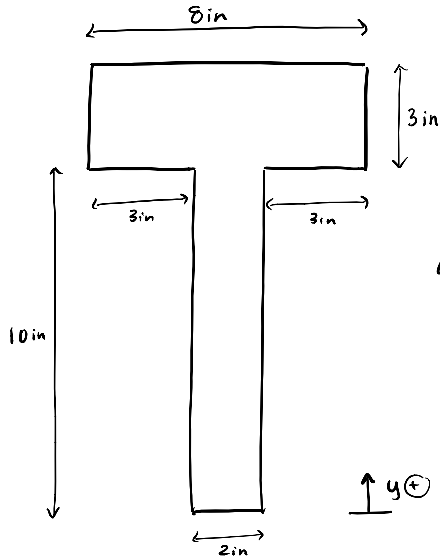
Materials Lecture #15

Hibbeler 6.4 → start from centroid of cross-sectional area

Example 1: Find centroid of cross-sectional area

ΣA = sum of areas of each smaller part

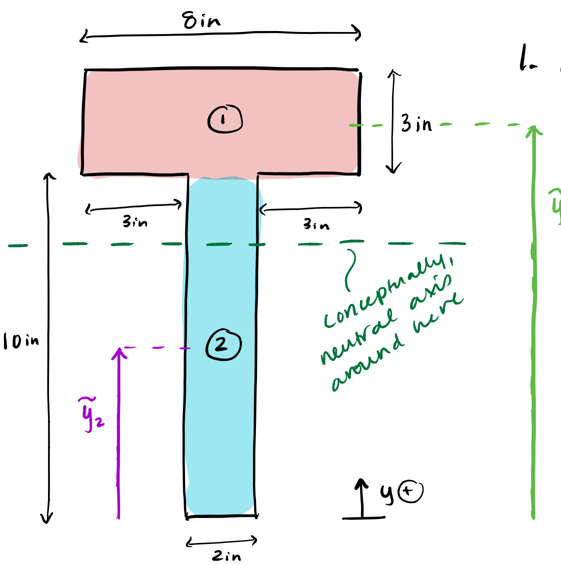
$\tilde{y}$ = distance from a reference pt to centroid of each smaller part



we'll need to figure out where the neutral axis is

3 ways to solve this ...

1. section into 2 smaller parts ...



1. Pick reference point (bottom of cross-section)

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A}$$

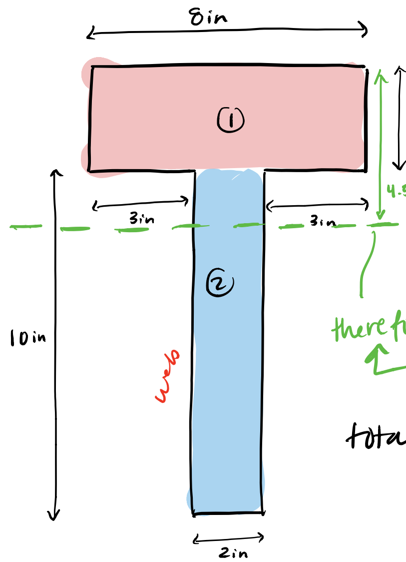
$$\bar{y} = \frac{\tilde{y}_1 A_1 + \tilde{y}_2 A_2}{A_1 + A_2}$$

$$\bar{y} = \frac{\overbrace{(10+1.5)}^{\tilde{y}_1}\overbrace{(3)(8)}^{A_1} + \overbrace{(5)}^{\tilde{y}_2}\overbrace{(2)(10)}^{A_2}}{\underbrace{(3)(8)}_{A_1} + \underbrace{(2)(16)}_{A_2}}$$

$$\bar{y} = 8.55 \text{ in}$$

Now, change origin of reference coordinate

↓ y(+)



$\downarrow y(+)$

$$\bar{y} = \frac{\Sigma y A}{\Sigma A} = \frac{\underbrace{(10in)(2in)}_{\text{Area}_2}\underbrace{(3+5in)}_{\substack{y_2 \\ \text{top to center} \\ \text{of web}}} + \underbrace{(8)(3)}_{A_1}\underbrace{(1.5in)}_{y_1}}{\underbrace{(10)(2)}_{A_2} + \underbrace{(8)(3)}_{A_1}}$$

