An Investigation into the Bending Behavior of Different Materials

Leah Gaeta
ME 305: Mechanics of Materials
ltgaeta@bu.edu

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Contents

1 Pre-lab

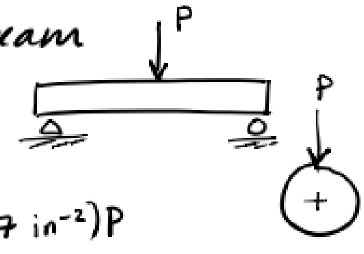
Lab 3: Bending Lab Pre-Lab

(1)

Material	Young's Modulus (E)	Yield Strength (σ)
2011-T3 Aluminum	10200000 psi	43000 psi
360 Brass	14100000 psi	52200 psi
304 stainless steel	28000000 psi	31200 psi

matweb.com

(2) $L = 18$ in $\quad d = 0.125$ in $\quad$ Simply-supported beam



$$P = k_{bending}\delta = \frac{48EI}{L^3}\delta \qquad I = \frac{\pi d^4}{64}$$

$$\sigma_{max} = \frac{Mc}{I} = \frac{P(L/2)(d/2)}{\pi d^4/64} = \frac{16PL}{\pi d^3} = (46936.7 \text{ in}^{-2})P$$

M = internal moment $\quad$ c = ⊥ distance from NA to where σ acts $\quad$ I = mass moment of inertia

(3)

$$Al: \frac{43000 \text{ lb/in}^2}{46936.7 \text{ 1/in}^2} = P_{Al} = 0.9161 \text{ lb}$$

$$Br: \frac{52200 \text{ lb/in}^2}{46936.7 \text{ 1/in}^2} = P_{Br} = 1.1121 \text{ lb}$$

$$St: \frac{31200 \text{ lb/in}^2}{46936.7 \text{ 1/in}^2} = P_{St} = 0.6647 \text{ lb}$$

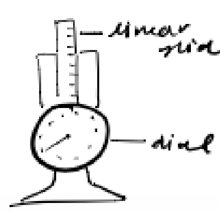
(4) mass in grams: $1g * 0.00220462 = $ lb

$m_{Al} = 7257.5$ g $\qquad m_{Br} = 504.5$ g $\qquad m_{st} = 301.5$ g

(5) max deflection $\delta = \frac{PL^3}{48EI}$ $\quad L = 18 \quad I = 1.1984(10^{-5})$

$\delta_{Al} = 0.9106$ in $\qquad \delta_{Br} = 0.7996$ in $\qquad \delta_{St} = 0.2407$ in

(6)



vernier depth gauge measures relative change in height (so does this based on previous measurement), not total change in height

2 Introduction

In many engineering applications involving the use of or construction with various materials, one often needs to know the strength of such materials when exposed to certain loads. In particular, an engineering may want to know the bending behavior of different forms of beams comprised of these materials. The aim of this experiment was to explore the bending response of three different materials when subjected to different loadings. Using a Megazord apparatus and Vernier depth gauge at Boston University to study bending behavior, the deflections of 2011-T3 Aluminum, 360 Brass, and 304 Stainless Steel were measured using different loadings and under two conditions: as cantilever and simply supported beams.

The ultimate goal of this investigation was to study beam bending by applying Euler-Bernoulli theory to these materials and analyzing any deviations from the theory (Euler-Bernoulli will be discussed further in the following section). The hope is that these long slender beams will validate the theory so that it can be applied to real-world engineering applications.

