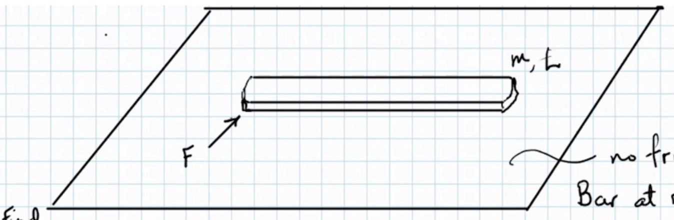
Dynamics – Lecture #19

HW #7 : ch. 17 due Sat. 4/11/2020
HW # 8 : ch. 19 due Wed. 4/15/2020

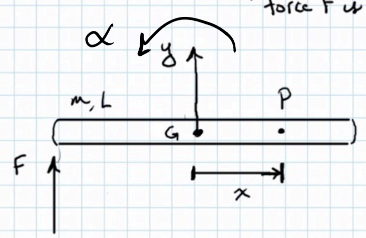
Rigid rod on horizontal table w/no friction and force is applied (initially at rest)



no friction

Bar at rest when force F suddenly applied.

Find initial acceleration of point P when force F is applied.



Want to find initial acceleration as a function of x (distance from the center of mass)

I. $\underset{\sim}{\Sigma F} = m \underset{\sim}{a_G} \Rightarrow \begin{cases} 0 = m a_{Gx} & [a_{Gx} = \phi] \\ F = m a_{Gy} & [a_{Gy} = F/m] \end{cases}$ — linear momentum principle

II. $\Sigma M_G = I_G \alpha$ — angular momentum principle

$$-F\left(\frac{L}{2}\right) = I_G \alpha, \text{ or } \left[\alpha = \frac{-FL}{2 I_G}\right]$$

What about $\underset{\sim}{a_P}$? Use kinematic relation among $\underset{\sim}{a_G}$, α, and $\underset{\sim}{a_P}$.

kinematics:

$\underset{\sim}{\omega} = 0$ initially

$$\underset{\sim}{a_P} = \underset{\sim}{a_G} + \underset{\sim}{\alpha} \times \underset{\sim}{r_{GP}} + \cancel{\underset{\sim}{\omega} \times (\underset{\sim}{\omega} \times \underset{\sim}{r_{GP}})}$$

$$\underset{\sim}{a_P} = \frac{F}{m}\underset{\sim}{j} + \left(-\frac{F\ell}{2 I_G}\underset{\sim}{k}\right) \times \left(x \underset{\sim}{i}\right)$$

$$= \frac{F}{m}\underset{\sim}{j} - \frac{F\ell}{2 I_G} x \underset{\sim}{j}$$

$$\underset{\sim}{i} \to \underset{\sim}{j} \to \underset{\sim}{k}$$

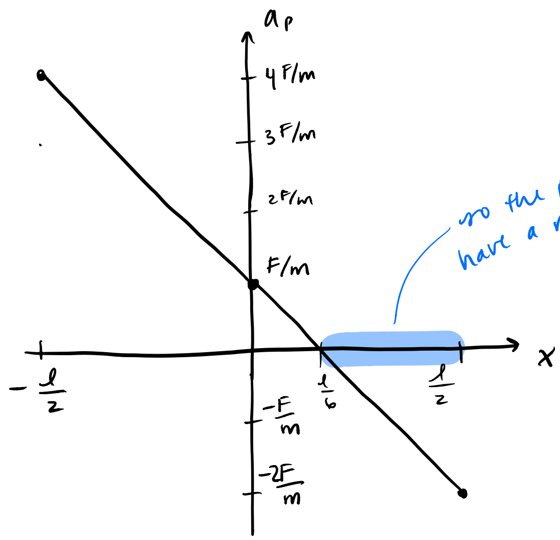
$$\underset{\sim}{a_P} = \left(\frac{F}{m} - \frac{F\ell}{2 I_G} x\right)\underset{\sim}{j}$$

so the bar doesn't have uniform acceleration, it varies w/ x

For a thin rod, $I_G = \frac{m\ell^2}{12}$ then

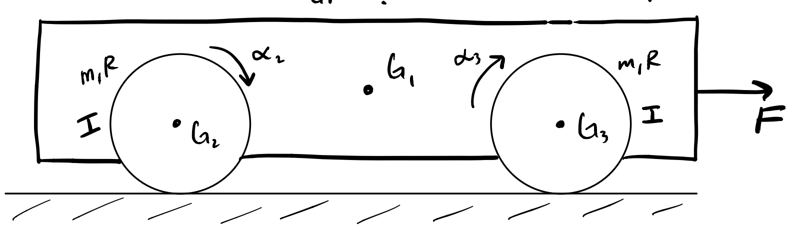
$$\underset{\sim}{a_P} = \frac{F}{m} - \frac{F\ell \cdot 12}{2 m \ell^2}$$

$$= \frac{F}{m}\left(1 - 6\frac{x}{\ell}\right)\underset{\sim}{j}$$



so the points between $\frac{l}{6}$ and $\frac{l}{2}$ have a negative (downward) acceleration

