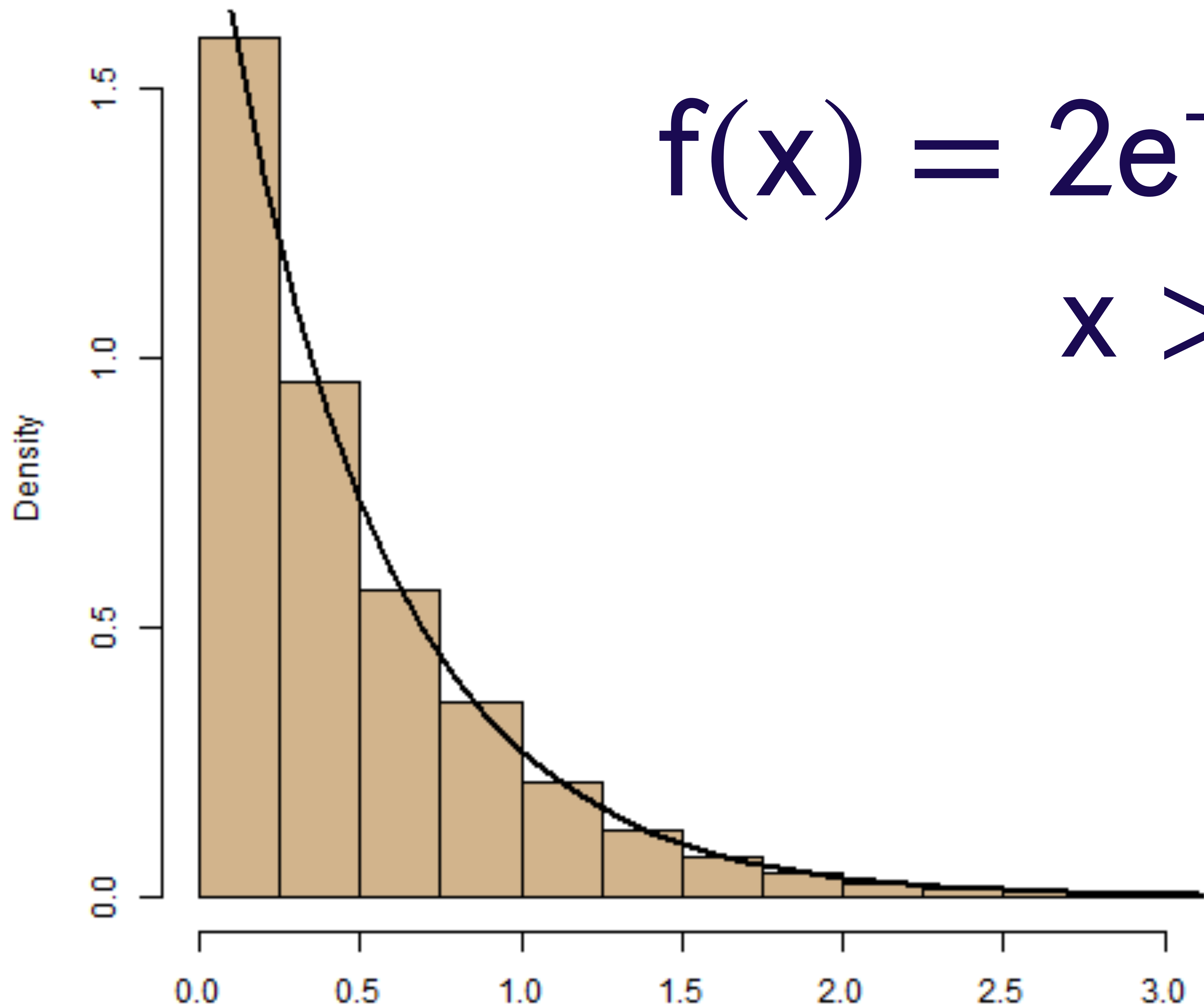
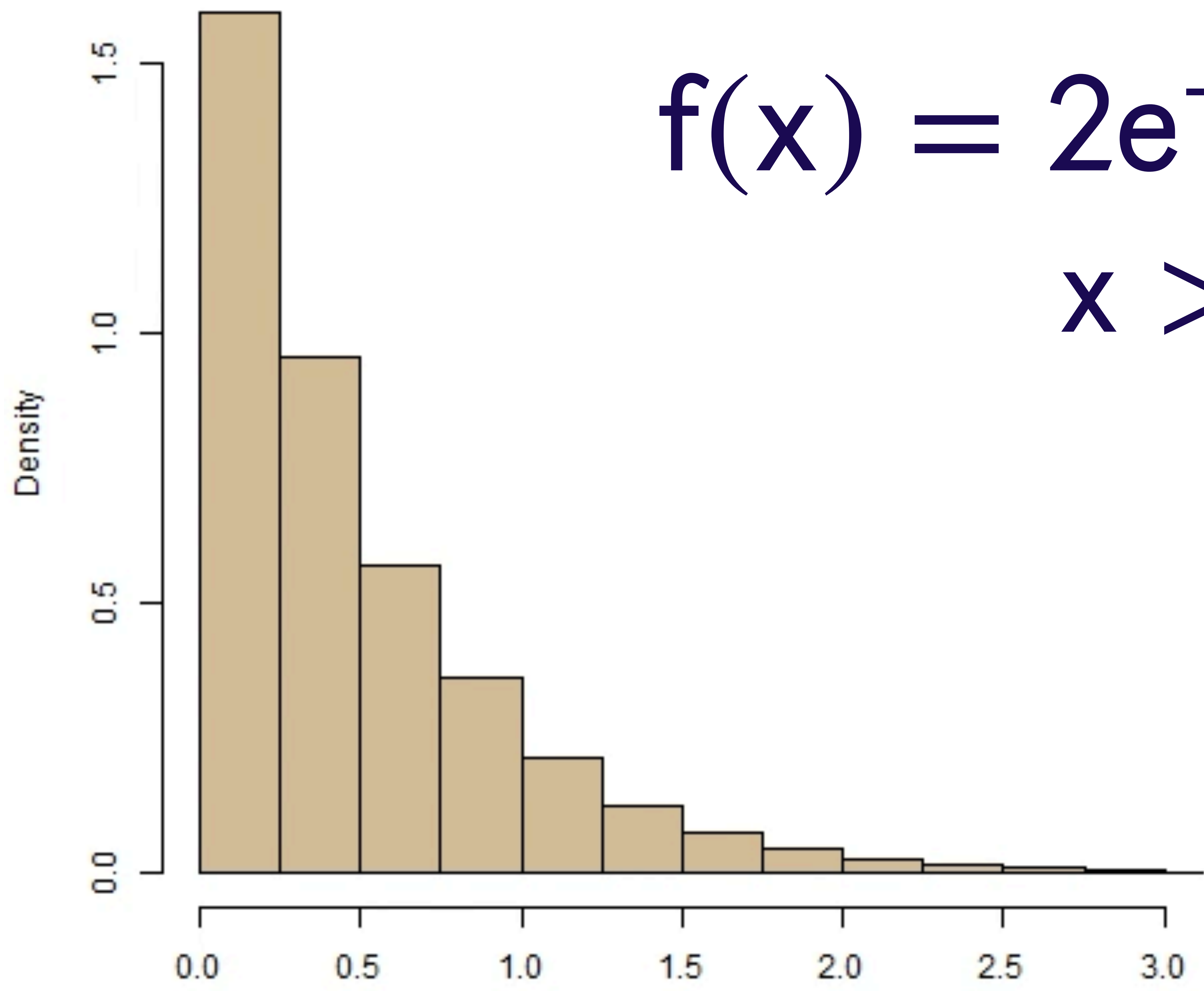
Let’s sample values from the exponential distribution with rate 2.

