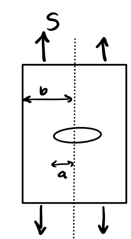
Fracture of Cracked Members

- K_{Ic} = fracture toughness, measure of a material's ability to resist failure due to a crack
- K = stress intensity factor, measure of the severity of a crack situation as affected by crack size, stress, and geometry
- k_t = stress [ ] factor, defined as ratio of max stress to remote stress, i.e. $k_t = \sigma_y / S$
- K_c = fracture toughness, a give material can resist a crack without brittle fracture as long as this K is below a critical value K_c
  - → values of K_c vary widely for different materials and are affected by temperature and loading rate, and secondarily by the thickness of the member
  - → ⊛ thicker members have lower K_c values until a worst case value is reached, which is denoted K_{Ic} and called the plane strain fracture toughness
    - ↳ hence, K_{Ic} is a measure of a given material's ability to resist fracture in the presence of a crack (Tables 8.1, 8.2)

ex. crack in center of wide plate of stressed material. K depends on the remotely applied stress S and the crack length a (measured from the centerline)

$$K = S\sqrt{\pi a} \qquad a << b$$



for a given material + thickness w/ fracture toughness K_c, the critical value of remote stress necessary to cause fracture is $S_c = \dfrac{K_c}{\sqrt{\pi a}}$

∴ longer cracks have a more severe effect on strength than do shorter ones

crack length where failure stress = yield strength, a_t (substitute $S_c = \sigma_o$)

$$a_t = \frac{1}{\pi}\left(\frac{K_c}{\sigma_o}\right)^2$$ * transition crack length

→ cracks longer than this transition crack length will cause the strength to be limited by brittle fracture rather than by yielding

→ crack lengths below a_t, yielding dominated behavior is expected (so little or no strength reduction due to crack)

ex. 2 materials:

Material I: low strength
- low σ_o (high ductility)
- high K_c
- → relatively large a_t

Material II: high strength
- high σ_o (low ductility)
- low K_c
- → relatively small a_t

* thus, cracks of moderate size may not affect the low-strength material, but they may severely limit the usefulness of the high strength one

Mathematical Form to Express K

$K = FS_g\sqrt{\pi a}$ ← crack length

S_g: applied stress

$F = f(\text{geometry}, a/b)$ ... dimensionless

pg. 290, 292, 293, 295 (Fig 8.12, Fig 8.13, Fig 8.14) applied forces or bending moments are often characterized by determining a nominal or average stress. In fracture mechanics, it's conventional to use the gross section nominal stress S_g, calculated under the assumption that no crack is present (done for 8.12-14)

* stress intensity factor K is a measure of the severity of a crack, as affected by crack length, geometry, and the applied stress
  ↳ for a given material, fracture is expected if this value reaches the fracture toughness K_{Ic}

Safety factor against brittle fracture: $X_K = \frac{K_{Ic}}{K} = \frac{K_{Ic}}{F S_g \sqrt{\pi a}}$ where S_g and a are what's expected to occur in actual service

Sometimes convenient to work directly with applied loads (forces)
↳ see Fig. 8.15 p. 295

a_c = critical crack length (equate K w/ K_{Ic})

To compare service crack length a with the crack length a_c that is expected to cause failure at the service stress S_g, note that the value of a_c is available from:

$$K_{Ic} = F_c S_g \sqrt{\pi a_c}$$

where F_c is evaluated at a_c

Safety factor on crack length: $X_a = \frac{a_c}{a} = \left(\frac{F}{F_c} X_K\right)^2$ ← note that safety factors on crack length must be rather large to achieve reasonable safety factors on K & stress

if crack length expected to occur in actual service is relatively small, safety factors against yielding may be calculated simply by comparing the service stress S_g with the material's yield strength σ_o

$$X_o = \sigma_o / S_g$$

... however, for applied stresses that are multiaxial, S_g must be replaced by an effective stress $\bar{\sigma}$

a more advanced method for calculating the safety factor against yielding is to compare the applied load with the fully plastic limit load

$X_o' = P_o / P$ $\quad X_o' = M_o / M$ see p. 299

Cracks having shapes that approximate a circle, half-circle, or quarter-circle may occur.
See Fig. 8.17 p. 303 * (b) and (d) especially common

