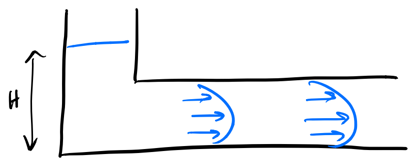
Fluids – Dimensional Analysis → CHAPTER 7: DIMENSIONAL ANALYSIS, SIMILITUDE, AND MODELING (7.1–10)

Our equation toolbox:

Conservation of mass
Conservation of linear momentum
Conservation of energy

— CV analysis
— Differential analysis

Now, experimental analysis



← example where we have a flow but we are unable to use differential analysis

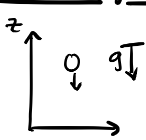
so we make a model to study pressure drop changes

$$\frac{\Delta P}{L} = f(D, V, \rho, \mu, H)$$

↳ ex. find relationship $\frac{\Delta P}{L}$ and V ... will need to test all of the variables that are in function
... difficult to have to run all of these experiments and controlling for several variables

Dimensional Analysis allows us to organize our variables

Ex. $\frac{d^2 z}{dt^2} = -g$



initial condition (ic) $= z_0$, initial vel $= V_0$

$$t = z_0 + V_0 t + -\frac{1}{2} g t^2$$

how do we change this in dimensional form?

$z = [L]$ $t = [T]$ $z_0 = [L]$ $V_0 = \left[\frac{L}{T}\right]$ $g = \left[\frac{L}{T^2}\right]$

z^* = undimensional variable $= \frac{z}{z_0}$ $\Rightarrow$ $z = z^* z_0$

t^* = undimensional time $= \frac{V_0 t}{z_0}$ $\Rightarrow$ $t = t^* \frac{z_0}{V_0}$

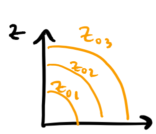
$$\frac{d^2 z}{dt} = \frac{d^2\left[z^* z_0\right]}{d\left[\frac{z_0 t^*}{V_0}\right]^2} = \frac{V_0^2}{z_0}\left[\frac{d^2 z^*}{dt^{*2}}\right] = -g$$

$$\boxed{\frac{V_0^2}{g z_0}} \frac{d^2 z^*}{dt^{*2}} = -1 \rightarrow \boxed{\frac{d^2 z^*}{dt^{*2}} = \frac{-1}{F_R^2}} \rightarrow z^* = 1 + t^* - \frac{1}{2F_R^2} t^{*2}$$

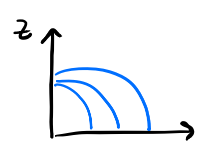
↑ FROUDE # $= \frac{V_0}{\sqrt{g z_0}}$ = studies effect of gravitational forces

Now running experiments...

① $t = z_0 + V_0 t - \frac{1}{2} g t^2$



$V_0 = C$
– keep initial velocity, varies initial heights



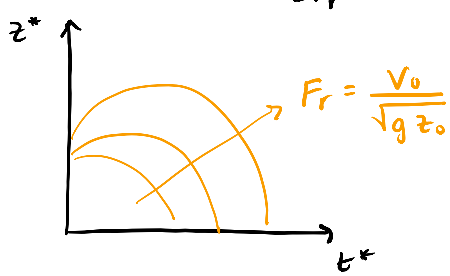
$z_0 = C$
– keep initial height same, but vary init. velocity

Why we want to use Dimensional Form:

→ allows us to simplify more

- To generate non-dimensional parameters that help in the design of experiments (physical and/or numerical) and in the reporting of experimental results
- To obtain scaling laws so that prototype performance can be predicted from model performance
- To (sometimes) predict trends in the relationship between parameters

Use $z^* = 1 + t^* - \frac{1}{2F_r^2} t^{*2}$



- Can we always use dimensional analysis to reduce the number of variables?
- How many dimensionless products are necessary to replace the original list of variables?

If an equation involving *k* variables is dimensionally homogeneous, it can be reduced to a relationship among *k* − *r* independent dimensionless products, where *r* is the minimum number of reference dimensions required to describe the variables.

How do we organize variables?

