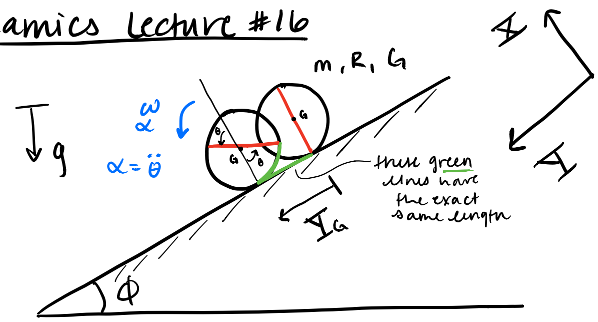
Dynamics Lecture #16

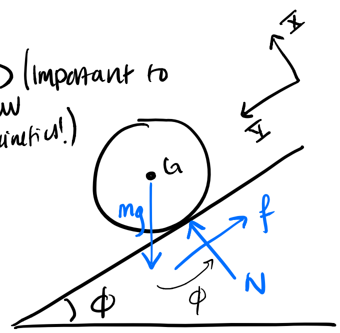
(Test #2: 4/1/2020 during class) → Ch. 14, 15, 16



Ex. Rolling Wheel, no slip

Calculate a_G and angular acceleration α of wheel.

FBD (Important to draw for kinetics!)



Rolling Constraint : $x_G = R\theta$

Take $\frac{d^2}{dt^2}$: $a_{G_x} = R\alpha$ (4)

Two Momentum Principles:

I. $\sum \underset{\sim}{F} = m\underset{\sim}{a}_G$ (linear momentum)

$$\begin{cases} N - mg\cos\phi = m a_{G_y} = 0 & (1) \text{ bc G can only move in x direction} \\ -f + mg\sin\phi = m a_{G_x} \neq 0 & (2) \end{cases}$$

don't need to write vectors since 2D so choose cw or ccw → choose $\curvearrowleft M_G \oplus$ bc consistent with the α

II. $\sum M_G = I_G \alpha$ (angular momentum
— just the friction has a moment about point G

$$fR = I_G\alpha \quad (3)$$

(1), (2), (3), (4) : unknowns a_{G_x}, α, f, N

From (2) and (4) $\quad f = mg\sin\phi - mR\alpha \quad$ (5)

From (3), $\quad f = \frac{I_G}{R}\alpha \quad$ (6)

use (5) & (6) to get α: $\quad mg\sin\phi - mR\alpha = \frac{I_G}{R}\alpha$

$$\alpha\left(\frac{I_G}{R} + mR\right) = mg\sin\phi$$

$$\alpha = \frac{mg\sin\phi}{\frac{I_G}{R} + mR}$$

from (4) $\quad a_{G_x} = R\alpha$

$$a_{G_x} = \frac{mgR\sin\phi}{\frac{I_G}{R} + mR}$$

If the wheel is solid disc then $I_G = \frac{1}{2}mR^2$ [moment of inertia of solid disc about its center]

$$\frac{I_G}{R} + mR = \frac{1}{2}mR + mR$$
$$= \frac{3}{2}mR$$

so if have solid wheel, then

$$\alpha = \frac{mg\sin\phi}{\frac{3}{2}mR} \Rightarrow \alpha = \frac{2}{3}\frac{g}{R}\sin\phi$$

$$a_{GI} = R\alpha = \frac{2}{3}g\sin\phi$$

find f, N:

from (1), $N = mg\cos\phi$

from (3), $f = \frac{I_G}{R}\alpha = \frac{I_G}{R}\frac{mg\sin\phi}{I_G/R + mR}$

for a solid wheel:

mg makes up the force!

$$f = \frac{1}{2}mR\,\frac{2}{3}\frac{g}{R}\sin\phi \rightarrow f = \frac{1}{3}mg\sin\phi$$

no slip condition: $f < \mu_s N$

for solid wheel, no slip if

$$\frac{1}{3}mg\sin\phi < \mu_s mg\cos\phi$$

or

$$\mu_s > \frac{1}{3}\tan\phi$$ ... this must be true if the wheel is not going to slip!

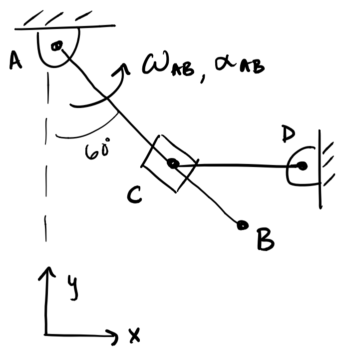
if there's no friction, the wheel will not roll, it will slide!

$\mu = 0$

∴ Rolling without slip requires friction. [why tires made of sticky rubber] [train wheels are steel, why rail vehicles cannot go uphill!]

