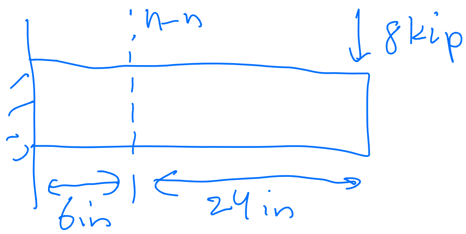
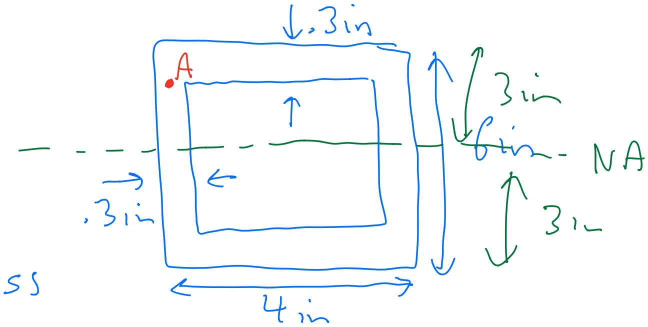


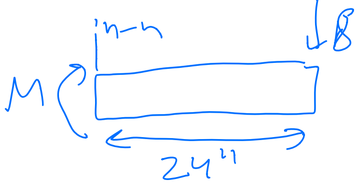
Cross Section at n-n:



Find: (at n-n)

1. Largest normal stress
2. Shear stress at A
3. Maximum shear stress

Moment at n-n:



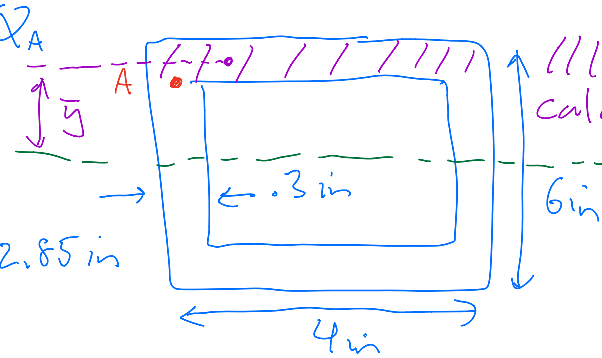
$\Sigma M = 0 = M + 8(24)$

$M = -192$ kip-in

– Moment of inertia = 27.39 in^4

σ_{max} at top/bottom $= \dfrac{Mc}{I} = \dfrac{(192)(3\text{ in})}{27.385} = 21$ ksi

→ τ_A : need Q_A



/// → used to calculate Q_A

$Q_A = \bar{y}A$

$A = (4\text{ in})(.3\text{ in})$

$y = 3\text{ in} - \dfrac{.3\text{ in}}{2} = 2.85$ in

$Q_A = (2.85)(4)(.3)$

$Q_A = 3.42$ in^3

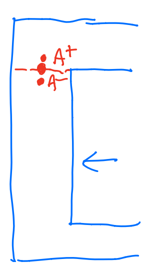
$\tau_{A^+} = \dfrac{VQ_A}{It_+} = \dfrac{(8\text{ kips})(3.42\text{ in}^3)}{(27.39\text{ in}^4)(4\text{ in})}$

$= 0.25$ ksi

$\tau_{A^-} = \dfrac{VQ_A}{It_-} = \dfrac{(8\text{ kips})(3.42\text{ in}^3)}{(27.39\text{ in}^4)(0.6\text{ in})}$

Thickness of A?

– 2 possible values



thick at A^+: 4 in

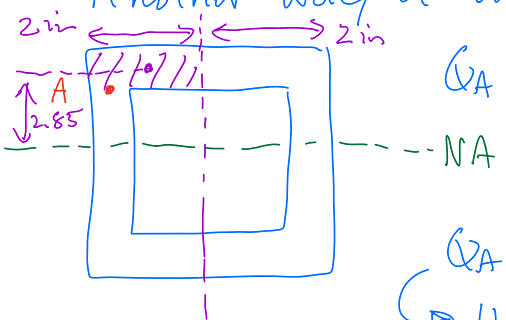
thick at A^-:

$(2)(.3) = 0.6$ in

→ So τ is discontinuous at A

$= 1.67$ ksi

Another way of thinking about Q_A and thickness



$Q_A = \bar{y}A$

$\bar{y} = 2.85$ in

$A = (2\text{ in})(0.3\text{ in})$

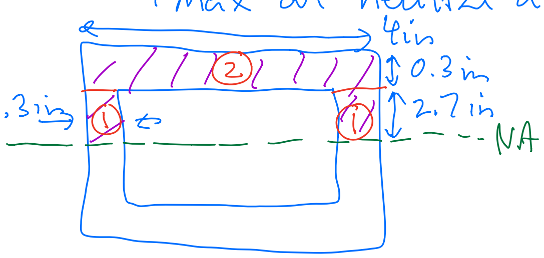
$Q_A = (2.85)(2)(0.3) = 1.71\text{ in}^3$

→ Half of previous Q because used half of area above A → so used half thickness

$$\tau_A^+ = \frac{VQ_A}{It} = \frac{(8\text{ kip})(1.71\text{ in}^3)}{(27.39\text{ in}^4)(2\text{ in})} = 0.25\text{ ksi}$$

$$\tau_A^- = \frac{VQ_A}{It} = \frac{(8\text{ kips})(1.71\text{ in}^3)}{(27.39\text{ in}^4)(0.3\text{ in})} = 1.67\text{ ksi}$$

- τ_{max} at neutral axis



$Q_{TOT} = Q_1 + Q_2 = 5.61\text{ in}^3$

$Q_1 = (2)\underbrace{(.3)(2.7)}_{A_1}\underbrace{\left(\frac{2.7}{2}\right)}_{\bar{y}}$

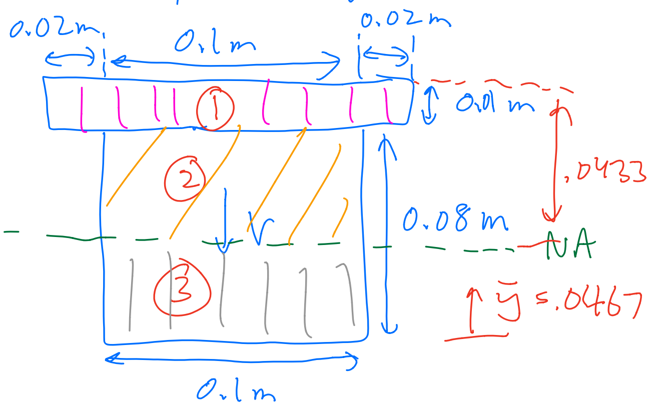
$Q_2 = \underbrace{(4)(0.3)}_{A_2}\underbrace{(2.85)}_{\bar{y}}$

$$\tau_{max} = \frac{VQ_{TOT}}{It} = \frac{(8\text{ kips})(5.61)}{(27.39)(0.6)}$$

→ Use thickness of 0.6in because calculated Q_{TOT} for entire Area above NA

→ Could use $\frac{1}{2}Q_{TOT}$, then use thickness of 0.3 in

– Physical significance of Q



$$\bar{y} = \frac{(0.1)(0.08)(0.04) + (.01)(.14)(.085)}{(.14)(.01) + (.08)(.1)}$$

$\bar{y} = .0467$ m from bottom

$= .0433$ m from top

$$Q_1 = A_1\bar{y}_1 = (.01\,m)(.14\,m)\left(.0433 - \frac{.01}{2}\right) = 5.32 \times 10^{-5}\,m^3$$

$$Q_2 = A_2\bar{y}_2 = (.0433 - .01)(0.1)\left(\frac{.0433 - .01}{2}\right) = 5.46 \times 10^{-5}\,m^3$$

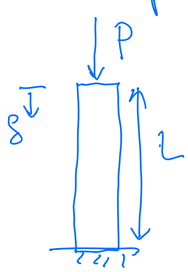
$$Q_3 = A_3\bar{y}_3 = (0.1)(.0467)\left(\frac{.0467\,m}{2}\right) = 1.08 \times 10^{-4}\,m^3$$

$\boxed{Q_1 + Q_2 = Q_3}$ → Q is same above and below the neutral axis

→ First moment of area is same above and below the NA

→ Q is largest at NA so τ is max at neutral axis

Chapter 13: Column Buckling



– Previously: assumed stable equilibrium for all loading types (axial).