→ use capital pi Π

$$\underset{\text{dependent variable}}{\Pi} = \underset{\text{function of independent pi's}}{\phi(\Pi_1, \Pi_2, \Pi_3, \dots)}$$

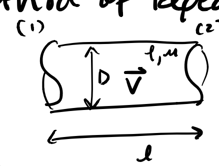
ϕ is found via experiments!

so the final aim of DA is finding all of these different pi's

How to build Π? → 2 ways

1. method of repeating variables (very long, but very systematic)
2. by inspection (much shorter)

Method of Repeating Variables ①



pipe flow that's steady, incompressible Newtonian, horizontal circular pipe

step 1

$$\frac{\Delta P}{\ell} = f(D, \rho, \mu, V)$$

step 2

express as dimensions using F L T or M L T

$\frac{\Delta P}{L} = FL^{-3}$ $\quad D = L$ $\quad \rho = FL^{-4}T^2$ $\quad \mu = FL^{-2}T$ $\quad V = LT^{-1}$

step 3

How many Π terms do need?

$$\#\ \Pi \text{ terms} = \underbrace{\#\text{ of variables}}_{k} - \underbrace{\#\text{ of dimensions we need to describe var.}}_{R}$$

here: $k = 5$ … 5 variables

$R = 3$ … only 3 units to describe all of the variables

$5 - 3 =$ 2 Π terms

step 4

Find the repeating variables

$R = \#$ of rep. var $= 3 \Rightarrow F, L, T$

step 5

always start w/ dependent variable

$$\Pi_1 = \frac{\Delta P}{L} D^a V^b \rho^c$$

$$(FL^{-3}) L^a (LT^{-1})^b (FL^{-4}T^2)^c = F^0 L^0 T^0$$

$F^0 \Rightarrow 1 + c = 0 \rightarrow c = -1$

$L^0 \Rightarrow -3 + a + b - 4c = 0 \rightarrow a = 1$

$T^0 \Rightarrow -b + 2c = 0 \rightarrow b = -2$

$$\Pi_1 = \frac{\Delta P/L \; D}{\rho V^2}$$

step 6

take the remaining variable (μ here) to build Π_2

$$\Pi_2 = \mu D^a V^b \rho^c$$

$$(FL^{-2}T) L^a (LT^{-1})^b (FL^{-4}T^2)^c$$

$F^0 \Rightarrow 1 + c = 0 \rightarrow a = -1$

$L^0 \Rightarrow -2 + a + b - 4c = 0 \rightarrow b = -1$

$T^0 \Rightarrow 1 - b + 2c = 0 \rightarrow c = -1$

$$\Pi_2 = \frac{\mu}{DV\rho}$$

step 7

check they are dimensional

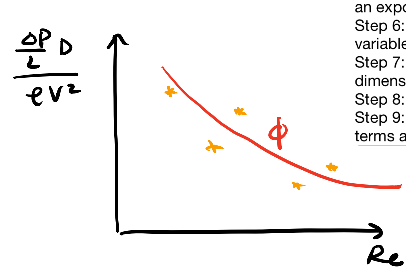
step 8

rearrange if have special #s:

for ex. $\Pi_2 = \frac{\mu}{DV\rho} = \frac{1}{Re}$

step 9

$$\frac{\frac{\Delta P}{L} D}{\rho V^2} = \phi(Re)$$



Determination of Pi terms: method of repeating variables

Step 1: list all the variables that are involved in the problem
Step 2: express each of the variables in terms of basic dimensions
Step 3: Determine the required number of pi terms
Step 4: Select a number of repeating variables, where the number required is equal to the number of reference dimensions
Step 5: Form a pi term by multiplying one of the non-repeating variables by the product of the repeating variables, each risen to an exponent that will make the combination dimensionless
Step 6: Repeat step 5 for each of the remaining non repeating variables
Step 7: check all the resulting pi terms to make sure they are dimensionless
Step 8: Rearrange the pi terms to simplify the problem
Step 9: Express the final form as a relationship among the pi terms and think about what it means

Last Week

4/22/20

- what is dimensional analysis, why and when we need it
- what is the purpose of dimensional analysis
- Buckingham PI Theorem
- method of repeating variables

$$\frac{\Delta P}{L} = \phi(D, \rho, \mu, V) \rightarrow \frac{\frac{\Delta P}{L} D}{\rho V^2} = \phi(Re)$$

Determination of PI terms by inspection

pressure drop per unit length

$$\frac{\Delta P}{L} = \phi(D, \rho, \mu, V)$$

diameter, density, viscosity, velocity

$$\frac{\Delta P}{L} = FL^{-3} \qquad D = L \qquad \rho = FL^{-4}T \qquad \mu = FL^{-2}T \qquad V = LT^{-1}$$

