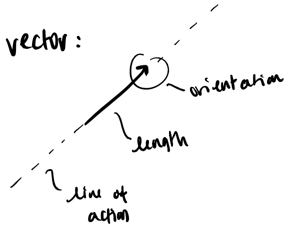
Tensors are basically higher order vectors

vector:



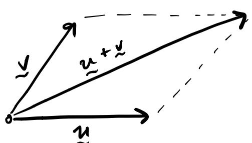
$\underset{\sim}{0}$ = $\emptyset$ vector : vector whose length = 0

$\underset{\sim}{\hat{e}}$ = unit vector : vector whose length is unity

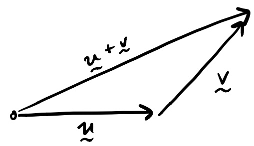
length (magnitude, norm) of a vector : $\| \cdot \| \neq | \cdot |$ absolute value

$\|\underset{\sim}{0}\| = 0 \quad \|\underset{\sim}{\hat{e}}\| = 1$

parallelogram (aka "head-to-tail") rule



parallelogram



head-to-tail

COMMUTATIVE $\underset{\sim}{u} + \underset{\sim}{v} = \underset{\sim}{v} + \underset{\sim}{u}$

ASSOCIATIVE $\underset{\sim}{u} + (\underset{\sim}{v} + \underset{\sim}{w}) = (\underset{\sim}{u} + \underset{\sim}{v}) + \underset{\sim}{w}$

if k is a scalar & $\underset{\sim}{u}$ is a vector, then $k\underset{\sim}{u}$ whose magnitude is $|k|\|\underset{\sim}{u}\|$

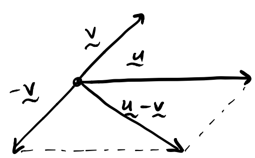
if k, m = scalars & $\underset{\sim}{u}$ is a vector, then...

ASSOCIATIVE $k(m\underset{\sim}{u}) = (km)\underset{\sim}{u}$

DISTRIBUTIVE $(k+m)\underset{\sim}{u} = k\underset{\sim}{u} + m\underset{\sim}{u}$

if take $\frac{\underset{\sim}{u}}{\|\underset{\sim}{u}\|} = \underset{\sim}{\hat{u}}$ a unit vector

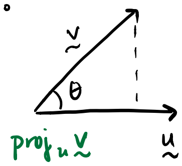
what is $\underset{\sim}{u} - \underset{\sim}{v}$?



Scalar (dot, inner) product:

$\underset{\sim}{u} \circ \underset{\sim}{v} = \|\underset{\sim}{u}\| \underbrace{\|\underset{\sim}{v}\| \cos\theta}_{proj_u \underset{\sim}{v}}$

for $0 \leq \theta \leq \pi$



$\underset{\sim}{\hat{u}} \cdot \underset{\sim}{\hat{v}} = \pm 1$ parallel and same/opposite sense

$\underset{\sim}{u} \circ \underset{\sim}{v} = 0$ orthogonal vectors

$\underset{\sim}{u} \circ \underset{\sim}{v} = \underset{\sim}{v} \circ \underset{\sim}{u}$ commutative

$\underset{\sim}{u} \circ (\underset{\sim}{v} + \underset{\sim}{w}) = \underset{\sim}{u} \circ \underset{\sim}{v} + \underset{\sim}{u} \circ \underset{\sim}{w}$ distributive

$(k\underset{\sim}{u}) \cdot \underset{\sim}{v} = k(\underset{\sim}{u} \cdot \underset{\sim}{v})$ associative

Based on associative and distributive properties:
α, β scalars & $\mathbf{u}, \mathbf{v}, \mathbf{w}$ vectors

then, $\boxed{\mathbf{u}(\alpha\mathbf{v} + \beta\mathbf{w}) = \alpha(\mathbf{u}\cdot\mathbf{v}) + \beta(\mathbf{u}\cdot\mathbf{w})}$ dot product is a linear operator

$\mathbf{u}\cdot\mathbf{u} = \|\mathbf{u}\|\,\|\mathbf{u}\|\cos 0^\circ = \|\mathbf{u}\|^2 \Rightarrow \|\mathbf{u}\| = \sqrt{\mathbf{u}\cdot\mathbf{u}}$

$\boxed{|\mathbf{u}\cdot\mathbf{v}| = \|\mathbf{u}\|\,\|\mathbf{v}\|\,|\cos\theta| \le \|\mathbf{u}\|\,\|\mathbf{v}\|}$ Cauchy-Schwarz Inequality

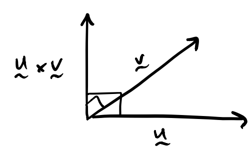
$\boxed{\|\mathbf{u}+\mathbf{v}\| \le \|\mathbf{u}\| + \|\mathbf{v}\|}$ ← clear from head-to-tail rule

Triangle Inequality

↑ when will they be equal? ... look this up

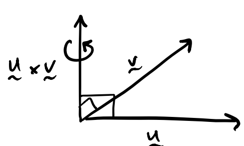
<u>Vector (cross) Product</u>

$\boxed{\|\mathbf{u}\times\mathbf{v}\| = \|\mathbf{u}\|\,\|\mathbf{v}\|\sin\theta}$ $0 \le \theta \le \pi$



↑ equal to magnitude of area spanned by $\mathbf{u}$ and $\mathbf{v}$

How do we determine <u>sense</u> of orientation $\mathbf{u}\times\mathbf{v}$?
By the Right Hand Rule



∴ not commutative, $\mathbf{u}\times\mathbf{v} \ne \mathbf{v}\times\mathbf{u}$

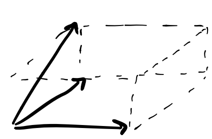
∴ it <u>is</u> distributive, $\mathbf{u}\times(\mathbf{v}+\mathbf{w}) = \mathbf{u}\times\mathbf{v} + \mathbf{u}\times\mathbf{w}$

∴ and it <u>is</u> associative, $(k\mathbf{u})\times\mathbf{v} = k(\mathbf{u}\times\mathbf{v})$

$\mathbf{u}\times(\alpha\mathbf{v} + \beta\mathbf{w}) = \alpha(\mathbf{u}\times\mathbf{v}) + \beta(\mathbf{u}\times\mathbf{w})$ ← linear operator

$\mathbf{u}\times\mathbf{v} = 0$ if $\mathbf{u} \parallel \mathbf{v}$

<u>Scalar Triple Product</u>
we have $\mathbf{u}, \mathbf{v}$, and $\mathbf{w}$ all non-coplanar ... form parallelopiped



height: $\mathrm{Proj}_{\mathbf{u}\times\mathbf{v}}\,\mathbf{w}$

$(\mathbf{u}\times\mathbf{v})\cdot\mathbf{w} = \text{volume}(\mathbf{u},\mathbf{v},\mathbf{w})$

$(\mathbf{v}\times\mathbf{w})\cdot\mathbf{u} = (\mathbf{w}\times\mathbf{u})\cdot\mathbf{v}$

but can't do $(\mathbf{v}\times\mathbf{u})\cdot\mathbf{w} = -\text{volume}(\mathbf{u},\mathbf{v},\mathbf{w})$
because always need ⊕