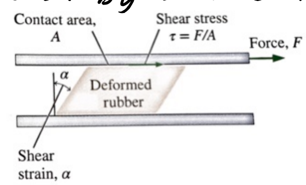
Fluid Mechanics

Hydrostatics — Kinematics — Fluid Dynamics

What is a fluid?

a substrate that deforms continuously when acted on by a shearing stress of any size

→ we treat fluid as a continuum



Characteristics of a fluid:

Density $\rho = \frac{m}{\forall} = \left[\frac{kg}{m^3}\right]$

specific weight $\gamma = \frac{W}{\forall} = \rho g = \left[\frac{N}{m^3}\right]$

specific gravity $SG = \frac{\rho}{\rho_{H_2O @ 4°C}} = \frac{\gamma}{\gamma_{water}}$

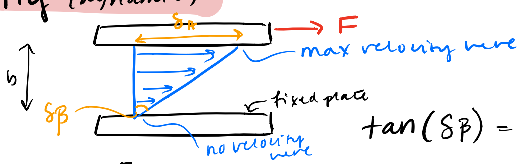
Ideal gas law:

$P = \rho R_s T$

speed of sound in ideal gas

$c = \sqrt{kRT}$

Viscosity (dynamic)



δ_A = displacement of top fluid

δ_β = change in angle

$\tan(\delta\beta) = \frac{\delta_A}{b} = \delta\beta$

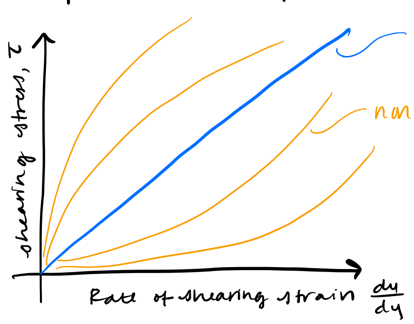
$\tau = \frac{\text{shear } F}{A} = \mu \frac{du}{dy}$ where u = velocity

μ units : $\left[\frac{kg}{m\,s}\right] = \left[\frac{N\,s}{m^2}\right] = [Pa\,s] = \text{Poise}$

*no slip condition (dye sticks to the bottom)

$F = A\mu \frac{du}{dy}$ (if remove kg by dividing by ρ get kinematic viscosity)

↑ rate of shearing strain



if $\mu \simeq 0$, then $\tau \simeq 0$ (no shear stress)

INVISCID FLUID

IDEAL FLUID IS INVISCID & INCOMP

∇ operator

$\nabla = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$

ex. $f(x,y) = x^2 - xy \rightarrow \frac{\partial f}{\partial x} = 2x - y \quad \frac{\partial f}{\partial y} = -x \quad \frac{\partial f}{\partial z} = 0$

$\nabla f = (2x-y)\hat{i} + (-x)\hat{j}$ at $\nabla f(2,1) = 3\hat{i} - 2\hat{j}$

Divergence

$\nabla \cdot \vec{V} = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right) \cdot \left(V_x\hat{i} + V_y\hat{j} + V_z\hat{k}\right) = \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z}$

ex. $\vec{V} = (4yx^{-2})\hat{i} + (\sin y)\hat{j} + 3\hat{k} \rightarrow \nabla \cdot \vec{V} = -8yx^{-3} + \cos y + 0$

Laplacian: $\nabla^2 f$

velocity profile

ex. $u = \frac{3V}{2}\left[1 - \left(\frac{y}{h}\right)^2\right]$ $\tau = \mu \frac{du}{dy}$ $\frac{du}{dy} = \frac{3V}{2}\left(-\frac{2y}{h^2}\right) = -\frac{3Vy}{h^2}$ $\rightarrow \tau = \frac{-\mu 3Vy}{h^2}$

Sea level atmospheric pressure

14.7 psi 101 kPa 1 atm 760 mmHg = 760 torr

Vapor Pressure

boiling is initiated when the absolute pressure in the fluid reaches the vapor pressure