Let's sample values from the exponential distribution with rate 2.



$$f(x) = 2e^{-2x}$$
$$x > 0$$

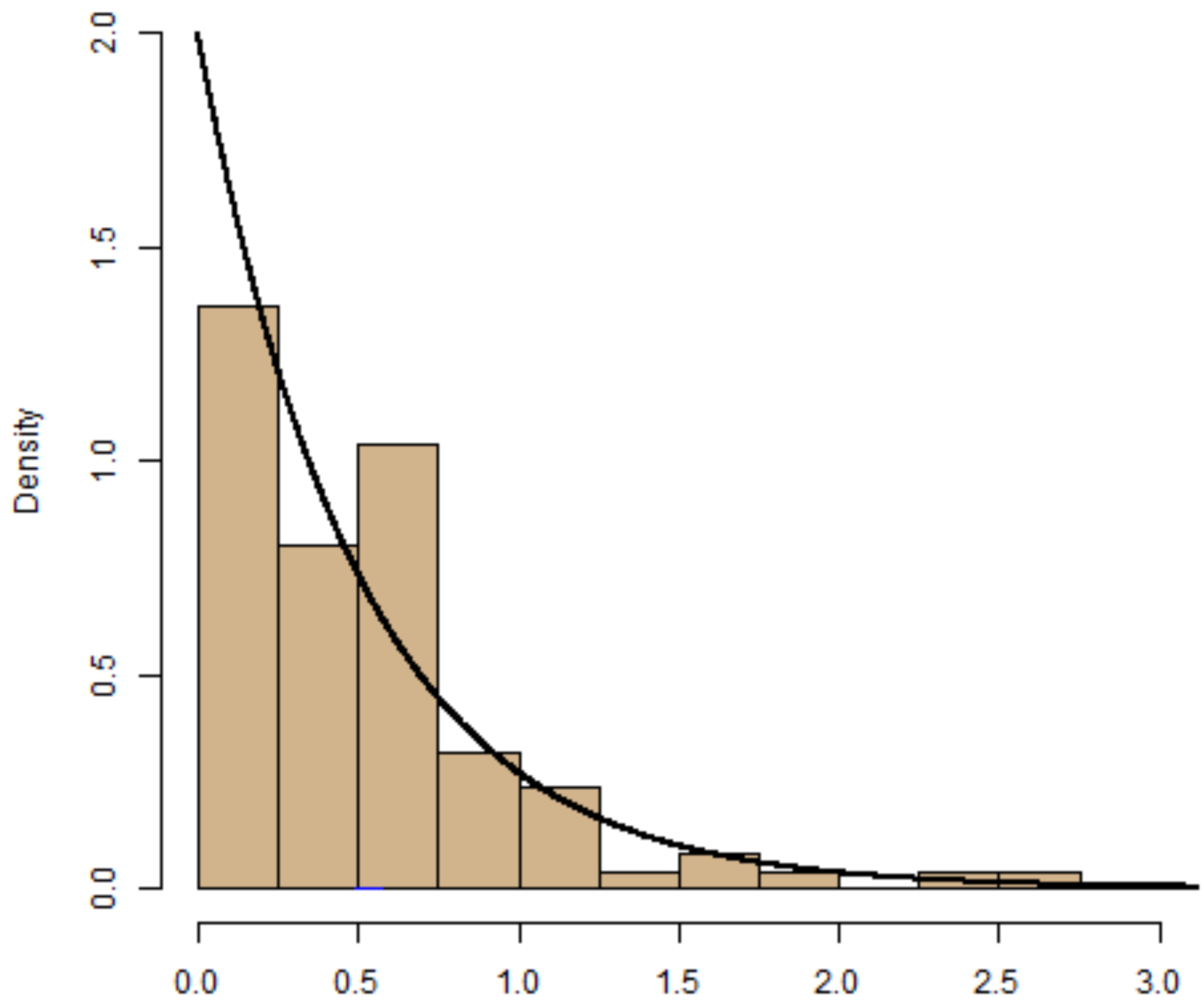
Let's sample values from the exponential distribution with rate 2.