5 variables, 3 ref dim. $\Rightarrow$ $5 - 3 = 2\pi$

$$\frac{\Delta P}{L} = FL^{-3} \rightarrow \text{divide by } \rho \rightarrow \frac{\Delta P}{L\rho} = \frac{FL^{-3}}{FL^{-4}T^2} = \frac{L}{T^2} \quad \text{(cancel F)}$$

$$\frac{\Delta P/L}{\rho}\left(\frac{1}{V^2}\right) = \frac{L}{T^2}\left(\frac{1}{L^2/T^2}\right) = \frac{1}{L} \quad \text{(cancel T)}$$

$$\boxed{\frac{\Delta P/L}{\rho}\left(\frac{1}{V^2}\right) D = F^0 T^0 L^0} \quad \pi \leftarrow \text{first pi}$$

π_2 : start w/ viscosity $\rightarrow \mu = FL^{-2}T$, want to get rid of force component but can't use dependent variable $\Delta P/L$!!!

$\rightarrow$ divide by ρ density

$$\pi_2 : \frac{\mu}{\rho} = \frac{FL^{-2}T}{FL^{-4}T^2} = \frac{L^2}{T}$$

... now want to get rid of velocity
$\leftarrow$ cancel F

$$\frac{\mu}{\rho}\left(\frac{1}{V}\right) = \frac{L^2}{T}\left(\frac{T}{L}\right) = L$$

... now want to get rid of length, divide by diameter

$$\frac{\mu}{\rho V D} = L^0 F^0 T^0 = \frac{1}{Re}$$

$$\boxed{\frac{\Delta P/L\ D}{\rho V^2} = \phi_1\left(\frac{\mu}{\rho V D}\right)} \qquad \boxed{\frac{\Delta P/L\ D}{\rho V^2} = \phi_2\left(\underbrace{\frac{\rho V D}{\mu}}_{Re\ \#}\right)}$$

The wing on the aircraft in Fig. 8–3 is subjected to a drag F_D created by airflow over its surface. It is anticipated that this force is a function of the density ρ and viscosity μ of the air, the "characteristic" length L of the wing, and the velocity V of the approaching flow. Show how the drag force depends on these variables.

$$F_D = f(\rho, \mu, L, V)$$

$F_D = F$

$\rho = F T^2 L^{-4}$

$\mu = F T L^{-2}$

$L = L$

$V = L T^{-1}$

5 variables, 3 reference dimensions : 5−3 = 2 π ... 1 dependent π, 1 independent π

$$\pi_1 = \phi(\pi_2)$$

dependent variable

π_1 : $F_D \rho^a L^b V^c = F^0 L^0 T^0$

$$F(F^a T^{2a} L^{-4a}) L^b (L^c T^{-c}) = F^0 L^0 T^0$$

$F^0 \rightarrow a + 1 = 0 \rightarrow a = -1$

$L^0 \rightarrow -4a + b + c = 0 \rightarrow b = -2$

$T^0 \rightarrow 2a - c = 0 \rightarrow c = -2$

$$\pi_1 = \rho^{-1} L^{-2} V^{-2} F_D = \frac{F_D}{\rho L^2 V^2}$$

π_2 : $\mu \rho^a L^b V^c$

$$FLT^{-2} (F^a T^{2a} L^{-4a})(L^b)(L^c T^{-c}) = F^0 L^0 T^0$$

$F^0 \rightarrow a + 1 = 0 \rightarrow a = -1$

$L^0 \rightarrow -4a + b + c - 2 = 0 \rightarrow b = -1$

$T^0 \rightarrow 2a - c + 1 = 0 \rightarrow c = -1$

$$\pi_2 = \frac{\mu}{\rho V L} = \frac{1}{Re}$$

$$\frac{F_D}{\rho L^2 V^2} = \phi(Re) \Rightarrow \boxed{F_D = \rho L^2 V^2 [\phi(Re)]}$$

<u>By inspection:</u>

$F^0 \rightarrow \frac{F_D}{\rho} = \frac{FL^4}{FT^2}$ $\quad T^0 \rightarrow \frac{F_D}{\rho V^2} = \frac{L^4}{T^2}\left(\frac{T^2}{L^2}\right)$ $\quad L^0 \rightarrow \frac{F_D}{\rho V^2 L^2} = L^2\left(\frac{1}{L^2}\right)$

... get same drag coefficient $\frac{F_D}{\rho V^2 L^2}$

Selection of Variables

$\pi_1 = \phi(\pi_2 \ldots)$

1. Clearly define the problem. What is the main variable of interest (the dependent variable)? Find dependent variable
2. Consider the basic laws that govern the phenomenon.
3. Start the variable selection process by grouping the variables into three broad classes: geometry, material properties, and external effects.
4. Consider other variables that may not fall into one of the above categories.
5. Be sure to include all quantities that enter the problem even though some of them may be held constant (e.g., the acceleration of gravity, g)
6. Make sure that all variables are independent