→ pressure that the vapor exerts on the liquid surface

Cavitation

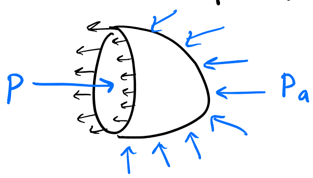
formation & subsequent collapse of vapor bubbles in a flowing fluid

→ when pressure in localized regions reach vapor pressure, causes cavitation

Surface Tension

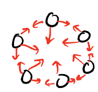
surface phenomena due to unbalanced cohesive forces acting on the liquid molecules at the fluid surface

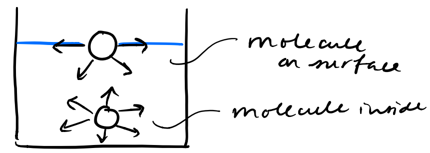
→ the intensity of the molecular attraction per unit length along any line in the surface, σ $\left[\frac{N \cdot m}{m^2}\right] = \left[\frac{J}{m^2}\right]$



$p = \frac{2\sigma}{R} + Pa$

"Force per unit length"





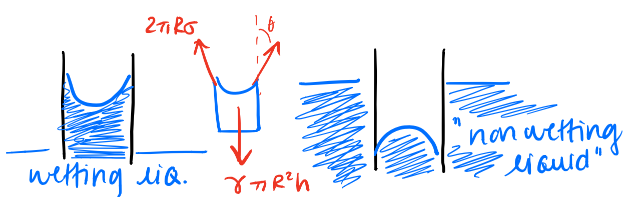
there is net attracting forces acting on the molecules at the surface that tends to pull molecules on surface toward interior of liquid

Capillary Rise

$\gamma \pi R^2 h = 2\pi R \sigma \cos\theta$

↑ force balance

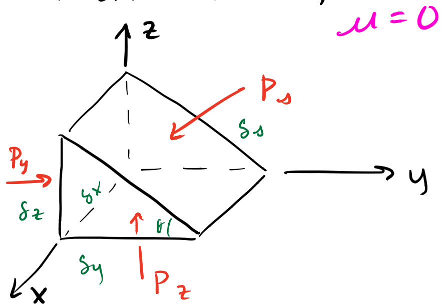
$h = \frac{2\sigma \cos\theta}{\rho g R}$



Pascal's Law

the pressure at a point in a fluid at rest is independent of direction (scalar)

$\mu = 0$



$\Sigma F_y = P_y \delta x \delta z - P_s \delta x \delta s \sin\theta = \rho \forall a_y$

$\Sigma F_z = P_z \delta x \delta y - P_s \delta x \delta s \cos\theta - \rho \forall g = \rho \forall a_z$

... basically, $P_y = P_s = P_z$

The pressure gradient

$F_{py} = -\frac{\partial P}{\partial y}\, dy\,dx\,dz \qquad F_{px} = -\frac{\partial P}{\partial x}\, dx\,dy\,dz \qquad F_{pz} = -\frac{\partial P}{\partial z}\, dz\,dx\,dy$

$$\vec{F}_p = F_{px}\hat{\imath} + F_{py}\hat{\jmath} + F_{pz}\hat{k}$$

$$= -\underbrace{\left(\frac{\partial P}{\partial x}\hat{\imath} + \frac{\partial P}{\partial y}\hat{\jmath} + \frac{\partial P}{\partial z}\hat{k}\right)}_{\text{pressure gradient}} dx\,dy\,dz$$

force per unit volume

$$\boxed{\frac{F_p}{dx\,dy\,dz} = -\nabla p}$$

forces divided by $\forall$

gravitational force

now all together...

$$\boxed{\frac{\Sigma F}{\forall} = -\nabla P - \rho g \hat{k} = \rho \vec{a}}$$

NEWTON'S SECOND LAW (INVISCID)

IDEAL FLUIDS

In statics, $a = 0$ so $\nabla p + \rho g \hat{k} = 0$

then
$$\begin{bmatrix} \frac{\partial p}{\partial x} \\ \frac{\partial p}{\partial y} \\ \frac{\partial p}{\partial z} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -\gamma \end{bmatrix}$$

we have linear relationship between pressure and depth
→ pressure will not be different as move along x and y