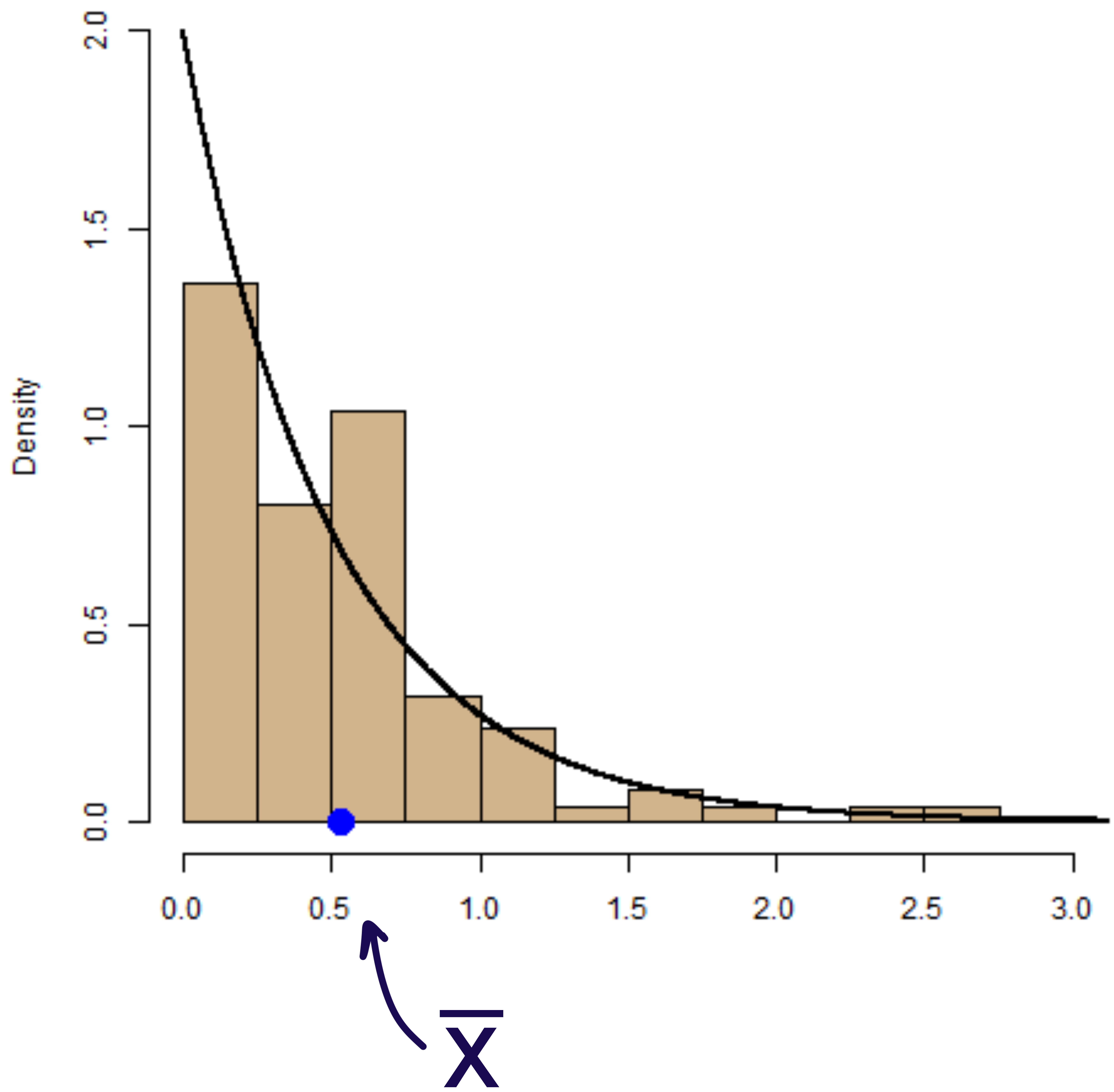
$$f(x) = 2e^{-2x}$$

$$x > 0$$

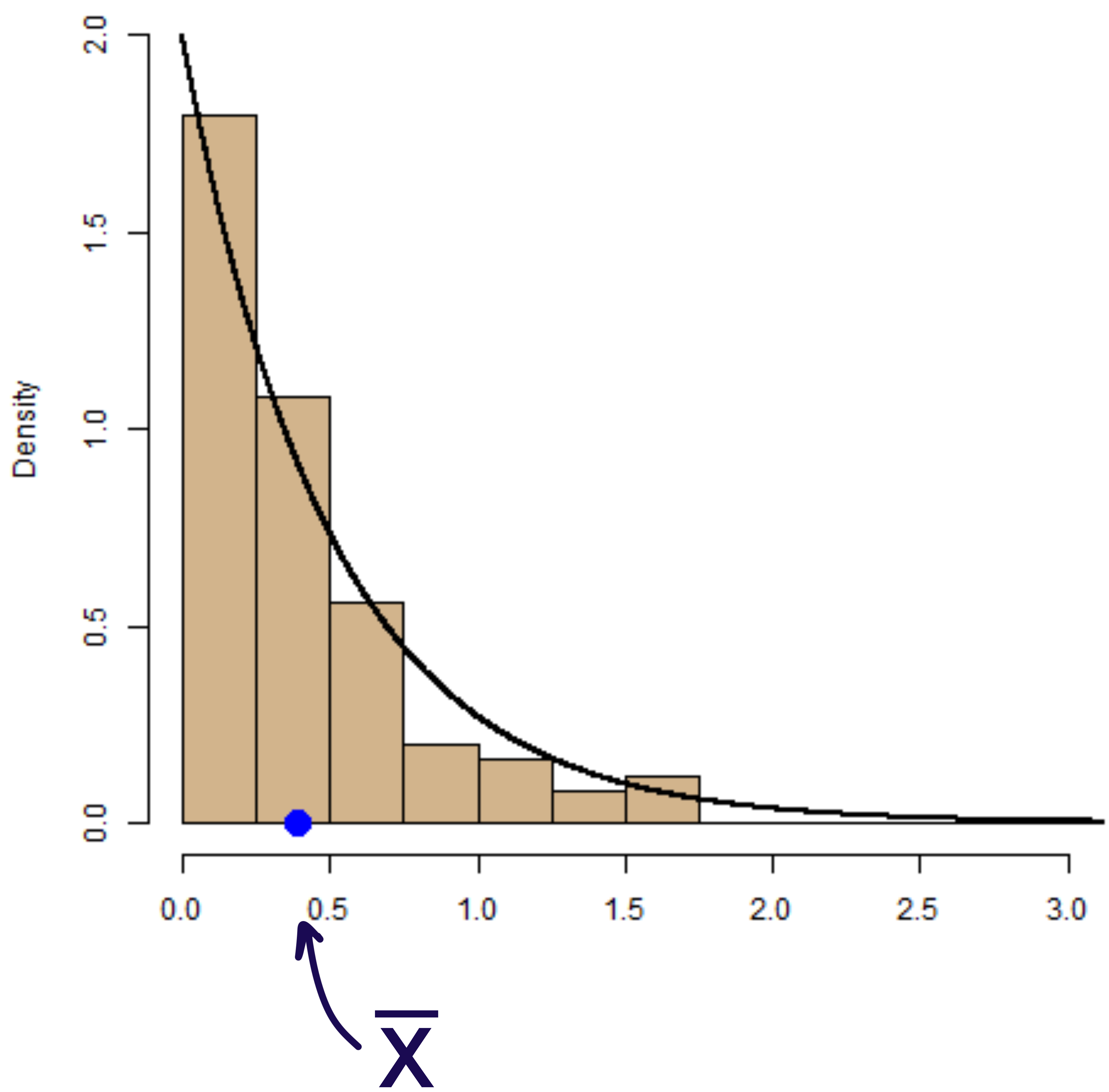
Let’s sample values from the