Common Dimensionless Groups

Reynolds Number

* seeing creeping flow videos

most important one!

$$\mathrm{Re} = \frac{\text{inertia force}}{\text{viscous force}} = \frac{\rho V L}{\mu}$$

Common Dimensionless Groups

Froude Number

- altitude

$$\mathrm{Fr} = \sqrt{\frac{\text{inertia force}}{\text{gravitational force}}} = \frac{V}{\sqrt{gL}}$$

Common Dimensionless Groups

Euler Number

$\frac{\Delta P}{L}$

$$\mathrm{Eu} = \frac{p}{\rho V^2}$$

closed conduit kind of problems

$$\mathrm{Eu} = \frac{\text{pressure force}}{\text{inertia force}} = \boxed{\frac{\Delta p}{\rho V^2}}$$

Common Dimensionless Groups

won't see in class as much → Strouhal Number

see how steady flow is

$$\mathrm{St} = \frac{\omega \ell}{V}$$

ratio of inertial forces due to the unsteadiness of the flow (local acceleration) to the inertial forces due to changes in velocity from point to point in the flow field (convective acceleration)

Common Dimensionless Groups

won't study this → **Mach Number** ← for compressibility, compressible flows

$$\mathrm{M} = \sqrt{\frac{\text{inertia force}}{\text{compressibility force}}} = \frac{V}{c}$$

$$c = \sqrt{E_v/\rho}$$

When the **Mach number is relatively small (less than 0.3) the compressibility of the fluid can be neglected**

Cauchy Number

$$\mathrm{Ca} = \frac{\rho V^2}{E_v}$$

Common Dimensionless Groups

won't study this → Weber Number

open channel kind of problems

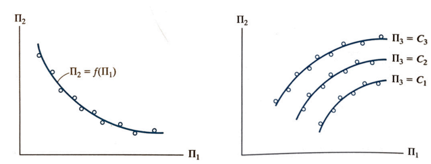
$$\mathrm{We} = \frac{\text{inertia force}}{\text{surface tension force}} = \frac{\rho V^2 L}{\sigma}$$

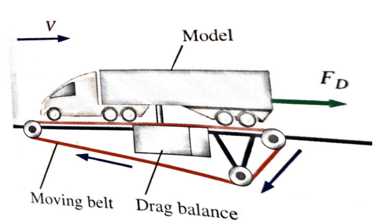
The equations of fluid mechanics are **dimensionally homogeneous**, which means that each term of an equation must have the same combination of units.

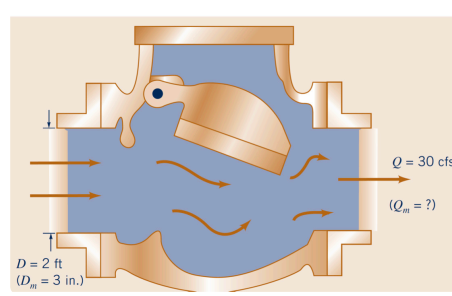
Dimensional analysis is used to **reduce the amount of data that must be collected from the variables in an experiment** in order to understand the behavior of a flow. This is done by arranging the variables into selected groups that are dimensionless. Once this is done, then it is only necessary to find one relationship among the dimensionless groups, and not to find several relationships among all the separate variables.

Five important dimensionless ratios of force occur frequently in fluid mechanics. All involve a **ratio of the dynamic or inertia force to some other force**. These five "numbers" are the Euler number for pressure, the Reynolds number for viscosity, the Froude number for weight, the Weber number for surface tension, and the Mach number for the elastic force causing compression.

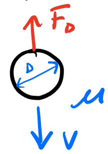
The Buckingham Pi theorem provides a systematic method for performing a dimensional analysis. T**he theorem indicates in advance how many unique dimensionless groups of variables (TI terms) to expect, and it provides a way to formulate a relationship among them.**







Example: at time 11:05 am



Re << 1 (laminar flow) ... creeping flow! (because Re # very small, so can neglect ρ)