3 Theory

When studying the bending and deflection of slender beams that homogeneous and uniform, Euler-Bernoulli theory is based upon Eq. 1 below in which E is the Young's Modulus of the material [force/area], I is the area moment of inertia of the cross-section [distance4], $v(x)$ is the upward deflection of the neutral axis [distance], and $M(x)$ is the internal bending moment of the cross-section at location x [force·distance]. For a circular cross-section, such as for these beams, I can be found by Eq.2 in which d is the diameter. In addition, of great importance is the assumption that these materials behave in a linear, elastic manner in order for Eq. 1 and subsequent equations to be valid. Essentially, Euler-Bernoulli theory describes how a beam's deflection is related to the load applied.(Mechanics of Materials, 2016)

$$EI\frac{d^2v}{dx^2} = M(x) \tag{1}$$

$$I = \frac{\pi d^4}{64} \tag{2}$$

Application of Euler-Bernoulli to cantilever bending is very different from when a simply-supported beam is loaded upon. Considering that cantilever beam is fixed upon one end and loaded at the other it will therefore have different boundary conditions. A general schematic of a typical cantilever beam loaded at its end can be seen in Figure 1. The bending moment distribution is described by Eq. 3 below in which P is the load [force], x is the location along the beam [distance], and L is the length of the beam [distance].

Integrating Eq. 1 twice and replacing the $M(x)$ term with Eq. 3, reveals Eq. 4 below which describes the deflection. Note that at location $x = 0$, v and the change in v with respect to x are both zero; these are therefore the boundary conditions of the cantilever beam. As mentioned, $v(x)$ is the positive, upward deflection and therefore the downward (negative) deflection can be described by $\delta = -v(x)$. In summary, the downward deflection at the end of a cantilever beam can be simplified further (from Eq. 4) to Eq. 5 below.

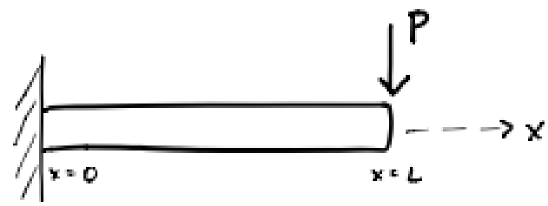


Figure 1: A general schematic of cantilever beam loading.

$$M(x) = P(x - L) \tag{3}$$

$$v(x) = \frac{P}{EI}\left[\frac{1}{6}(x - L)^3 - \frac{1}{2}L^2 x + \frac{1}{6}L^3\right] \tag{4}$$

$$\delta = \frac{PL^3}{3EI} \tag{5}$$

In the case of a simply-supported beam, Euler-Bernoulli is applied slightly differently. The study of deflection of these types of beams is referred to as "three-point bending," in which the beam is supported at its two ends and then acted upon by some load P at some location x along its length. In this case, the bending moment distribution is described by Eq. 6 in conjunction with Figure 2, which visually describes three-pointing. Applying Eq. 6 to Eq. 1 and integrating twice as with the cantilever, the equation for the upward deflection v at some location x can be described by Eq. 7 below. Noting that there cannot be any

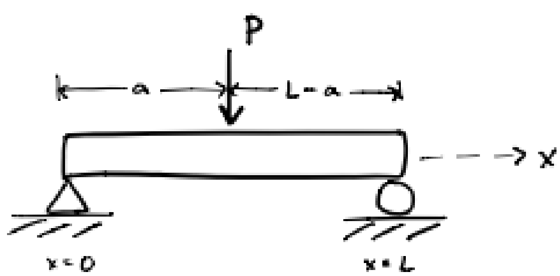


Figure 2: A general schematic of the three-point bending configuration.

deflection at the beam's ends provides the boundary conditions in which $v(0) = 0$ and $v(L) = 0$. As with the cantilever, v describes the upward, positive deflection so the downward deflection is therefore negative. In this investigation, the loading was conducted at the beam length's center, or a/2. Therefore, given that the downward deflection is $\delta = -v(L/2)$, Eq. 8 describes the three-point bending deflection in the same terms as Eq. 5.

$$M(x) = (L-a)\frac{Px}{L} - \frac{1}{2}P[|x-a| + (x-a)] \tag{6}$$

$$v(x) = \frac{P}{EI}\left[\frac{(L-a)}{6L}(x^2 - 2aL + a^2)x - \frac{1}{12}[|x-a|^3 + (x-a)^3]\right] \tag{7}$$

$$\delta = \frac{PL^3}{48EI} \tag{8}$$

Finally, Eq. 5 and Eq. 8 can be generally described below by Eq. 9 in which P is the force load, $k_{bending}$ is the spring constant, and δ is the deflection. Since the material is assumed to behave linear, elastically, we can treat $k_{bending}$ as a "spring constant" that is dependent on material properties, beam geometry, and boundary conditions. Specifically, $k_{bending}$ is defined below for both the cantilever and three-point bending conditions in Eq. 10 and Eq. 11 respectively.

