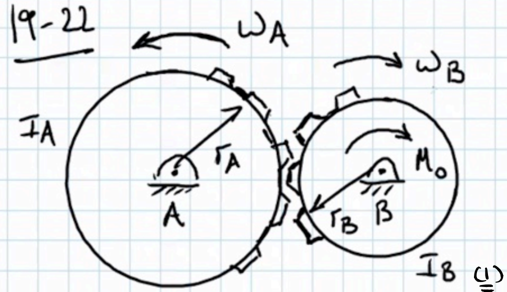
Dynamics Lecture #21

19-22



Homework 9 : 19-31, 19-33, 19-46
Read Ch. 22 Due W 4/22/2020

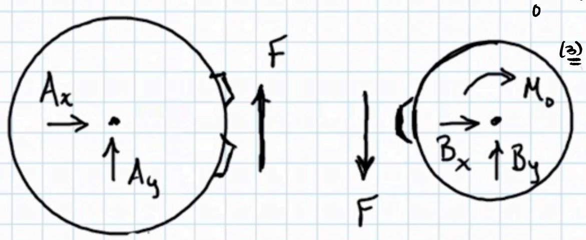
System initially at rest.
Constant moment M_o applied at $t=0$.
Find ω_B and ω_A for time $t>0$.

(1) $\int_0^t M_o\,dt - \int_0^t F r_B\,dt = I_B\,\omega_B(t) - \cancel{I_B\,\omega_B(0)}^{\,0}$

(2) $\int_0^t F r_A\,dt = I_A\,\omega_A(t) - I_A\,\omega_A(0)$

(3) Kinematic equation: $\omega_A r_A = \omega_B r_B$

Free-body diagrams



Rewrite (1) and (2) : take (1) and divide by r_B

$$\frac{1}{r_B} M_o t - \int_0^t F\,dt = \frac{1}{r_B} I_B\,\omega_B(t) \qquad (4)$$

$$\int_0^t F\,dt = \frac{1}{r_A} I_A\,\omega_A(t) \qquad (5)$$

Add (4) and (5)

$$\frac{1}{r_B} M_o t = \frac{1}{r_B} I_B \omega_B + \frac{1}{r_A} I_A\,\omega_A \qquad (6)$$

... leaving out t since ω is function of time

→ solve using (6) and (3)

$$\frac{1}{r_B} M_o t = \frac{1}{r_B} I_B\,\omega_B + \frac{1}{r_A} I_A \left(\frac{r_B}{r_A}\,\omega_B\right)$$

... multiply throughout by r_B

$$M_o t = I_B \omega_B + \frac{r_B}{r_A} I_A \frac{r_B}{r_A}\,\omega_B$$

$$\boxed{\omega_B = \frac{M_o t}{I_B + \left(\frac{r_B}{r_A}\right)^2 I_A}}$$

→ use this to solve $\omega_A = \frac{r_B}{r_A}\,\frac{M_o t}{I_B + \left(\frac{r_B}{r_A}\right)^2 I_A}$

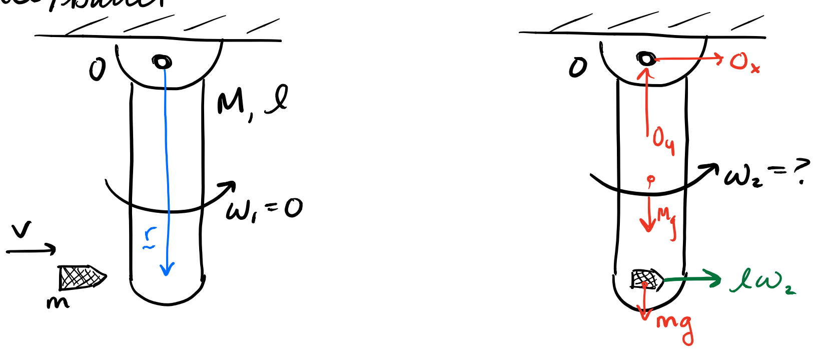
If $I_A = 0$ (gear A not present) then just have motor driving gear B
where $\omega_B = M_o t / I_B$

So when A *is* present,
the denominator changes

$$I_B + \left(\frac{r_B}{r_A}\right)^2 I_A = \text{the effective moment of a inertia of the two gears, as seen by the motor.}$$

Collision problem w/ rigid body
→ collision between pendulum (which we'll model as thin bar) and small projectile/bullet

we'll assume projectile becomes embedded into pendulum



just before collision

just after collision

During collision process, no external moments about O.