$F_D = \phi(D, V, \mu, \cancel{\rho})$

$F_D = F$; $D = L$; $V = LT^{-1}$; $\mu = FL^{-2}T$

4 variables, 3 reference dimensions $\Rightarrow$ 4 − 3 = 1 π ... $\pi_1 = C_1$

π_1 (by inspection) $\Rightarrow \dfrac{F_D}{\mu} = \dfrac{F}{FL^{-2}T} \Rightarrow \dfrac{F_D}{\mu} = \dfrac{L^2}{T} \Rightarrow \dfrac{F_D}{\mu}\left(\dfrac{1}{V}\right) = \dfrac{L^2}{T}\left(\dfrac{T}{L}\right) \Rightarrow$

$\Rightarrow \dfrac{F_D}{\mu V} = L \Rightarrow \dfrac{F_D}{\mu V D} = \dfrac{L}{L} \Rightarrow \dfrac{F_D}{\mu V D} = C_1$ or $F_D = C_1 \mu V D$

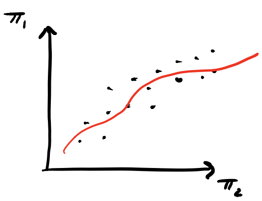
$C_1 = 3\pi$ ...

so drag force is directly related to velocity for creeping flow

Re << 1 ... $\boxed{F_D = 3\pi\mu V D \text{ Stoke's Law}}$

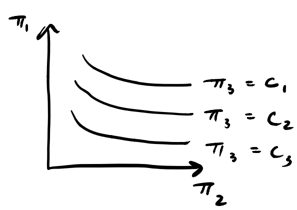
$C_1 = 3\pi$, found in experiments

$\pi_1 = \phi(\pi_2)$



* so have to do a lot of experiments!

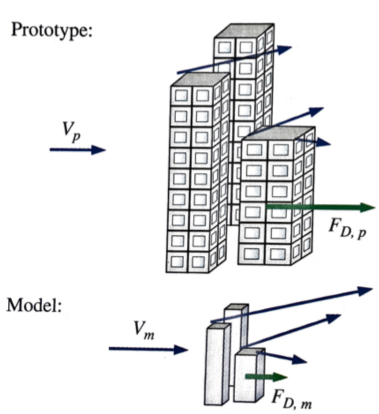
$\pi_1 = \phi(\pi_2, \pi_3, \ldots)$



Modeling and Similitude

A **model** is a representation of a physical system that may be used to predict the behavior of the system in some desired respect.

The physical system for which the predictions are to be made is called the **prototype**



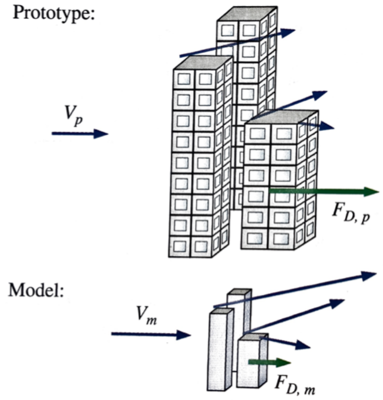
DA allows us to make sure that model is good representation of system

Modeling and Similitude

Three necessary conditions to complete similarity:

- Geometric similarity: the model must be the same shape as the prototype, but may be scaled by some constant factor

$$\frac{L_m}{L_p}$$



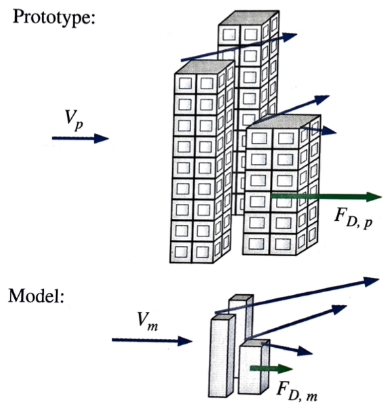
Modeling and Similitude

Three necessary conditions to complete similarity:

- Geometric similarity
- Kinematic similarity:

The velocity at any point in the model flow must be proportional to the velocity at the corresponding point in the prototype flow

$$\frac{V_m}{V_p} = \frac{L_m T_p}{L_p T_m}$$



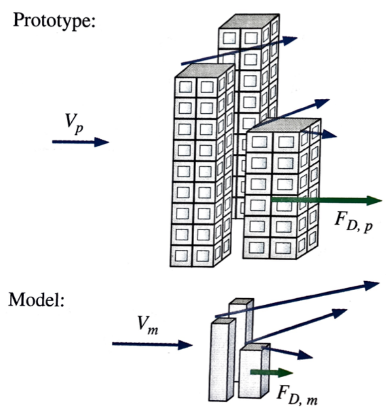
Modeling and Similitude

Three necessary conditions to complete similarity:

- Geometric similarity
- Kinematic similarity
- Dynamic similarity:

All forces in the model flow scale by a constant factor to corresponding forces in the prototype flow

$$\frac{F_m}{(F_i)_m} = \frac{F_p}{(F_i)_p}$$



Modeling and Similitude

To ensure complete similarity the model and prototype must be geometrically similar, and all independent Π groups must match between model and prototype

$$\Pi_1 = f(\Pi_2, \Pi_3, \ldots, \Pi_k)$$

If $\Pi_{2,m} = \Pi_{2,p}$ and $\Pi_{3,m} = \Pi_{3,p} \cdots$ and $\Pi_{k,m} = \Pi_{k,p}$,

then $\Pi_{1,m} = \Pi_{1,p}$

$\pi_1 = \phi(\pi_2) \rightarrow$ prototype $\qquad \pi_{1m} = \phi(\pi_{2m}) \rightarrow$ model

if $\pi_2 = \pi_{2m}$ then $\pi_1 = \pi_{1m}$

... can infer dependent variable from model

The aerodynamic drag of a new sports car is to be predicted at a speed of 50.0 mi/h at an air temperature of 25°c. Automotive engineers build a one-fifth scale model of the car to test in a wind tunnel. It is winter and the wind tunnel is located in an unheated building; the temperature of the wind tunnel air is only about 5°C. Determine how fast the engineers should run the wind tunnel in order to achieve similarity between the model and the prototype.

ON THE FINAL!!!

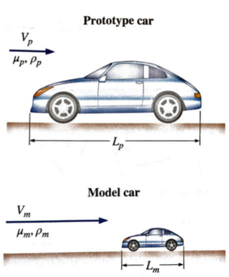
F_D car @ speed 50 mi/hr
T = 25°C

Model $\frac{1}{5}$ to test in wind tunnel T = 5°C

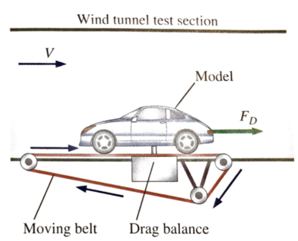
How fast should the flow be in wind tunnel to ensure similarity?

car T = 20°C, $\rho = 1.184 \frac{kg}{m^3}$
$\mu = 1.849 (10^{-5}) \frac{kg}{m s}$

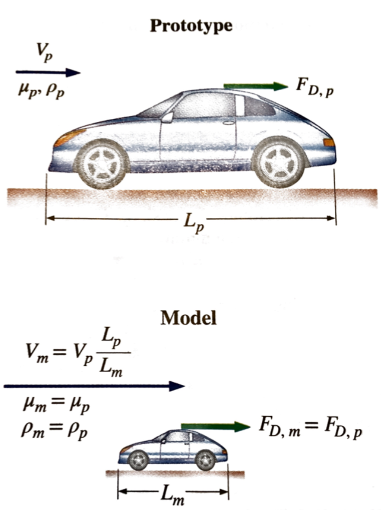
model/prototype @ T = 5°C, $\rho_m = 1.269 \frac{kg}{m^3}$
$\mu_m = 1.754 (10^{-5}) \frac{kg}{m s}$



Geometric similarity



Kinematic similarity



$F_D = \phi(\rho, \mu, L, V)$

$F_D = F$; $\rho = FT^2L^{-4}$; $\mu = FTL^{-2}$; $L = L$
$V = LT^{-1}$

$\frac{F_D}{\rho} = \frac{L^4}{T^2} \rightarrow \frac{F_D}{\rho V^2} = L^2 \rightarrow \boxed{\frac{F_D}{\rho V^2 L^2}} = L^0 T^0 F^0$

↑ Drag coefficient

2nd #: look @ viscosity
$\mu = FTL^{-2}$ $\quad \frac{\mu}{\rho} = \frac{L^2}{T}$ $\quad \frac{\mu}{V\rho} = L$

$\frac{\mu}{\rho V L} = \frac{1}{Re} \rightarrow \boxed{\frac{F_D}{\rho V^2 L^2} = \phi\left(\frac{\rho V L}{\mu}\right)}$

now have to make sure independent π is same
so make sure π_2 is same as π_1 (model)

$$\Pi_2 = Re = \Pi_{2m} = Re_m$$

$$\left\{ Re_m = \frac{\rho_m V_m L_m}{\mu_m} = \frac{\rho V L}{\mu} \quad \rightarrow L = 5 L_m \right\}$$