$$P = k_{bending}\delta \tag{9}$$

$$k_{bending} = \frac{3EI}{L^3} \tag{10}$$

$$k_{bending} = \frac{48EI}{L^3} \tag{11}$$

Again, E is the material's Young's Modulus, I is the area moment of inertia of the circular cross-section, L is the beam length, and the constants 3 and 48 are derived from the specific boundary conditions previously defined.

4 Measurement Methods

The experimental setup consisted of the Megazord apparatus, a micrometer, and vernier depth gauge, to test 30-inch long cylindrical shafts with a 0.125-inch diameter of materials 2011-T3 Aluminum, 360 Brass, and 304 Stainless Steel. The dimensions were all measured to be used for calculations and analysis later. The Megazord was then delicately balanced before the samples were inserted. The cantilever setting was used first and the length between the central collar and hole were measured (this was the length L of the cantilever used for calculations. After zeroing the gauge dial, a 20g mass was added to the free tip using a paperclip and the deflection was measured using the gauge. This was repeated by adding 20g masses at a time until a total of 80g of loading at the free end was reached. This was completed for all materials.

Without changing out the material in the Megazord, the three-point arrangement was configured. This meant that the U-bolt bar was set to 12 inches from the wall support and the gauge mount bar 6 inches from the wall support. The gauge was then zeroed and a 50g mass was added to the load wire and the new gauge position was recorded. The mass was increased by 50g and new readings taken until reaching 200g total. This procedure was repeated for the U-bolt bar at 15 inches and mount bar at 7.5 inches, and for the U-bolt bar at 18 inches and mount bar at 9 inches. However, with the mount bar at 9 inches the deflection was measured for the 100g and 200g loadings by adjusting the gauge slide and measuring along the shaft sample in 1-inch increments until the midpoint was reached. As with the cantilever testing, this procedure was completed for all three materials.

5 Results & Analysis

Prior to conducting the actual experimentation, the dimensions of each beam were measured and are tabulated in Table 1 below.("MATWEB2020", n.d.) In addition, while the loading was performed by adding various masses in units of grams, these were converted to into pound-force (lb) and can be seen in Table 2. The Data Collection Worksheet that was used to note all observations can be found in the Appendix.

Table 1: Measurements & Material Properties of Samples.

Material	**Diameter (in.)**	**Mass Moment of Inertia, I (in^4)**	**Length (in.)**	**Young's Modulus (psi)**
2011-T3 Aluminum	0.125	$1.1841 \cdot 10^{-5}$	30	10200000
360 Copper Brass	0.125	$1.1841 \cdot 10^{-5}$	30	14100000
304 Stainless Steel	0.125	$1.1841 \cdot 10^{-5}$	30	28000000

Table 2: Loadings in Grams & Pounds.

20g	**40g**	**50g**	**60g**	**80g**	**100g**	**150g**	**200g**
0.0440924 lb	0.0881848 lb	0.1102310 lb	0.1322772 lb	0.1763696 lb	0.2204620 lb	0.3306930 lb	0.440924 lb

After converting the loading into pounds and calculating the deflections for each loading scenario, the loading vs. deflection was plotted for each material and can be seen in Figure 3 below. While deflection was the variable that was measured in response to loading, the deflection is plotted on the x-axis so that a line can be determined based on the relationship described by Eq. 9, in which the loading P can be determined from the spring constant $k_{bending}$ and deflection δ. In order to better understand Figure 3 and its application to Euler-Bernoulli, linear fit lines in the form of Eq. 9 to describe the raw data were found and can be seen in Figure 4; these lines describe the relationship between each material's deflection and the loadings.