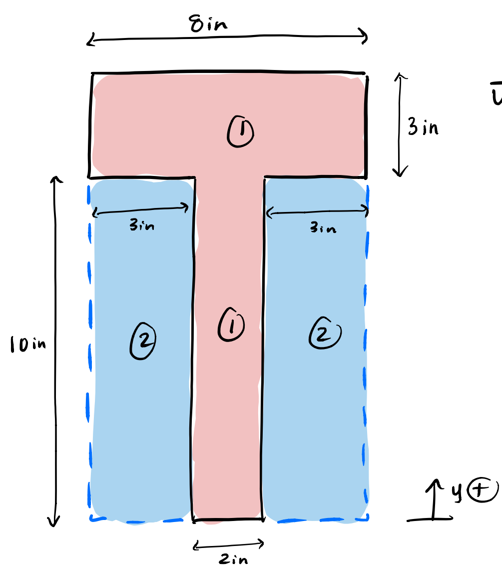
therefore NA

$\bar{y} = 4.45$ in

total height is 13 inches...

$13 - 4.45 = 8.55$

Approach #3: large area minus small area



$$\bar{y} = \frac{\underbrace{(13in)(8in)}_{\text{large Area}}\underbrace{(\tfrac{1}{2})(3in)}_{y\text{ large}} - \underbrace{(2)(3)(10)}_{\text{small area}}\underbrace{(5)}_{\text{small } y}}{\underbrace{(13)(8)}_{\text{large area}} - \underbrace{2(10)(3)}_{\text{small area}}}$$

$\bar{y} = 8.55$ in

same as before

large area = area$_1$ + area$_2$

What does the NA represent physically?

→ before we said that it represents the geometric center of a structure (bc that's where centroid is)

What doesn't it represent?

→ calculate the areas above and below

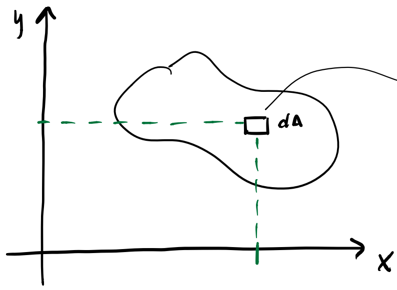
Area above = $(8)(3) + (2)(1.45) = 26.9\ in^2$ above NA

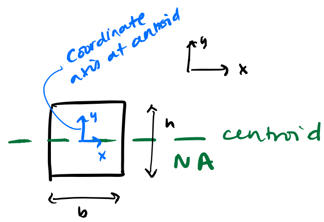
Area below = $(2)(8.55) = 17.1\ in^2$ below NA

... shows that NA doesn't split the structure into two equal areas!!!

area above ≠ area below ⇒ from $\int y\,dA$ which is moment of area, NOT area itself

Moment of Inertia





we want to calculate moment of inertia about x-axis, it's the second moment of the area

$$I_x = \int y^2 \, dA$$

If rectangular: use coordinate axis of centroid

$$I_x = \int_{-\frac{b}{2}}^{b/2} \int_{-h/2}^{h/2} y^2 \, dy \, dx$$

$$= \int_{-\frac{b}{2}}^{\frac{b}{2}} \frac{1}{3} y^3 \Big|_{-\frac{h}{2}}^{\frac{h}{2}} dx = \int_{-\frac{b}{2}}^{\frac{b}{2}} \frac{h^3}{12} \, dx$$

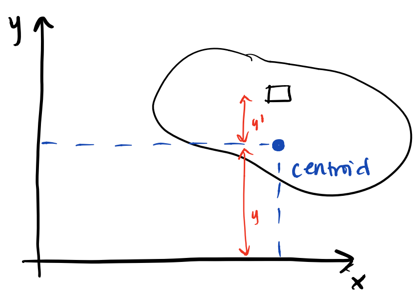
$$= \frac{x h^3}{12} \Big|_{-b/2}^{b/2}$$

$$\boxed{I_x = \frac{1}{12} b h^3}$$

← when we can use this:

1. works if centroid lies in the middle of the cross section, and cross-section is symmetric about x-axis
2. units are length to the fourth power, $\therefore I_x > 0$

- If want I_x at some distance from the centroid?



calculate moment of inertia for this addnl area...

