Materials – Lecture #16

→ download new syllabus
→ next Thur, will post exam on blackboard (ch.5) and do during class period

Hibbeler Section 6.6

Composite Beams → wikipedia

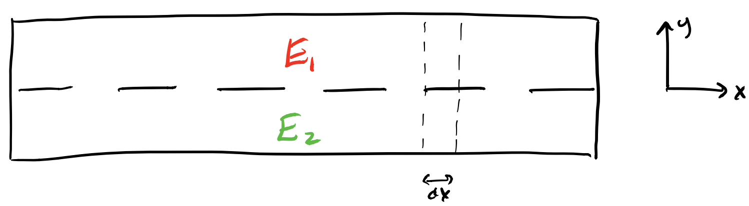
- trying to get better properties than would just one material alon
  ↳ beams made of two or more materials

→ idea: get superior properties than is possible using a single material

ex. wood/steam beam
- steel reinforces wood
  1. steel is much stiffer & stronger
  2. steel doesn't degrade in environment as quickly
  3. steel is less flammable

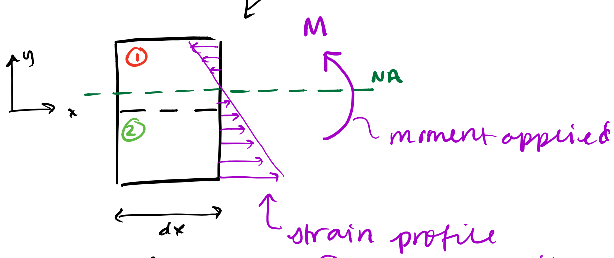
⇒ **Flexure Formula only good for a single material**

↳ so need to transform all of the materials of the composite into a single material.



Youngs Moduli:

$E_1 > E_2$

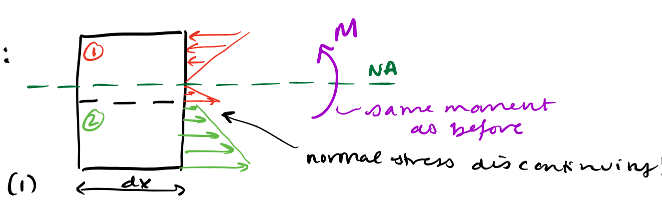


1. strain distribution is continuous along the beam height (y-direction)
2. weaker material (#2) deforms more

Normal strain
(∅ at NA, distribution varies linearly w/ height)

↓ vs.

Normal stress distribution:
- compression above NA
- tension below NA
  1. $E_1 > E_2$, most load carried by stiffer material (1)



② stress discontinuity at interface between materials 1 and 2
→ ϵ at interface is continuous (ϵ_{int})

at the interface $\begin{cases} \sigma_1 = E_1 \epsilon_{int} \\ \sigma_2 = E_2 \epsilon_{int} \end{cases}$ ... same strain but different Youngs Moduli!

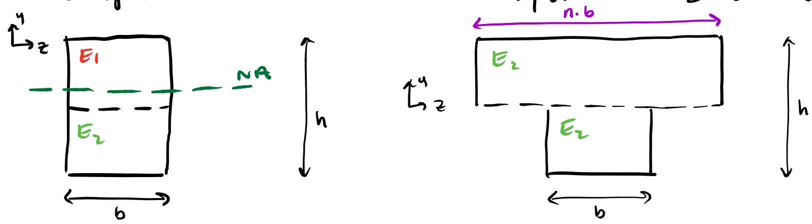
↳ $\therefore \sigma_1 > \sigma_2$

How to resolve this issue? Transform beam into one material made of a single material

↳ keep beam height the same (bc keeps the strain distribution the same as before)
→ assume beam is made of only less stiff material (2)
↳ implies that changing material (1) into (2)
... so then will need more of material (2) to carry load
→ so we widen the upper portion of the beam to carry load equivalent to that carried by material (1)

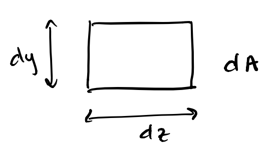
original

Transformed Beam (height stays the same)



- transform beam to nb → what is the transformation factor n?

original material 1:



$dF = \sigma dA$
$= E_1 \epsilon \, dy \, dz$

transformed material:

$dF' = \sigma' dA$
$= E_2 \epsilon \, n \, dy \, dz$

Equate the forces: $dF = dF'$

$dF = dF'$

$E_1 \epsilon \, dy \, dz = E_2 \epsilon \, n \, dy \, dz$

$$\boxed{n = \frac{E_1}{E_2}}$$

n = "transformation factor"

here: $E_1 > E_2$ → so need more of material 2 to carry equiv. load!

