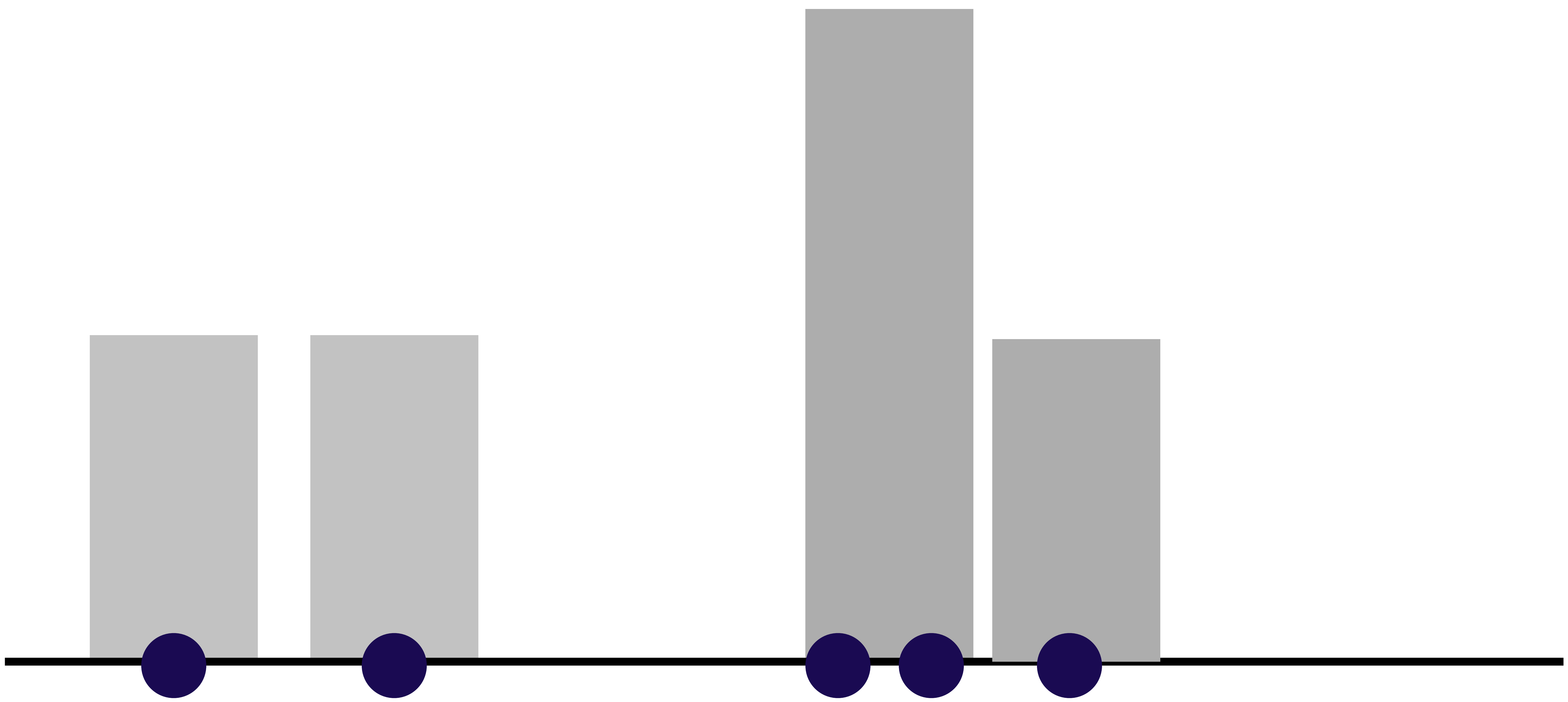
Let $X_1, X_2, ..., X_n$ be a random sample from the normal distribution with mean μ and variance σ^2.



