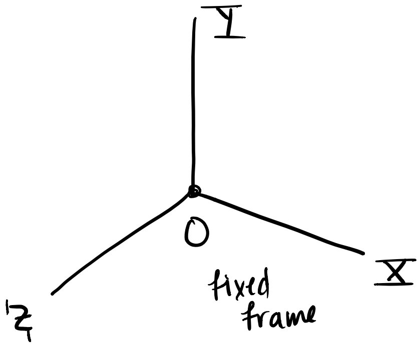
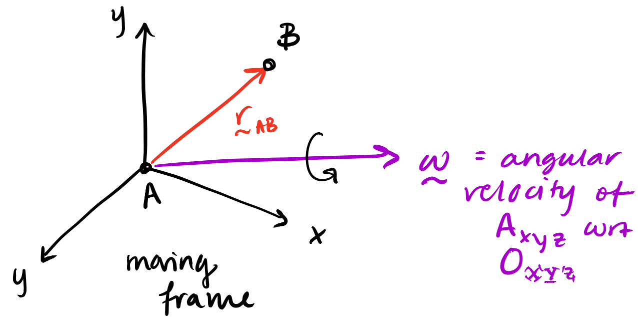
Dynamics – Lecture #14

HW#6 posted on Blackboard





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$\mathbf{v}_B$ = velocity of B wrt O_{XYZ}

$\mathbf{a}_B$ = acceleration of B wrt O_{XYZ}

$(\mathbf{v}_B)_{rel}$ = velocity of B wrt A_{xyz}

$(\mathbf{a}_B)_{rel}$ = acceleration of B wrt A_{xyz}

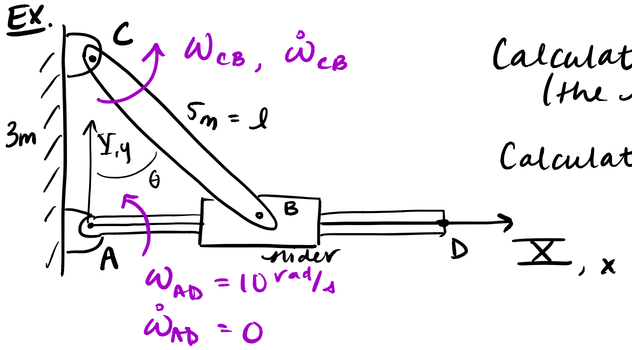
$\mathbf{v}_A$ = velocity of A wrt O_{XYZ}

$\mathbf{a}_A$ = acceleration of A wrt O_{XYZ}

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega} \times \mathbf{r}_{AB} + (\mathbf{v}_B)_{rel}$$

$$\mathbf{a}_B = \mathbf{a}_A + \dot{\boldsymbol{\omega}} \times \mathbf{r}_{AB} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_{AB}) + 2\boldsymbol{\omega} \times (\mathbf{v}_B)_{rel} + (\mathbf{a}_B)_{rel}$$

EX.



Calculate ω_{CB}, v_B relative to AD (the sliding velocity)

Calculate $\dot{\omega}_{CB}$, $\underset{\sim}{a}_B$ relative to AD

... at the particular configuration shown

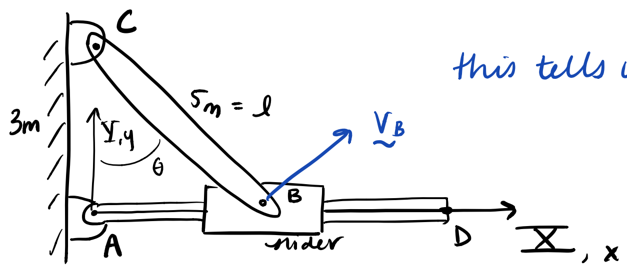
A $\underline{XYZ}$ fixed
A xyz attached to AD

... @ this particular time, the frames match but if waited a little longer then x would move up but X would stay the same

