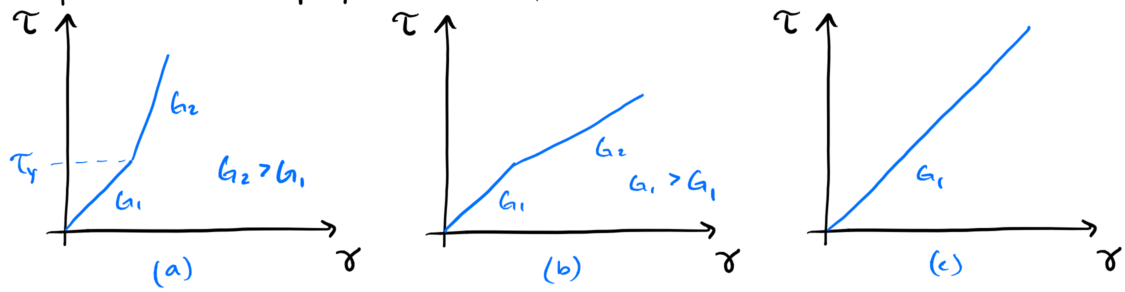
Materials – Lecture #18

Quiz Review:

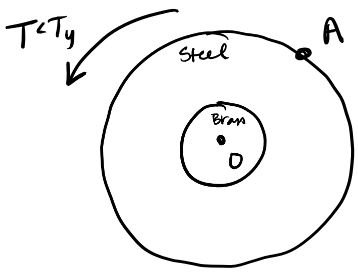
2nd problem: (most people did well)



$\Phi_b > \Phi_c > \Phi_c$

After yielding, the slope of B is the least so it's the easiest to deform (least resistance to twisting) since G_2 smaller
→ b is least stiff, c in middle, a is stiffest

1st problem: composite cylinder



$G_{st} > G_{Br}$

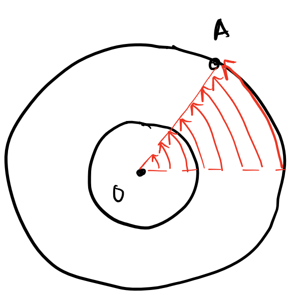
Sketch shear stress & strain distributions

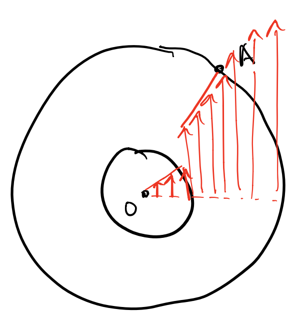
Strain distribution

→ continuous strain, increases linearly
(continuous bc brass & steel are bonded together, which means that they must move together)

Shear stress distribution

→ γ is continuous, $\tau = G\gamma$ so shear stress is going to ↑ until get to boundary
→ once get to boundary, going to get jump in stress
→ at interface $\gamma_{st} = \gamma_{Br}$ so $\tau_{st} > \tau_{Br}$
→ slope of τ_{st} > slope of τ_{Br}





Back to Ch.12: Deflection of Beams / Deflection of Elastic Curve

we wrote the deflection of the beam in 3 different ways

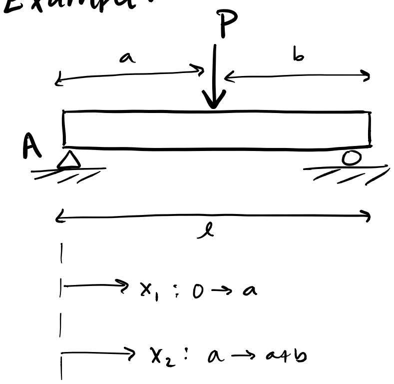
$$EI \frac{d^4 v}{dx^4} = \omega(x)$$

$v(x)$ = elastic curve (beam) deflection

$$EI \frac{d^3 v}{dx^3} = V(x)$$

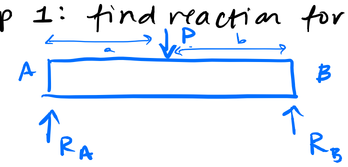
$$EI \frac{d^2 v}{dx^2} = M(x) \longleftarrow$$ we'll typically use this approach

Example:



