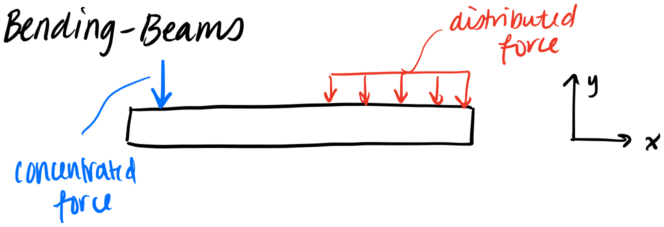
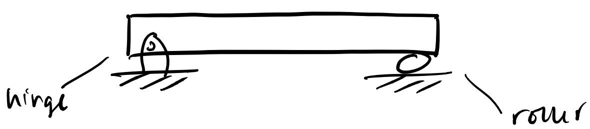
Materials - Lecture #13

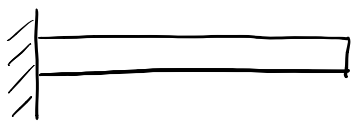
Bending - Beams



- beams are long, straight bars w/ constant area
- beams are classified according to support



Simply Supported

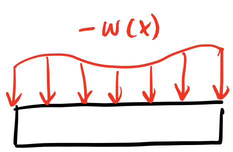


Cantilever

Issue: ① Beams develop internal shear and moment, varies from pt to pt.

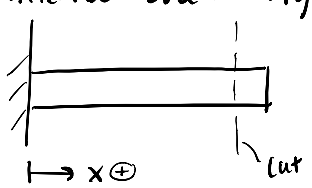
② where is shear force (V), moment (M) maximized along beam length?

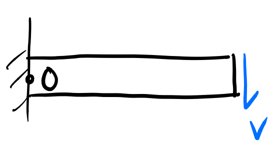
Beam sign convention



distributed load downward is negative

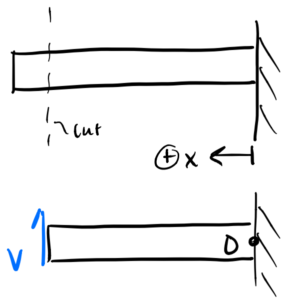
Internal shear sign convention





positive V causes clockwise rotation of the beam

moment about O is clockwise

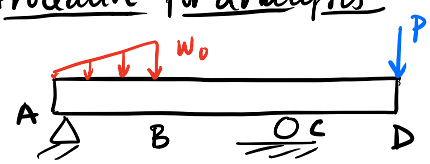


V still causes clockwise rotation

Sign Convention for internal moments

Both positive moments cause compression of top surface, tension of bottom surface

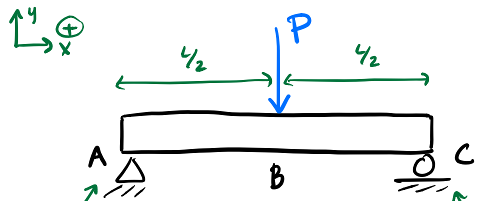
Procedure for analysis



- Use method of sections to find V and M at any point
  → BUT, shear/moments are discontinuous
- Find shear/moment diagram for each region between the discontinuities

1. Calculate support reactions (force + moments)
2. Start at left end of beam, specify separate coordinates extending to regions of beam between concentrated loads (forces/moments)
3. Section beam according to (2), draw FBDs, find internal shear and moment

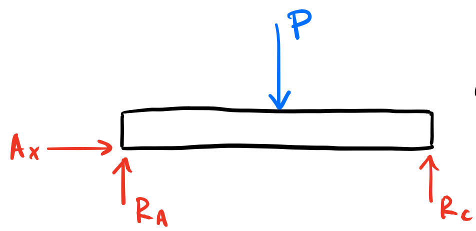
Example #1 (by Prof. Park)



beam is fixed at A, so $dx = dy = 0$

roller, prevents beam moving in y-direction $\begin{bmatrix} dy = 0 \\ dx \neq 0 \end{bmatrix}$

Step 1: Reaction forces



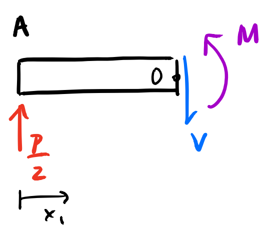
$\xrightarrow{\oplus} \Sigma F_x = 0 = A_x$ $\qquad A_x = 0$

$\oplus \circlearrowleft \Sigma M_A = R_C L - P\left(\frac{L}{2}\right) = 0$ $\qquad R_C = \frac{P}{2}$

$\oplus \uparrow \Sigma F_y = 0 = R_A + R_C - P$ $\qquad R_A = \frac{P}{2}$

Step 2: Internal shear/moment

→ first section: A to B



$x_1 : 0 \to L/2$

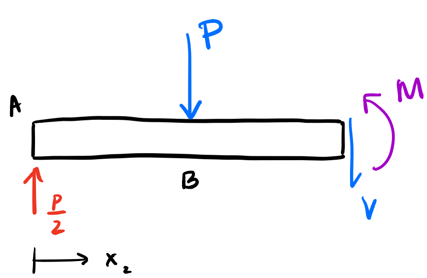
$\oplus \uparrow \Sigma F_y = 0 = \frac{P}{2} - V$ $\qquad V = P/2$

$\oplus \circlearrowleft \Sigma M_o = 0 = M - \left(\frac{P}{2}\right)(x_1)$ $\qquad M = \frac{P}{2} x_1$

Relationship between V and M?