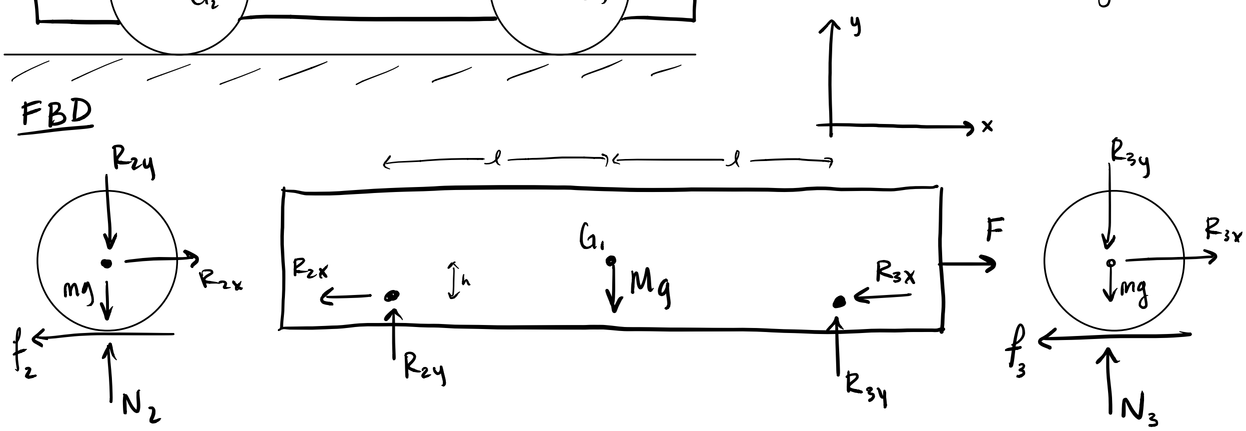
ex. (not in textbook) vehicle



3 rigid bodies (2 wheels and 1 vehicle)

<u>note</u>: this is <u>not</u> one rigid body

<u>FBD</u>



There will be three separate sets of equations.

note that α_1 (α of body) is zero!
$\alpha_1 = \emptyset$

<u>kinematics</u>

$a_{G2} = a_{G1}$

$a_{G3} = a_{G1}$

$a_{G2} = \alpha_2 R$

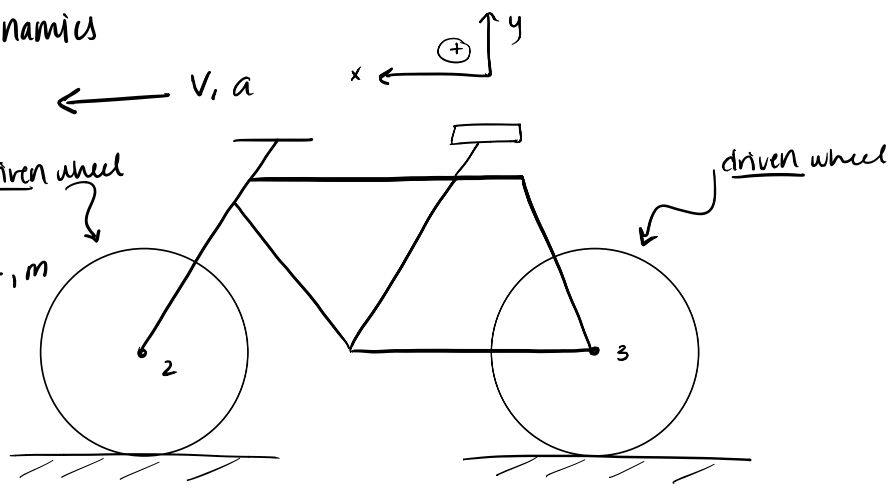
$a_{G3} = \alpha_3 R$

$\therefore \alpha_2 = \alpha_3$

	WHEEL 2	BODY 1	WHEEL 3
x direction:	$-f_2 + R_{2x} = m a_{G2}$	$-R_{2x} - R_{3x} + F = m a_{G1}$	$-f_3 + R_{3x} = m a_{G3}$
y direction:	$N_2 - R_{2y} - mg = \emptyset$	$-Mg + R_{2y} + R_{3y} = \emptyset$	$N_3 - R_{3y} - mg = \emptyset$
$\Sigma M_{G2/G1/G3}$:	$f_2 R = I\alpha_2$	$R_{2y} l - R_{3y} l + R_{2x} h + R_{3x} h = \emptyset$	$f_3 R = I\alpha_3$

Now there are enough equations to calculate a_{G1} in terms of M, m, R, I, F.

