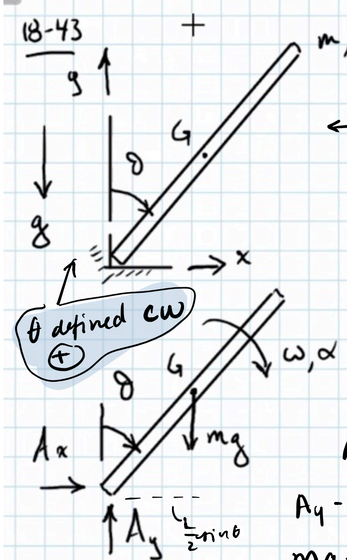
Dynamics – Lecture #20

Homework 8 due W 4/15/2020 ①

18-43



Find reaction forces A_x, A_y in terms of θ as rod falls.

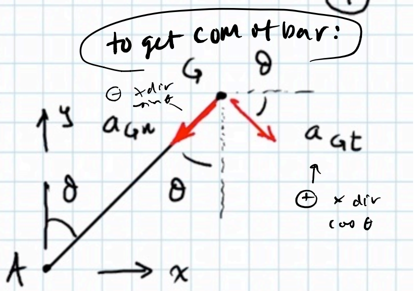
← bar that is falling from vertical position

initial state: $\theta = 0,\ \dot\theta = 0$

$\Sigma F_x = m a_{Gx}$ (1)

$\Sigma F_y = m a_{Gy}$ (2)

$\Sigma M_A = I_A \alpha = I_A \ddot\theta$ (3)



$a_{Gt} = \frac{L}{2}\ddot\theta$

$a_{Gn} = \frac{L}{2}\dot\theta^2$ ↖ centripetal component

$A_x = m\left(\frac{L}{2}\ddot\theta\cos\theta - \frac{L}{2}\dot\theta^2\sin\theta\right)$ (4) (from (1))

$A_y - mg = m\left(-\frac{L}{2}\ddot\theta\sin\theta - \frac{L}{2}\dot\theta^2\cos\theta\right)$ (5) (from (2))

$mg\frac{L}{2}\sin\theta = I_A\ddot\theta \Rightarrow \ddot\theta = \frac{mgL\sin\theta}{2\ I_A} = \frac{mgL\sin\theta}{2\,mL^2(\frac{1}{3})}$ I_A for thin rod $= mL^2\frac{1}{3}$

$$\ddot\theta = \frac{3}{2}\frac{g}{L}\sin\theta \quad (6)$$

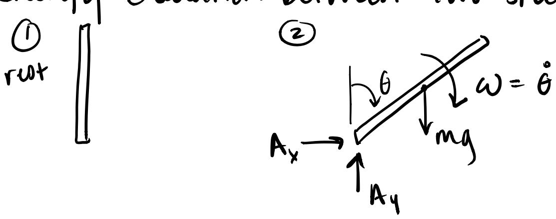
(6) into (4) & (5)

$$A_x = m\left(\frac{L}{2}\frac{3}{2}\frac{g}{L}\sin\theta\cos\theta - \frac{L}{2}\dot\theta^2\sin\theta\right) \quad (7)$$

$$A_y = mg + m\left(-\frac{L}{2}\frac{3}{2}\frac{g}{L}\sin^2\theta - \frac{L}{2}\dot\theta^2\cos\theta\right) \quad (8)$$

still need $\dot\theta^2$ in terms of θ. get $\dot\theta^2$ from energy equation

Energy Equation between Two States



$$\cancel{T_1} + V_1 + \cancel{U^{nc}_{1\to2}} = T_2 + V_2$$

gravity is conservative force so goes into potential function
non forces are nonconservative but they don't do any work
→ treating A as fixed pt.

$$V_1 = T_2 + V_2$$

$$mg\frac{L}{2} = \frac{1}{2}I_A\dot\theta^2 + mg\frac{L}{2}\cos\theta$$

$$\cancel{m}g\frac{L}{2} = \frac{1}{2}\frac{\cancel{m}L^2}{3}\dot\theta^2 + \cancel{m}g\frac{L}{2}\cos\theta \Rightarrow \frac{1}{6}L^2\dot\theta^2 = \frac{gL}{2}(1-\cos\theta) \Rightarrow \dot\theta^2 = \frac{3g}{L}(1-\cos\theta) \quad (9)$$

(9) into (7) & (8)