$$I_x' = \int_A (y + y')^2 \, dA$$

$$= \int_A (y^2 + 2yy' + y'^2) \, dA$$

(1) (2) (3)

$$I_x' = \int_A y^2 \, dA + 2y' \int_A y \, dA + y'^2 \int dA$$

(1) the original moment of inertia, I_x $\left[\frac{1}{12} b h^3\right]$

(2) $= 0$ b/c integrating about the centroid

(3) the product of the area, square of distance between NA and center of the area

moment of inertia of something itself

area of piece × distance between NA and center of area squared

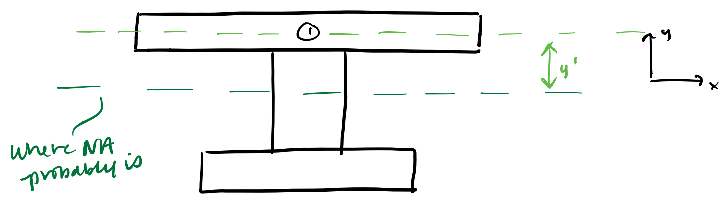
$$\boxed{I_x' = I_x + Ay'^2}$$

y' = distance from NA to centroid of other geometry

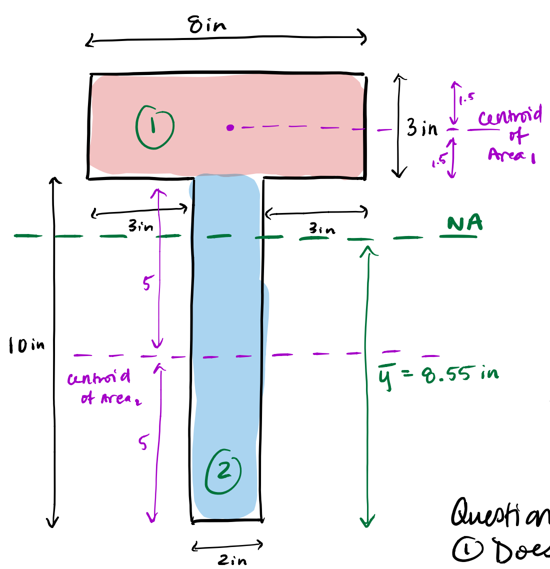
how to calc moment of inertia for something that's some distance away from centroid

"PARALLEL AXIS THEOREM"

Example



Example: Find moment of Inertia



$$I_x = \frac{1}{12} bh^3 \text{ if symmetric}$$

or

$$I_x' = \underbrace{\Sigma\left(I_x + Ad^2\right)}_{\text{sum of all sections that are not symmetric wrt neutral axis}}$$

Calculate I_x for both sections...

$$I_{total} = I_1 + I_2$$

Questions to think about:

1. Does the NA pass through centroid of $Area_1$?
   NO.
2. Does the NA pass through centroid of $Area_2$?
   NO.

→ so we'll have to use parallel axis theorem for both of these

$$I_1 = \frac{1}{12} b h^3 + A d^2$$

distance from NA to centroid

$$I_1 = \underbrace{\tfrac{1}{12}(8)(3)^3}_{I_x} + \underbrace{(8)(3)}_{A_1}\underbrace{(2.95)^2}_{d_1 = 4.45 - 1.5} = 226.86 \text{ in}^4$$

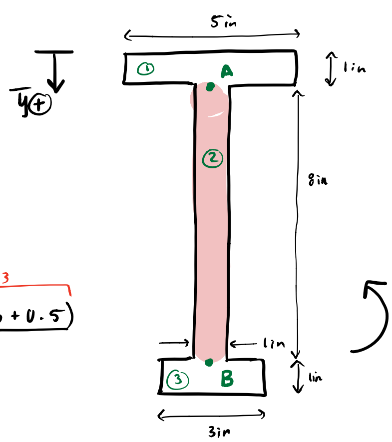
$$I_2 = \underbrace{\tfrac{1}{12}(2)(10)^3}_{I_x} + \underbrace{(2)(10)}_{A_2}\underbrace{(3.55)^2}_{d_2 = 8.55 - 5} = 418.72 \text{ in}^4$$

$$I_{total} = I_1 + I_2 = \boxed{645.58 \text{ in}^4}$$

[More complicated] example:

$M = 15$ kip·ft

Find: % of moment resisted by the web

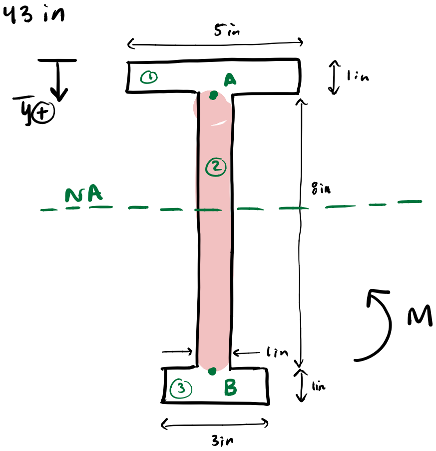


Proceed in steps:

1. NA location
2. Moment of inertia

① $$\bar{y} = \frac{\overbrace{(1)(5)}^{A_1}\overbrace{(0.5)}^{y_1} + \overbrace{(1)(8)}^{A_2}\overbrace{(1+4)}^{y_2} + \overbrace{(3)(1)}^{A_3}\overbrace{(1+8+0.5)}^{y_3}}{\underbrace{(1)(5)}_{A_1} + \underbrace{(8)(1)}_{A_2} + \underbrace{(3)(1)}_{A_3}}$$

$\bar{y} = 4.43$ in



top part of I beam has more area than bottom (more mass @ top)

② $$I_{total} = I_1 + I_2 + I_3$$

for all 3 have to use PAT bc NA doesn't pass through any centroid

$$I_1 = \underbrace{\tfrac{1}{12}(5)(1)^3}_{I_1} + \underbrace{(5)(1)}_{A_1}\underbrace{(4.4375 - 0.5)^2}_{d_1} = 77.94 \text{ in}^4$$

$$I_2 = \underbrace{\tfrac{1}{12}(1)(8)^3}_{I_2} + \underbrace{(1)(8)}_{A_2}\underbrace{(1+4-4.4375)^2}_{d_2} = 45.2 \text{ in}^4$$

$$I_3 = \underbrace{\tfrac{1}{12}(3)(1)^3}_{I_3} + \underbrace{(3)(1)}_{A_3}\underbrace{(9.5 - 4.4375)^2}_{d_3} = 77.14 \text{ in}^4$$

$$I_{total} = 200.3 \text{ in}^4$$

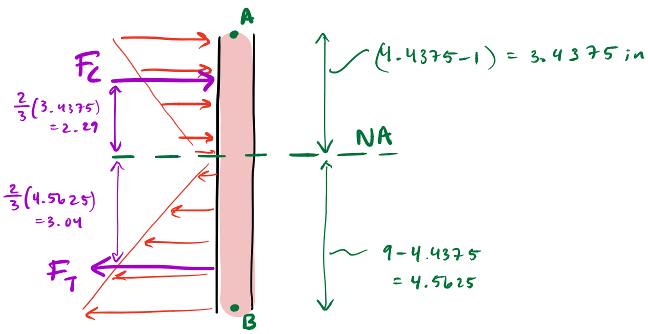
we've got I_x, NA, now can calculate stresses at different pts [A & B]

↳ this way can see what's happening @ web

$$\sigma_A = -\frac{My}{I} = \frac{-(15\text{ kip ft})(12\text{ in/ft})(4.375-1)}{200.3\text{ in}^4} = -3.09\text{ ksi} \quad \text{(compression)}$$

negative b/c below NA

$$\sigma_B = -\frac{My}{I} = \frac{-(15\text{ kip ft})(12\text{ in/ft})(-9+4.375)}{200.3\text{ in}^4} = 4.1\text{ ksi} \quad \text{(tension)}$$



width of web

$$F_C = \tfrac{1}{2}\sigma_A(3.4375\text{ in})(1\text{ in})$$
$$= 5.31\text{ kips}$$

$$F_T = \tfrac{1}{2}\sigma_B(4.5625\text{ in})(1\text{ in})$$
$$= 9.35\text{ kips}$$

$$\circlearrowright\oplus \quad \Sigma M_{NA} = 0 = F_C(2.29) + F_T(3.04) = M_{web}$$

$$M_{web} = 40.62\text{ kip}\cdot\text{in} \quad \text{or} \quad M_{web} = 3.38\text{ kip}\cdot\text{ft}$$

% of internal moment that's carried by the web:

$$\frac{3.38}{15}\times 100 \Rightarrow \boxed{22.6\%}$$