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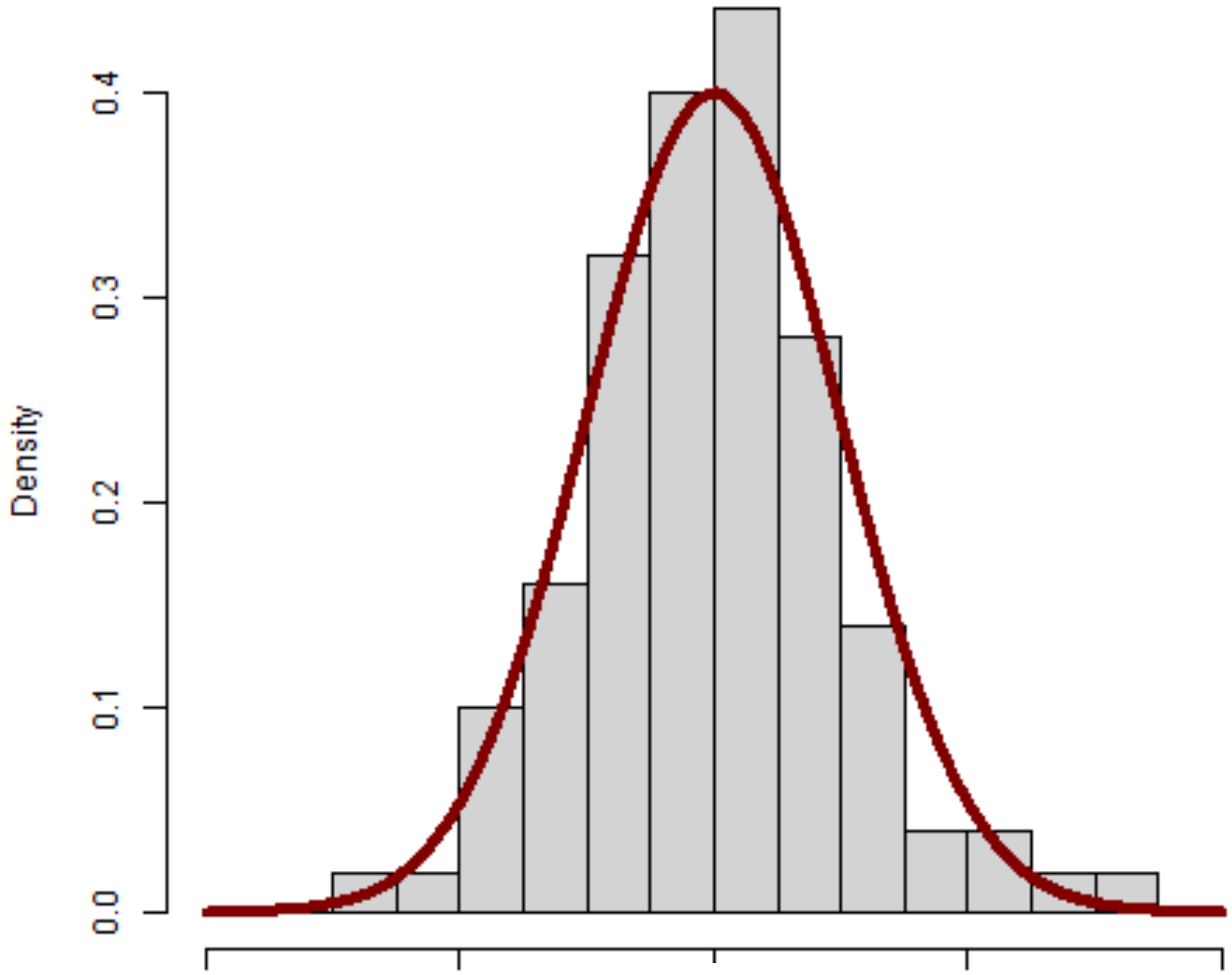

random sample

independent and identically distributed (iid)

Example of random sample after it is observed:

$$2.73, 1.14, 3.98, 2.15, 5,85, 1.97, 2.54, 2.03$$

Example of random sample before it is observed:

$$X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8$$

Example of random sample after it is observed:

$$2.73, 1.14, 3.98, 2.15, 5,85, 1.97, 2.54, 2.03$$

Based on what you are seeing, do you believe that the true population mean μ is

$$\mu <= 3$$

or

$$\mu > 3$$

The sample mean is $\bar{x} = 2.799.$

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This is below 3, but can we say that $\mu < 3$?

This seems awfully dependent on the random sample we happened to get!

Let's try to work with the most generic random sample of size 8:

$$X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8$$

Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from the $N(\mu, \sigma^2)$ distribution.

We say that

$$X_1, X_2, \ldots, X_n \overset{iid}{\sim} N(\mu, \sigma^2)\,.$$

The sample mean is

$$\overline{X} = \frac{1}{n}\sum_{i=11}^{n} X_i$$

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- We're going to tend to think that $\mu < 3$ when $\overline{X}$ is "significantly" smaller than 3.
- We're going to tend to think that $\mu > 3$ when $\overline{X}$ is "significantly" larger than 3.
- We're never going to observe $\overline{X} = 3$, but we may be able to be convinced that $\mu = 3$ if $\overline{X}$ is not too far away.

Terminology/Notation

Hypotheses:

$$H_0 : \mu \leq 3$$ null hypothesis

$$H_1 : \mu > 3$$ alternate hypothesis

(Also denoted by H_a.)