(7)

$$A_x = m\left(\frac{L}{2}\frac{3}{2}\frac{g}{L}\sin\theta\cos\theta - \frac{L}{2}\dot{\theta}^2\sin\theta\right)$$

$$= m\left(\frac{3}{4}g\sin\theta\cos\theta - \frac{L}{2}\frac{3g}{L}(1-\cos\theta)\sin\theta\right) = mg\left(\frac{3}{4}\sin\theta\cos\theta - \frac{3}{2}\sin\theta + \frac{3}{2}\cos\theta\sin\theta\right)$$

(8)

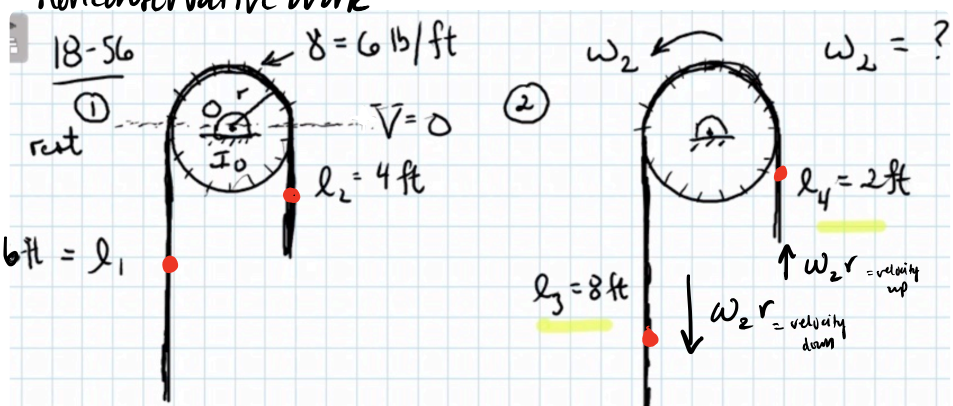
$$A_y = mg + m\left(-\frac{L}{2}\frac{3}{2}\frac{g}{L}\sin^2\theta - \frac{L}{2}\dot{\theta}^2\cos\theta\right)$$

$$= mg + m\left(-\frac{3g}{4}\sin^2\theta - \frac{L}{2}\frac{3g}{L}(1-\cos\theta)\cos\theta\right) = mg\left(-\frac{3}{4}\sin^2\theta - \frac{3}{2}\cos\theta + \frac{3}{2}\cos^2\theta + 1\right)$$

Can A_x or A_y go to zero as the bar falls?

18.56 pulley w/a fixed center and around the pulley is a chain and the chain wraps around (heavy chain will have mass, a weight per unit length $\gamma = 6$ lb/ft) remember $KE = \frac{1}{2}mv^2$

→ we'll assume that pulley is free to spin, no friction, so no nonconservative work



no need to put potential energy of curved part in bc it's the same on both sides of the equation so it will cancel out anyway

$$T_1 + V_1 = T_2 + V_2$$

$$0 - \frac{l_1}{2}(\gamma l_1) - \frac{l_2}{2}(\gamma l_2) = T_2 - \frac{l_3}{2}(\gamma l_3) - \frac{l_4}{2}(\gamma l_4) \quad (1)$$

(2)

$$T_2 = \frac{1}{2}I_O\omega_2^2$$ ... KE of rigid body rotating about fixed pt.

$$+\frac{1}{2}\left(\frac{\gamma l_3}{g}\right)(\omega_2 r)^2$$ ... KE of left hand chain in state 2

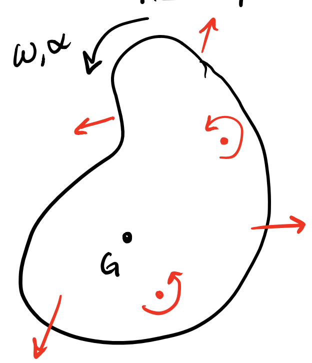
$$+\frac{1}{2}\left(\frac{\gamma l_4}{g}\right)(\omega_2 r)^2$$ ... KE of right hand chain in state 2

$$+\frac{1}{2}\left(\frac{\gamma \pi r}{g}\right)(\omega_2 r)^2$$ ... KE of top half of pulley in state 2

now substitute (2) into (1), solve for ω_2

CHAPTER 19 MATERIAL NOW

Momentum & Impulse for a Rigid Body



$$\sum \underset{\sim}{F} = \frac{d}{dt}(m \underset{\sim}{v}_G) \quad (1)$$

$$\sum M_G = \frac{d}{dt}(I_G \omega) \quad (2)$$ angular momentum of rigid body

since keeping problem in the plane (don't need vector notation... z comp)