Cracks growing from a stress raiser, such as a hole, notch, or fillet (p. 307)

Leak before break design of pressure vessels — 2 possibilities
(1) crack may gradually extend and penetrate the wall, causing leak before sudden brittle fracture can occur ← (1) is preferred!
(2) sudden brittle fracture may occur prior to vessel leaking

$c_c \geq t \qquad c_c = \frac{1}{\pi}\left(\frac{K_{Ic}}{\sigma_t}\right)^2$ p. 311

Fatigue of Materials: Introduction and Stress-Based Approach

→ study cyclic fatigue behavior of materials as a process of progressive damage leading to cracking and failure

→ Description of Cyclic Loading ←

- stress range, $\Delta\sigma = \sigma_{max} - \sigma_{min}$
- mean stress, σ_m = average of σ_{max} and σ_{min}
- stress amplitude, σ_a = 1/2 the range $\Delta\sigma$ (variation about the mean)

$$\sigma_a = \frac{\Delta\sigma}{2} = \frac{\sigma_{max} - \sigma_{min}}{2} \qquad \sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$$

$$\sigma_{max} = \sigma_m + \sigma_a \qquad \sigma_{min} = \sigma_m - \sigma_a$$

$\sigma_a, \Delta\sigma$ always ⊕

$\sigma_{max}, \sigma_{min}, \sigma_m$ ⊕ or ⊖

⊕ == tension

$$R = \frac{\sigma_{min}}{\sigma_{max}} \qquad A = \frac{\sigma_a}{\sigma_m}$$

stress ratio

amplitude ratio

Add'tn'l relationships:

$$\sigma_a = \frac{\Delta\sigma}{2} = \frac{\sigma_{max}}{2}(1-R) \qquad \sigma_m = \frac{\sigma_{max}}{2}(1+R) \qquad R = \frac{1-A}{1+A} \qquad A = \frac{1-R}{1+R}$$

COMPLETELY REVERSED CYCLING: $\sigma_m = 0$, $R = -1$

ZERO-TO-TENSION CYCLING: $\sigma_{min} = 0$, $R = 0$

Point Stresses vs. Nominal Stresses

σ

S, calculated from a force or moment or their combination, only equal to σ in certain situations (such as simple axial loading $\sigma = S = P/A$)

- bending $S = Mc/I$

for stress raisers such as notches, S needs to be multiplied by an elastic stress [ ] factor k_t to obtain the peak stress at the notch $\sigma = k_t S$ (values for k_t in Appendix, Fig. A11, A12)

→ note that k_t is based on linear-elastic behavior so not applicable if there's yielding

Stress-Life Curve, aka S-N curve (σ_a or S_a vs. N_f the # of cycles to failure)

- # of cycles to failure changes rapidly w/stress level and may range over several orders of magnitude ⟹ logarithmic scale (can also be used on stress axis)

- log-linear plot: $\sigma_a = C + D\log N_f$

- log-log plot: $\sigma_a = A N_f^{B}$ <=> $\sigma_a = \sigma_f'(2N_f)^b$, $A = 2^b\sigma_f'$ & $B = b$ (Table 9.1, p. 365)

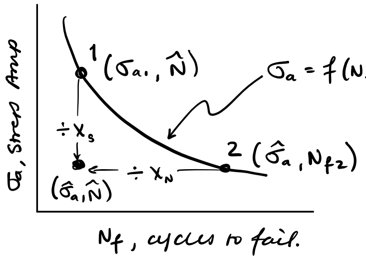
Fatigue limit aka endurance limit: where S-N curve appears to become flat and asymptotically approach the stress amplitude S_e

Safety factor for S-N curves

$\hat{\sigma}_a$ = stress level expected to occur in actual service

$\hat{N}$ = number of cycles expected to occur in actual service

} this combo must fall below the stress-life curve $\sigma_a = f(N_f)$ that corresponds to failure, so that there is an adequate safety factor



pt. 1: stress amp σ_{a1} corresponds to failure at desired service life $\hat{N}$

