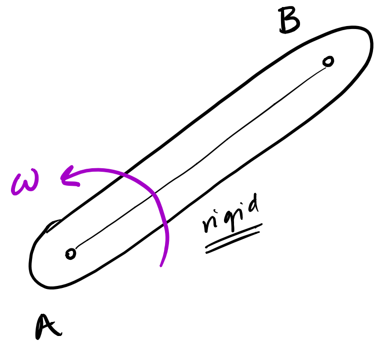
Dynamics - Lecture #12



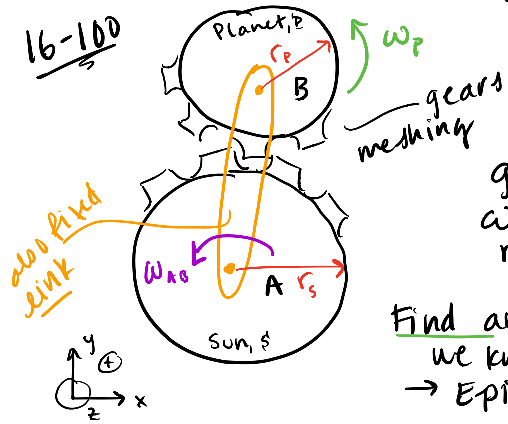
General Motion of a Rigid Body

(1) $\underset{\sim}{V}_B = \underset{\sim}{V}_A + \underset{\sim}{\omega} \times \underset{\sim}{r}_{AB}$

(2) $\underset{\sim}{a}_B = \underset{\sim}{a}_A + \underset{\sim}{\dot{\omega}} \times \underset{\sim}{r}_{AB} + \underset{\sim}{\omega} \times (\underset{\sim}{\omega} \times \underset{\sim}{r}_{AB})$

Another gear Problem: gear A has fixed center, gear B is not fixed

16-100



gear connected to Link@ A is sun gear
gear connected to Link@ B is planetary gear

gear S is fixed.
ω_{AB} = angular velocity of link (given)
r_P, r_S (given) = radii of gears

Find angular velocity of gear P ω_P.
we know that ω_P must be counterclockwise
→ Epicyclic gear

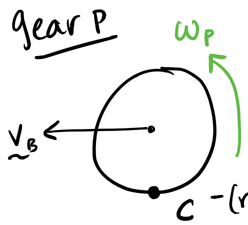
Link AB

$\underset{\sim}{V}_B = \underset{\sim}{V}_A + \underset{\sim}{\omega}_{AB} \times \underset{\sim}{r}_{AB}$ $V_A = \emptyset$ since A is fixed.

$= 0 + \omega_{AB} \underset{\sim}{k} \times (r_S + r_P) \underset{\sim}{j}$

$\underset{\sim}{V}_B = -(r_S + r_P)\omega_{AB} \underset{\sim}{i}$ ... makes sense since B must move sideways when link is straight up.

Gear P



pt. C is where meshes w/ sun gear

$\underset{\sim}{V}_B = \underset{\sim}{V}_C + \underset{\sim}{\omega}_P \times \underset{\sim}{r}_{CB}$ $V_C = \emptyset$ where meshes w/ sun gear bc doesn't slip here at meshing pt and sun gear has $V_A = \emptyset$

$-(r_S + r_P)\omega_{AB} \underset{\sim}{i} = 0 + \omega_P \underset{\sim}{k} \times r_P \underset{\sim}{j}$

$-(r_S + r_P)\omega_{AB} \underset{\sim}{i} = -\omega_P r_P \underset{\sim}{i}$

$$\boxed{\omega_P = \frac{(r_S + r_P)\omega_{AB}}{r_P}}$$

angular velocity of the arm

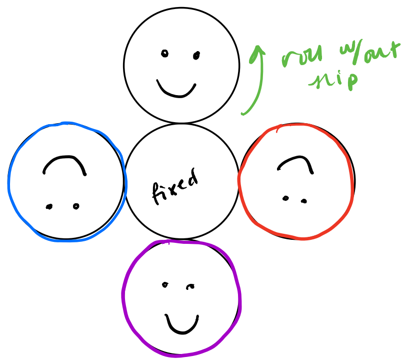
this tells us that planetary gears rotates faster than link AB

If $r_S = r_P$, $\omega_P = 2\omega_{AB}$ (planetary gear rotates twice as fast!)