Integrate (1) over time between $t = t_1$ and $t = t_2$

$$\int_{t_1}^{t_2} \sum \underset{\sim}{F} \, dt = \int_{t_1}^{t_2} \frac{d}{dt}(m \underset{\sim}{v}_G) \, dt$$

$$\underbrace{\sum \int_{t_1}^{t_2} \underset{\sim}{F} \, dt}_{\text{net impulse on body}} = \underbrace{m \underset{\sim}{v}_G(t_2) - m \underset{\sim}{v}_G(t_1)}_{\text{change in the linear momentum of the body}}$$

Integrate (2) over time between $t = t_1$ and $t = t_2$

$$\int_{t_1}^{t_2} \sum M_G \, dt = \int_{t_1}^{t_2} \frac{d}{dt}(I_G \omega) \, dt$$

$$\underbrace{\sum \int_{t_1}^{t_2} M_G \, dt}_{\text{net angular impulse on body}} = \underbrace{I_G \omega(t_2) - I_G \omega(t_1)}_{\text{change in angular momentum}}$$

Special Case: if the net angular impulse $\sum \int_{t_1}^{t_2} M_G \, dt = 0$, then $I_G \omega(t_2)$ must equal $I_G \omega(t_1)$ $\Leftarrow$ "conservation of angular momentum"

19-9 Pulley system w/a fixed center and rope wrapped around it many times (coiled up)... assume rope is massless

19-9

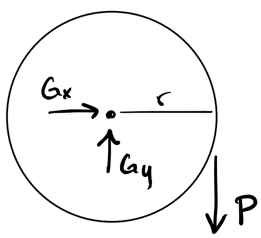
$P = 10$ lb

At $t_1 = 0$ pulley at rest

Force P is applied

What is angular velocity ω_2 of disk at $t = t_2 = 4$ sec?

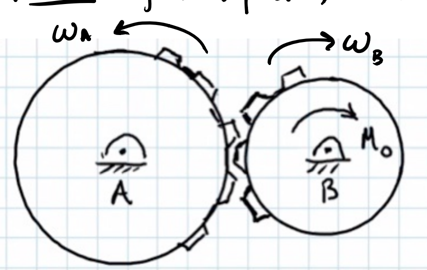
$$\int_{t_1}^{t_2} \Sigma M_G \, dt = I_G \omega_2 - \cancel{I_G \omega_1}$$



$$\int_0^{t_2} (Pr)\,dt = I_G \omega_2$$

$$P r t_2 = I_G \omega_2 \quad \therefore \boxed{\omega_2 = \frac{P r t_2}{I_G}}$$

19-22 – gear system, meshing gears

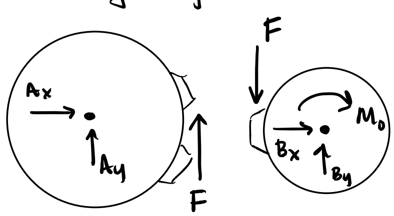


M_0 = constant system at rest at $t = 0$ then M_0 moment applied to B.

Find ω_B and ω_A at time t_2.

B moves clockwise, A moves counterclockwise

Free Body Diagram



(1), (2), & (3) are 3 equations for ω_{B2}, ω_{A2}, $\int_0^{t_2} F\,dt$

$$\int_{t_1}^{t_2} \Sigma M_B \, dt = I_B \omega_{B2} - \cancel{I_B \omega_{B1}}$$

$$\int_{t_1}^{t_2} \Sigma M_A \, dt = I_A \omega_{A2} - \cancel{I_A \omega_{A1}}$$

$$\int_0^{t_2} (M_0 - F r_B)\,dt = I_B \omega_{B2}$$

$$\int_0^{t_2} F r_A \, dt = I_A \omega_{A2}$$

$$M_0 t_2 - r_B \int_0^{t_2} F\,dt = I_B \omega_{B2} \quad (1)$$

$$+ r_A \int_0^{t_2} F\,dt = I_A \omega_{A2} \quad (2)$$

kinematics: $\omega_B r_B = \omega_A r_A$ (3)