Find: equation of elastic curve in terms of x_1, x_2

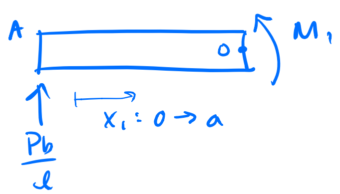
step 1: find reaction forces

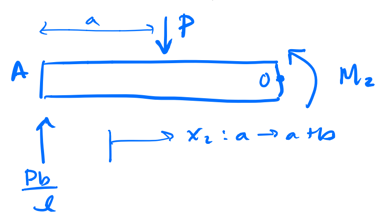


$\circlearrowleft^{+} \sum M_A = 0 = -Pa + R_B l$

$R_B = \frac{Pa}{l}$ $\quad R_A = P - \frac{Pa}{l} = \frac{P(l-a)}{l} = \frac{Pb}{l}$

step 2: calculate internal moments in terms of x_1 and x_2





$\circlearrowleft^{+} \sum M_0 = 0 = M_1 - \frac{Pb}{l} x_1$

$M_1 = \frac{Pb}{l} x_1$

$\circlearrowleft^{+} \sum M_0 = 0 = M_2 + P(x_2 - a) - \frac{Pb}{l} x_2$

$M_2 = x_2\left(\frac{Pb}{l} - P\right) + Pa = Pa - \frac{Pa}{l} x_2$

step 3: write/integrate equations of the elastic curve

Part 1: $x_1 : 0 \to a$

$$EI \frac{d^2 v_1}{dx_1^2} = \frac{Pb}{l} x_1$$

$$EI \frac{dv_1}{dx_1} = \frac{Pb}{l} \frac{x_1^2}{2} + C_1$$

$$EI v_1 = \frac{Pb}{l} \frac{x_1^3}{6} + C_1 x_1 + C_2$$

$\rightarrow$ 2 unknowns (C_1, C_2)

Part 2: $x_2 : a \to a+b$

$$EI \frac{d^2 v_2}{dx_2^2} = Pa - \frac{Pa}{l} x_2$$

$$EI = \frac{dv_2}{dx_2} = Pa x_2 - \frac{Pa}{l} \frac{x_2^2}{2} + C_3$$

$$EI v_2 = Pa \frac{x_2^2}{2} - \frac{Pa}{l} \frac{x_2^3}{6} + C_3 x_2 + C_4$$

$\rightarrow$ 2 unknowns (C_3, C_4)

we have these unknowns, now need to solve

step 4: solve for integration constants (C_1, C_2, C_3, C_4)
→ How?

Apply boundary conditions
① At $x_1 = 0$, beam is constrained and can't deflect downwards so $v_1 = 0$
② At $x_2 = \ell$, beam can't move in vertical direction so can't deflect down, $v_2 = 0$
→ this will help us define two of the unknowns
→ still need two more equations/constraints for other two constants
→ need to impose continuity conditions
- beam deflection is continuous: $v_1 \big|_{x_1 = a} = v_2 \big|_{x_2 = a}$
- beam slope is continuous: $\frac{dv_1}{dx_1}\Big|_{x_1=a} = \frac{dv_2}{dx_2}\Big|_{x_2=a}$

→ now we have four constraints to solve for four unknown constants

Apply Boundary Conditions:

$v_1 = 0$ @ $x_1 = 0 \Rightarrow EIv_1 = \frac{Pb}{\ell}\frac{x_1^3}{6} + C_1 x_1 + C_2 \Rightarrow 0 + 0 + C_2 = 0 \quad \therefore \underline{\underline{C_2 = 0}}$

$v_2 = 0$ @ $x_2 = \ell \Rightarrow EIv_2 = Pa\frac{x_2^2}{2} - \frac{Pa}{\ell}\frac{x_2^3}{6} + C_3 x_2 + C_4$

$$0 = Pa\frac{\ell^2}{2} - \frac{Pa\,\ell^2}{6} + C_3\ell + C_4 \quad \underline{\underline{(1)}}$$

Apply Continuity Conditions:

$$v_1\big|_{x_1=a} = v_2\big|_{x_2=a}$$

$$\frac{Pb}{\ell}\frac{x_1^3}{6} + C_1 x_1 + \cancel{C_2}^{0} = Pa\frac{x_2^2}{2} - \frac{Pa}{\ell}\frac{x_2^3}{6} + C_3 x_2 + C_4$$

$$\frac{Pb}{\ell}\frac{a^3}{6} + C_1 a = Pa\frac{a^2}{2} - Pa\frac{a^3}{6} + C_3 a + C_4 \quad \underline{\underline{(2)}}$$

$$\frac{dv_1}{dx_1}\Big|_{x_1=a} = \frac{dv_2}{dx_2}\Big|_{x_2=a}$$

$$\frac{Pb}{\ell}\frac{x_1^2}{2} + C_1 = Pa\,x_2 - \frac{Pa}{\ell}\frac{x_2^2}{\ell} + C_3$$

$$\frac{Pb}{\ell}\frac{a^2}{2} + C_1 = Pa^2 - \frac{Pa}{\ell}\frac{a^2}{\ell} + C_3 \quad \underline{\underline{(3)}}$$