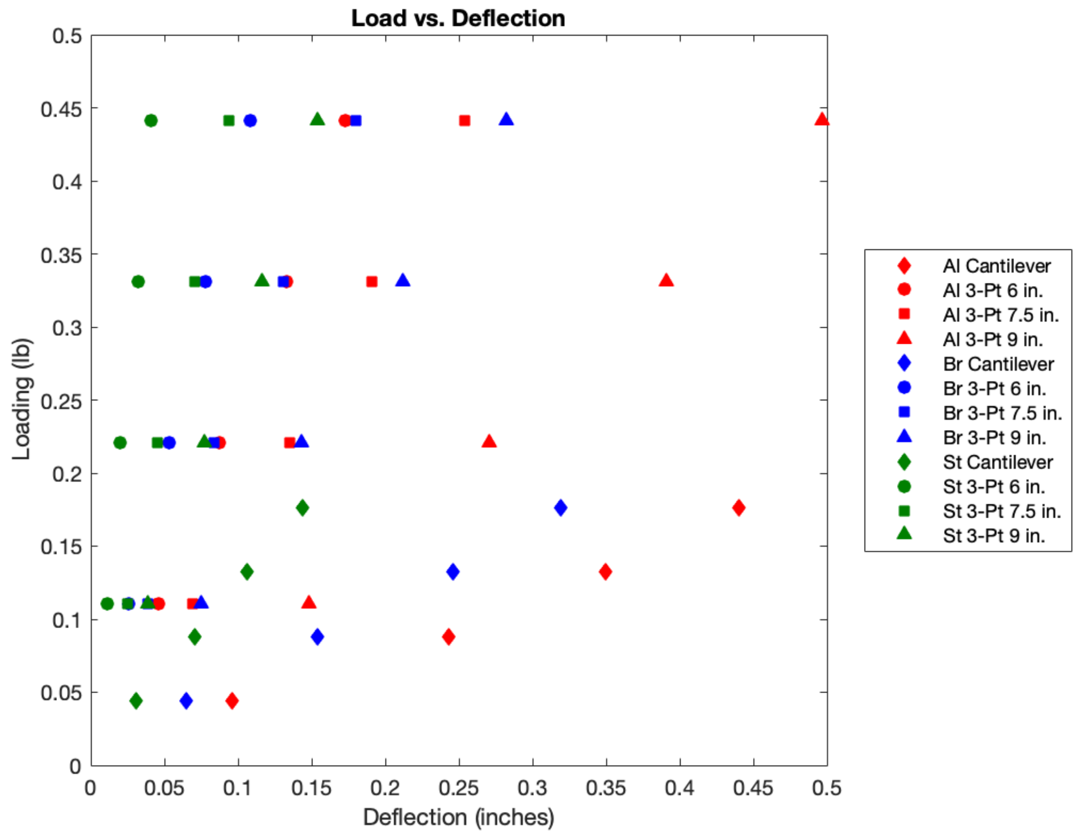


Figure 3: Raw data for each material of all loading experiments.

