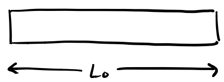
Thermal Stresses

→ due to changes in temperature ΔT

simple case: bar with initial length L_0



$\Delta L = \alpha \Delta T L_0$

α = coefficient of thermal expansion $\left(\frac{1}{{}^\circ F} \text{ or } \frac{1}{{}^\circ C}\right)$

axial strain due to $\Delta T = \frac{\Delta L}{L_0} = \frac{\alpha \Delta T L_0}{L_0} = \alpha \Delta T = \varepsilon_T$

total strain $\varepsilon_{tot} = \varepsilon_{mech} + \varepsilon_{thermal}$

$$\varepsilon_{tot} = \frac{\sigma}{E} + \alpha \Delta T$$

solve for stress: $\sigma = E(\varepsilon_{tot} - \alpha \Delta T)$

→ impact on the weak form? ... remember that considering linear problems here

governing DE:

$$\underbrace{\frac{d}{dx}(A\sigma)}_{\text{get standard weak form term}} + \underbrace{bA}_{\text{ignore as unaffected by thermal strain}} = 0$$

$$= -\int_0^L (\sigma A)\frac{d}{dx}(\delta u)\,dx$$

$$= -\int_0^L A(E\varepsilon - E\alpha\Delta T)\frac{d}{dx}(\delta u)\,dx$$

$$= \underbrace{-\int_0^L AE\varepsilon \frac{d}{dx}(\delta u)\,dx}_{\text{becomes } K} + \underbrace{\int_0^L AE\alpha\Delta T \frac{d}{dx}(\delta u)\,dx}_{\text{thermal strain term}}$$

evaluate as usually do: shape function, weight ...

$$= \int_0^L AE\alpha\Delta T\left[\frac{d}{dx}(N\delta u)\right]^T dx$$

$$= \delta u^T \int_0^L B^T AE\alpha\Delta T\,dx$$

$$Kd = F^{ext} + F^{body} + F^{thermal} \underbrace{+ \left(F^{electrical} + F^{magnetic} + F^{\cdots}\right)}_{\text{other stuff that could show up (if linear)}}$$

$$\Rightarrow F^{thermal} = \int_0^L B^T AE\alpha\Delta T\,dx$$

⋮

$$= \int_{-1}^{1}\left(\frac{dN}{d\xi}\cancel{\frac{d\xi}{dx}}\right)^T AE\alpha\Delta T \cancel{\frac{dx}{d\xi}}\,d\xi \Rightarrow \int_{-1}^{1}\begin{bmatrix} -\frac{1}{2} \\ \frac{1}{2} \end{bmatrix} AE\alpha\Delta T\,d\xi \Rightarrow F^{thermal}_{\text{for 1 element}} : \begin{bmatrix} -AE\alpha\Delta T \\ AE\alpha\Delta T \end{bmatrix}$$

↑ notice that forces sum to zero for each elem.