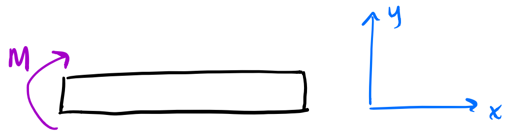
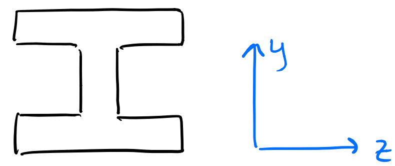
Materials – Lecture #14

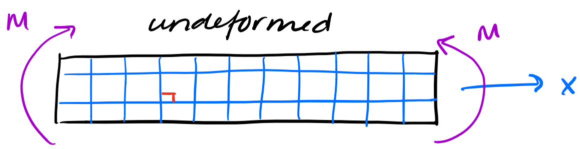
- Final project due during finals week (will be assigned mid-April) ~ big chunk of grade
- Small quiz on Ch. 5 in couple weeks (~10% of grade)

Section 6.3 ... Bending of a straight member



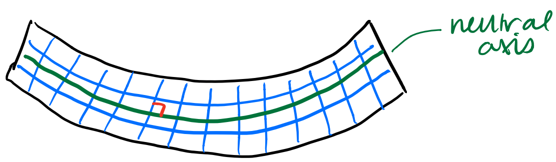
cross-sectional view of beam





$\Downarrow$

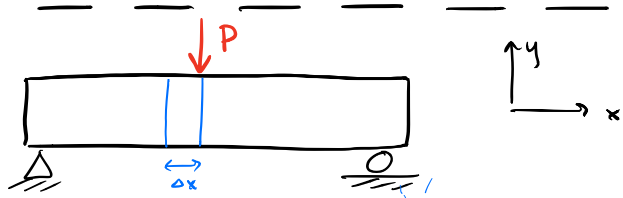
deformed

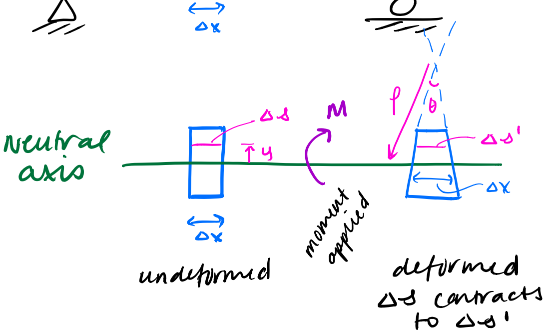


- horizontal lines become curved
- vertical lines remain straight but they rotate

key idea:

1. Top surface → shorter (compression)
2. Bottom surface → expands/elongates (tension)
3. Implies existence of "neutral axis" between top and bottom with no change in length ($\sigma_{NA} \leq 0$, $\epsilon_{NA} = 0$)





– Normal strain along Δs

$$\epsilon = \lim_{\Delta s \to 0} \frac{\Delta s' - \Delta s}{\Delta s}$$

$$\Delta x = \rho \Delta \theta = \Delta s$$

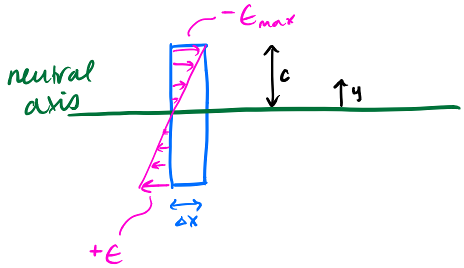
$$\Delta s' = (\rho - y)\Delta \theta$$

$$\epsilon = \frac{(\rho - y)\Delta\theta - \rho\Delta\theta}{\rho\Delta\theta}$$

$$\lim_{\Delta\theta \to 0} \epsilon = \frac{(\rho - y)\Delta\theta - \rho\Delta\theta}{\rho\Delta\theta} \Rightarrow \epsilon = \frac{-y\Delta\theta}{\rho\Delta\theta} \Rightarrow \boxed{\epsilon = -\frac{y}{\rho}}$$

this means that the strain in the beam varies linearly w/height from the neutral axis (?)

y = distance from neutral axis
$y = 0$ = on neutral axis $\rightarrow \epsilon = 0$



strain is maximized when you're at one of the surfaces

ϵ is maximized when $y = c$

$$\epsilon_{max} = \frac{c}{\rho}$$

$$\frac{\epsilon}{\epsilon_{max}} = \frac{(-y/\rho)}{(c/\rho)}$$

- elongation below neutral axis ($+\epsilon$)
- contraction above neutral axis ($-\epsilon$)