↳ Safety factor in Stress: $X_s = \frac{\sigma_{a1}}{\hat{\sigma}_a}$ $(N_f = \hat{N})$

pt. 2: failure life N_{f2} corresponds to service stress $\hat{\sigma}_a$

↳ Safety factor in Life: $X_N = \frac{N_{f2}}{\hat{N}}$

Safety factors in stress for fatigue should be similar in magnitude to other stress-based safety factors

$$\left.\begin{aligned} \sigma_{a1} &= A\hat{N}^B \quad \text{pt. 1} \\ \hat{\sigma}_a &= AN_{f2}^B \quad \text{pt. 2} \end{aligned}\right\} \Rightarrow X_s = \frac{A\hat{N}^B}{AN_{f2}^B} = \left(\frac{1}{X_N}\right)^B = X_N^{-B}$$

$$X_N = X_s^{-1/B}$$

safety factor in life — safety factor in stress

Sources of cyclic loading:
- static loads
- working loads
- vibratory loads
- accidental loads

Presenting Mean-Stress Data
- → plot as family of S-N curves (p. 393)
- → constant-life diagram (p. 394)
- → stress-life curves for different R's (p. 395)

Normalized Amplitude-Mean Diagrams

σ_{ar} = stress amplitude for the particular case of zero mean stress

↳ on a constant-life diagram, σ_{ar} is thus the intercept at $\sigma_m = 0$ of the curve for any particular life

→ graph can then be normalized by plotting values of σ_a/σ_{ar} vs. σ_m (Fig. 9.39, p. 395)

→ this normalized amplitude-mean diagram forces agreement at $\sigma_m = 0$ where $\sigma_a/\sigma_{ar} = 1$, and tends to consolidate data at various mean stresses and lives onto a single curve

- for values of stress amplitude approaching zero, mean stress should approach the ultimate strength of the material

* Modified Goodman Eq. & Line

$$\frac{\sigma_a}{\sigma_{ar}} + \frac{\sigma_m}{\sigma_u} = 1$$

← very conservative, error is such that it causes extra safety in life estimates

* Gerber Parabola

$$\frac{\sigma_a}{\sigma_{ar}} + \left(\frac{\sigma_m}{\sigma_u}\right)^2 = 1 \quad (\sigma_m \geq 0)$$

← fits little more closely but limited to tensile mean stress (incorrect for compressive)

* Morrow Equation — improved agreement for ductile metals by replacing σ_u with either corrected true fracture strength $\tilde{\sigma}_{fB}$ from a tension test or constant σ_f' from the unnotched axial S-N curve for $\sigma_m = 0$

$$\frac{\sigma_a}{\sigma_{ar}} + \frac{\sigma_m}{\tilde{\sigma}_{fB}} = 1 \qquad \frac{\sigma_a}{\sigma_{ar}} + \frac{\sigma_m}{\sigma_f'} = 1$$

aluminum alloys — steels

* SWT Equation

$$\sigma_{ar} = \sqrt{\sigma_{max}\sigma_a} \quad (\sigma_{max} > 0) \qquad \text{or} \qquad \sigma_{ar} = \sigma_{max}\sqrt{\frac{1-R}{2}} \quad (\sigma_{max} > 0)$$

equivalent equations

advantage: does not rely on a material constant!

* Walker Equation:

$$\sigma_{ar} = \sigma_{max}^{1-\gamma}\sigma_a^{\gamma} \quad (\sigma_{max} > 0) \qquad \text{or} \qquad \sigma_{ar} = \sigma_{max}\left(\frac{1-R}{2}\right)^{\gamma} \quad (\sigma_{max} > 0)$$

equivalent equations

γ = material constant

SWT relationship is a special case of Walker where $\gamma = 0.5$

SWT and Walker are plotted on σ_a/σ_{ar} vs. σ_m/σ_{ar}

Walker > SWT > Morrow > Gerber, Goodman (accuracy)

