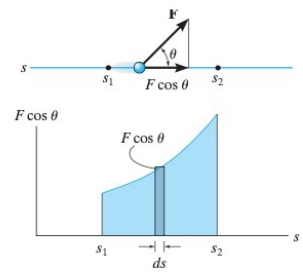
Dynamics Midterm II Study Guide

CHAPTER 14: KINETICS OF A PARTICLE: WORK AND ENERGY

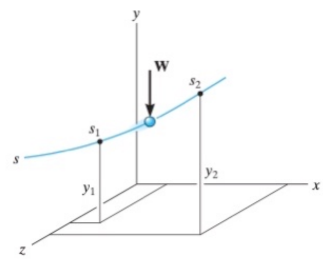
Work of a force

- a force does work when it undergoes a displacement along its line of action. If the force varies with the displacement, then the work is $U = \int F\cos\theta \, ds$

graphically, this represents the area under the F-s diagram

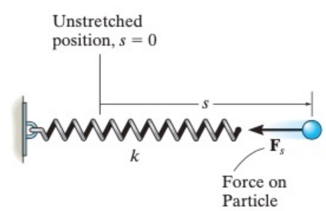


- if the force is constant, then for a displacement Δs in the direction of the force, $U = F_c \Delta s$. A typical example of this case is the work of a weight, $U = -W\Delta y$. Here, Δy is the vertical displacement



- the work done by a spring force, $F = ks$, depends upon the stretch or compression s of the spring.

$$U = -\left(\tfrac{1}{2}ks_2^2 - \tfrac{1}{2}ks_1^2\right)$$



The Principle of Work and Energy

- if the equation of motion in the tangential direction, $\Sigma F_t = ma_t$, is combined w/ the kinematic equation, $a_t\, ds = v\, dv$, we obtain the principle of work and energy. This equation states that the initial kinetic energy T_1 plus the work done ΣU_{1-2} is equal to the final kinetic energy

$$T_1 + \Sigma U_{1-2} = T_2$$

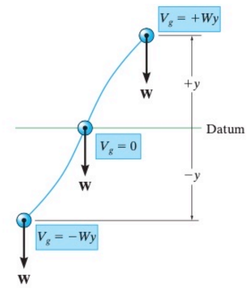
- the principle of work and energy is useful for solving problems that involve force, velocity, and displacement. For application, the FBD of the particle should be drawn in order to identify the forces that do work

Conservation of Energy

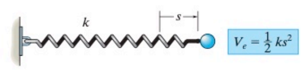
- a conservative force does work that is independent of its path. Two examples are the weight of a particle and the spring force

- friction is a nonconservative force since the work depends upon the length of the path. The longer the path, the more work done.

- the work done by a conservative force depends upon its position relative to a datum. When this work is referenced from a datum, it is called potential energy. For a weight, it is $V_g = \pm W y$, and for a spring it is $V_e = + \frac{1}{2} k y^2$



Gravitational potential energy



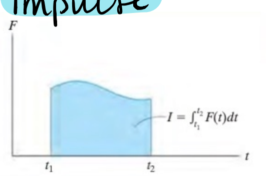
Elastic potential energy

$$T_1 + V_1 = T_2 + V_2$$

- mechanical energy consists of kinetic energy T and gravitational and elastic potential energies V. According to the conservation of energy, this sum is constant and has the same value at any position on the path. If only gravitational and spring forces cause motion of the particle, then the conservation-of-energy equation can be used to solve problems involving these conservative forces, displacement, and velocity.