$\rightarrow \frac{dM}{dx_1} = V$

→ section A to C



$x_2 : \frac{L}{2} \to L$

$\oplus \uparrow \Sigma F_y = 0 = \frac{P}{2} - P - V$ $\qquad V = -\frac{P}{2}$

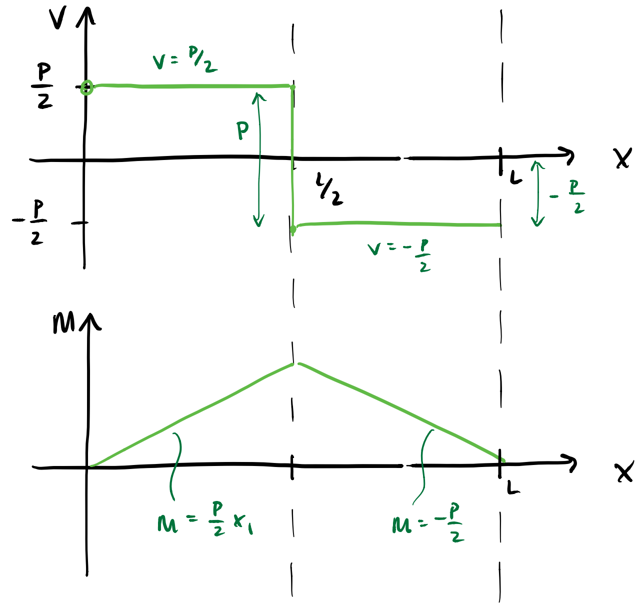
note that this differs from section AB by P

$\oplus \circlearrowleft \Sigma M_o = 0 = M - \frac{P}{2} x_2 + P\underbrace{\left(x_2 - \frac{L}{2}\right)}_{\text{moment arm for conc force P}}$

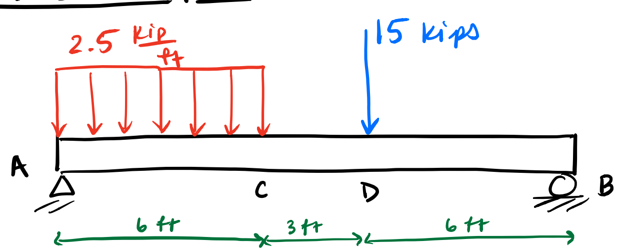
$M = \frac{P}{2}(L - x_2)$

$\frac{dM}{dx_2} = -\frac{P}{2} = V$

Step 3: Plot Shear/Moment Diagrams



In-class Example

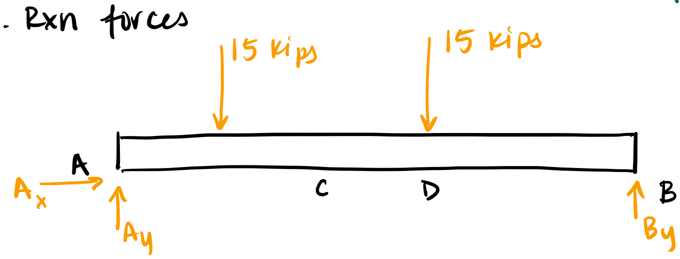


Find: Maximum absolute value of shear.

Maximum absolute value of bending moment

How many beam sections? ① AC ② CD ③ DB

1. Rxn forces

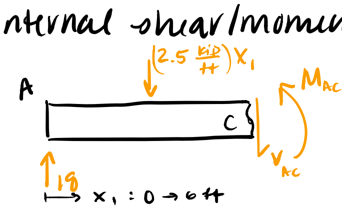


$\overset{+}{\rightarrow} \Sigma F_x = 0, \ A_x = 0$

$\oplus \circlearrowleft \Sigma M_A = 0 = -15\text{ kip}(3\text{ ft}) - 15\text{ kip}(9\text{ ft}) + B_y(15\text{ ft}) \Rightarrow \quad B_y = 12\text{ kip}$

$\oplus \uparrow \Sigma F_y = 0 = A_y - 15\text{ kips} - 15\text{ kips} + 12\text{ kip} \Rightarrow \quad A_y = 18\text{ kip}$

2. Internal shear/moment



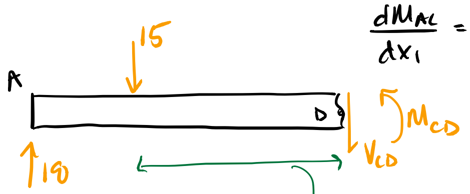
$\oplus \uparrow \Sigma F_y : 18 - 2.5x_1 - V_{AC} = 0 \qquad V_{AC} = 18 - 2.5x_1$

$\oplus \circlearrowleft \Sigma M_c : M_{AC} + 2.5x_1(x_1/2) - 18(x_1) = 0$

$M_{AC} = 18x_1 - 1.25x_1^2$

check M_{AC} by taking derivative:

$$\frac{dM_{AC}}{dx_1} = 18 - 2.5x_1 = V_{AC}\ !$$



$\mapsto x_2 : 6\text{ ft} \rightarrow 9\text{ ft}$

what is this distance?