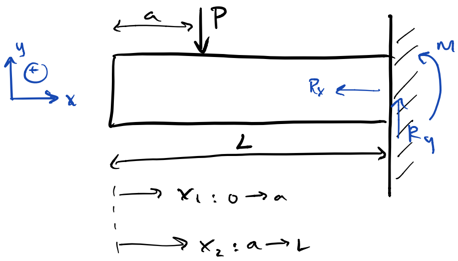
$\underline{\underline{(1)}}, \underline{\underline{(2)}}, \underline{\underline{(3)}}$: 3 equations, 3 unknowns (C_1, C_3, C_4)
solve simultaneously

$$C_1 = -\frac{Pb}{6\ell}(\ell^2 - b^2), \quad C_3 = \frac{-Pa}{6\ell}(2\ell^2 + a^2), \quad C_4 = \frac{Pa^3}{6}$$

→ now have equations for v_1, v_2 in terms of x_1 and x_2
→ units of the constants are different

C_3 : force · length^2 $\qquad$ C_4 : force · length^3

Example #2: Cantilever



Find: equations of elastic curve

step 1: reaction forces

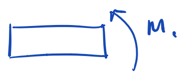
$\Sigma F_y : -P + R_y = 0$ $\qquad R_x = 0 \qquad \underline{R_y = P}$

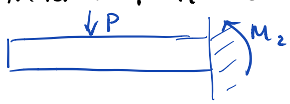
$\cancel{\Sigma M : -Pa + R_y L = 0 \quad R = \frac{Pa}{L}}$

$M + P(L-a) = 0$

$M = -P(L-a)$

step 2: internal moments in terms of x_1 and x_2





$M_2 + P(x_2 - a) = \cancel{M}$

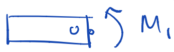
$M_1 = 0$

$\cancel{M_2 = -P(x_2-a) + -P(L-a)}$

$\cancel{= -Px_2 + Pa - PL + Pa \quad -P(2a - x_2 - L)}$

For $x_1 : 0 \to a$

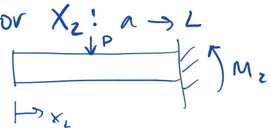
→ no force applied



$\oplus \circlearrowleft \Sigma M_o = 0 = M_1$

$\therefore M_1 = 0$

For $x_2 : a \to L$



$\oplus \circlearrowleft \Sigma M_o = 0 = M_2 + P(x_2 - a)$

$\therefore M_2 = -P(x_2 - a)$

step 3: write/integrate equations of elastic curve

For x_1:

$$EI \frac{d^2 v_1}{dx^2} = 0$$

$$EI \frac{d v_1}{dx} = C_1$$

$$EI\, v_1 = C_1 x_1 + C_2$$

For x_2:

$$EI = \frac{d^2 v_2}{dx^2} = -P(x_2 - a)$$

$$EI = \frac{d v_2}{dx_2} = -\frac{P}{2} x_2^2 + Pa x_2 + C_3$$

$$EI = v_2 = \frac{-P}{6} x_2^3 + \frac{Pa}{2} x_2^2 + C_3 x_2 + C_4$$

step 4: solve for constants using boundary conditions + continuity conditions

(2) $x_2 = L \quad \frac{dv_2}{dx_2} = 0$

→ bc beam is cantilevered

(1) $x_2 = L \quad v_2 = 0$

bc fixed so can't be any deflection here

(4) $x_1 = x_2 = a, \quad \frac{dv_1}{dx_1} = \frac{dv_2}{dx_2}$ → beam slope is continuous

(3) $x_1 = x_2 = a, \quad v_1 = v_2$

→ beam deflection is continuous

ALL CORRECT !!! YES!

(1) & (2) are boundary conditions

(3) & (4) are continuity conditions

$C_3 = \frac{PL^2}{2} - PaL \qquad C_1 = \frac{Pa^2}{2} - PaL + \frac{PL^2}{2}$

$C_4 = \frac{PaL^2}{2} - \frac{1}{3}PL^3 \qquad C_2 = -\frac{Pa^3}{6} + \frac{PaL^2}{2} - \frac{PL^3}{3}$

⇐ Do this at home later, find more "HW" problems on blackboard