$\rightarrow$ we can rewrite this relationship & find velocity

$$V_m = V\left(\frac{\mu_m}{\mu}\right)\left(\frac{\rho}{\rho_m}\right)\left(\frac{L_p}{L_m}\right) = 221$$

$$\underline{V_m = 221 \text{ mi/hr}}$$

Part II: $F_D = 21.2$ lbf
$\rightarrow$ Find F_D at 50 mi/hr and 25°C

Since $\pi_{2m} = \pi_2$ then $\pi_{1m} = \pi_1$

$$\frac{F_{Dm}}{\rho_m V_m^2 L_m^2} = \frac{F_D}{\rho V^2 L^2} \Rightarrow F_D = F_{Dm}\left(\frac{\rho}{\rho_m}\right)\left(\frac{V}{V_m}\right)^2\left(\frac{L}{L_m}\right)^2 = \boxed{25.3 \text{ lbf}}$$

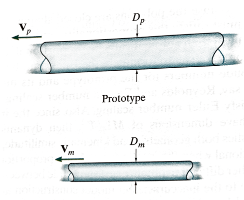
Flows through closed conduits

$$\frac{\ell_{im}}{\ell_i} = \frac{\varepsilon_m}{\varepsilon} = \frac{\ell_m}{\ell} = \lambda_\ell \qquad \frac{\rho_m V_m \ell_m}{\mu_m} = \frac{\rho V \ell}{\mu}$$

Flows through closed conduits

$$\frac{\ell_{im}}{\ell_i} = \frac{\varepsilon_m}{\varepsilon} = \frac{\ell_m}{\ell} = \lambda_\ell$$

$$\frac{\rho_m V_m \ell_m}{\mu_m} = \frac{\rho V \ell}{\mu}$$



$$\left(\frac{\rho V D}{\mu}\right)_m = \left(\frac{\rho V D}{\mu}\right)_p$$

$$V_m D_m = V_p D_p$$

Flows around immersed bodies

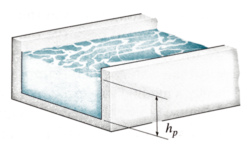
$$\frac{\ell_{im}}{\ell_m} = \frac{\ell_i}{\ell} \qquad \frac{\varepsilon_m}{\ell_m} = \frac{\varepsilon}{\ell}$$

$$\frac{\rho_m V_m \ell_m}{\mu_m} = \frac{\rho V \ell}{\mu}$$

drag coeff?

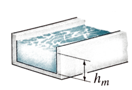
$$\frac{\mathscr{D}}{\frac{1}{2}\rho V^2 \ell^2} = \frac{\mathscr{D}_m}{\frac{1}{2}\rho_m V_m^2 \ell_m^2}$$

Flows in open channels (with a free surface)



Prototype

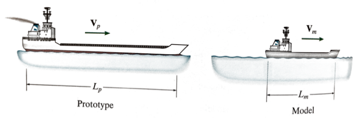
$$\frac{V_m}{\sqrt{h_m}} = \frac{V_p}{\sqrt{h_p}}$$



Model

Flows with a free surface

$$F_D = \rho L^2 V^2 f[\mathrm{Re}, \mathrm{Fr}]$$



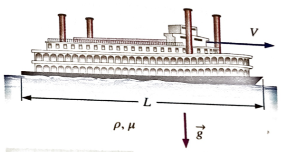
beyond scope of this class

$$\left(\frac{\rho V L}{\mu}\right)_m = \left(\frac{\rho V L}{\mu}\right)_p \rightarrow \frac{V_p}{V_m} = \frac{\nu_p L_m}{\nu_m L_p}$$

$$\left(\frac{V}{\sqrt{gL}}\right)_m = \left(\frac{V}{\sqrt{gL}}\right)_p \rightarrow \frac{V_p}{V_m} = \left(\frac{L_p}{L_m}\right)^{1/2}$$

$$\Rightarrow \frac{\nu_p}{\nu_m} = \left(\frac{L_p}{L_m}\right)^{3/2}$$

Flows with a free surface



$$\mathrm{Re} = \frac{\rho V L}{\mu} = \frac{VL}{\nu} \qquad \mathrm{Fr} = \frac{V}{\sqrt{gL}}$$

$$\frac{\ell_{im}}{\ell_m} = \frac{\ell_i}{\ell} \qquad \frac{\varepsilon_m}{\ell_m} = \frac{\varepsilon}{\ell}$$

$$\frac{\rho_m V_m \ell_m}{\mu_m} = \frac{\rho V \ell}{\mu}$$

$$\frac{V_m}{\sqrt{g_m \ell_m}} = \frac{V}{\sqrt{g \ell}}$$

surface tension complicates more!

