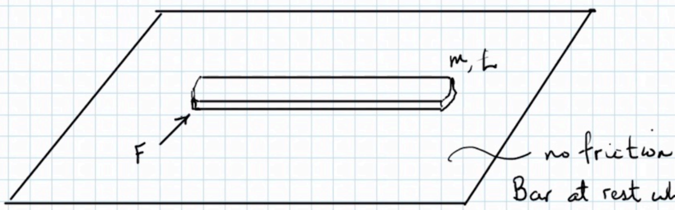
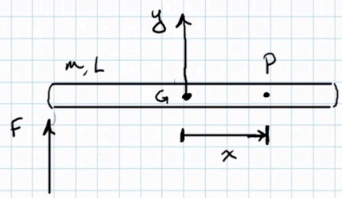
Dynamics - Lecture #17

Zoom meeting open for asking questions

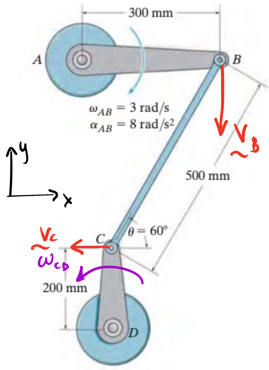
Check blackboard for procedure for exam



Find initial acceleration of point P when force F is applied.



16-123 Determine velocity and acceleration of pt. C at instant shown.



Link AB: $\underset{\sim}{V_B} = \cancel{\underset{\sim}{V_A}} + \underset{\sim}{\omega_{AB}} \times \underset{\sim}{r_{AB}}$ (1)

Link CD: $\underset{\sim}{V_C} = \cancel{\underset{\sim}{V_D}} + \underset{\sim}{\omega_{CD}} \times \underset{\sim}{r_{DC}}$ (2)

Link BC: $\underset{\sim}{V_B} = \underset{\sim}{V_C} + \underset{\sim}{\omega_{BC}} \times \underset{\sim}{r_{BC}}$ (3)

In (3), unknowns: $\underset{\sim}{V_C}$, $\underset{\sim}{\omega_{BC}}$

$\underset{\sim}{V_C} = -V_C \underset{\sim}{i}$ and $\underset{\sim}{\omega_{BC}} = \omega_{BC} \underset{\sim}{k}$ — we don't know exact direction yet

In (3), two scalar unknowns

Find V_C and ω_{BC} from (3)

Link AB: $\underset{\sim}{a_B} = \cancel{\underset{\sim}{a_A}}^{0} + \underset{\sim}{\alpha_{AB}} \times \underset{\sim}{r_{AB}} + \underset{\sim}{\omega_{AB}} \times (\underset{\sim}{\omega_{AB}} \times \underset{\sim}{r_{AB}}) \Rightarrow \underset{\sim}{a_B} = \checkmark$

Direction of acceleration of C?

→ C moves in circle since D fixed pt.

– so there's tangential + normal acceleration

$\underset{\sim}{a_C} = \underbrace{-r_{CD}\alpha_{CD}\underset{\sim}{i}}_{\text{tangential}} - \underbrace{\omega_{CD}^2 r_{CD}\underset{\sim}{j}}_{\text{normal}}$ $\qquad \omega_{CD} = \frac{V_C}{r_{CD}}$ from (2)

Link BC: $\underset{\sim}{a_B} = \underset{\sim}{a_C} + \underset{\sim}{\alpha_{BC}} \times \underset{\sim}{r_{BC}} + \underset{\sim}{\omega_{BC}} \times (\underset{\sim}{\omega_{BC}} \times \underset{\sim}{r_{BC}}) \longrightarrow$ unknowns: α_{CD}, α_{BC}

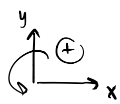
16-77

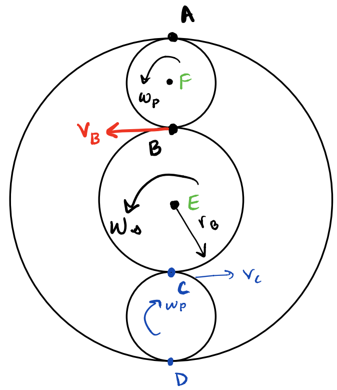
ω_s given

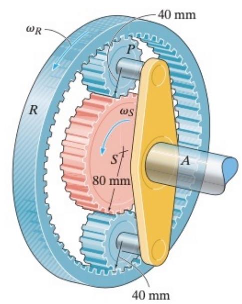
$V_A = 0$

b/c rolls on fixed surface

$V_B = \omega_s r_B$







planetary gear:

ω_P = angular velocity of planetary gear

$$\mathbf{V}_B = \mathbf{V}_A + \boldsymbol{\omega}_P \times \mathbf{r}_{AB}$$

$$-\omega_s r_B \,\mathbf{i} = \omega_P \mathbf{k} \times -2r_P \mathbf{j}$$

$$= 2 r_P \omega_P \mathbf{i}$$

$$\omega_P = -\omega_s \frac{r_B}{2r_P} \quad (\omega_P \text{ is } \underline{\text{clockwise}})$$

which way is pt C moving? to the right!