$$\epsilon = -\left(\frac{y}{c}\right)\epsilon_{max}$$

* note that the neutral axis is <u>not</u> always in the middle

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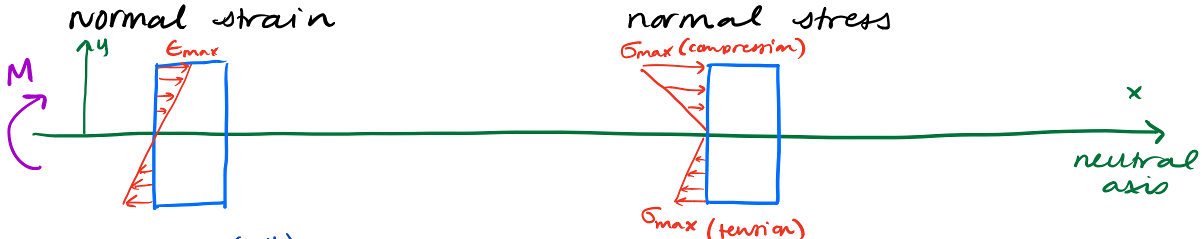
6.4 → Flexure Formula

goal: develop an equation that relates the longitudinal stress distribution in a beam to resulting internal moment at beam cross-section

* bending moment (M) causes normal stresses and strains
* shear force (V) causes shear stresses and shear strains

Assume Hooke's Law: $\sigma = E\epsilon$, linear elastic



$$\epsilon = \left(-\frac{y}{c}\right)\epsilon_{max}$$

$$\frac{\sigma}{E} = \left(-\frac{y}{c}\right)\frac{\sigma_{max}}{E} \longrightarrow \sigma = \left(-\frac{y}{c}\right)\sigma_{max}$$

where have compressive strain, will also have compressive stress
where have tensile strain, will also have tensile stress

→ σ linearly away from neutral axis

* How do we find position of the neutral axis?
→ satisfy condition that resultant force caused by stress distribution over cross section = 0

so the resultant forces due to compression and tension have to cancel out somehow

$$dF = \sigma dA$$

force distribution

$$\Rightarrow \int_A dF = 0 = \int_A \sigma dA = 0$$

$$= \int_A -\left(\frac{y}{c}\right)\sigma_{max} dA$$

$$= -\frac{\sigma_{max}}{c}\int_A y\, dA$$

this term cannot equal ϕ

therefore this integral must equal zero

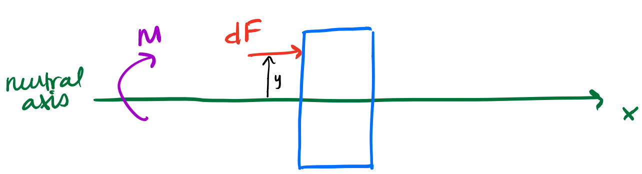
$$\int_A y\, dA = 0$$

→ the first moment of area about the neutral axis = 0

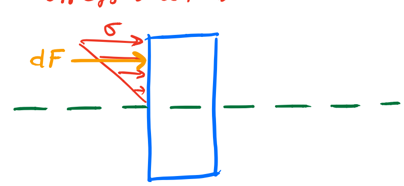
→ this condition is only satisfied if the neutral axis is also the horizontal centroidal axis

→ so we need to find the centroid of the cross-sectional area

→ find expression for stress in beam
idea: require resultant internal moment to equal the moment produced by stress distribution about neutral axis



dF = resultant force caused by stress distribution



$$\Sigma M = 0 = M + \int_A y\, dF \qquad \text{note that } dF = \sigma dA$$

$$0 = M + \int_A y(\sigma dA) \Rightarrow \text{solving for } M, \quad M = -\int_A y\sigma dA$$

then can write

$$M = -\int_y \left(-\frac{y}{c}\right) \sigma_{max}\, dA$$

$$M = \frac{\sigma_{max}}{c} \int_A y^2\, dA$$

note that

$\int_A y^2\, dA$ = moment of inertia of cross section computed about the neutral axis (I)

$$M = \frac{\sigma_{max} I}{c}$$

rearrange to solve for σ_{max}

$$\sigma_{max} = \frac{Mc}{I}$$

more simplifying ...

$$\frac{M}{I} = -\frac{\sigma}{y} \Rightarrow \frac{\sigma_{max}}{c} = -\frac{\sigma}{y}$$

$$\Downarrow$$

$$\boxed{\sigma = \frac{-My}{I}}$$

stress depends on three things...