Bicycle Dynamics

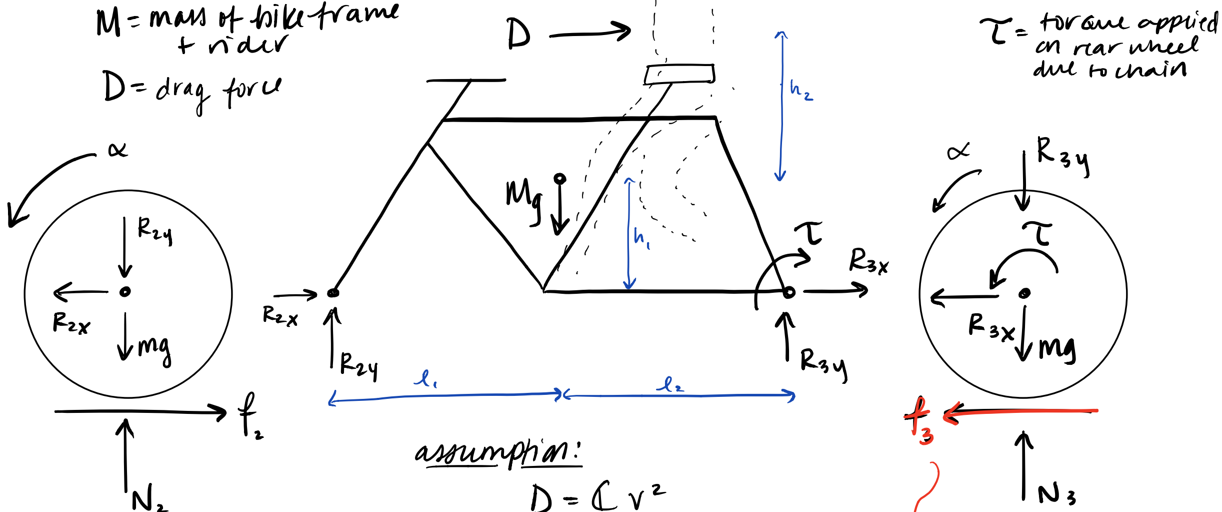


- different from vehicle in that no applied force
- we can't neglect mass of rider bc that would be a bad approximation

M = mass of bike frame + rider

D = drag force

τ = torque applied on rear wheel due to chain



assumption:

$$D = C v^2$$

(depends on Re #)

C = drag coefficient (in practice, want to make this small)

only thing that can propel bicycle (rear wheel drive)

→ in beginning not much drag but really peddling to get acceler.
→ then hit constant speed and work to reduce drag

[Problem: given a, v, and the properties of the bicycle, what torque τ is necessary? [more of a practical problem]]

Same kinematics from before apply:

$$a = \alpha R$$

FRONT WHEEL

$$ma = -f_2 + R_{2x}$$
$$0 = -mg - R_{2y} + N_2$$
$$\Sigma M: \quad I\alpha = f_2 R$$

REAR WHEEL

$$ma = +f_3 + R_{3x}$$
$$0 = -mg - R_{3y} + N_3$$
$$\Sigma M: \quad I\alpha = \tau - f_3 R$$

angular momentum for undriven wheel / driven wheel

FRAME

$$Ma = R_{2x} + R_{3x} - D$$
$$0 = -Mg + R_{2y} + R_{3y}$$
$$0 = R_{3y} l_2 - R_{2y} l_1 - \tau - R_{2x} h_1 - R_{3x} h_1 - D h_2$$

Results

$$\tau = R\left(M + 2m + 2\frac{I}{R^2}\right)a + DR$$

mass of frame + body; 2 wheels; 2 moments of inertia wheel

INTERNAL FORCES ARE IMPORTANT!

... so this is how hard have to pedal!

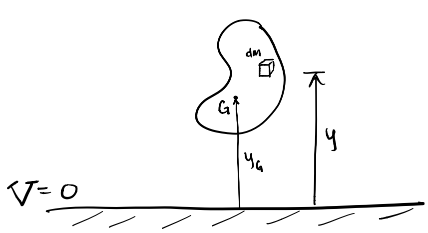
$$f_2 = \frac{I}{R^2} a \quad \left[\text{undriven wheel}\right] \qquad f_3 = \left(M + 2m + \frac{I}{R^2}\right)a + D$$

note: friction at driven wheel is much larger than friction at undriven wheel
... makes sense since rear tires in rear wheel drive wear out much quicker due to larger friction

CH. 18 Energy Methods for Rigid Body Dynamics

→ need potential energy V and kinetic energy T for a rigid body.

Potential Energy for Rigid Body (no elastic energy since not deformable)



For element dm,

$$dV = dm\, g\, y$$

For entire body,

$$V = \int_{body} g y\, dm = g \int_{body} y\, dm = g(M y_G)$$

therefore,

$$\boxed{V = M g y_G}$$ (potential energy for a rigid body of total mass M)