Regular Perturbation Method: an Initial Value Problem

Recall:

- we assumed a guess for the solution of x for Projectiles

$$\ddot{x}(t) = -\frac{1}{(1+\epsilon x)^2} \qquad x(0)=0 \qquad \dot{x}(0)=1$$

x = vertical ht of the ball

ϵ = small parameter, ratio of ht of ball to radius of Earth

→ our goal: $x(t) = f(\epsilon, t)$

→ make a guess: using Taylor Series Expansion

$$x(t) = \underbrace{x_0}_{x_0(t)} + \epsilon^\alpha x_1 + \dots$$

... take guess and start truncated for every order, starting at 1

$-\frac{1}{(1+\epsilon x)^2}$ ...we can make this simpler by computing a Taylor S. Exp.

Taylor Series Expansion:

$$\ddot{x}(t) = -1 + 2\epsilon x - 3\epsilon^2 x^2 + \dots$$

ODE with our guess:

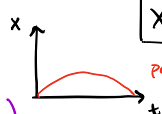
$$\boxed{\ddot{x}_0} + \epsilon^\alpha \ddot{x}_1 + \dots = \boxed{-1} + 2\epsilon(x_0 + \epsilon^\alpha x_1 + \dots)$$

I.C. $\begin{cases} \boxed{x_0(0)} + \epsilon^\alpha x_1(0) + \dots = \boxed{0} \\ \boxed{x_0'(0)} + \epsilon^\alpha x_1'(0) + \dots = \boxed{1} \end{cases}$

order 1, $O(1)$: $\ddot{x}_0 = -1$, $x_0(0) = 0$, $\dot{x}_0(0) = 1$ $\xrightarrow{\text{integration}}$ $\dot{x}_0(t) = -t + \boxed{c_1} \rightarrow c_1 = 1$

$x_0(t) = -\frac{1}{2}t^2 + t + \boxed{c_2} \rightarrow c_2 = 0$

$$\boxed{x_0 = t\left(1 - \tfrac{1}{2}t\right)}$$



nondimensionalized so looks slightly different than what seen before

look at, by inspection, what is happening 1st order:

$$\ddot{x}_0 + \boxed{\epsilon^\alpha \ddot{x}_1} + \dots = -1 + \boxed{2\epsilon}\boxed{x_0} + \epsilon^\alpha x_1 + \dots)$$

$2\epsilon^{1+\alpha} x_1$

I.C. $\begin{cases} x_0(0) + \boxed{\epsilon^\alpha x_1(0)} + \dots = 0 \\ x_0'(0) + \boxed{\epsilon^\alpha x_1'(0)} + \dots = 1 \end{cases}$ $\boxed{\alpha = 1}$

$O(\epsilon)$: $x_1'' = 2x_0 \Rightarrow \cancel{\epsilon}\ddot{x}_1 = 2\cancel{\epsilon}x_0$ $\xrightarrow{\text{integration}}$

$x_1(0) = 0$ ⇒ bc if adding something that needs to equal zero or 1, then must be zero

$x_1'(0) = 0$

$\rightarrow x_1 = \frac{1}{12}t^3(4-t)$

↳ so now we have $x(t) \approx \underbrace{t\left(1 - \tfrac{1}{2}t\right)}_{O(1)} + \underbrace{\tfrac{1}{12}\epsilon t^3(4-t)}_{O(\epsilon)} + O(\epsilon^2)$

See corresponding Mathematica notebook

New problem: Thermokinetic

-concentration u

-temperature q

$$\boxed{\begin{aligned}\dot{u} &= 1 - u e^{\epsilon(q-1)} \\ \dot{q} &= u e^{\epsilon(q-1)} - q\end{aligned}}$$

$u(0) = 0$

$q(0) = 0$

looking for 2 solutions:

$$u \sim u_0(t) + \epsilon u_1(t) + \dots$$

$$q \sim q_0(t) + \epsilon q_1(t) + \dots$$

→ going into breakout rooms, don't go to order ϵ^2