M = resultant internal moment

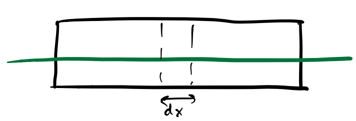
y = distance from neutral axis

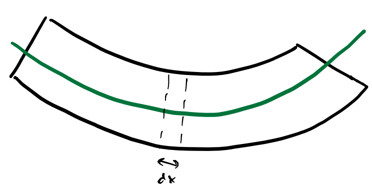
- y is positive towards compression
- y is negative toward tensile side of beam

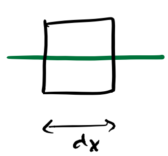
I = moment of inertia of cross-section

Summary for solving bending stress problems

1. Calculate resultant internal moment
2. Calculate neutral axis of cross-section
3. Calculate cross sectional moment of inertia (I)
4. Find bending (normal) stress distribution along cross section



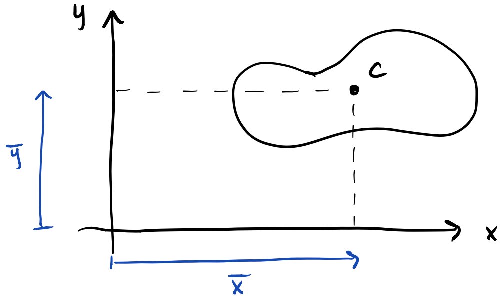




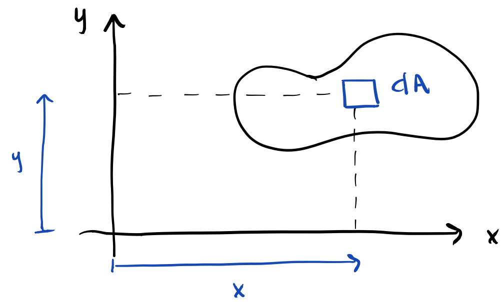
note that our examples deal w/ straight neutral axis bc this is such a small length dx

Neutral Axis/Moment of Inertia

Centroid of an Area: point that defines geometric center of the area



$$\bar{x} = \frac{\int_A x\,dA}{\int_A dA} \qquad \bar{y} = \frac{\int_A y\,dA}{\int_A dA}$$



composite areas: cross sectional area can be divided into several parts having smaller shapes

goal is to find location of centroid of each smaller shape, can avoid doing integration

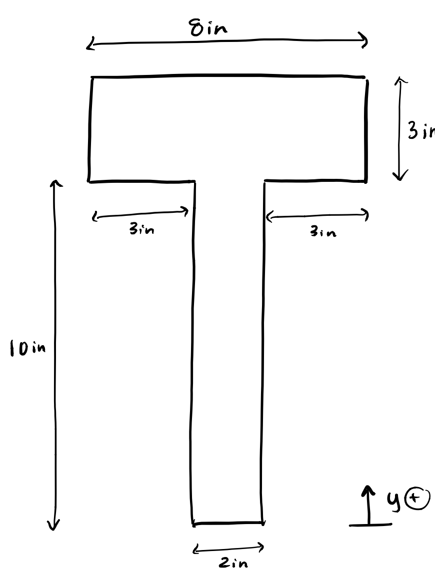
$$\bar{x} = \frac{\Sigma \tilde{x} A}{\Sigma A} \qquad \boxed{\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A}}$$

typically only going to look at this one

ΣA = sum of areas of each smaller part

$\tilde{y}$ = distance from a reference pt to centroid of each smaller part

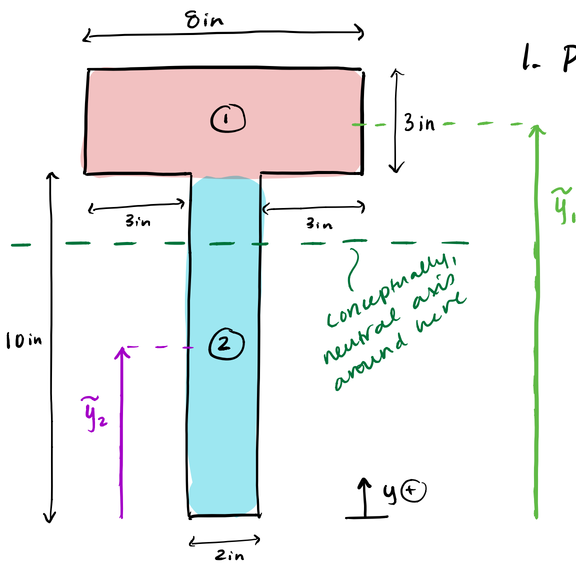
Example 1: Find centroid of cross-sectional area



we'll need to figure out where the neutral axis is

3 ways to solve this ...

1. section into 2 smaller parts ...



1. Pick reference point (bottom of cross-section)

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A}$$

$$\bar{y} = \frac{\tilde{y}_1 A_1 + \tilde{y}_2 A_2}{A_1 + A_2}$$

$$\bar{y} = \frac{\overbrace{(10 + 1.5)}^{\tilde{y}_1}\overbrace{(3)(8)}^{A_1} + \overbrace{(5)}^{\tilde{y}_2}\overbrace{(2)(10)}^{A_2}}{\underbrace{(3)(8)}_{A_1} + \underbrace{(2)(16)}_{A_2}}$$

$$\bar{y} = 8.55 \text{ in}$$