Measurement of pressure

$P_g = P_{abs} - P_{atm} \qquad P_{vac} = P_{atm} - P_{abs}$

$\gamma = \rho g$ — specific weight, density, acceleration due to grav

INCOMPRESSIBLE: FLUID W/CONSTANT DENSITY

Manometer Rule
1. write pressure at either ends of manometer
2. proceed along manometer adding γh if moving to greater depth or subtracting it if going to a lesser depth
3. stop at far end or any point in between and set the expression equal to the local pressure

Fluid Statics
- there's no relative motion between adj fluid layers, and thus no shear stresses
- only stress we deal with is normal (pressure) and the variation of pressure is due only to the wt of the fluid

HYDROSTATIC FORCE ON PLANE SURFACE: $\nabla p + \gamma \hat{k} = 0 \quad \frac{\partial P}{\partial x} = 0 \quad \frac{\partial P}{\partial y} = 0 \quad \frac{\partial P}{\partial z} = -\gamma$

Hydrostatic Force on a Plane Surface
pressure at centroid of a plane surface is equal to average pressure of surface
→ resultant force on a plane surface = pressure at centroid of surface area × line of action passes through the center of pressure

$F_R = \gamma \bar{h} A$ — γ: specific wt.; $\bar{h}$: depth of centroid; A: submerged area of plate

$y_p = \bar{y} + \frac{I_x}{\bar{y} A}$ — y_p: distance along plate to center of pressure; $\bar{y}$: distance along plate to centroid of submerged area; I_x: moment of inertia of submerged area about centroidal axis

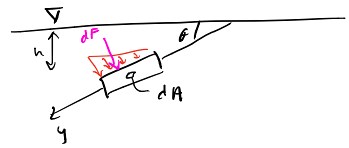
$x_p = \bar{x} + \frac{I_{xy}}{\bar{y} A}$ — x_p: location of center of pressure; I_{xy}: product of inertia of submerged area relative to centroidal axes

$\bar{x}, \bar{y}$ = coordinate distance to centroid of submerged area submerged plate: $F_R = PA = \int_A \rho g h \, dA$

$F_R = \int_A \rho g y \sin\theta \, dA \rightarrow \rho g \sin\theta \int_A y \, dA$

$y_c = \frac{1}{A} \int_A y \, dA$ (coordinate for centroid)

$F_R = \gamma A y_c \sin\theta \rightarrow F_R = \gamma A h_c$



$y_R = \frac{I_{xx,c}}{y_c A} + y_c$ (y-coordinate of force)

$x_R = \frac{I_{xy,c}}{y_c A} + x_c$ (x-coordinate of result force)

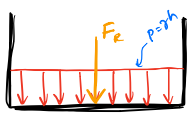
Hydrostatic Force on Curved Surface

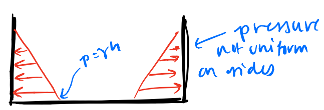
$dF = P\,dA$

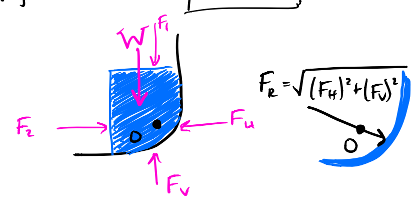
$h \, dA \sin\theta$ = volume of col. of fluid on top of horizontal $= d\forall$

$dF_H = P\,dA\cos\theta = P\,dA_V \rightarrow \boxed{F_H = PA_V}$

$dF_V = P\,dA\sin\theta = P\,dA_H \rightarrow \underbrace{\rho g h}_{P} dA \sin\theta \rightarrow dF_V = \rho g \, d\forall = dW \rightarrow \boxed{F_V = W}$