$\longrightarrow V_C$

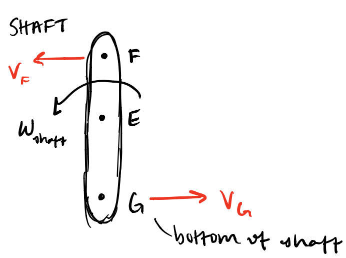
... ∴ planetary gear at bottom also clockwise

... and pt D @ the bottom is instantaneously fixed

planetary gear: $\mathbf{V}_F = \mathbf{V}_A + \boldsymbol{\omega}_P \times \mathbf{r}_{AF}$

$$= -\omega_s \frac{r_B}{2r_P} \mathbf{k} \times -r_P \mathbf{j} = -\frac{1}{2}\omega_s r_B \,\mathbf{i}$$

why is $\mathbf{V}_F$ ½ as much as $\mathbf{V}_B$? because B is twice as far as F from pt. A

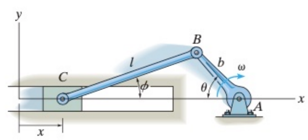


$$\omega_{shaft} = \frac{V_F}{r_{EF}} = \frac{V_F}{r_s + r_P}$$

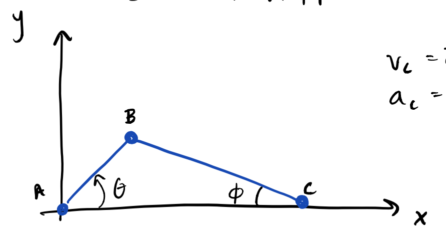
14-21 → see homework #3

16-52

***16–52.** The crank AB has a constant angular velocity $\boldsymbol{\omega}$. Determine the velocity and acceleration of the slider at C as a function of θ. *Suggestion:* Use the x coordinate to express the motion of C and the ϕ coordinate for CB. $x = 0$ when $\phi = 0°$.



Adjust coordinate system:
AB = crankshaft



$v_c = ?$
$a_c = ?$

$$x_c = b\cos\theta + l\cos\phi$$

$$v_c = \frac{dx_c}{dt} = -b\sin\theta\,\dot{\theta} - l\sin\phi\,\dot{\phi}$$

$\dot{\theta} = \omega$ law of sines: $\frac{\sin\theta}{l} = \frac{\sin\phi}{b} \Rightarrow \sin\phi = \frac{b}{l}\sin\theta$

take $\frac{d}{dt}$

$$\frac{d}{dt}(\sin\phi) = \frac{d}{dt}\left(\frac{b}{l}\sin\theta\right)$$

$$\cos\phi\,\dot{\phi} = \frac{b}{l}\cos\theta\,\dot{\theta}$$

$$\dot{\phi} = \frac{b}{l}\dot{\theta}\frac{\cos\theta}{\cos\phi}$$

$$\dot{\phi} = \frac{(b/l)\,\dot{\theta}\cos\theta}{\sqrt{1-\frac{b^2}{l^2}\sin^2\theta}}$$

$\sin\phi$ $\dot{\phi}$

$$v_c = -b\sin\theta\,\omega - l\left(\frac{b}{l}\sin\theta\right)\left(\frac{(b/l)\cos\theta\,\omega}{\sqrt{1-\frac{b^2}{l^2}\sin^2\theta}}\right)$$

$$a_c = -b\sin\theta\,\ddot{\theta} - b\cos\theta\,\dot{\theta}^2 - l\cos\phi\,\dot{\phi}^2 - l\sin\phi\,\ddot{\phi}$$

if $\omega = \dot{\theta} = \text{constant}$ then $\ddot{\theta} = \phi$ but can't assume $\ddot{\phi} = \phi$

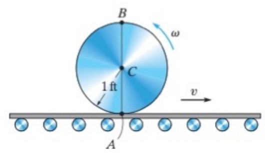
from before . . .

$$\cos\phi\,\ddot{\phi} - \sin\phi\,\dot{\phi}^2 = -\frac{b}{l}\sin\theta\,\dot{\theta}^2 + \frac{b}{l}\cos\theta\,\ddot{\theta}$$