exponential distribution with rate 2.



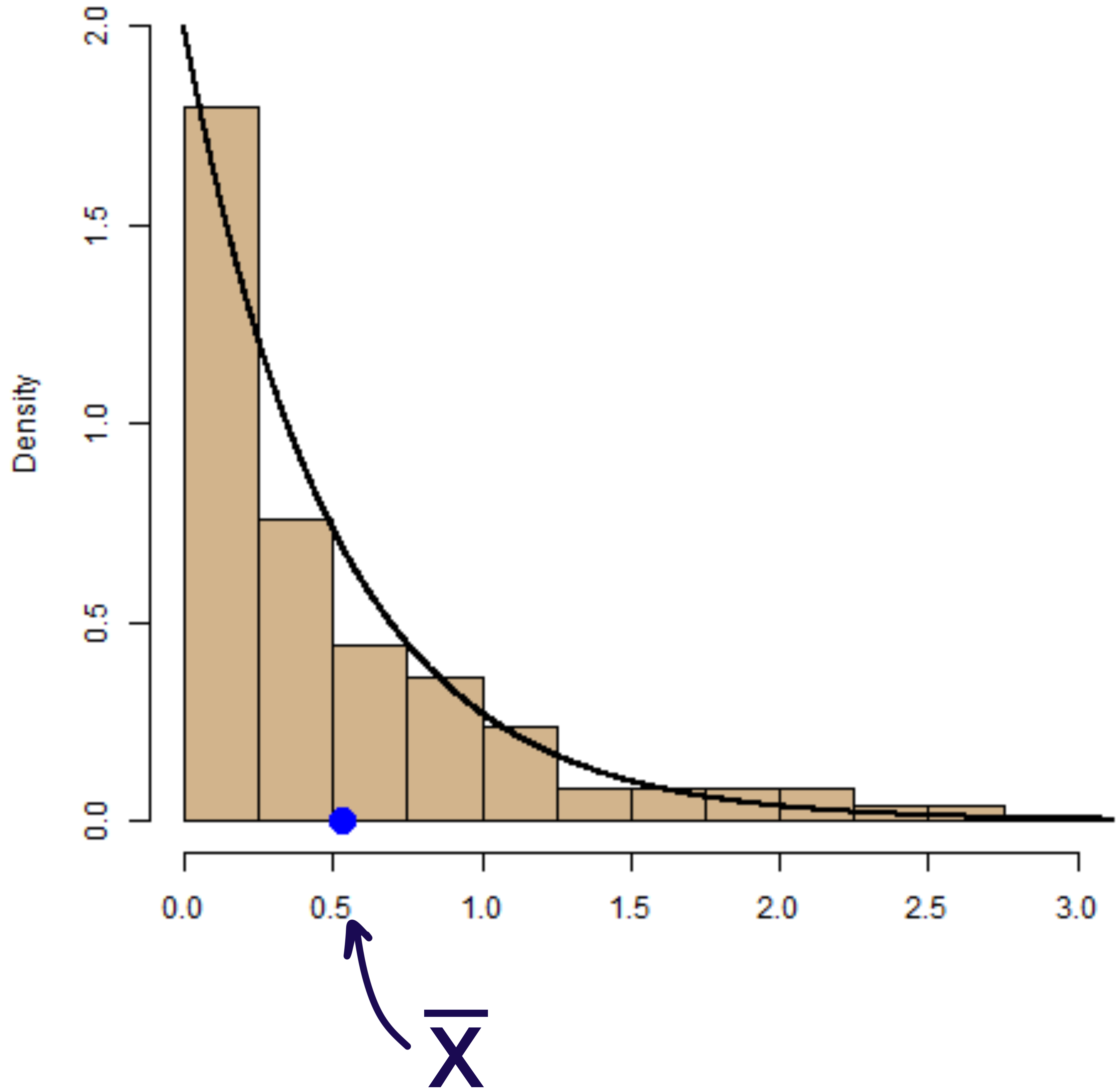
Let’s sample values from the exponential distribution with rate 2.