$$H_0 : \mu \leq 3$$ null hypothesis

$$H_1 : \mu > 3$$ alternate hypothesis

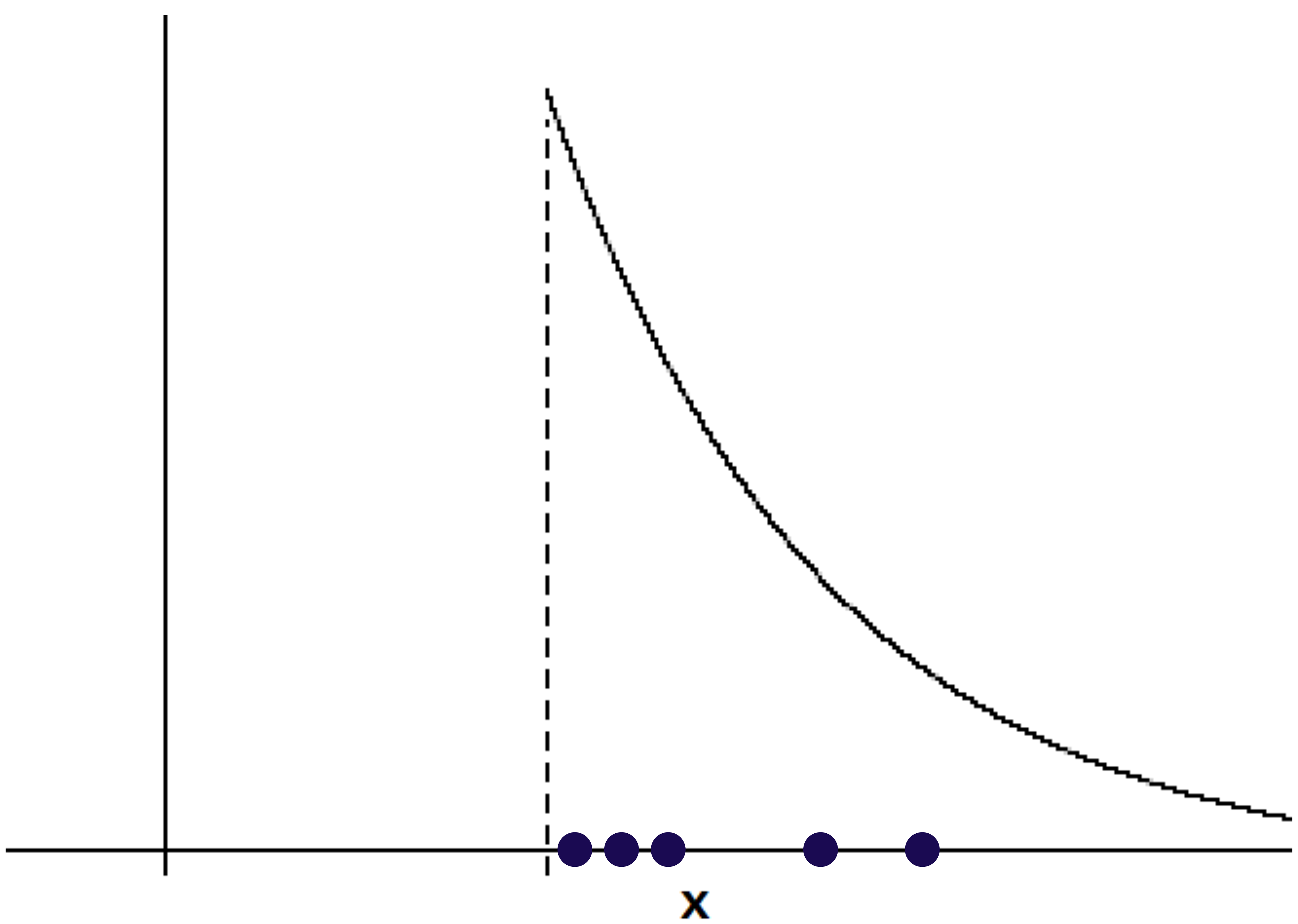
- The null hypothesis is assumed to be true.
- The alternate hypothesis is what we are out to show.

Conclusion is either:

$\boxed{\text{Reject } H_0}$ OR $\boxed{\text{Fail to Reject } H_0.}$

Suppose that $X_1, X_2, \ldots, X_n$ is a random sample from a continuous distribution with probability density function (pdf)

$$f(x;\theta) = \begin{cases} e^{-(x-\theta)} & , x \geq \theta \\ 0 & , x < \theta \end{cases}$$



Want to test

$$H_0 : \theta \geq 1$$

versus

$$H_1 : \theta < 1$$

A simplified set of hypotheses:

$$\left.\begin{aligned} H_0 &: \mu = 3 \\ H_1 &: \mu > 3 \end{aligned}\right\} \text{all possibilities in the parameter space}$$

Suppose you observe

$$\bar{x} = -59,349,348$$

Then you probably will fail to reject H_0.

Let $X_1, X_2, \ldots, X_n$ be a random sample from the normal distribution with mean μ and variance σ^2.

Suppose that the variance σ^2 is known.

$$H_0 : \mu = 3$$

is called a simple hypothesis.

$$H_0 : \mu \leq 3$$

is called a composite hypothesis.

Let $X_1, X_2, \ldots, X_n$ be a random sample from the normal distribution with mean μ and variance σ^2.

$$H_0 : \mu = 3$$

is a composite hypothesis!