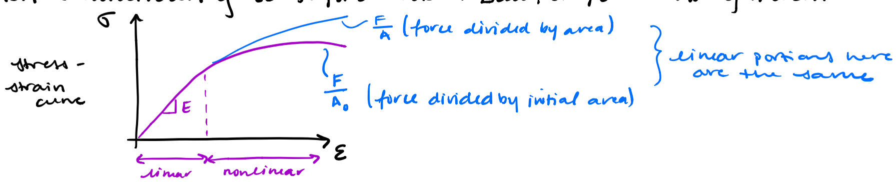
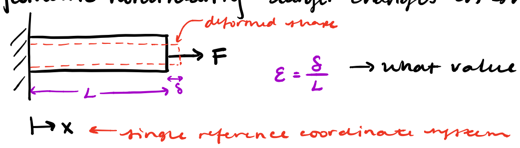
Nonlinear FEM/Nonlinear Continuum Mechanics

→ goal: ① understand when linear approximations breakdown
② see similarities to linear FEM

→ reasons to use nonlinear FEM vs. linear FEM
① material nonlinearity: ie beyond Hooke's Law, or for rubbery materials



② geometric nonlinearity: larger changes in shape (think stretching rubber band)

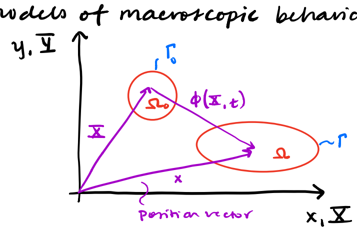


$\varepsilon = \frac{\delta}{L}$ → what value of ε makes the deformation geometrically nonlinear?

underlying assumption?
① geometry doesn't change much
② history of deformation doesn't matter

Define Motion & Kinematics

→ models of solids & fluids where properties can be characterized by smooth functions of spatial variables → means we ignore inhomogeneities like microstructure, grains, etc...
→ models of macroscopic behavior of matter



• volume ... Ω_0 = reference volume, Ω = current volume
• boundary ... Γ_0 = reference area, Γ = current area
$\phi(\mathbf{X}, t)$ = "motion"

$\mathbf{X}$ = material, or Lagrangian coordinates = $\sum_{i=1}^{NSD} \mathbf{X}_i e_i$ (# spatial dimensions; unit vector)
x = Eulerian coordinates = $\sum_{i=1}^{NSD} x_i e_i$
or current configuration

Can describe deformation in 2 ways:
$(\mathbf{X}, t)$ → material/Lagrangian → $\mathbf{X}_i$ are fixed in time
(x, t) → spatial/Eulerian → x_i coordinates do change with time

2 Classes of problems:
→ solids: Lagrangian
① stresses generally depend on deformation history
② knowing reference/undeformed configuration is useful

→ fluids: Eulerian
① stresses are history independent (in general, for most fluids)
② coordinates can move/convert with fluid flow

Lagrangian & Eulerian FEM is equivalent (can convert between the two)

Motion: $x = \phi(\mathbf{X}, t)$

→ basically maps reference configuration to current configuration at time t

$$\mathbf{X} = x(\mathbf{X}, 0) = \phi(\mathbf{X}, 0)$$

→ using this motion, can measure displacement: difference between current and reference configurations

$$u(\mathbf{X}, t) = \phi(\mathbf{X}, t) - \phi(\mathbf{X}, 0) = \phi(\mathbf{X}, t) - \mathbf{X}$$
$$= x - \mathbf{X}$$

$$u_i = x_i - \mathbf{X}_i$$

→ velocity will just be time derivative of displacement

$$\frac{\partial u(\mathbf{X}, t)}{\partial t} = v$$

→ acceleration: $\dfrac{\partial^2 u(\mathbf{X}, t)}{dt^2} = a$

Deformation Gradient (F): use to characterize deformation and strain

initial configuration current configuration

$\Rightarrow \boxed{F = \dfrac{dx}{d\mathbf{X}}}$ or: $dx = F\,d\mathbf{X}$

→ does F look like something we've used before? like Jacobian $J = \dfrac{dx}{d\xi}$

↳ $J = \det(F) \rightsquigarrow F$ in 2D: $F = \begin{bmatrix} \frac{\partial x}{\partial \mathbf{X}} & \frac{\partial x}{\partial \mathbf{Y}} \\ \frac{\partial y}{\partial \mathbf{X}} & \frac{\partial y}{\partial \mathbf{Y}} \end{bmatrix}$

- strain units: so why not use this deformation gradient F as a strain measure?
- what would strain be if nothing changed shape?
  - → strain should be zero if no deformation
  - → if no deformation, $d\mathbf{X} = dx$, then $F = 1$ which is nonsensical for a strain, so need to define a different strain measure!