CHAPTER 15: KINETICS OF A PARTICLE: IMPULSE AND MOMENTUM

Impulse



an impulse is defined as the product of force and time. graphically it represents the area under the F-t diagram. If the force is constant, then the impulse becomes $I = F_c(t_2 - t_1)$

Principle of Impulse and Momentum

- when the equation of motion $\Sigma \underset{\sim}{F} = m\underset{\sim}{a}$ and the kinematic equation $a = dv/dt$ are combined, we obtain the principle of impulse and momentum. This is a vector equation that can be resolved into rectangular components and used to solve problems that involve force, velocity, and time. For application the FBD should be drawn in order to account for all of the impulses that act on a particle

$$m\underset{\sim}{v_1} + \Sigma \int_{t_1}^{t_2} \underset{\sim}{F}\,dt = m\underset{\sim}{v_2}$$

Conservation of Linear Momentum

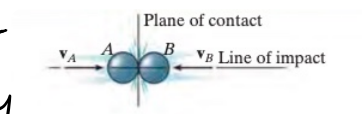
- if the principle of impulse and momentum is applied to a system of particles, then the collisions between the particles produce internal impulses that are equal, opposite, and collinear, and therefore cancel from the equation. Furthermore, if an external impulse is small, that is, the force is small and the time is short, then the impulse can be classified as nonimpulsive and can be neglected. Consequently, momentum for the system of particles is conserved.

- the conservation of momentum equation is useful for finding the final velocity of a particle when internal impulses are exerted between two particles and the initial velocities of the particles is known. If the internal impulse is to be determined, then one of the particles is isolated and the principle of impulse and momentum is applied to this particle.

$$\Sigma m_i (\underset{\sim}{v_i})_1 = \Sigma m_i (\underset{\sim}{v_i})_2$$

Impact

- when two particles A and B have a direct impact, the internal impulse between them is equal, opposite, and collinear. Consequently the conservation of momentum for this system applies along the line of impact.

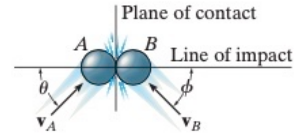


$$m_A(v_A)_1 + m_B(v_B)_1 = m_A(v_A)_2 + m_B(v_B)_2$$

- if the final velocities are unknown, a second equation is needed for solution. We must use the coefficient of restitution, e. This experimentally determined coefficient depends upon the physical properties of the colliding particles. It can be expressed as the ratio of their relative velocity after collision to their relative velocity before collision. If the collision is elastic, no energy is lost and $e=1$. For a plastic collision $e=0$.

$$e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$$

- if the impact is oblique, then the conservation of momentum for the system and the coefficient-of-restitution equation apply along the line of impact. Also, conservation of momentum for each particle applies perpendicular to this line (plane of contact) because no impulse acts on the particles in this direction.

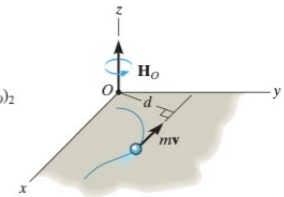


Principle of Angular Impulse and Momentum

- the moment of the linear momentum about an axis (z) is called the angular momentum

$$(H_O)_z = (d)(mv)$$

$$(\mathbf{H}_O)_1 + \Sigma \int_{t_1}^{t_2} \mathbf{M}_O \, dt = (\mathbf{H}_O)_2$$



- the principle of angular impulse and momentum is often used to eliminate unknown impulses by summing the moments about an axis through which the lines of action of these impulses produce no moment. For this reason, a FBD should accompany the solution.

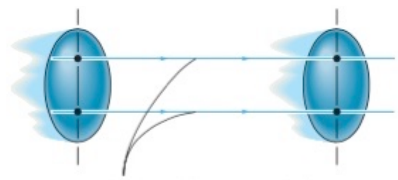
CHAPTER 16: PLANAR KINEMATICS OF A RIGID BODY

Rigid-Body Planar Motion

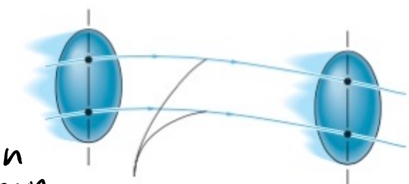
- a rigid body undergoes three types of planar motion: translation, rotation about a fixed axis, and general plane motion

Translation

- when a body has rectilinear translation, all the particles of the body travel along parallel straight-line paths. If the paths have the same radius of curvature, then curvilinear translation occurs. Provided we know the motion of one of the particles, then the motion of all of the others is also known



Path of rectilinear translation



Path of curvilinear translation