* one implication from this:
→ basically changing mass of part of beam ...
⇒ so centroid and neutral axis location change after transformation
⇒ moment of inertia changes also

– Stress Distribution in transformed beam is equivalent to stress in same material of actual beam

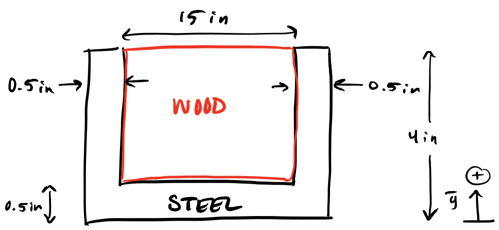
$$dF = \sigma dA = \sigma' dA'$$

$$\sigma\, dy\, dz = n \sigma' dy\, dz \quad \text{because } dA' = n\, dy\, dz$$

$$\boxed{\sigma = n\sigma'}$$

→ Problem Solving steps
1. Calculate n
2. calculate new NA location
3. calculate new moment of inertia (I)
4. analyze as before

Example #1



$E_{st} = 29(10^3)$ ksi
$E_{wd} = 1600$ ksi
$M = 850$ ft·lbs
(compression @ top, tension @ bottom)

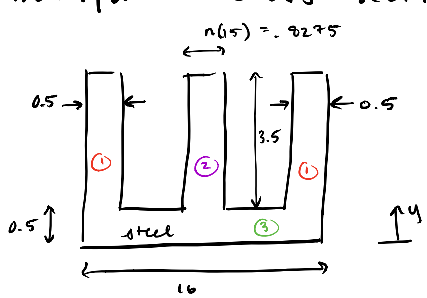
Find:
1. $(\sigma_{st})_{max}$
2. $(\sigma_{wd})_{max}$

do this first!
Find:
1. n
2. Find $\bar{y}$ (from bottom)
3. Find I

2 options for n:
1. transform steel to wood
2. transform wood to steel ← choose this, why?
   - would need a lot more wood
   - xform to steel then shrink down

$$n = \frac{E_{wd}}{E_{st}} = \frac{1600 \text{ ksi}}{29(10^3) \text{ ksi}} = .055$$

Transformed cross section



$$\bar{y} = \frac{\overbrace{(0.5)(16)}^{A_3}\overbrace{(0.25)}^{y_3} + \overbrace{(3.5)(0.5)}^{A_1}\overbrace{(2.25)}^{y_1 = 0.5 + \frac{3.5}{2}}(2) + \overbrace{(3.5)(.8275)}^{A_2}\overbrace{(2.25)}^{y_2 = 0.5 + \frac{3.5}{2}}}{(16)(0.5) + (2)(0.5)(3.5) + (3.5)(.8275)}$$

$$\bar{y} = \frac{16.3876}{14.39625} = 1.139$$

$$I = 2I_1 + I_2 + I_3$$

$$I_1 = \frac{1}{12}(0.5)(3.5)^3 + (0.5)(3.5)\left(\frac{3.5}{2} + 0.5 - 1.139\right)^2 = 3.95 \text{ in}^4$$

$$I_2 = \frac{1}{12}(0.8275)(3.5)^3 + (0.8275)(3.5)\left(\frac{3.5}{2} + 0.5 - 1.139\right)^2 = 6.53 \text{ in}^4$$

$$I_3 = \frac{1}{12}(16)(0.5)^2 + (16)(0.5)(1.139 - 0.25)^2 = 6.49 \text{ in}^4$$

$$I = (3.95 \text{ in}^4)(2) + 6.53 + 6.49 = 20.91 \text{ in}^4$$

max stresses...

$$\cancel{\sigma_{st\,max} = n\sigma'_{st}} \qquad \cancel{\sigma_{wd,max} = n\sigma'_{wd}}$$

$$\cancel{\sigma_{st} = \frac{850 \text{ ft}\cdot\text{lbs}(1.139 \text{ in})\left(\frac{1 \text{ ft}}{12 \text{ in}}\right)}{20.91 \text{ in}^4\left(\frac{1 \text{ ft}^4}{(12 \text{ in})^4}\right)} = 80008 \frac{\text{lb}}{\text{ft}^2}}$$

$$\sigma_{st\,max} = \frac{Mc}{I} = \frac{(850 \text{ ft}\cdot\text{lb})\left(\frac{12 \text{ in}}{\text{ft}}\right)(4 \text{ in} - 1.139 \text{ in})}{20.91 \text{ in}^4} = 1395 \text{ psi} = \boxed{1.4 \text{ ksi} \text{ (compression)}}$$

$$(\sigma_{st})_{wood} = n(\sigma_{st})_{max} = (0.055)(1395 \text{ psi}) = \boxed{77 \text{ psi}}$$

wood doesn't carry much stress since it's less stiff than steel

How do you assume top part in compression?
- assumption is that internal moment is bending beam upwards