① Link CB $\quad \underset{\sim}{v}_B = \cancel{\underset{\sim}{v}_C}^{\,0} + \underset{\sim}{\omega}_{CB} \times \underset{\sim}{r}_{CB} + \cancel{(\underset{\sim}{v}_B)_{rel}}^{\,0}$

$$\underset{\sim}{v}_B = \omega_{CB}\,\underset{\sim}{k} \times l(\sin\theta\,\underset{\sim}{i} - \cos\theta\,\underset{\sim}{j})$$

$$\underset{\sim}{v}_B = l\omega_{CB}\sin\theta\,\underset{\sim}{j} + l\omega_{CB}\cos\theta\,\underset{\sim}{i}$$



this tells us...

② Link AD $\quad \underset{\sim}{v}_B = \cancel{\underset{\sim}{v}_A}^{\,0} + \underset{\sim}{\omega}_{AD} \times \underset{\sim}{r}_{AB} + (\underset{\sim}{v}_B)_{rel}$

note the different subscripts!

now this is not zero bc B can slide along the AD bar

$$\underset{\sim}{v}_B = \omega_{AD}\,\underset{\sim}{k} \times \tfrac{4}{5} l\,\underset{\sim}{i} + (v_B)_{rel}\,\underset{\sim}{i}$$

we know the slider constrains the motion to along x-axis only

$$\underset{\sim}{v}_B = \tfrac{4}{5} l\omega_{AD}\,\underset{\sim}{j} + (v_B)_{rel}\,\underset{\sim}{i}$$

③ Equate (1) + (2)

$$\omega_{CB} l(\sin\theta\,\underset{\sim}{j} + \cos\theta\,\underset{\sim}{i}) = \omega_{AD}\tfrac{4}{5} l\,\underset{\sim}{j} + (v_B)_{rel}\,\underset{\sim}{i}$$

$$\omega_{CB} = \frac{4}{5}\frac{\omega_{AD}}{\sin\theta} = \frac{4}{5}\left(\frac{10\text{ rad/s}}{4/5}\right) \Rightarrow \boxed{\omega_{CB} = 10\text{ rad/s}}$$

$$(v_B)_{rel} = \omega_{CB} l\cos\theta = (10\text{ rad/s})(5\text{ m})\left(\tfrac{3}{5}\right) \Rightarrow \boxed{(v_B)_{rel} = 30\text{ m/s}}$$

↳ this means it slides to the right, ⊕ number

④ Link CB $\quad \underset{\sim}{a}_B = \cancel{\underset{\sim}{a}_C}^{\,0} + \dot{\underset{\sim}{\omega}}_{CB} \times \underset{\sim}{r}_{CB} + \underset{\sim}{\omega}_{CB} \times (\underset{\sim}{\omega}_{CB} \times \underset{\sim}{r}_{CB})$ $\quad \underline{(1)}$

no relative terms b/c B is fixed on the bar

⑤ Link AD

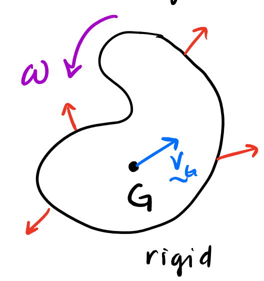
$$\underset{\sim}{a}_B = \cancel{\underset{\sim}{a}_A}^{\,0} + \cancel{\dot{\underset{\sim}{\omega}}_{AD}}^{\,0} \times \underset{\sim}{r}_{AB} + \underset{\sim}{\omega}_{AD} \times (\underset{\sim}{\omega}_{AD} \times \underset{\sim}{r}_{AB}) + \underbrace{2\,\underset{\sim}{\omega}_{AD} \times (\underset{\sim}{v}_B)_{rel}}_{\text{coriolis acceleration}} + (\underset{\sim}{a}_B)_{rel} \quad \underline{(2)}$$

Equate $\underline{(1)}$ and $\underline{(2)}$; unknowns $\dot{\omega}_{CB}$ and $(a_B)_{rel}$