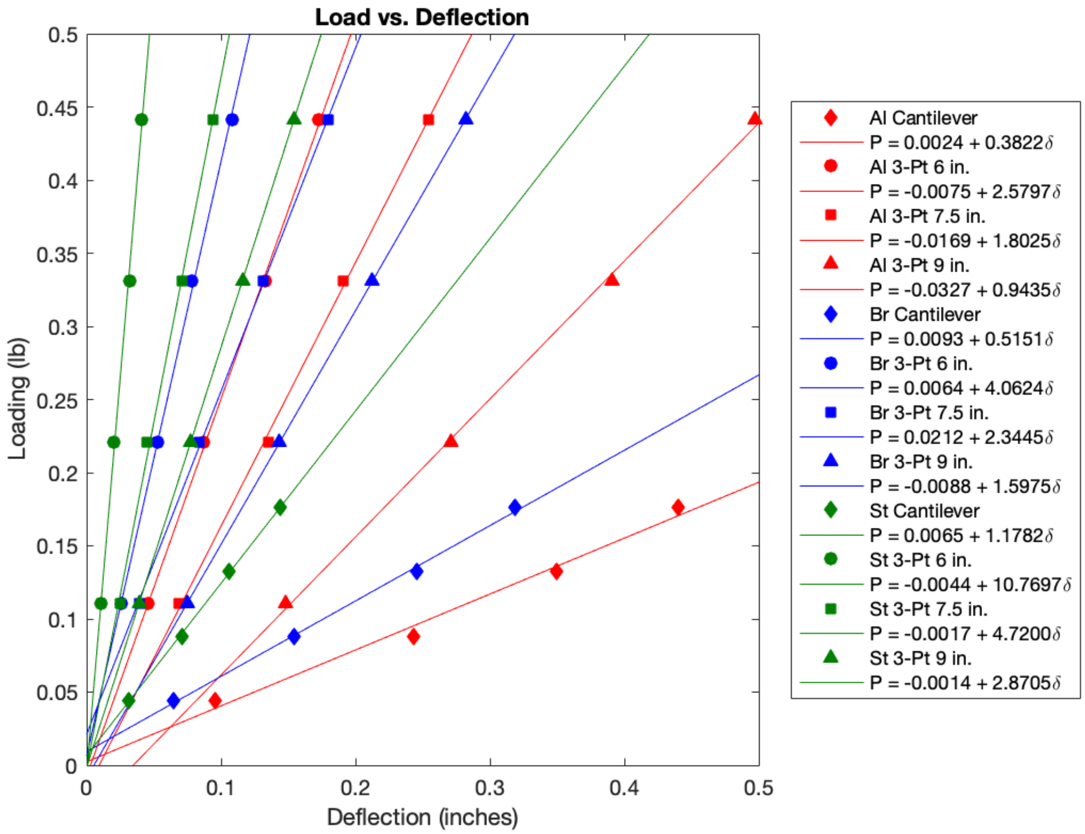


Figure 4: Trends for each material of all loading experiments.

The lines of Figure 4 reflect the loading as a linear function of the deflection, in which slope is the spring constant $k_{bending}$. The fit lines have the general form of Eq. 12, indicating that when no load is applied there should be no deflection and P_0 should also be zero. Offset errors will cause P_0 to not be zero so these deflections were also calculated based on the linear fits, using $\delta_{offset} = -P_0/k_{bending}$. In addition, using $k_{bending}$, the reduced bending stiffness $\tilde{k}_{bending}$ were also calculated for each material and loading situation and compared to the theoretical values. These values are described by Eq. 13 and Eq. 14, and their rearrangement in Eq. 15. All of these values are tabulated in Table 3 below.

$$P = P_0 + k_{bending}\delta \tag{12}$$

$$\tilde{k}_{bending} = \frac{1}{3}k_{bending} = \frac{EI}{L^3} \tag{13}$$

$$\tilde{k}_{bending} = \frac{1}{48}k_{bending} = \frac{EI}{L^3} \tag{14}$$

$$\sqrt[3]{\frac{1}{EI}} = \frac{1}{L}\sqrt[3]{\frac{1}{\tilde{k}_{bending}}} \tag{15}$$

Table 3: Calculated Values for Each Material.