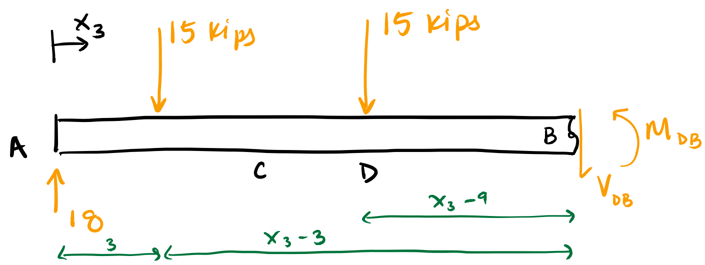
→ once gone past pt. C, gone over whole distributed load so this distance can be written as $x_2 - 3$

$$\oplus\uparrow \Sigma F_y = 0 = 18 - 15 - V_{CD} \Rightarrow \quad V_{CD} = 3$$

$$\oplus\circlearrowleft \Sigma M_D = 0 = M_{CD} + 15(x_2 - 3) - 18x_2 \Rightarrow \quad M_{CD} = 3x_2 + 45$$

check by taking derivative...

$$\frac{dM_{CD}}{dx_2} = 3 = V_{CD}$$

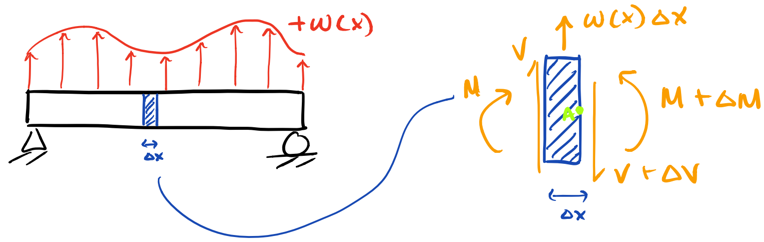


$$\oplus\uparrow \Sigma F_y = 0 = 18 - 15 - 15 - V_{DB} \Rightarrow \quad V_{DB} = -12$$

$$\oplus\circlearrowleft \Sigma M_B = 0 = M_{DB} - 18x_3 + 15(x_3 - 3) + 15(x_3 - 9) \Rightarrow \quad M_{DB} = -12x_3 + 180$$

check... $\frac{dM_{DB}}{dx_3} = -12 = V_{DB}$

Hibbeler 6.2: Graphical Shear/Moment Diagrams



$\oplus\uparrow \sum F_y = 0 = V - w(x)\Delta x - (V+\Delta V)$

$\Delta V = w(x)\Delta x$

$\lim_{\Delta x \to 0} \frac{\Delta V}{\Delta x} = w(x)$

so... $\frac{dV}{dx} = w(x)$

$\oplus\circlearrowleft \sum M_A = 0 = -V\Delta x - M + M + \Delta M - w(x)\Delta x\left(\frac{1}{2}\right)\Delta x$

$\Delta M = V\Delta x + w(x)\Delta x\left(\frac{1}{2}\right)\Delta x$

$\lim_{\Delta x \to 0} \frac{\Delta M}{\Delta x} = V + \frac{1}{2} w(x)\Delta x$

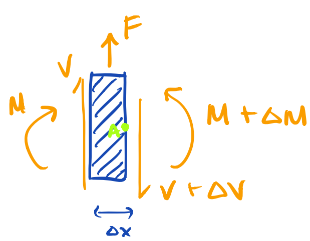
so . . . $\frac{dM}{dx} = V$

we can also rewrite eauations...

$\underbrace{\Delta V}_{\text{change in shear}} = \underbrace{\int w(x)\,dx}_{\text{area under distributed loading}}$

$\underbrace{\Delta M}_{\text{change in moment}} = \underbrace{\int V(x)\,dx}_{\text{area under shear diagram}}$

Regions of concentrated force/moment

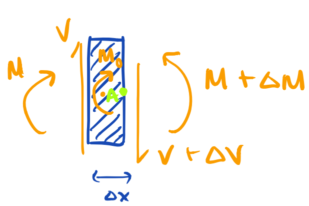


$\oplus\uparrow \sum F_y = 0 = F + V - (V+\Delta V)$

$\Delta V = F$

→ this means that if have ⊕ F applied, ΔV jumps upward

→ if have ⊖ F applied, ΔV jumps downward



$\oplus\circlearrowleft \sum M_A = 0 = M + \Delta M - M_0 - M - V\Delta x$

$\Delta M = M_0$

→ so concentrated moment gives jump in moment diagram