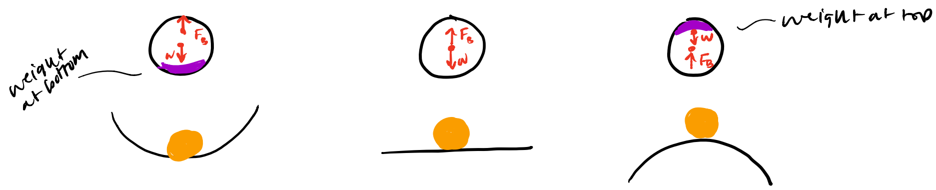


Buoyancy Force

$\Sigma F_y = F_{buoy} - W = F_{balloon}$ $\quad F_{buoy} = \gamma_{\substack{fluid \\ \text{surrounding object}}} \forall_{submerged}$ $\quad W = mg = \rho \forall g_{balloon}$

$F_{balloon} = \gamma_{H_2O} \forall_{submerged} - \rho_{He} \forall g$ $\quad F_{Buoy} = \rho g \forall = \gamma \forall$

ARCHIMEDE'S PRINCIPLE : F_B acting on body of uniform density immersed in a fluid is equal to the weight of the fluid displaced by the body. It acts upward through the centroid of the displaced volume



stable $\quad$ neutrally stable $\quad$ unstable

Pressure Variation in a Fluid w/ Rigid Body Motion

$-\nabla P - \gamma \hat{k} = \rho a$ $\quad -\frac{\partial P}{\partial x} = \rho a_x \quad -\frac{\partial P}{\partial y} = \rho a_y \quad -\frac{\partial P}{\partial z} = \gamma + \rho a_z$

$-\frac{\partial P}{\partial s} - \gamma \frac{\partial z}{\partial s} = \rho \frac{dV}{ds} V \rightarrow \frac{1}{2} \rho \frac{dV^2}{ds}$

$+\frac{\partial P}{\partial s} + \gamma \frac{\partial z}{\partial s} + \frac{1}{2} \rho \, dV^2 = 0$ ... just using linear momentum along a streamline

↳ then integrate... $P + \frac{1}{2} \rho V^2 + \gamma z = C$ BERNOULLI
conservation of linear momentum along a streamline (INVISCID)

$P + \frac{1}{2}\rho V^2 + \gamma z = C$ BERNOULLI ASSUMPTIONS

1. inviscid ($\mu \approx 0$)
2. steady
3. incompressible ($\rho = C$)
4. streamline
5. gravity constant

BERNOULLI'S IN UNIT OF LENGTH

$$\frac{p}{\gamma} + \frac{V^2}{2g} + z = C_1$$

pressure head — velocity head — elevation head

BERNOULLI'S DIVIDED BY DENSITY

(only for incompressible)

$$\frac{p}{\rho} + \frac{1}{2}V^2 + gz = C_1$$

BERNOULLI'S CONSTANT ALONG A STREAMLINE

$$p + \frac{1}{2}\rho V^2 + \gamma z = C_1$$

STATIC P TERM — DYNAMIC P TERM — HEAD/ ELEVATION P

* hairdryer w/ping pong ball example

BERNOULLI'S CONSTANT ACROSS A STREAMLINE

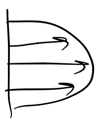
$$p + \rho \int \frac{V^2}{R}\,dn + \gamma z = C_1$$

streamline: line tangent to velocity vector at every pt.

steady flow: no changes w/ time at given location (steady streamline field)

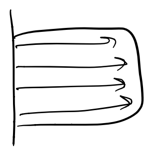
Laminar flow

fluid particles follow straight line paths



Turbulent flow

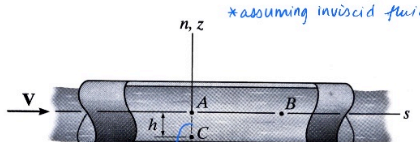
fluid particles follow erratic paths which change direction in space & time



ave velocity profile of turbulent flow

Steady horizontal flow of an ideal fluid

*assuming inviscid fluid



$V_A = V_C$

so even if go across streamline, the difference is still $\rho g h$ if no acceleration ... can basically treat like hydrostatics problem

