Coursework for
4G3 Computational Neuroscience
Assignment 2

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assigned: March 14, 2025
due: April 4, 2025, 4pm
submit through Moodle
don't forget to attach the correct cover page

In this handout,

- text in blue corresponds to things you have to do / implement. To perform your simulations, you may use any programming language of your choice.
- text in orange corresponds to questions you need to answer in some form in your report.

Please write up your findings in a report to be submitted on Moodle in PDF format, and *include all your code in the Appendix*. Please do not forget to include the correct cover page (which depends on your course, see the cover page section on Moodle) and clearly include either your candidate number or your name on it as required by the form.

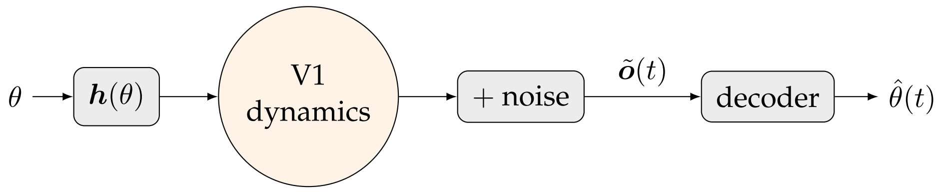
Your report should address all the questions raised in this handout, be structured around the Sections herein, and ****be a maximum of five A4 pages**** excluding any Appendix (minimum font size 11pt, minimum margins 1.5cm on each side), and you are encouraged to make your submission shorter than this maximum as long as you solve all exercises). You are very much encouraged to think of data/results visualisations to best support the exposition and interpretation of your results.

Please make sure to include a statement at the beginning of your report declaring whether you used generative AI tools (e.g. ChatGPT) for your simulations and / or writing your report. If you did, prepare an Appendix including all prompts you entered while using these tools in connection with this Assignment (including for literature search, explaining scientific concepts, help with code writing, help with writing / polishing the text or creating / polishing the figures of the report). When using Copilot (or similar), please report the fact that you used it but you do not need to report individual auto-fill prompts. As a word of caution, use these tools with great care: you will be responsible for any information you present and, if asked, you will need to be able to give evidence that you understand what is in your report.

Question I. Recurrent neural network dynamics **[30 points]**

Setup

This coursework is a basic study of how well a recurrent network model of primary visual cortex (V1) can represent a specific feature (in this case, the orientation) of a brief visual stimulus, in the face of readout noise, depending on its connectivity. The basic setup you will investigate is depicted in the following diagram:



In broad strokes, the stimulus orientation θ is encoded into V1 activity, which is then decoded into an estimate $\hat{\theta}$ which we hope to be as close as possible to θ. Noise is injected in the output, thus corrupting the representation of θ and giving rise to reconstruction errors ($\hat{\theta} \neq \theta$). The question we ask here is: how does the structure of recurrent connectivity in the V1 network affect the reconstruction error?

Details The default values of the various parameters used in the following are given in Table 1. Let $\{\phi_i = \frac{2\pi i}{m}\}$ be a grid of m regularly spaced orientations[1]. The stimulus orientation θ, also measured in radians between 0 and 2π, is first encoded into a vector $\boldsymbol{h}(\theta) \in \mathbb{R}^m$ with the i^{th} element given by

$$h_i(\theta) = \mathcal{V}(\phi_i - \theta) \quad \text{with} \quad \mathcal{V}(z) = \exp\left(\frac{\cos(z) - 1}{\kappa^2}\right). \tag{1}$$

This vector is then used as a pulse of input driving the dynamics of V1[2]:

$$\tau \frac{d\boldsymbol{r}}{dt} = -\boldsymbol{r} + W\boldsymbol{r} + B\boldsymbol{h}(\theta)\delta(t) \tag{2}$$

where $\boldsymbol{r}(t) \in \mathbb{R}^n$ is a vector whose i^{th} element $r_i(t)$ represents the momentary firing rate of V1 neuron i (relative to some positive baseline), $B \in \mathbb{R}^{n \times m}$ is a matrix of feedforward input weights, W is a matrix of recurrent connection strengths, τ is the characteristic neuronal time constant, and $\delta(t)$ is the standard Dirac delta function. Thus, the input pulse is delivered at time $t = 0$, with $\boldsymbol{r}(t < 0) = 0$.

A noisy readout of V1 activity can be obtained at any time t in the form

$$\tilde{\boldsymbol{o}}(t) = C\boldsymbol{r}(t) + \sigma\boldsymbol{\varepsilon}(t) \tag{3}$$

[1] Even though an "orientation" technically spans the $[0, \pi]$ range and wraps around at π, here – for mathematical and programming convenience – we work with the full $[0 : 2\pi]$ range of "directions" but still call them "orientations" to facilitate biological interpretation.