Material of Shaft	Loading Condition	P_0 $(lbs.)$	δ_{offset} $(in.)$	$\tilde{k}_{bending}$ (lb/in)	$\frac{1}{L}\sqrt[3]{\frac{1}{\tilde{k}_{bending}}}$ $(lb \cdot in)^{-2/3}$	$\sqrt[3]{\frac{1}{EI}}$ $(theor.)$
Aluminum	Cantilever	0.0024	-0.0062	0.1274	0.2208 ± 9.6%	0.2015
	Three-Pt (6 in)	-0.0075	0.0029	0.0537	0.2208 ± 9.6%	
	Three-Pt (7.5 in)	-0.0169	0.0094	0.0376	0.1991 ± 1.2%	
	Three-Pt (9 in)	-0.0327	0.0347	0.0197	0.2059 ± 2.2%	
Brass	Cantilever	0.0093	-0.0180	0.1717	0.1999 ± 10.5%	0.1809
	Three-Pt (6 in)	0.0064	-0.0016	0.0846	0.1898 ± 4.9%	
	Three-Pt (7.5 in)	0.0212	-0.0090	0.0488	0.1824 ± 0.8%	
	Three-Pt (9 in)	-0.0088	0.0055	0.0333	0.1727 ± 4.5%	
Steel	Cantilever	0.0065	-0.0056	0.3927	0.1517 ± 5.4%	0.1439
	Three-Pt (6 in)	-0.0044	0.0004	0.2244	0.1371 ± 4.7%	
	Three-Pt (7.5 in)	-0.0017	0.0004	0.0983	0.1444 ± 0.4%	
	Three-Pt (9 in)	-0.0014	0.0005	0.0598	0.1421 ± 1.3%	

Finally, the deflection curves for each material with theoretical curves are pictured in Figures 5 through 7 below for each material. The shapes and differences between the data and theoretical curve will be discussed further in the following Discussion section. In addition, the maximum deflections for each beam and their respective fitting errors were calculated and are also tabulated in Table 4.

Table 4: Calculated Deflections of Three-Point Bending for Each Material.

Material	δ_{max} (in.)	$\Delta\delta(in.)$
Aluminum	0.2131	0.0056
Brass	0.1179	0.0031
Steel	0.0644	0.0017

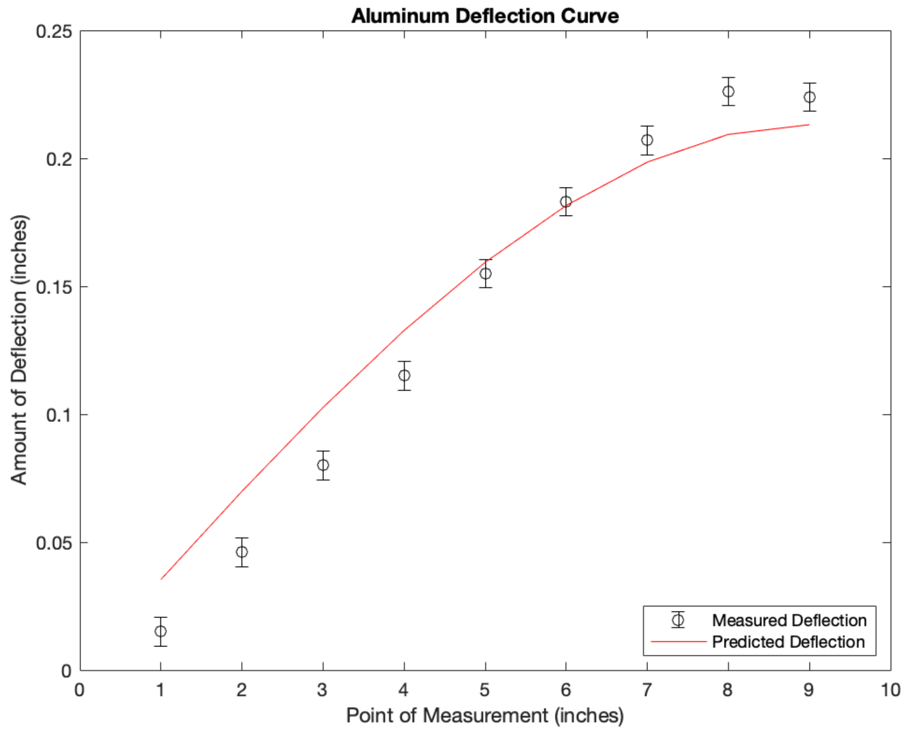


Figure 5: Deflection curve for Aluminum.