$$\therefore \begin{pmatrix}\text{angular}\\ \text{momentum}\end{pmatrix}_1 = \begin{pmatrix}\text{angular}\\ \text{momentum}\end{pmatrix}_2$$

$$lmv = I_o\omega_2 + l(ml\omega_2) = (I_o + ml^2)\omega_2$$

$$\omega_2 = \frac{lmv}{I_o + ml^2}$$

Is energy conserved in collision?
... analagous to plastic collision, energy is <u>not</u> conserved

energy of system 1 (of particle): $E_1 = \frac{1}{2}mv^2$

energy after collision:

$$E_2 = \frac{1}{2}I_o\omega_2^2 + \frac{1}{2}m(l\omega_2)^2$$

$$= \frac{1}{2}(I_o + ml^2)\omega_2^2$$

$$= \frac{1}{2}(I_o + ml^2)\frac{l^2m^2v^2}{(I_o + ml^2)^2}$$

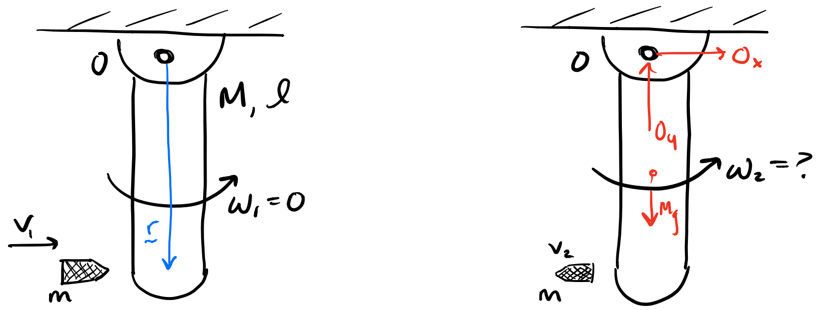
$$= \frac{1}{2}\frac{l^2m^2v^2}{(I_o + ml^2)} = \frac{1}{2}\frac{ml^2}{I_o + ml^2}mv^2$$

$$E_2 = \frac{1}{2}\left(\frac{ml^2}{I_o + ml^2}\right)mv^2$$

$$E_2 < E_1$$

$\therefore$ energy is <u>not</u> conserved

Now, let's see what happens when we assume energy is conserved ... elastic collision



just before collision   just after collision

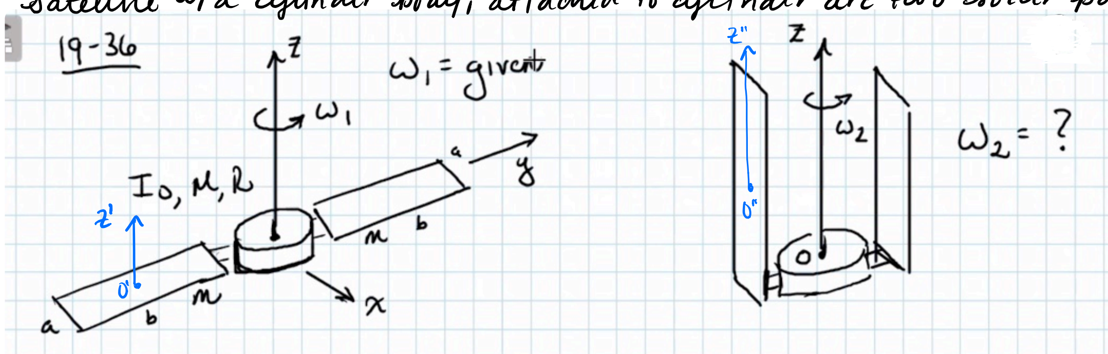
Angular momentum about O: $l m v_1 = I_o \omega_2 + l m v_2$ (1)

energy equation: $\cancel{\tfrac{1}{2}} m v_1^2 = \cancel{\tfrac{1}{2}} I_o \omega_2^2 + \cancel{\tfrac{1}{2}} m v_2^2$ (2)

→ two equations for ω_2 and v_2

19-36 SIMILAR TO HW PROBLEM

satellite w/ a cylinder body, attached to cylinder are two solar panels



angular velocity is expected to ↑ when panels fold in (like figure skater going faster when fold arms in)