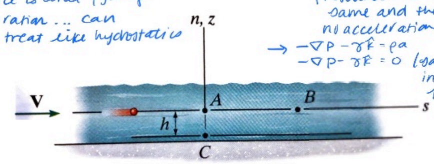
Closed conduit

$P_A - P_B = \frac{1}{2}\rho(V_B^2 - V_A^2)$

if $V_B = V_A$ then pressures are the same and there's no acceleration

$\rightarrow -\nabla p - \gamma\hat{k} = \rho a$

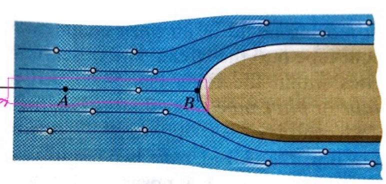
$-\nabla p - \gamma\hat{k} = 0$ (same thing in open plane, too)



Open conduit

Stagnation pressure

-have a certain flow and it's heating an object

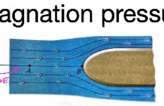


streamline between A & B, particle here gets stuck b/c V goes from max to $\emptyset$ then pressure force is @ maximum at pt. B

Stagnation point

Stagnation pressure

velocity from A to B will gradually ↓ until hits $\emptyset$ @ B



BERNOULLI ALONG THIS STREAMLINE

same equation as before just dividing by γ

Stagnation point

this will be zero

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B$$

no change in altitude so they're the same value

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + \underbrace{0}_{z_A} = \frac{p_B}{\gamma} + \underbrace{0}_{V_B^2} + \underbrace{0}_{z_B}$$

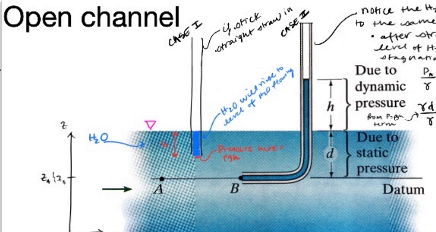
$$p_B = p_A + \rho\frac{V_A^2}{2}$$

[Stagnation pressure] = [Static pressure] + [Dynamic pressure]

energy crashed into object and has to go somewhere

*DEMO FOR THIS

Open channel



notice the H_2O level doesn't rise to the same level as flowing H_2O

- after straw becomes filled to level of H_2O, becomes a stagnation pt

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B$$

from $P = \rho g h$ term

$$\frac{\gamma d}{\gamma} + \frac{V_A^2}{2g} + \emptyset = \frac{\gamma(h+d)}{\gamma} + \emptyset + \emptyset$$

z_A & z_B are same height

⇓

$$\frac{\gamma d}{\gamma} + \frac{V_A^2}{2g} = \frac{\gamma(h+d)}{\gamma}$$

⇓

$$d + \frac{V_A^2}{2g} = h + d$$

$$\frac{V_A^2}{2g} = h$$

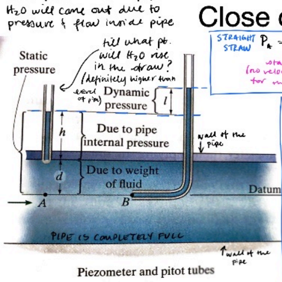
so now we can estimate the velocity of the river (again assuming inviscid, steady, etc) → then $V_A^2 = 2gh$

$$\boxed{V_A = \sqrt{2gh}}$$

this is for CASE II w/curved straw

when poke hole into pipe: H_2O will come out due to pressure & flow inside pipe

Close channel



Piezometer and pitot tubes

STRAIGHT STRAW $P_A = \gamma(d+h)$

static (no velocity involved for this distance)

CURVED STRAW

Bernoulli Between A & B:

$$\frac{P_A}{\gamma} + \frac{V_A^2}{2g} + z_A = \frac{P_B}{\gamma} + \frac{V_B^2}{2g} + z_B$$

$$\frac{\gamma(h+d)}{\gamma} + \frac{V_A^2}{2g} + \phi = \frac{\gamma(\underbrace{h+d}_{\text{static}}+\underbrace{l}_{\text{dynamic}})}{\gamma} + \phi + \phi$$