$$\frac{\sigma_m/\rho_m}{\sigma/\rho} = (\lambda_\ell)^2$$

- *Inertia, pressure, and viscous forces* predominate within the flow around cars, slow-flying airplanes, and the flow through pipes and ducts. **Reynolds** number similitude.
- *Inertia, pressure, and gravity forces* predominate within the flow along open channels, over dams and spillways, or the wave action on structures. **Froude** number similitude.
- *Inertia, pressure, and surface tension forces* predominate within the flow of liquid films, bubble formation, and the passage of liquids through capillarity or small-diameter tubes. **Weber** number similitude.
- *Inertia, pressure, and compressibility forces* predominate within the flow around high-speed aircraft, or the high-speed flow of gas through jet or rocket nozzles, and through pipes. **Mach** number similitude.

DIMENSIONALIZE N-S EQUATION!!!

Nondimentionalized equations of motion

$$\vec{\nabla}\cdot\vec{V} = 0 \qquad \rho\frac{D\vec{V}}{Dt} = \rho\left[\frac{\partial \vec{V}}{\partial t} + (\vec{V}\cdot\vec{\nabla})\vec{V}\right] = -\vec{\nabla}P + \rho\vec{g} + \mu\nabla^2\vec{V}$$

Scaling parameters used to nondimensionalize the continuity and momentum equations, along with their primary dimensions

Scaling Parameter	Description	Primary Dimensions
L	Characteristic length	{L}
V	Characteristic speed	{Lt^{-1}}
f	Characteristic frequency	{t^{-1}}
$P_0 - P_\infty$	Reference pressure difference	{mL^{-1}t^{-2}}
g	Gravitational acceleration	{Lt^{-2}}

$$t^* = ft \qquad \vec{x}^* = \frac{\vec{x}}{L} \qquad \vec{V}^* = \frac{\vec{V}}{V}$$

$$P^* = \frac{P - P_\infty}{P_0 - P_\infty} \qquad \vec{g}^* = \frac{\vec{g}}{g} \qquad \vec{\nabla}^* = L\vec{\nabla}$$

Next time... pipe flow

11:45 AM mark

Maybe extra credit here?

Nondimentionalized equations of motion

$$\frac{V}{L}\vec{\nabla}^*\cdot\vec{V}^* = 0 \qquad \vec{\nabla}^*\cdot\vec{V}^* = 0$$

$$\rho(\vec{V}\cdot\vec{\nabla})\vec{V} = \rho\left(V\vec{V}^*\cdot\frac{\vec{\nabla}^*}{L}\right)V\vec{V}^* = \frac{\rho V^2}{L}\left(\vec{V}^*\cdot\vec{\nabla}^*\right)\vec{V}^*$$

$$\rho V f\frac{\partial \vec{V}^*}{\partial t^*} + \frac{\rho V^2}{L}\left(\vec{V}^*\cdot\vec{\nabla}^*\right)\vec{V}^* = -\frac{P_0 - P_\infty}{L}\vec{\nabla}^* P^* + \rho g\vec{g}^* + \frac{\mu V}{L^2}\nabla^{*2}\vec{V}^*$$

$$\left[\frac{fL}{V}\right]\frac{\partial \vec{V}^*}{\partial t^*} + \left(\vec{V}^*\cdot\vec{\nabla}^*\right)\vec{V}^* = -\left[\frac{P_0 - P_\infty}{\rho V^2}\right]\vec{\nabla}^* P^* + \left[\frac{gL}{V^2}\right]\vec{g}^* + \left[\frac{\mu}{\rho V L}\right]\nabla^{*2}\vec{V}^*$$

Nondimentionalized equations of motion

$$\vec{\nabla}^*\cdot\vec{V}^* = 0$$

$$[\mathrm{St}]\frac{\partial \vec{V}^*}{\partial t^*} + (\vec{V}^*\cdot\vec{\nabla}^*)\vec{V}^* = -[Eu]\vec{\nabla}^* P^* + \left[\frac{1}{\mathrm{Fr}^2}\right]\vec{g}^* + \left[\frac{1}{\mathrm{Re}}\right]\nabla^{*2}\vec{V}^*$$

St (Struhal) = characteristic flow time/Period of oscillation (steadiness of the flow)
Eu (Euler) = Pressure forces/inertia forces
Fr (Froude) = inertial force/gravitational force
Re (Reynolds) =inertial forces/viscous forces

When building a model we need to ensure similarity for all of these parameters