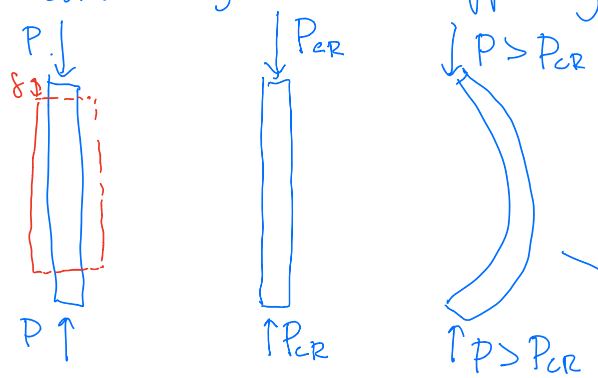
→ $\delta = \frac{PL}{AE}$

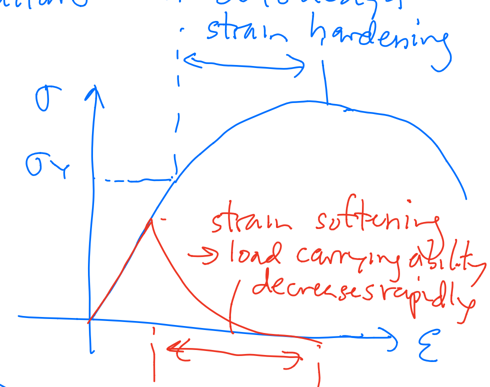
— If compress a long/slender member, it may deflect sideways as well as axially

– Long, slender member subject to axial compression – "column"

– Lateral deflection called buckling

- Buckling leads to sudden, dramatic failure → need to design columns against this happening

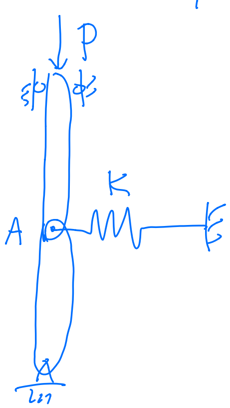




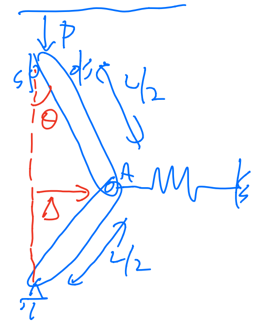
column deflects laterally, i.e. starts to bend

- P_{CR} = maximum axial load column can take/support before it buckles
- When load $P > P_{CR}$, column will buckle

→ Analysis

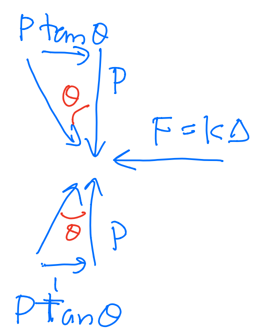


Deformed



$\Delta = \theta \frac{L}{2}$

FBD



Restoring force of spring is $k\Delta$

$= \frac{k\theta L}{2}$

Disturbing force is $2P\tan\theta$

$\approx 2P\theta$

(assume θ is small)

Equate forces in x: $2P\theta = \frac{k\theta L}{2}$

$P_{CR} = \frac{kL}{4}$ ⟹ neutral equilibrium

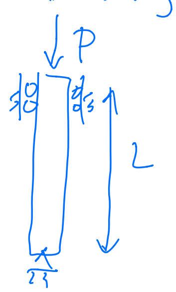
- If $P < \frac{kL}{4}$ ⟹ stable equilibrium → force developed in spring is greater than applied force, so spring force can push bars back to vertical positions

- If $P > \frac{kL}{4}$ $\Rightarrow$ unstable equilibrium $\rightarrow$ spring force not enough to resist buckling, so bars will move out of equilibrium, can't be restored.
- If $P = P_{CR}$ $\Rightarrow$ bifurcation point $\rightarrow$ bar mechanism is unstable to any small perturbation (ie small increase in force).

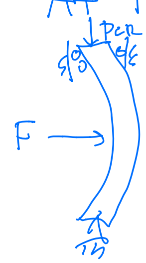
"Bifurcation Point" $\rightarrow$ small change in system causes large, nonlinear response.