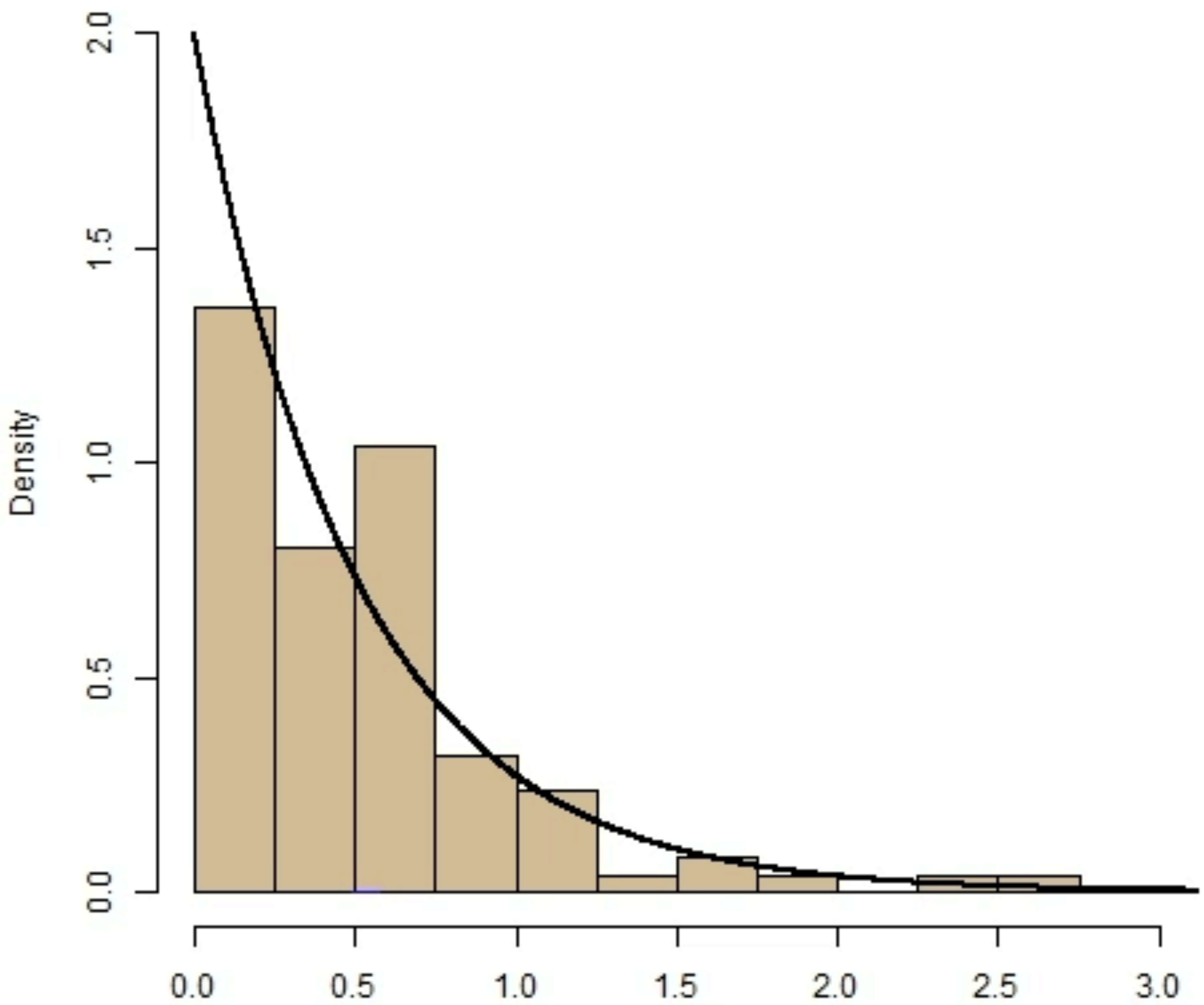
Let’s sample values from the exponential distribution with rate 2.