Life Estimates with Mean Stress

→ from Morrow for steels: $\sigma_{ar} = \dfrac{\sigma_a}{1 - \sigma_m/\sigma_f'}$ ... "equivalent completely reversed stress amplitude"

ex. assume S-N curve for completely reversed loading is known, tests at $\sigma_m = 0$ employed to obtain constants σ_f' and b, then the stress amplitude σ_a corresponds to the special case denoted σ_{ar} so:

$$\sigma_{ar} = \sigma_f'(2N_f)^b$$

can be combined w/SWT too

then can get more general stress-life equation that applies for nonzero mean stress:

$$\sigma_a = (\sigma_f' - \sigma_m)(2N_f)^b$$

Safety Factors with Mean Stress

$$X_S = \left.\frac{\sigma_{ar1}}{\hat{\sigma}_{ar}}\right|_{N_f = \hat{N}} \qquad X_N = \left.\frac{N_{f2}}{\hat{N}}\right|_{\sigma_{ar} = \hat{\sigma}_{ar}}$$

value of $\hat{\sigma}_{ar}$ is calculated from the stress amplitude $\hat{\sigma}_a$ and mean stress $\hat{\sigma}_m$ expected to occur in actual service

another option? multiply by load factors $Y_a + Y_m$

↳ see slides + text for these equations

Multiaxial Fatigue (p.406) ← ex. in slides, also p.408-409

Stress-Based Approach to Fatigue: Notched Members

fatigue notch factor $k_f = \frac{\sigma_{ar}}{S_{ar}}$

where k_f is formally defined only for completely reversed stresses, σ_{ar} for the smooth member, S_{ar} for the notched member

- if notch has large radius ρ at its tip $k_f \approx k_t$
- for small ρ, $k_f \ll k_t$

notch sensitivity: $q = \frac{k_f - 1}{k_t - 1}$

- if notch has its maximum possible effect, so that $k_f = k_t$, then $q = 1$
- minimum value of $q = 0$ where $k_f = 1$, corresponding to notch having no effect
- value of q between 0 and 1 is therefore a convenient measure of how severely a given member is affected by a notch

↳ Peterson: $q = \frac{1}{1 + \alpha/\rho}$ α = material constant w/ dimensions of length

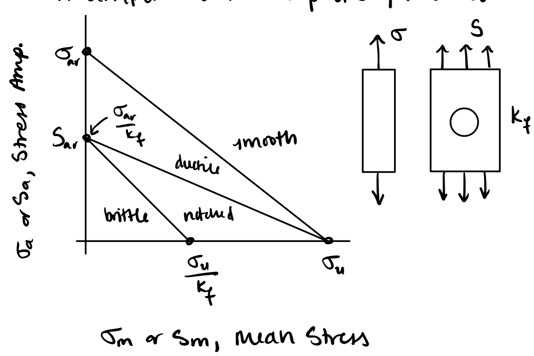
$\alpha = 0.51$ mm (Al alloys)
$\alpha = 0.25$ mm (annealed or normalized low-carbon steels)
$\alpha = 0.064$ mm (quenched and tempered steels)

variation of α for steels:
$\log \alpha = 2.654(10^{-7})(\sigma_u^2) - 1.309(10^{-3})\sigma_u + 0.01103$ $\alpha, mm = 10^{\log \alpha}$ $(345 \le \sigma_u \le 2070 \text{ MPa})$

$$k_f = 1 + \frac{k_t - 1}{1 + \alpha/\rho}$$

Combined Effects of Notches and Mean Stress

* Goodman amplitude-mean plots for smooth and notched members of brittle and ductile materials



$S_{ar} = \frac{\sigma_{ar}}{k_f} = \frac{S_a}{1 - S_m/\sigma_u}$ (ductile materials)

↳ compared w/ unnotched material, equivalent completely reversed nominal stress S_{ar} is reduced by a notch factor k_f (also implies that the notch factor for the mean stress is $k_{fm} = 1$)

$S_{ar} = \frac{\sigma_{ar}}{k_f} = \frac{S_a}{1 - \frac{k_{fm} S_m}{\sigma_u}}$ (brittle materials)

↳ low ductility, redistribution of stress doesn't occur and σ_u for notched member is reduced ... notch factor k_{fm} for the mean stress is generally taken to be the same value as for the stress amp, that is, $k_{fm} = k_f$

