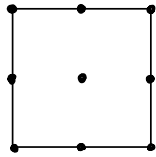


* Discussion of Final Project in beginning *

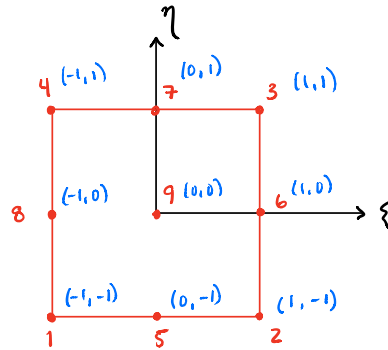
* note that this page was not recorded *

9-Node Quadratic Element

x, y space



isoparametric mapping



Shape Functions:

$$\begin{aligned} N_1 &= \frac{1}{2} \xi(\xi-1) \frac{1}{2} \eta(\eta-1) & N_6 &= \frac{1}{2} \xi(\xi+1)(1-\eta^2) \\ N_2 &= \frac{1}{2} \xi(\xi+1) \frac{1}{2} \eta(\eta-1) & N_7 &= (1-\xi^2) \frac{1}{2} \eta(\eta+1) \\ N_3 &= \frac{1}{2} \xi(\xi+1) \frac{1}{2} \eta(\eta+1) & N_8 &= \frac{1}{2} \xi(\xi-1)(1-\eta^2) \\ N_4 &= \frac{1}{2} \xi(\xi-1) \frac{1}{2} \eta(\eta+1) & N_9 &= (1-\xi^2)(1-\eta^2) \\ N_5 &= (1-\xi^2) \frac{1}{2} \eta(\eta-1) \end{aligned}$$

characteristics of shape function:

$$\rightarrow \sum_{i=1}^9 N_i(\xi, \eta) = 1$$

→ shape functions are 1 at node, 0 at all other nodes, vary quadratically in between

2D Error

$$\text{remainder in 1D: Error} = \left[\int |u_{\text{exact}} - u_{\text{FEM}}|^2 d\Omega \right]^{1/2}$$

in 2D: → convert to isoparametric space

$$= \left[\int_{-1}^1 \int_{-1}^1 [(error_x)^2 + (error_y)^2] \text{Det}(J) d\xi d\eta \right]^{1/2}$$

error_x: x component of analytical - FEM

$$u_{\text{exact}-x} - u_{\text{FEM}-x}$$

error_y: y component of analytical - FEM

$$u_{\text{exact}-y} - u_{\text{FEM}-y}$$

double summation:

$$= \left[\sum_{a=1}^{\text{dim}} \left[\sum_{i=1}^{NQ} \sum_{j=1}^{NA} (error_x^2 + error_y^2) w(\xi_i) w(\eta_j) \text{Det}(J) \right] \right]^{1/2} = \text{total error}$$

convergence

$\log_{10}(L^e)$

$\log_{10}(\text{error})$

for L^e : pick element length in one-direction (say x)