[2] In your simulations, you will need to discretize time using a time step Δ (I suggest $\Delta = 0.001$ seconds). Simple Euler integration of Equation 2 should suffice.

τ	m	n	B	C	σ	κ	α	α'
20 ms	200	200	I	I	1	$\pi/4$	0.9	0.9

Table 1. Default parameters to be used unless otherwise noted. I denotes the identity matrix.

where $C \in \mathbb{R}^{m\times n}$ is a matrix of output weights and each element of $\boldsymbol{\varepsilon}(t)$ is drawn from $\mathcal{N}(0,1)$ independently (across vector elements, time, and trials). Finally, this noisy readout is decoded into a momentary estimate of θ according to[3]

$$\hat{\theta}(t) = \text{atan}\left(\frac{\sum_i \tilde{o}_i(t)\sin\phi_i}{\sum_i \tilde{o}_i(t)\cos\phi_i}\right) \tag{4}$$

and the circular distance $d(\hat{\theta},\theta) = \cos^{-1}(\cos(\hat{\theta}-\theta))$ is used to measure the decoding error.

We will consider four models for V1 connectivity. In the following description, we denote by $\mathcal{R}(W,\alpha)$ the rescaling of W by a scalar that sets its spectral abscissa (largest real part in its eigenvalue spectrum) to a desired α.

Model 1: no recurrence For this model, $n = m$ and $W^{(1)} = 0_m$ (the $m \times m$ matrix full of zeros).

Model 2: random symmetric connectivity For this model, $n = m$ and $W^{(2)} = \mathcal{R}(\tilde{W} + \tilde{W}^\top, \alpha)$ with $\tilde{W}_{ij} \sim \mathcal{N}(0,1)$ i.i.d. Note that those random weights should be generated once and for all before any simulation of the network dynamics with various θ's.

Model 3: symmetric ring structure For this model, $n = m$ and $W^{(3)} = \mathcal{R}(\tilde{W},\alpha)$ with $W_{ij} = \mathcal{V}(\phi_i - \phi_j)$, where $\{\phi_i\}$ and $\mathcal{V}(\cdot)$ have been defined previously (Equation 1 and above).

Model 4: balanced ring structure For this model, $n = 2m$, and $W^{(4)} = \begin{pmatrix} \tilde{W} & -\tilde{W} \\ \tilde{W} & -\tilde{W} \end{pmatrix}$ with $\tilde{W}_{ij} = \mathcal{R}(W^{(3)},\alpha')$. Moreover, given that $n = 2m$, one can no longer use $B = C = I$ as in Table 1. Instead, you will use

$$B = \begin{pmatrix} I_m \\ 0_m \end{pmatrix} \text{ and } C = (I_m, 0_m)\,. \tag{5}$$

where I_m denotes the $m \times m$ identity matrix.

Questions

Question 1. Integrate the dynamics of each of the 4 models with their default parameters for $\theta = \pi$, and show the corresponding response $\boldsymbol{r}(t)$ at $t = \{0^+, 20, 60\}$ ms.

[3] You might want to look up the standard `atan2` function.

Question 2. The responses of Model 2 should look a lot "noisier" across the V1 population than in the other 3 models. By using a combination of analytical derivations (including the eigendecomposition of the $W^{(2)}$ matrix) and any additional numerics you deem useful, provide a rational explanation for this phenomenon.

Question 3. You should also find that the response of Model 3 at 60 ms is stronger than for the other models. By using a combination of analytical derivations (including the eigendecomposition of the $W^{(3)}$ matrix) and any additional numerics you deem useful, provide a rational explanation for this phenomenon. In particular, why would you expect Model 3's response to be larger than Model 2's at 60 ms?

Question 4. For each model, compute and show the time course (60 ms) of the corresponding decoding error, averaged over repeated trials, with $\theta = \pi$. Based on your answers so far, explain your decoding accuracy results and in particular how accuracy differs across models.

Question 5. Revisit the behaviour of Model 4, now setting $\alpha' = 5$ (instead of 0.9 previously). You should find that the reconstruction of θ is now the best amongst the four models in the $[0 : 60]$ ms time interval. Explain this phenomenon.

Question 6. What happens in Models 2 and 3 if you similarly set $\alpha = 5$? (instead of 0.9 previously). Why?

Question 7. How could Model 4's input weights B be chosen differently from Equation 5 (other than through mere rescaling) to further improve reconstruction accuracy in that model? Justify your suggestion and try it out.

Question 8. It appears that the coding fidelity of Model 4 can be made arbitrarily good by increasing α'. Where is the catch?

Question 9. Briefly conclude by discussing the biological implications of your results, in particular in regard to the putative role of excitation-inhibition balance in the speed and precision of orientation coding in V1.