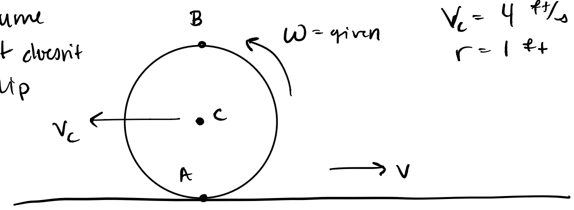
$$\ddot{\phi} = \frac{-b/l\,\sin\theta\,\dot{\theta} + \sin\phi\,\dot{\phi}^2}{\cos\phi}$$

16-85

16–85. The conveyor belt is moving to the right at $v = 12$ ft/s, and at the same instant the cylinder is rolling counterclockwise at $\omega = 6$ rad/s while its center has a velocity of 4 ft/s to the left. Determine the velocities of points A and B on the disk at this instant. Does the cylinder slip on the conveyor?



do not assume that doesn't slip



$V_c = 4$ ft/s
$r = 1$ ft

$$\underset{\sim}{V}_B = \underset{\sim}{V}_c + \underset{\sim}{\omega} \times \underset{\sim}{r}_{CB} = -4\,\tfrac{ft}{s}\,\underset{\sim}{i} + 6\,\tfrac{rad}{s}\,\underset{\sim}{k} \times 1\text{ ft}\,\underset{\sim}{j}$$
$$= -4\,\tfrac{ft}{s}\,\underset{\sim}{i} + -6\,\tfrac{ft}{s}\,\underset{\sim}{i}$$

$$\underset{\sim}{V}_B = -10\,\tfrac{ft}{s}\,\underset{\sim}{i}$$

$$\underset{\sim}{V}_A = \underset{\sim}{V}_c + \underset{\sim}{\omega} \times \underset{\sim}{r}_{CA} = -4\,\tfrac{ft}{s}\,\underset{\sim}{i} + 6\,\tfrac{rad}{s}\,\underset{\sim}{k} \times -1\text{ ft}\,\underset{\sim}{j}$$
$$= -4\,\tfrac{ft}{s}\,\underset{\sim}{i} + 6\,\tfrac{ft}{s}\,\underset{\sim}{i}$$

$$\underset{\sim}{V}_A = 2\,\tfrac{ft}{s}\,\underset{\sim}{i}$$

slip on conveyor?

velocity of conveyor belt is 12 ft/s
velocity of A is 2 ft/s

A is in contact w/ belt

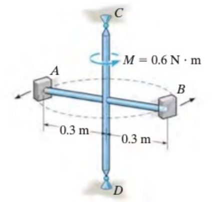
$V_A \neq$ vel of conveyor belt
... if two things are touching and don't have same velocity, then there's slipping!

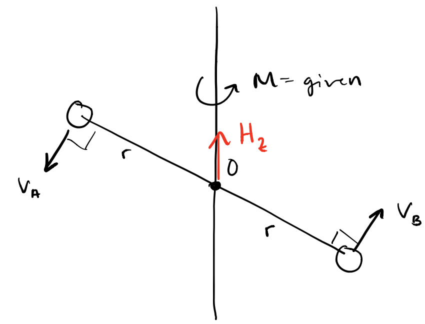
Yes, this disc is slipping!

... in order for there to not be any slip, V_A would need to equal velocity of conveyer

15-105

15–105. The two blocks A and B each have a mass of 400 g. The blocks are fixed to the horizontal rods, and their initial velocity along the circular path is 2 m/s. If a couple moment of $M = (0.6)$ N·m is applied about CD of the frame, determine the speed of the blocks when $t = 3$ s. The mass of the frame is negligible, and it is free to rotate about CD. Neglect the size of the blocks.





$V_{A0} = V_{B0} = 2$ m/s

Find V_A, V_B when $t = 3$ seconds

H_z points up for cc rotation

$$\Sigma M_{Oz} = \frac{d}{dt}(H_{Oz})$$

$$= \frac{d}{dt}\Big[\underbrace{r}_{\text{angular}}\underbrace{mV_B}_{\text{linear momentum}} + rmV_A\Big]$$

= total angular momentum of system in z-direction
(since 90° no need for x product)

$$M_O = \frac{d}{dt}(2mrV_A)$$

$$M_O t = 2mrV_A + C_1$$

$V_A = V_{A0}$ at $t = 0$

$$0 = 2mrV_{A0} + C_1$$

$$\therefore C_1 = -2mrV_{A0}$$

$$M_O t - C_1 = 2mrV_A$$

$$M_O t + 2mrV_A = 2mrV_A$$

$$V_A = V_{A0} + \frac{M_O}{2mr}t$$

... then plug in $t = 3$ seconds

Ch. 14 except 14.4

Ch. 15.1 – 15.7

Ch. 16 all of it