Experiment w/2 coins:

when coin gets to this position →

→ coin rolls 90° but spins 180° degrees



Epicyclic gear is one that rolls along outside of another circle

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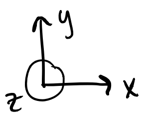
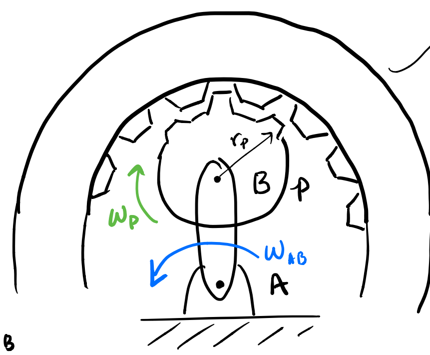




Ring gear (fixed)

A is center of Ring gear (fixed)

ω_{AB} given

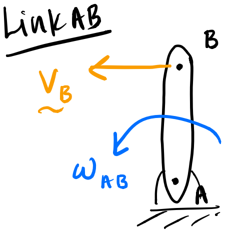
l = length of link AB

if the arm rotates counter clockwise, then the small gear rotates clockwise so that it can walk around inside of outer gear

$r_R = r_P + l_{AB}$

$\omega_P = ?$

Link AB



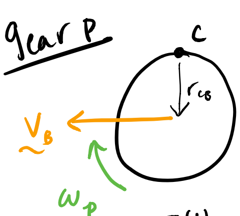
$$\underset{\sim}{V}_B = \underset{\sim}{V}_A + \underset{\sim}{\omega}_{AB} \times \underset{\sim}{r}_{AB}$$

$$= 0 + \omega_{AB} \underset{\sim}{k} \times l_{AB} \underset{\sim}{j}$$

$$\underset{\sim}{V}_B = -\omega_{AB} l_{AB} \underset{\sim}{i}$$

... notice the similarities w/circular motion

gear P



C is the fixed point, doesn't slip on the ring and the ring doesn't move so $\underset{\sim}{V}_C = 0$

$$\underset{\sim}{V}_B = \underset{\sim}{V}_C + \underset{\sim}{\omega}_P \times \underset{\sim}{r}_{CB}$$

notice that we're going down now from c to B

$$-\omega_{AB} l_{AB} \underset{\sim}{i} = 0 + (-\omega_P \underset{\sim}{k} \times -r_P \underset{\sim}{j})$$

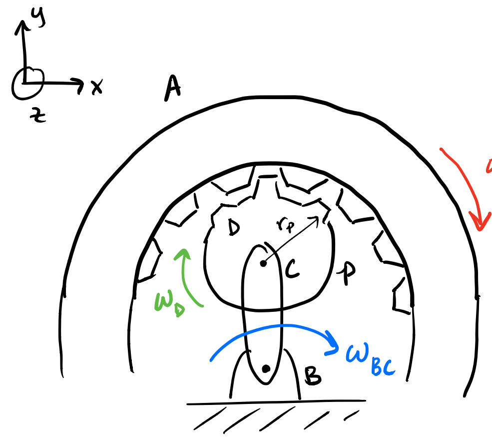
$$-\omega_{AB} l_{AB} \underset{\sim}{i} = -\omega_P r_P \underset{\sim}{i} \quad \Rightarrow \quad \boxed{\omega_P = \frac{l_{AB}}{r_P}\omega_{AB}}$$

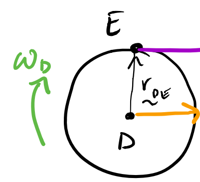
... can also look at the length of the link as $r_R - r_p$

so, $\boxed{\omega_p = \frac{r_R - r_p}{r_p}\,\omega_{AB}}$ Hypocyclic gear

Same problem but now ring gear <u>is not</u> fixed

$\ell_{BC} = 250\,mm$ $\quad r_A = 300\,mm$





$\underset{\sim}{V_E} = 9\,m/s$

$\underset{\sim}{V_D} = (250\,mm)(15\,rad/s)$

$\underset{\sim}{V_D} = 3.75\,m/s$

(velocity of <u>center</u> of small gear)

$\underset{\sim}{V_E}$ = NOT ZERO BECAUSE RING MOVES

$\underset{\sim}{V_E} = (300\,mm)(30\,rad/s) = 9\,m/s$

Which way does upper gear rotate? Top (E) going faster than middle (D) so must be <u>clockwise</u>

$r_{DE} = 50\,mm$

$$\underset{\sim}{V_E} = \underset{\sim}{V_D} + \underset{\sim}{\omega_D} \times \underset{\sim}{r_{DE}}$$

$$(9\,m/s)\,\underset{\sim}{i} = (3.75\,m/s)\,\underset{\sim}{i} + (-\omega_D\,\underset{\sim}{k}) \times (50\,mm\,\underset{\sim}{j})$$

$$5.25\,m/s\,\underset{\sim}{i} = \omega_D (50\,mm)\,\underset{\sim}{i}$$

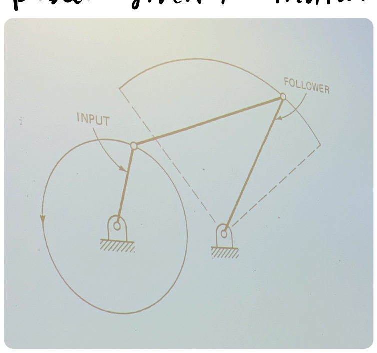
$$\omega_D = \frac{5.25\,m/s}{(0.05\,m)} \Rightarrow \boxed{\omega_D = 105\,\frac{1}{s} = 105\,\frac{rad}{s}}$$

Linkage System

- input link makes a circle
- follower just wipes back and forth

* way to translate circular motion to wiping motion

Common problem: given the motion of the input link, find that for the follower.