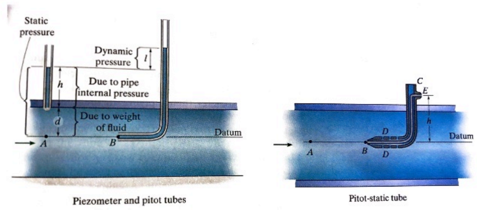
$z_A = z_B$

$$h+d+\frac{V_A^2}{2g} = h+d+l$$

$$\Downarrow$$

$$V_A^2 = 2gl \Rightarrow \boxed{V_A = \sqrt{2gl}}$$

Close channel: the Pitot tube



$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B$$

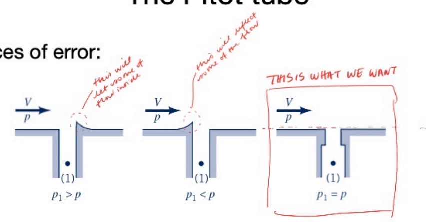
$$\frac{p_C + \rho g h}{\rho g} + \frac{V_A^2}{2g} + 0 = \frac{p_E + \rho g h}{\rho g} + 0 + 0$$

$$V_A = \sqrt{\frac{2}{\rho}(p_E - p_C)}$$

ISSUES WITH

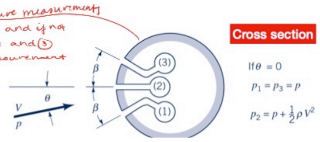
The Pitot tube

Sources of error:



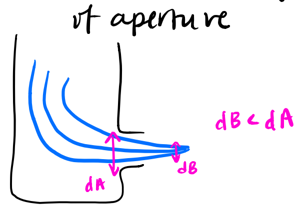
Flow misalignments

this allows multiple static pressure measurements
→ should measure same in ① and ③ and if not then can rotate it so that ① and ③ do then have the same measurement



VENA CONTRACTA

- reduction in diameter
- get contraction coefficient, ratio of cross-section of jet to cross-section of aperture



THE CONTINUITY EQUATION

$A_1V_1 = A_2V_2 \rightarrow Q_1 = Q_2$ (1-dimensional, steady, incomp. flow)

- combined flows  syringe example

Eulerian method (what happens to volume in time)

fluid motion is given by completely prescribing the necessary properties (pressure, density, velocity, etc) as functions of space and time

Lagrangian method

following individual fluid particles as they move about and determining how the fluid properties associated with these particles change as a function of time

pathline - path of particle for $0 \le t \le t_1$

→ line that is traced out by a given particle as it flows from one pt to another

$$u = \frac{dx}{dt} \qquad v = \frac{dy}{dt} \qquad w = \frac{dz}{dt}$$

streakline - path of many particles at instant $t = t_1$

→ consists of all particles in a flow that have previously passed thru common pt.

streamline - line that is everywhere tangent to the velocity vector

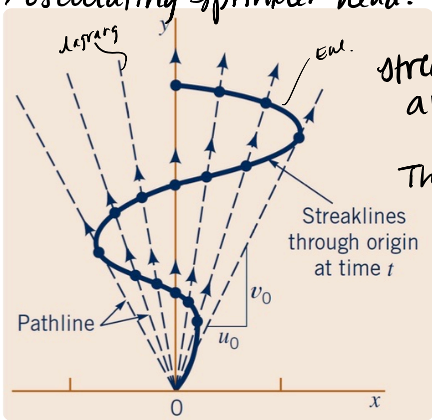
$$\frac{dy}{dx} = \frac{v}{u} \quad \text{(2D case)} \qquad \frac{dz}{dx} = \frac{w}{u} \quad \frac{dz}{dy} = \frac{w}{v} \quad \text{(3D case)}$$

* streamline is EUL

If the flow is steady, streamlines, pathlines, and streaklines always coincide

ex. - H_2O ejected from fixed nozzle (*)

→ oscillating sprinkler head:



streaklines & pathlines are different here!