CHAPTER 17 ... KINETICS!

before was kinematics

Kinetics of Rigid Body



I. $\Sigma \underset{\sim}{F} = \frac{d}{dt}(m\,\underset{\sim}{v}_G) = m\,\underset{\sim}{a}_G$

[linear momentum principle of mechanics]

II. $\Sigma \underset{\sim}{M}_G = \frac{d}{dt}(\underset{\sim}{H}_G)$ $\quad$ H = angular momentum

[angular momentum principle]

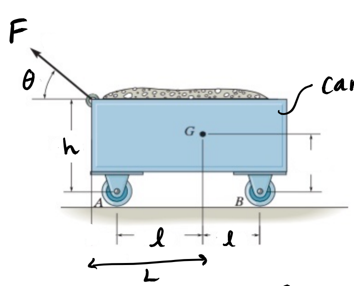
Special Case: translation only (no spinning/rotation)

Then...

I. $\Sigma \underset{\sim}{F} = m\,\underset{\sim}{a}_G$ $\quad$ translational motion

II. $\Sigma \underset{\sim}{M}_G = 0$ $\quad$ b/c the time derivative of ang. momentum is zero if the body does not rotate

Ex. 17.33 from the text (simplified)



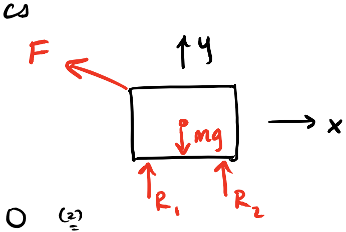
cart mass = m

neglect mass of wheels (for now)

Calculate the rxn forces at the wheels.

... need to draw FBD since back to kinetics

given: F, l, h, θ



I. $\Sigma \underset{\sim}{F} = m\,\underset{\sim}{a}_G \Rightarrow \begin{cases} -F\cos\theta = m\,a_{Gx} & \underline{(1)} \\ F\sin\theta - mg + R_1 + R_2 = m\,a_{Gy} = 0 & \underline{(2)} \end{cases}$

since the cart doesn't tip over or move in the y-direction

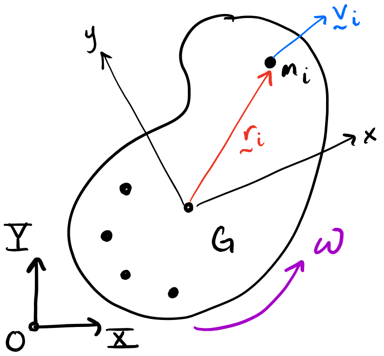
unknowns:

R_1, R_2, a_{Gx} $\quad$ ... so we need the angular momentum principle!

II. $\Sigma M_G = 0$ (no vector symbol since 2D and can take of like from statics)

$G \oplus \quad = 0 = -R_1 l + R_2 l + F\cos\theta\, h - F\sin\theta\, L \quad \underline{(3)}$

$\underline{(1)}, \underline{(2)}, \underline{(3)}$ can be used to solve for R_1, R_2, a_{Gx}



$$\Sigma M_G = \frac{d}{dt}(H_G)$$

Q. How is H_G (ang momentum) related to ω (ang velocity)?

A. H_G and ω related by the motion of inertia.

G_{xyz} is body frame: frame attached to body

Body frame moves with body

Discrete Math modeling

- imagine rigid body made of discrete masses

$$v_i = v_G + \omega \times r_i$$

since body is rigid, there are no relative terms (vector stuck in body essentially)

$$H_G = \Sigma r_i \times m_i v_i = \Sigma r_i \times m_i (v_G + \omega \times r_i)$$

↑ sum all of the masses

$$H_G = \Sigma m_i r_i \times v_G + \Sigma m_i r_i \times (\omega \times r_i)$$

$$= (\cancel{\Sigma m_i r_i}) \times v_G + \Sigma m_i (x_i i + y_i j) \times [\omega k \times (x_i i + y_i j)]$$