SWT Equation w/ Nominal Stresses

$S_{ar} = \sqrt{S_{max} S_a}$ $S_{ar} = S_{max}\sqrt{\frac{1-R}{2}}$

Walker Equation w/ Nominal Stresses

$S_{ar} = S_{max}^{1-\gamma} S_a^{\gamma}$ $S_{ar} = S_{max}\left(\frac{1-R}{2}\right)^{\gamma}$

ex. 10.3 p. 455-456

FATIGUE CRACK GROWTH

- apply stress intensity factor K of fracture mechanics to fatigue crack growth
- explore fatigue crack growth rate curves, da/dN vs. ΔK, including fitting common equations and evaluating R-ratio (mean stress) effects
- calculate the life to grow a fatigue crack to failure

$K = FS\sqrt{\pi a}$ K quantifies the severity of a crack situation, K depends on combination of crack length, loading, and geometry

worst-case crack of initial length a_d then grows until it reaches a critical length a_c, where brittle fracture occurs after N_{if} cycles of loading, if the number of cycles expected in actual service is $\hat{N}$, then the safety factor on life is $X_N = \frac{N_{if}}{\hat{N}}$

critical strength for brittle fracture of the member is determined by the current crack length and the fracture toughness K_c for the material and thickness involved:

$$S_c = \frac{K_c}{F\sqrt{\pi a}}$$

as the worst-case crack grows, its length increases, causing the worst-case strength S_c to decrease, with failure occurring when S_c reaches S_{max}, the maximum value for the cyclic loading applied in actual service. the safety factor on stress against sudden brittle fracture due to the applied cyclic load is: $X_c = \frac{S_c}{S_{max}}$

periodic inspections for cracks: inspections done @ intervals of N_p cycles, the length of the worst-case crack ↑ due to growth between inspections, and the safety factor on life is then determined by the inspection period: $X_N = \frac{N_{if}}{N_p}$

fatigue crack growth rate: da/dN, slope at any point on a a vs. N curve

assume that the applied loading is cyclic, with constant values of the loads P_{max} and P_{min}; the corresponding gross section nominal stresses S_{max} and S_{min} are also constant.

stress range: $\Delta S = S_{max} - S_{min}$ stress ratio: $R = \frac{S_{min}}{S_{max}}$

the primary variable affecting growth rate of a crack is the range of the stress intensity factor $\underline{\Delta K = F \Delta S \sqrt{\pi a}}$ => $K_{max} = FS_{max}\sqrt{\pi a}$ $K_{min} = FS_{min}\sqrt{\pi a}$
$\Delta K = K_{max} - K_{min}$ $R = K_{min}/K_{max}$

crack growth behavior can be described by the relationship between cyclic crack growth rate da/dN and stress intensity range ΔK

- log log plot
- @ intermediate values of ΔK, there is often a straight line

$$\frac{da}{dN} = C(\Delta K)^m$$

C = constant
m = slope

- at low growth rates, curve generally becomes steep and appears to approach a vertical asymptote denoted ΔK_{th} (fatigue crack growth threshold)

applying the Walker relationship to the stress intensity factor K : $\overline{\Delta K} = K_{max}(1-R)^{\gamma}$
- → γ is material constant
- → $\overline{\Delta K}$ equivalent zero-to-tension ($R=0$) stress intensity that causes same growth rate as the actual K_{max} and R combination : $\Delta K = K_{max}(1-R)$

$$\overline{\Delta K} = \frac{\Delta K}{(1-R)^{1-\gamma}}$$

C_0 = constant C for special case $R=0$

$$\frac{da}{dN} = C_0 (\Delta K)^m \qquad (R=0)$$

since $\overline{\Delta K}$ equivalent ΔK for $R=0$, can substitute $\overline{\Delta K}$ for ΔK

$$\frac{da}{dN} = C_0 \left[\frac{\Delta K}{(1-R)^{1-\gamma}}\right]^m$$

this represents a family of da/dN vs. ΔK curves which on a log-log plot are all parallel straight lines of slope m. some manipulation gives :

$$\frac{da}{dN} = \frac{C_0}{(1-R)^{m(1-\gamma)}} (\Delta K)^m$$

↳ can see that m is not expected to be affected by R, but C becomes a function of R :

$$C = \frac{C_0}{(1-R)^{m(1-\gamma)}}$$

see ex. 11.3 p. 517-519