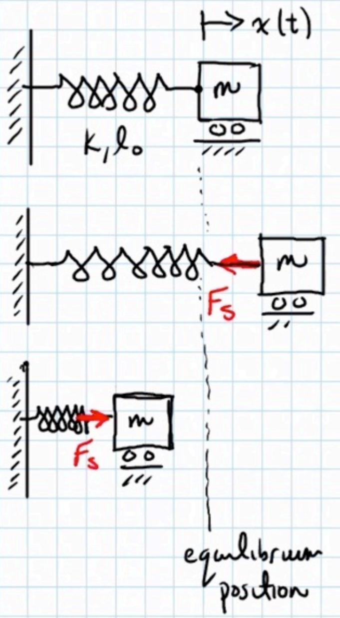
no external moments about O: $(\text{Ang. mom})_1 = (\text{Ang. mom})_2$

$$I_o \omega_1 + 2\left[I_{o'} + m\left(R + \tfrac{b}{2}\right)^2\right]\omega_1 = I_o \omega_2 + 2\left[I_{o''} + mR^2\right]\omega_2$$

for plates, have to use parallel axis theorem   have to use parallel axis th. for plates

$$\left\{I_o + 2\left[\tfrac{1}{12} m(a^2+b^2) + m\left(R+\tfrac{b}{2}\right)^2\right]\right\}\omega_1 = \underbrace{\left\{I_o + 2\left[\tfrac{1}{12} m a^2 + m R^2\right]\right\}}_{\text{this is smaller than ... so } \omega_2 > \omega_1}\omega_2$$

CH. 22 — VIBRATION THEORY

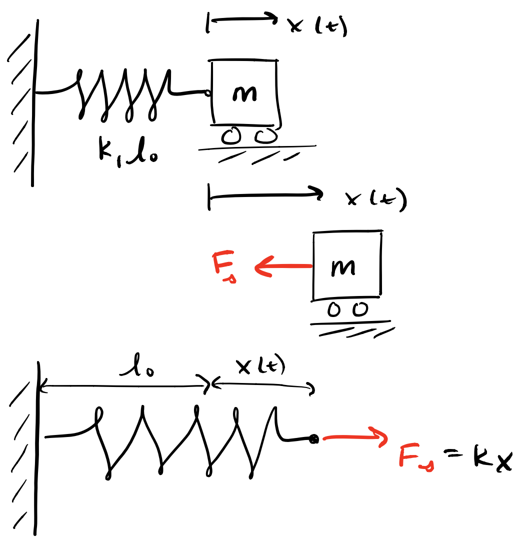


Ch. 22: Mechanical Vibrations (6)

Vibration: oscillatory (back-and-forth) motion about an equilibrium position.

If mass is disturbed from its equilibrium position, spring exerts a restoring force that pushes mass back toward its equilibrium position.

next, derive the equation of motion for displacement $x(t)$ of mass



For mass:

$$m\ddot{x} = -F_s$$

$$m\ddot{x} = -kx$$

$$m\ddot{x} + kx = 0 \qquad (1)$$

equation of motion for displacement $x(t)$ of mass

solution of (1) is

$$x(t) = A\sin\omega_n t + B\cos\omega_n t \quad \text{where } \omega_n = \sqrt{\frac{k}{m}}$$

→ 2nd order, linear DE

ω_n = natural frequency of the spring mass system

can also write: $x(t) = C\sin(\omega_n t + \phi)$

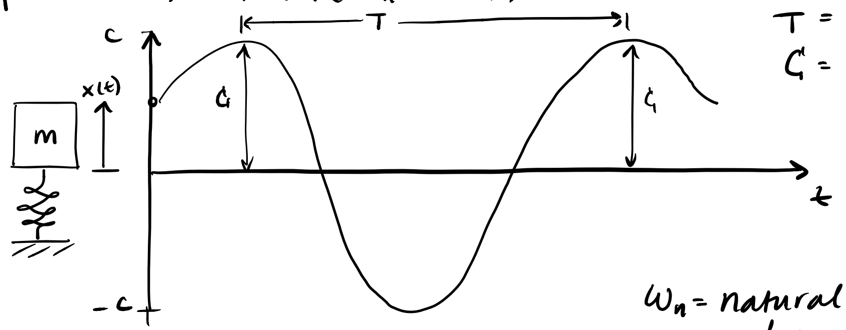
phase angle

where $C = \sqrt{A^2 + B^2}$

$\phi = \tan^{-1}(B/A)$

A & B, or C and ϕ are determined by initial conditions

plot $x(t) = C \sin(\omega_n t + \phi)$ vs. t



T = period of oscillation

C = amplitude of oscillation

$$T = \frac{2\pi}{\omega_n}$$

ω_n = natural frequency (or <u>radian</u> frequency)

... units are radians/second

f_n = cyclic frequency

... units are cycles/second or Hz

$$f_n = \frac{1}{T} \quad \text{(1)}$$

$$\omega_n = \frac{2\pi}{T} \quad \text{(2)}$$

$$\text{(1) \& (2)} \Rightarrow \underset{\text{rad/sec}}{\omega_n} = 2\pi \underset{\text{Hz}}{f_n}$$

READ VIBRATIONS CHAPTER