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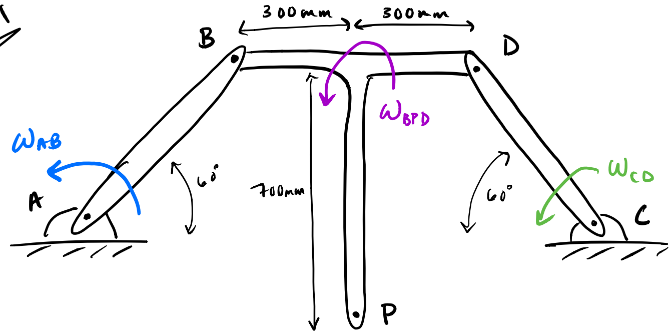


$l_{AB} = 300$ mm $= l_{CD}$

$\omega_{AB} = 8$ rad/s

$\omega_{CD} = ?$ $\omega_{BPD} = ?$ $\underset{\sim}{v}_P = ?$

we don't know ω_{CD} but guessing this direction

we don't know ω_{BPD} but guessing this direction

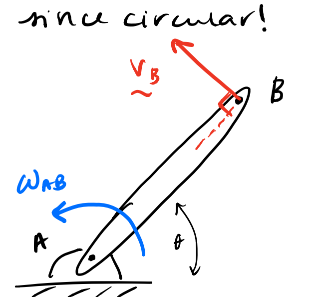
Link AB

$$\underset{\sim}{v}_B = \underset{\sim}{v}_A + (+\underset{\sim}{\omega}_{AB} \times \underset{\sim}{r}_{AB})$$

$$= 0 + \omega_{AB}\, \underset{\sim}{k} \times \left[l_{AB}(\cos\theta\, \underset{\sim}{i} + \sin\theta\, \underset{\sim}{j})\right]$$

$$\underset{\sim}{v}_B = l_{AB}\,\omega_{AB}\cos\theta\, \underset{\sim}{j} + - l_{AB}\,\omega_{AB}\sin\theta\, \underset{\sim}{i}$$

the velocity of B must be ⊥ to the motion since circular!

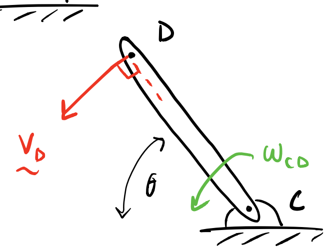


Link CD

$$\underset{\sim}{v}_D = \underset{\sim}{v}_C + (\underset{\sim}{\omega}_{CD} \times \underset{\sim}{r}_{CD})$$

$$= 0 + \omega_{CD}\, \underset{\sim}{k} \times \left[l_{CD}(-\cos\theta\, \underset{\sim}{i} + \sin\theta\, \underset{\sim}{j})\right]$$

$$= -l_{CD}\,\omega_{CD}\cos\theta\, \underset{\sim}{j} - l_{CD}\,\omega_{CD}\sin\theta\, \underset{\sim}{i}$$



Link BPD

bc 600mm $= 2 l_{AB}$

$$\underset{\sim}{v}_D = \underset{\sim}{v}_B + \underbrace{\underset{\sim}{\omega}_{BPD} \times \underset{\sim}{r}_{BD}}_{\omega_{BPD}\, \underset{\sim}{k} \times 2 l_{AB}\, \underset{\sim}{i} \;=\; 2 l_{AB}\,\omega_{BPD}\, \underset{\sim}{j}}$$

$$-\cancel{l_{CD}}\,\omega_{CD}\cos\theta\, \underset{\sim}{j} - \cancel{l_{CD}}\,\omega_{CD}\sin\theta\, \underset{\sim}{i} = \cancel{l_{AB}}\,\omega_{AB}\cos\theta\, \underset{\sim}{j} - \cancel{l_{AB}}\,\omega_{AB}\sin\theta\, \underset{\sim}{i} + 2\cancel{l_{AB}}\,\omega_{BPD}\, \underset{\sim}{j}$$

since $l_{CD} = l_{AB}$, all can cancel

separate the components . . .

i-component: $-\omega_{CD}\sin\theta = -\omega_{AB}\sin\theta \implies \boxed{\omega_{CD} = \omega_{AB} = 8 \text{ rad/s}}$

so yes, CD is rotating counterclockwise!

j-component: $-\omega_{CD}\cos\theta = \omega_{AB}\cos\theta + 2\omega_{BPD}$

$-2\omega_{AB}\cos\theta = 2\omega_{BPD}$

$-\omega_{AB}\cos\theta = \omega_{BPD} \implies \boxed{\omega_{BPD} = -\frac{1}{2}\omega_{AB} = -4 \text{ rad/s}}$

so, BPD goes clockwise and half as fast!

$\underset{\sim}{v}_P = \underset{\sim}{v}_B + \underset{\sim}{\omega}_{BPD} \times \underset{\sim}{r}_{BP}$ . . . these are all known so can calculate $\underset{\sim}{v}_P$!