The flow is unsteady so line equations are different

velocity field: $\vec{V} = \underbrace{u_0 \sin\theta\left[\omega\left(t - \frac{y}{V_0}\right)\right]}_{u}\hat{i} + \underbrace{V_0}_{v}\hat{j}$

streamline:

$$\frac{dy}{dx} = \frac{v}{u} = \frac{V_0}{u_0 \sin\theta\left[\omega\left(t - \frac{y}{V_0}\right)\right]}$$

$u_0 \sin\theta\left[\omega\left(t - \frac{y}{V_0}\right)\right] dy = V_0\, dx$ → then integrate both sides to get equation for x

pathlines (lagrangian) vs. streaklines (eulerian)

Velocity Field $\vec{V} = u(x,y,z)\hat{i} + v(x,y,z)\hat{j} + w(x,y,z)\hat{k}$

$$\vec{a} = \frac{d\vec{V}}{dt} = \frac{\partial V}{\partial x}u + \frac{\partial V}{\partial y}v + \frac{\partial V}{\partial z}w + \frac{\partial V}{\partial t}$$

$$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$$

$$a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + \frac{\partial v}{\partial t}$$

$$a_z = u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} + \frac{\partial w}{\partial t}$$

3×4 matrix that describes the acceleration field of a fluid particle

$$\frac{dV}{dt} = \boxed{V \cdot \nabla V} + \boxed{\frac{\partial V}{\partial t}}$$

convective term of acceleration

local term of acceleration

→ can then rewrite:

$$-\nabla p - \gamma\hat{k} = \rho a = \rho\left(V \cdot \nabla V + \frac{\partial V}{\partial t}\right)$$

material derivative of acceleration

$$\frac{D\vec{V}}{Dt} = \frac{d\vec{V}}{dt} = \frac{\partial\vec{V}}{\partial t} + (\vec{V} \cdot \vec{\nabla})\vec{V}$$

Material derivative of pressure

$$\frac{DP}{Dt} = \frac{dP}{dt} = \frac{\partial P}{\partial t} + (\vec{V} \cdot \vec{\nabla})P$$

$m = \rho\forall$ and $B = mb$

General RTT:

time rate of change of mass of system

time rate of change of mass of contents in cv

net flow of mass through cs

if $B = M$

$$\frac{DB_{sys}}{Dt} = \frac{\partial}{\partial t}\int_{cv} \rho b\, d\forall + \int_{cs} \rho b \vec{V} \cdot \hat{n}\, dA$$

Restricted RTT:

$$\frac{DB_{sys}}{Dt} = \frac{\partial B_{cv}}{\partial t} + \rho_2 A_2 V_2 b_2 - \rho_1 A_1 V_1 b_1$$

RTT (general)

*meaning of each term if B=M and then b=1

$$\frac{DB_{sys}}{Dt} = \frac{d}{dt}\int_{CV} \rho b \, d\forall + \int_{CS} \rho b \vec{V} \cdot \hat{n} \, dA$$

time rate of change of mass of sys

time rate of change of mass of contents in CV

net flow of mass thru CS

CONSERVATION OF MASS PRINCIPLE

(total mass entering CV during Δt) − (total mass leaving CV during Δt) = (net change of mass w/in CV during Δt)

$$m_{in} - m_{out} = \Delta m_{cv}$$

$$\dot{B}_{net} = \dot{B}_{out} - \dot{B}_{in} = \int_{cs} \rho b \vec{V} \cdot \hat{n} \, dA$$

Newton's 2nd Law can be described in terms of linear momentum

sum of ext. forces acting on system = time rate of change of linear momentum of the system

$$\frac{D}{Dt}\int_{sys} \vec{V} \rho \, d\forall = \frac{d}{dt}\int_{CV} \vec{V} \rho \, d\forall + \int_{CS} \vec{V} \rho \vec{V} \cdot \hat{n} \, dA$$

(time rate of change of linear momentum of the system) = (time rate of change of linear momentum of contents of CV) + (net rate of flow of linear momentum through the CS)

Linear Momentum Equation

$$\frac{d}{dt}\int_{CV} \vec{V} \rho \, d\forall + \int_{CS} \vec{V} \rho \vec{V} \cdot \hat{n} \, dA = \sum F_{\text{contents of CV}}$$