HW problem 16-140 Review

*** STUDY THIS ***



Link DC $\quad \underset{\sim}{V}_c = \underset{\sim}{V}_D + \underset{\sim}{\omega}_{CD} \times \underset{\sim}{r}_{DC}$ (1)

↑ unknown (under $\underset{\sim}{\omega}_{CD}$)

Link AB

$$\underset{\sim}{V}_c = \underset{\sim}{V}_A + \underset{\sim}{\omega}_{AB} \times \underset{\sim}{r}_{AC} + (\underset{\sim}{V}_c)_{rel} \quad (2)$$

From (1) and (2):

$$\underset{\sim}{\omega}_{CD} \times \underset{\sim}{r}_{DC} = \underset{\sim}{\omega}_{AB} \times \underset{\sim}{r}_{AC} + (\underset{\sim}{V}_c)_{rel} \quad (3)$$

unknowns $\omega_{CD}, (V_c)_{rel}$

↳ along AB direction

Link DC

$$\underset{\sim}{a}_c = \underset{\sim}{a}_D + \underset{\sim}{\alpha}_{CD} \times \underset{\sim}{r}_{DC} + \underset{\sim}{\omega}_{CD} \times (\underset{\sim}{\omega}_{CD} \times \underset{\sim}{r}_{DC}) \quad (4)$$

↑ unknown (under $\underset{\sim}{\alpha}_{CD}$)

Link AB

coriolis acceleration ↓ (above $2\underset{\sim}{\omega}_{AB} \times (\underset{\sim}{V}_c)_{rel}$)

$$\underset{\sim}{a}_c = \underset{\sim}{a}_A + \underset{\sim}{\alpha}_{AB} \times \underset{\sim}{r}_{AC} + \underset{\sim}{\omega}_{AB} \times (\underset{\sim}{\omega}_{AB} \times \underset{\sim}{r}_{AC}) + 2\underset{\sim}{\omega}_{AB} \times (\underset{\sim}{V}_c)_{rel} + (\underset{\sim}{a}_c)_{rel} \quad (5)$$

Equate (4), (5): unknowns $\alpha_{CD}, (a_c)_{rel}$

↳ along AB direction

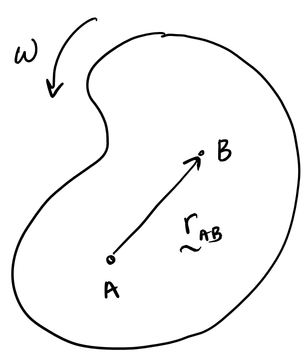
$(\underset{\sim}{V}_c)_{rel} = (V_c)_{rel}\, \underset{\sim}{u}_{AB}$

↑ magnitude of V_c (under $(V_c)_{rel}$)

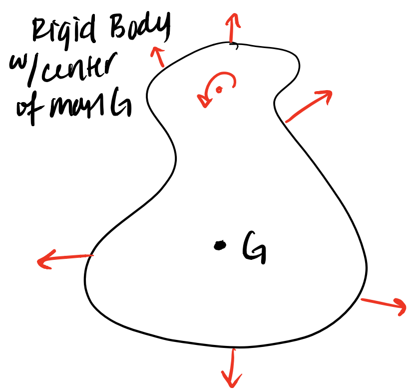
↑ unit vector along AB (under $\underset{\sim}{u}_{AB}$)

$(\underset{\sim}{a}_c)_{rel} = (a_c)_{rel}\, \underset{\sim}{u}_{AB}$

↑ unit vector in AB direction (direction of relative acceleration) (under $\underset{\sim}{u}_{AB}$)

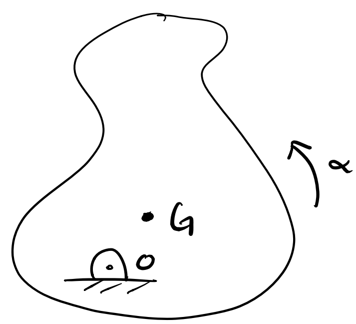


$\underset{\sim}{V}_B = \underset{\sim}{V}_A + \omega \times$



$$\text{I. } \Sigma \underset{\sim}{F} = m\underset{\sim}{a}_G$$
$$\text{II. } \Sigma M_G = I_G \alpha$$
} always valid equations

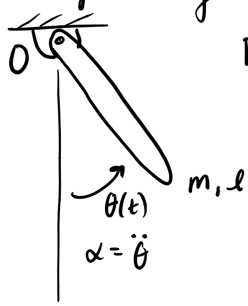
If the body has a fixed pt O,



Can use $\Sigma M_0 = I_0 \alpha$ instead of (II)
but only if O = fixed point

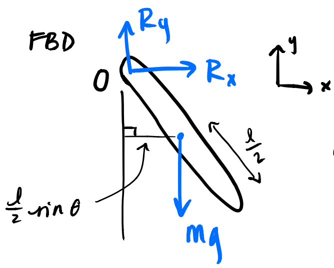
$\text{IIa. } \Sigma M_0 = I_0 \alpha$ } valid only if O is fixed pt.