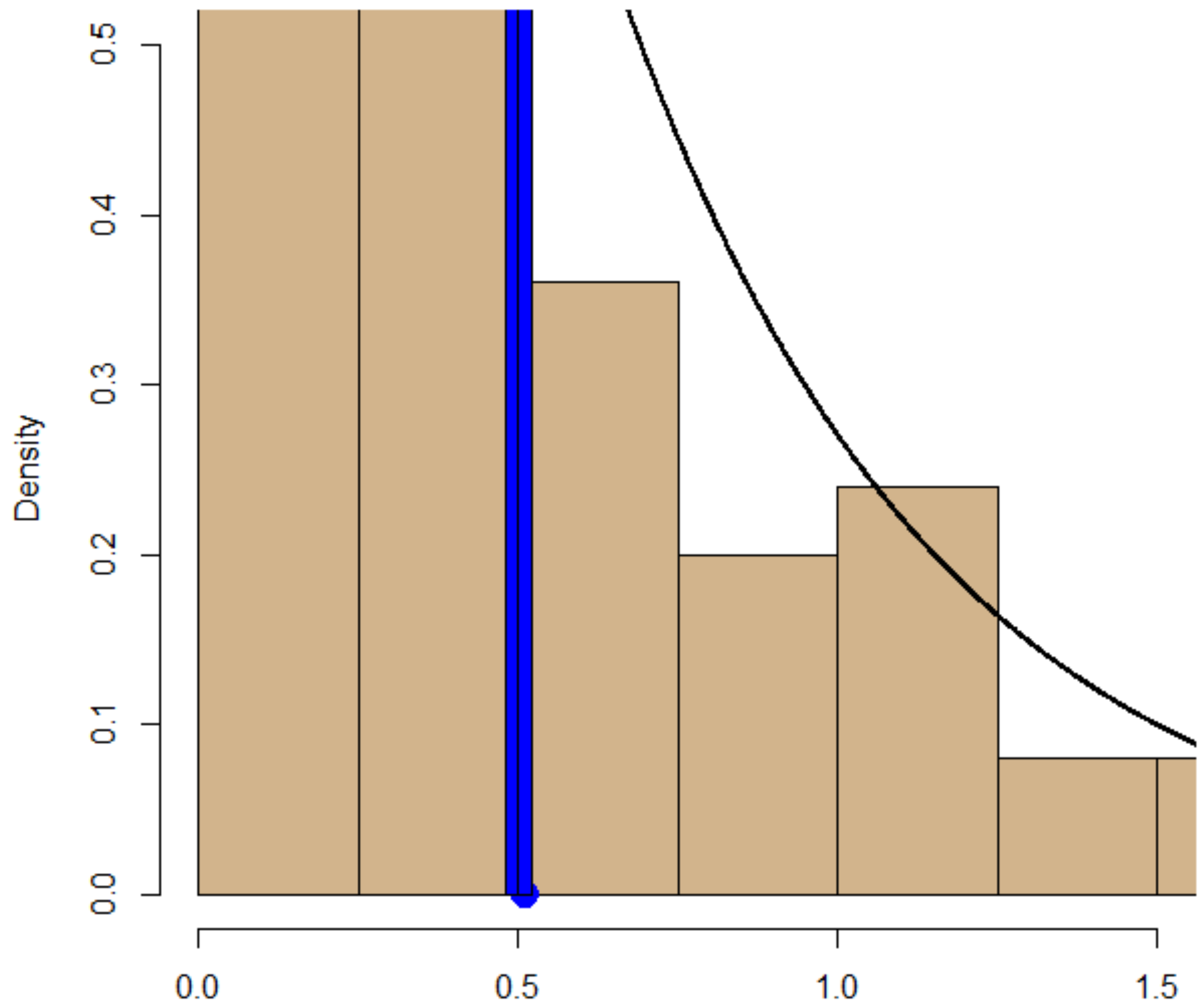
Let’s sample values from the exponential distribution with rate 2.



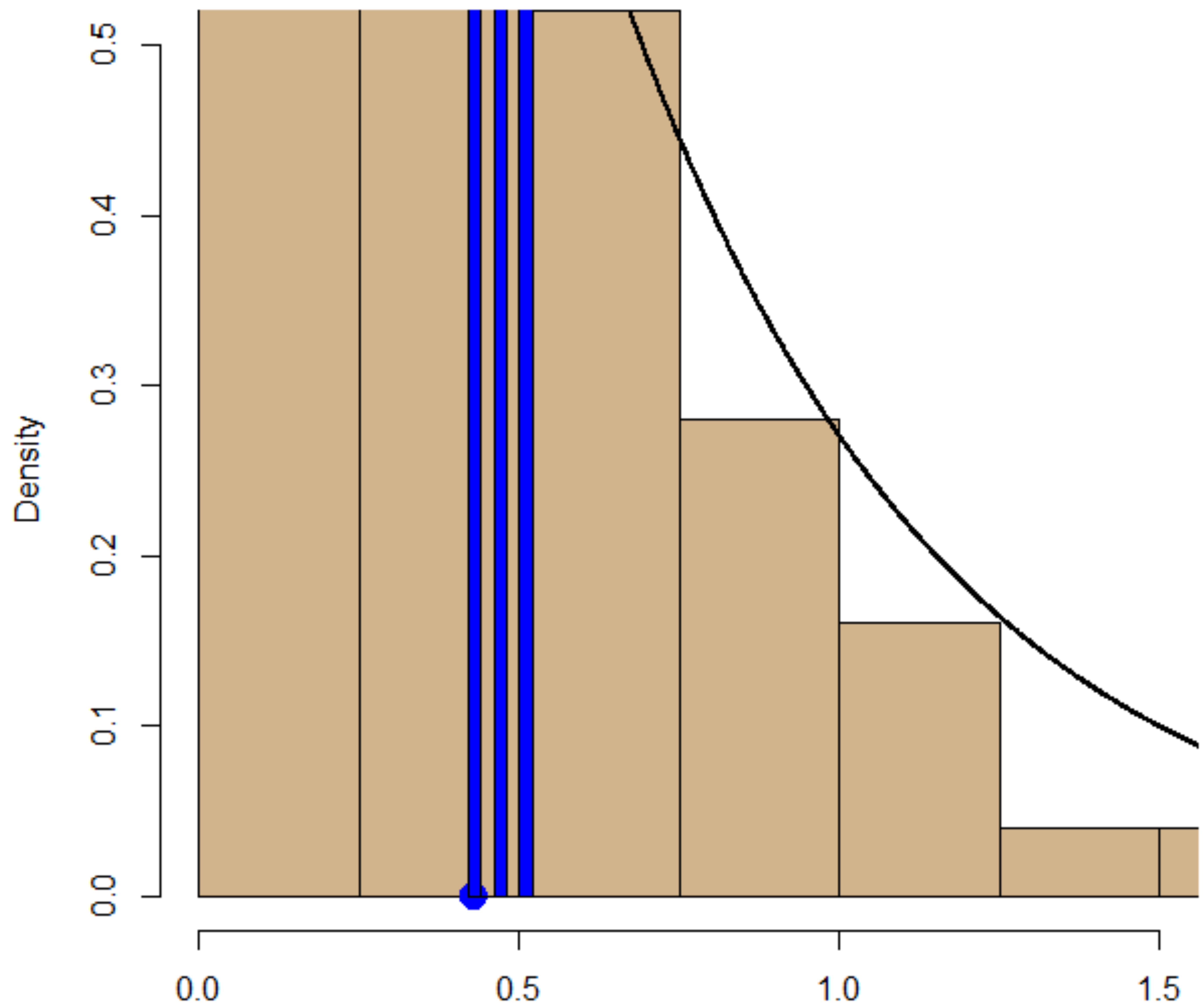
Let's sample values from the exponential distribution with rate 2.



