = 3.841459$$