$\cancel{\Sigma m_i r_i} \to 0$: this first term is zero bc it's a property of the com (think of shooting vectors out from center G)

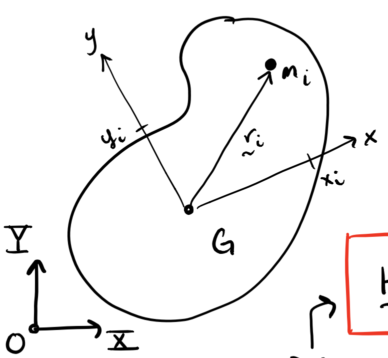
$$H_G = \Sigma m_i \omega (x_i i + y_i j) \times [x_i j - y_i i]$$

$$= \Sigma m_i \omega (x_i^2 k + y_i^2 k)$$

$$H_G = [\Sigma m_i (x_i^2 + y_i^2)] \omega k$$

⇓

the moment of inertia of the body



$$H_G = [\Sigma m_i (x_i^2 + y_i^2)] \omega k$$

I_G = moment of inertia of body wrt G

$= \Sigma m_i (x_i^2 + y_i^2)$

$= \Sigma m_i r_i^2$

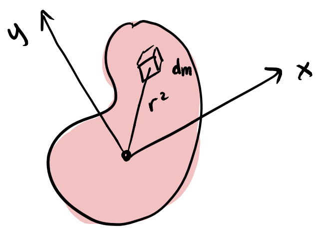
x_i, y_i are positions of particles

$r_i^2 = x_i^2 + y_i^2$ ... the vector

$$\boxed{H_G = I_G \omega}$$

* DISCRETE MASS MODEL

now we see how angular velocity is related to angular momentum using moment of inertia wrt G.

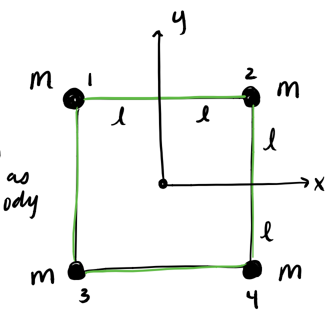


$$\boxed{I_G = \int (x^2+y^2)\,dm = \int r^2\,dm}$$

*CONTINUOUS MASS MODEL →

Ex.

masses linked as rigid body



First step, locate COM G and then draw coordinates

$$I_G = \sum_{i=1}^{4} m_i (x_i^2 + y_i^2)$$

$$= \underbrace{m((-l)^2 + l^2)}_{1} + \underbrace{m(l^2 + l^2)}_{2} + \underbrace{m((-l)^2 + (-l)^2)}_{3} + \underbrace{m(l^2 + (-l)^2)}_{4}$$

$$I_G = 8ml^2$$

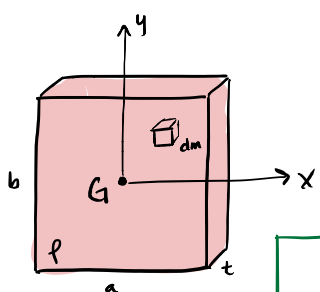
↑ moment of inertia of this body about its center of mass

units $[I_G]$ = mass · length²

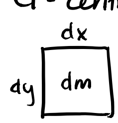
for example, $kg \cdot m^2$

or $slug \cdot ft^2$

Ex. continuous mass body ... solid slab of material (we'll say it's a uniform body so $\rho = G$)



G = centroid



$dm = \rho\, dx\, dy\, t$

dm = mass element

... since flat element

don't have to integrate over z, just use thickness

$$\boxed{I_G = \int x^2 + y^2\, dm = \int_{y=-\frac{b}{2}}^{y=\frac{b}{2}} \int_{x=-\frac{a}{2}}^{x=a/2} (x^2+y^2)\,\rho\, t\, dx\, dy}$$