Ex. Rigid Body Pendulum



Derive the equation of motion for angle $\theta(t)$.

FBD



only moment about O is the gravity force

O = fixed point
$\Sigma M_0 = I_0 \alpha$

$$\Sigma M_0 = I_0 \alpha$$
$$-mg\frac{l}{2}\sin\theta = I_0 \ddot{\theta}$$
or
$$I_0 \ddot{\theta} + mg\frac{l}{2}\sin\theta = 0$$
differential equation for θ over time

For a thin rod, $I_0 = \frac{1}{3}ml^2$

so $\frac{1}{3}ml^2\ddot{\theta} + mg\frac{l}{2}\sin\theta = 0$

or $\ddot{\theta} + \frac{3}{2}\frac{g}{l}\sin\theta = 0$

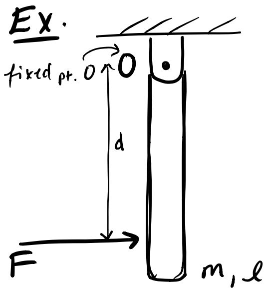
or $\ddot{\theta} + \frac{g}{\frac{2}{3}l}\sin\theta = 0$

} get the (3/2) b/c the distributed mass

mass is spread out along length of bar over entire

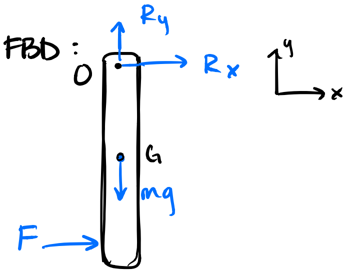
for pt. mass pendulum,
$$\ddot{\theta} + \frac{g}{l}\sin\theta = 0$$

Ex.



Pendulum initially at rest, then struck by force F.
Find the initial reaction forces at pivot pt. O.

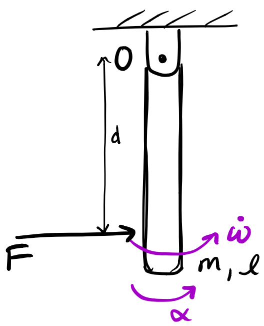
FBD:



since need to find reaction forces, no help to find sum of moments about pt. O. So, we'll use:

I. $\Sigma \underline{F} = m \underline{a}_G$

II. $\Sigma M_G = I_G \alpha$



draw in angular acceleration

Initially,

$\omega = 0$ [no angular velocity at rest]

$\alpha \neq 0$ [moment strike bar, going to cause angular acceleration]

linear momentum

I. $\begin{cases} F + R_x = m a_{Gx} = m \frac{\ell}{2} \ddot{\theta} = m \frac{\ell}{2} \alpha \quad \text{(1)} \\ R_y - mg = m a_{Gy} = m \frac{\ell}{2} \omega^2 = 0 \quad \text{(2)} \end{cases}$

[circular motion formula in the tangential direction] (x direction acceleration of G)

[centripetal acceleration, equal to ϕ since $\omega_i = 0$]

know $\frac{v^2}{R}$ vs $\omega^2 R$ side note

angular momentum

II. $-R_x \frac{\ell}{2} + F\left(d - \frac{\ell}{2}\right) = I_G \alpha$ (3)

from (2)

$$\boxed{R_y = mg}$$

lots of algebra!!

to get R_x, use (1) and (3)

$$\frac{2}{m\ell}(F + R_x) = \frac{1}{I_G}\left(-R_x \frac{\ell}{2} + F\left(d - \frac{\ell}{2}\right)\right)$$

$$F\left(\frac{2}{m\ell} - \frac{1}{I_G}\left(d - \frac{\ell}{2}\right)\right) = R_x\left(-\frac{\ell}{2}\frac{1}{I_G} - \frac{2}{m\ell}\right)$$

[... multiply by $\frac{m\ell}{2}$]

$$R_x\left(\frac{-m\ell^2/4}{I_G} - 1\right) = F\left(1 - \frac{m\ell/2}{I_G}\left(d - \frac{\ell}{2}\right)\right)$$

I_G for thin rod $= \frac{1}{12} m \ell^2$

$$R_x(-3 - 1) = F\left(1 - \frac{6}{\ell}\left(d - \frac{\ell}{2}\right)\right)$$

$$R_x = \frac{1}{4} F\left(\frac{6d}{\ell} - 3 - 1\right) \Rightarrow \boxed{R_x = \frac{1}{4} F\left(\frac{6d}{\ell} - 4\right)}$$

center of percussion is place where strike bar and get no rxn force in x-direction!!!

$\frac{d}{\ell} = \frac{2}{3}$ defines the center of percussion of the bar!!!

* Special case: $R_x = 0$ when $\frac{d}{\ell} = \frac{2}{3}$ * ← applications in sporting equipment, so if this bar is tennis racket and ball strikes in right place ... feel nothing!!! the sweet spot