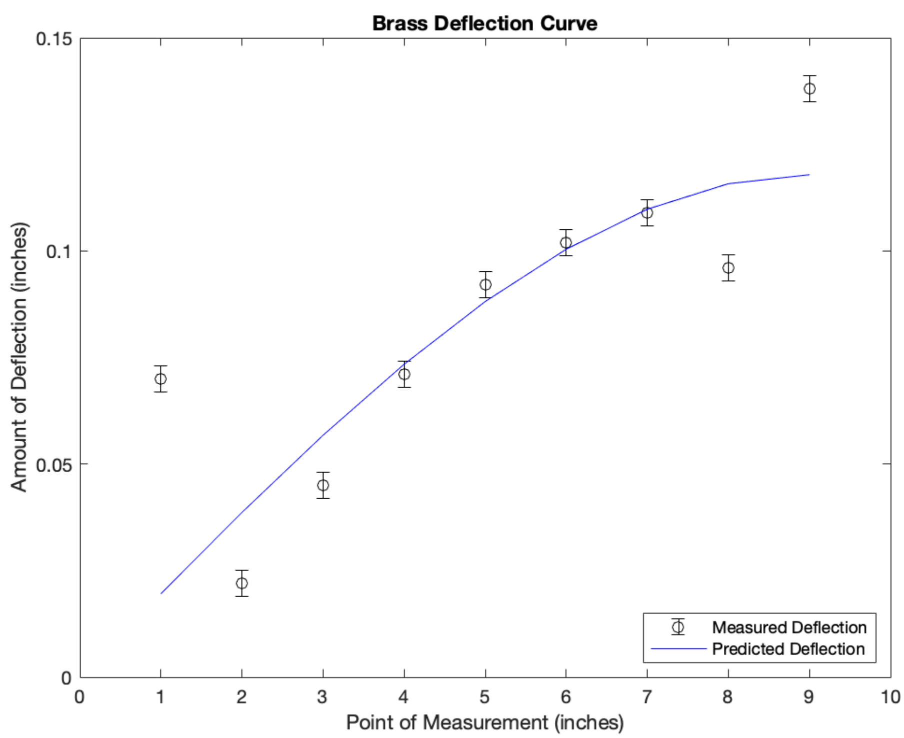


Figure 6: Deflection curve for Brass.

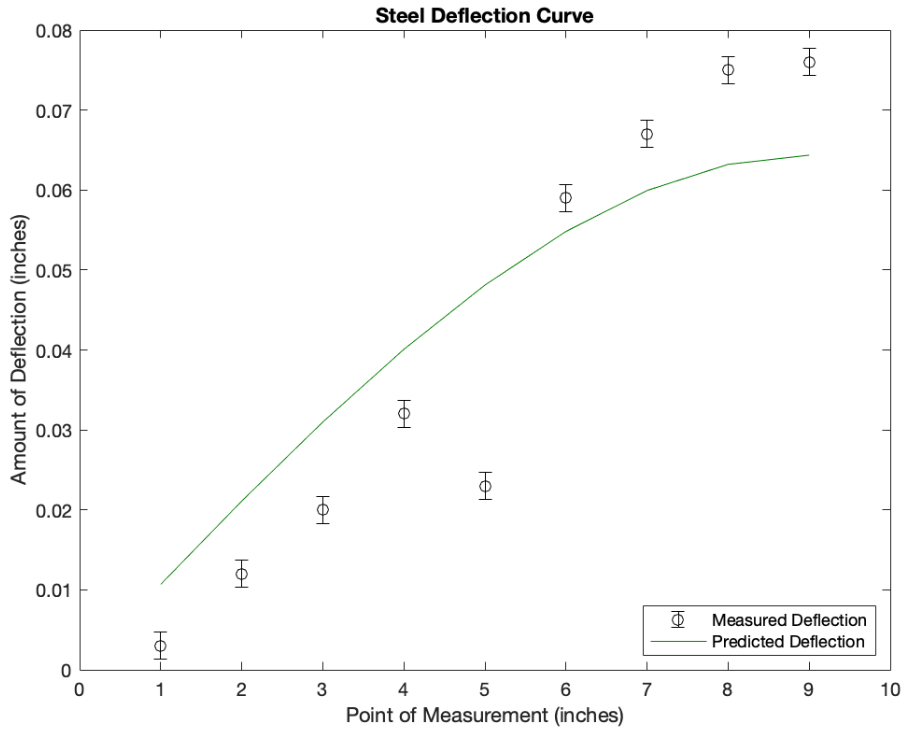


Figure 7: Deflection curve for Steel.

