Chapter 2

Yi Yao

September 21, 2023

1 Continuous Compounding

1.1 Notation

The following notation will be used:

- $f(t)$: Balance at time t
- r: Continuous compounding interest rate

1.2 Derivation of $f(t)$

At any given time t, accrued interest is $rf(t)$. Thus:

$$\frac{df(t)}{dt} = rf(t)$$

Therefore:

$$\begin{array}{lcl}
\frac{1}{f(t)}\frac{df(t)}{dt} & = & r \\
\int \frac{1}{f(t)}\frac{df(t)}{dt}dt & = & \int rdt \\
\int \frac{1}{f(t)}df(t) & = & rt \\
\ln f(t) & = & rt \\
f(t) & = & e^{rt}
\end{array}$$

2 Geometric Sequence Sum

2.1 Proof

Prove that

$$\sum_{t=a}^{b} z^t = \frac{z^a - z^{b+1}}{1-z}$$

Proof.

$$\sum_{t=a}^{b} z^t = z^a + z^{a+1} + \cdots + z^b$$

$$z\sum_{t=a}^{b} z^t = z^{a+1} + z^{a+2} + \cdots + z^{b+1}$$

$\therefore$

$$\begin{array}{cl}
 & z\sum_{t=a}^{b} z^t - \sum_{t=a}^{b} z^t \\
= & z^{b+1} + (z^b - z^b) + \cdots + (z^{a+1} - z^{a+1}) - z^a \\
= & z^{b+1} - z^a
\end{array}$$

$\therefore$

$$\begin{aligned}
(z-1)\sum_{t=a}^{b} z^t &= z^{b+1} - z^a \\
\sum_{t=a}^{b} z^t &= \tfrac{z^{b+1}-z^a}{z-1} \\
&= \tfrac{z^a-z^{b+1}}{1-z}
\end{aligned}$$

$\square$