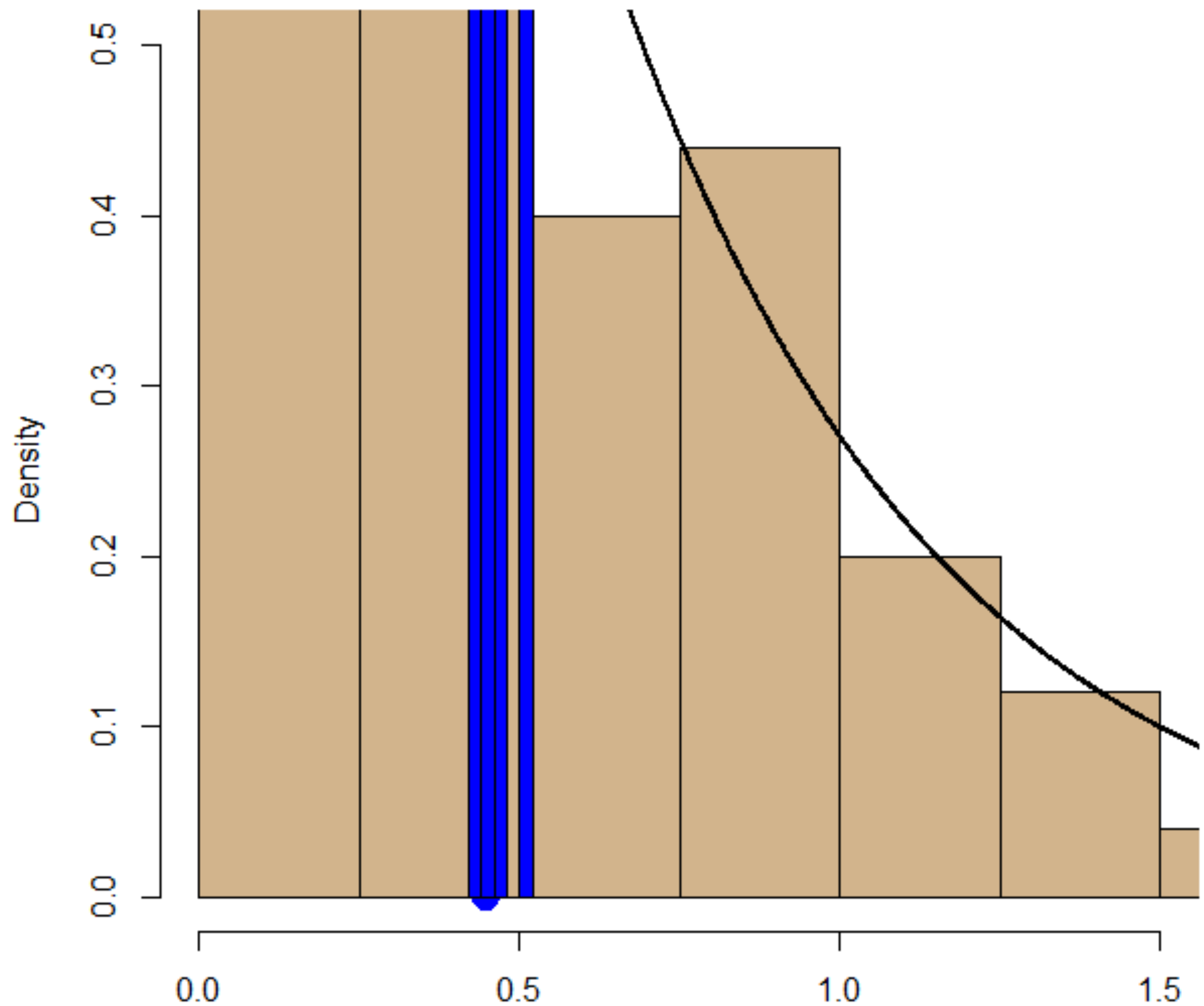
Let's sample values from the exponential distribution with rate 2.