– Buckling of ideal column with pin supports

Assumptions
- Perfectly straight column before loading
- Linear elastic material
- Buckling happens in a single plane



At P_{CR}



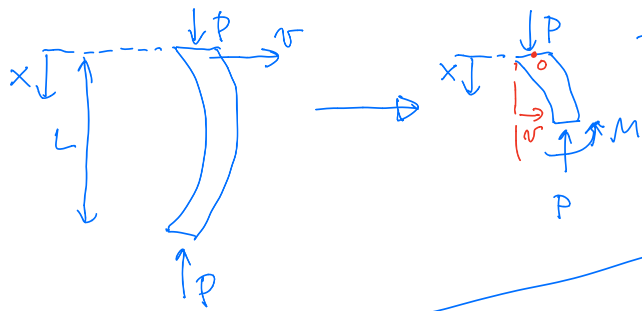
- Small lateral force F will cause column to remain in deflected / bent position after F is removed.

Remove F:
- If reduce load from P_{CR}, column can straighten out
- If increase load beyond P_{CR}, get more increase in lateral deflection

IDEA: whether column remains stable or not depends on its ability to restore itself, or, its resistance to bending

So: $EI \frac{d^2v}{dx^2} = M$ $\rightarrow$ relates internal bending moment to deflected shape

FBD of pin-connected column



$+\circlearrowleft \Sigma M_o = 0 = M + Pv$

$M = -Pv$

So $EI \frac{d^2v}{dx^2} = -Pv$

$$\frac{d^2v}{dx^2} + \left(\frac{P}{EI}\right)v = 0$$

→ Solution is: $v = C_1 \sin\left(\sqrt{\frac{P}{EI}}\,x\right) + C_2 \cos\left(\sqrt{\frac{P}{EI}}\,x\right)$

→ Need boundary conditions to solve for C_1 and C_2

① $v=0$ at $x=0$ → $0 = C_1 \cancel{\sin(0)}^{\,0} + C_2 \cos(0)$

$C_2 = 0$

② $v=0$ at $x=L$

→ $0 = C_1 \sin\left(\sqrt{\frac{P}{EI}}\,L\right)$

If $C_1 = 0$, is trivial solution because v always 0

Other possibility: $\sin\left(\sqrt{\frac{P}{EI}}\,L\right) = 0$

Satisfied when $\sqrt{\frac{P}{EI}}\,L = n\pi$, $n = 1, 2, 3 \ldots$

$\sin \pi = 0$, $\sin 2\pi = 0$, $\sin 3\pi = 0, \ldots$

$$\frac{P}{EI}L^2 = n^2\pi^2, \quad n = 1, 2, 3, \ldots$$

$$P = \frac{n^2\pi^2 EI}{L^2}$$

Smallest value for P is when $n=1$

So $\boxed{P_{CR} = \frac{\pi^2 EI}{L^2}}$

"Euler buckling load"

* P_{CR} expression depends on boundary conditions!!!!

Buckled shape given by:

$$v = C_1 \sin\left(\sqrt{\frac{P}{EI}}\,x\right) = C_1 \sin\left(\sqrt{\frac{\frac{\pi^2 \cancel{EI}}{L^2}}{\cancel{EI}}}\,x\right)$$

$$v = C_1 \sin\left(\sqrt{\frac{\pi^2}{L^2}}\,x\right)$$

$$\boxed{v = C_1 \sin \frac{\pi x}{L}}$$

C_1 represents maximum deflection v_{max}, which happens column midpoint

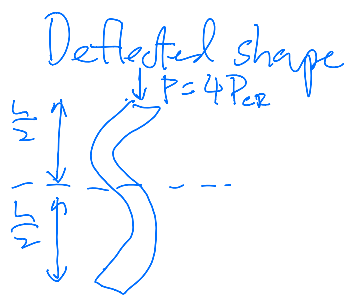
→ n represents # of waves in the deflected shape of the column

General solution for $P = \frac{\pi^2 n^2 EI}{L^2}$

$$v = C_1 \sin\left(\sqrt{\frac{P}{EI}}\,x\right) = C_1 \sin\left(\sqrt{\frac{\frac{\pi^2 n^2 \cancel{EI}}{L^2}}{\cancel{EI}}}\,x\right)$$

$$v = C_1 \sin \frac{n \pi x}{L}$$

– So if $n = 2$: →
$P = 4P_{cr}$



→ Since buckling load is 4 times P_{cr}, and deflected shape is unstable, this mode ($n=2$) will practically not exist

→ Same for all "higher order", ie $n \geq 2$, modes

$$P_{cr} = \frac{\pi^2 n^2 EI}{L^2} \qquad (P_{cr})_{n=1}: \frac{\pi^2 EI}{L^2}$$

$$(P_{cr})_{n=2} = \frac{4\pi^2 EI}{L^2}$$