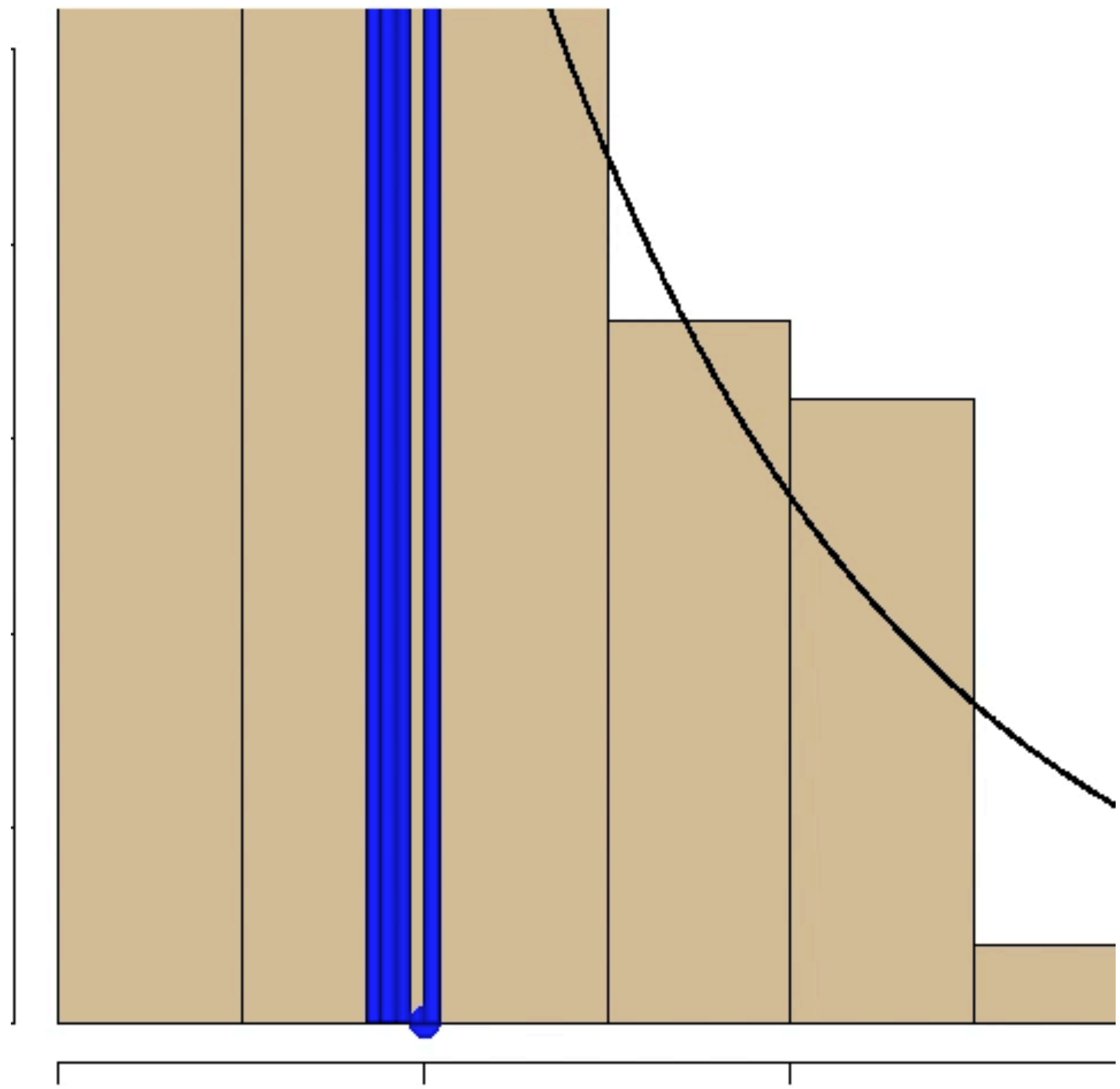
Let's sample values from the exponential distribution with rate 2.



Let’s sample values from the exponential distribution with rate 2.



Let’s sample values from the exponential distribution with rate 2.



Convergence in Distribution

Let $X_1, X_2, X_3, \ldots$ be a sequence of random variables and let X be another random variable.

If $$\lim_{n\to\infty} P(X_n \leq x) = P(X \leq x)$$

i.e. $$\lim_{n\to\infty} F_n(x) = F(x)$$

Then the sequence $\{X_n\}$ **converges in distribution** to X.

Notation: $X_n \xrightarrow{d} X$

The Central Limit Theorem

Let $X_1, X_2, X_3, \ldots$ be a sequence of random variables from any distribution with mean μ and variance $\sigma^2 < \infty$.

Let

$$\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i \qquad (\overline{X} = \overline{X}_n)$$

Then

$$\frac{\overline{X} - \mu}{\sigma/\sqrt{n}} \xrightarrow{d} N(0, 1)$$

$$\frac{\overline{X} - \mu}{\sigma/\sqrt{n}} \xrightarrow{d} N(0, 1)$$

We write

$$\overline{X} \overset{\text{asymp}}{\sim} N\left(\mu, \frac{\sigma^2}{n}\right)$$

In general, $X_n \sim N\left(a_n, b_n\right)$

means $$\frac{X_n - a_n}{\sqrt{b_n}} \xrightarrow{d} N(0, 1)$$

Note that $\dfrac{\overline{X} - \mu}{\sigma/\sqrt{n}} \xrightarrow{d} N(0, 1)$

$$\Rightarrow \quad \frac{\frac{1}{n}\sum_{i=1}^{n} X_i - \mu}{\sigma/\sqrt{n}} \xrightarrow{d} N(0, 1)$$

$$\Rightarrow \quad \frac{\sum_{i=1}^{n} X_i - n\mu}{\sigma\sqrt{n}} \xrightarrow{d} N(0, 1)$$

$$\Rightarrow \quad \sum_{i=1}^{n} X_i \overset{asymp}{\sim} N(n\mu, \sigma^2 n)$$

The Point?

Let $X_1, X_2, X_3, \ldots$ be a sequence of random variables from any distribution with mean μ and variance $\sigma^2 < \infty$.

An approximate, large sample, test of

$$H_0 : \mu \leq \mu_0 \qquad H_1 : \mu > \mu_0$$

is to reject H_0, in favor of H_1 if

$$\overline{X} > \mu_0 + z_\alpha \frac{\sigma}{\sqrt{n}}$$