6 Discussion

As displayed by the loading vs. deflection plots in Figures 3 and 4, there is an elastic, linear relationship between the applied loading of a beam and it's deflection, in which the slopes represent the bending spring constant $k_{bending}$. The raw data itself is very close to the respective fit lines for each material and each testing situation, suggesting that loading is in fact a linear function of a beam's deflection and thereby satisfying Euler-Bernoulli. However, it's interesting to note the different intercepts, or P_0 values for the different materials. aluminum had the highest P_0 values, followed by brass, and then steel, indicating that steel satisfies Euler-Bernoulli more closely than the others, followed by brass, and then aluminum. Further, this indicates why steel may be a preferred material to use in real-life applications as it's predicted deflection can be calculated based upon the theory. Also worth noting are the smaller δ_{offset} seen from steel and brass versus aluminum, with steel having the smaller values and therefore having very little offset error. Finally, from the plots it seems that the greatest offsets come from the cantilever loading and that in general, aluminum deflects the most.

In addition, of interest is the $k_{bending}$ dependence on the beam material properties, which can be seen in Table 3. From the theory, these bending spring constants should not depend on the boundary conditions so the theoretical values for each material were calculated for comparison. Though for each loading situation for all three materials the values found were close to the theoretical calculations, once again steel had experimental values closest to its theoretical 0.1439 with very little error (the error is presented as a $\pm$ percentage in the table). Also of interest, is how the percent error was always greatest for the cantilever situations for each material versus the three-point bending conditions. This could be due to the nature of the experiment, the way the samples were handled, among many other possibilities.

Another item of interest is the validation of Eq. 7 in the three-point bending experimentation. Though from the plots it looks as though aluminum had less variation between the measurements and predicted deflection due to the scaling of the axes, in reality both brass and steel actually varied the least. Again, the deflection measurements for steel were the closest to the predicted curve further supporting why many engineers may choose to work with steel as more accurate predictions can be made.

The discrepancies observed between the results of calculations from the raw data and the theoretical values for each material can be attributed to various sources of error. One possibility may come from the handling of the test equipment and calibration. It's possible that the apparatus was not properly balanced and that the samples weren't handled as gently as needed. In addition, there could have also been reporting errors of the measurements made which have subsequently impacted the analysis and the conclusions that could be drawn. It's also possible that the materials are not necessarily pure and uniform and as mentioned in the Theory section of this report, the analysis is based on the assumption that materials behave in elastic fashion, which could ultimately impact the true results.

Finally, given all of these potential sources of error, the experiment can be improved in a variety of ways for the future. Of most importance for future improvements, would be increasing the number of trials conducted for each material and loading situation. Increasing the different the number of intervals of measurement taking and loading could also be done. This would allow for a more coherent analysis further validate Euler-Bernoulli. In addition, it would also behoove future resources to not just implement these suggestions, but also discover a way to handle the materials and equipment in such a way that there are negligible or no effects on the analysis.

7 Conclusion

In conclusion, the goal of this laboratory exercise was to discover a relationship between applied force and deflection of cantilevered and simply-supported beams. This was done by conducting various loading experiments to the three different materials, measuring the deflections, and analyzing the results in light of Euler-Bernoulli beam theory. In both loading situations, steel varied the least from its predicted lines/curves, followed by brass, and then aluminum. In addition, the bending spring constant, $k_{bending}$ found from experimentation for steel was closer to the theoretical values with little percent error compared to the other materials. In turn, this gives great perspective on why engineers may choose to work with some materials over others, especially when considering the safety factor during construction, among other goals.

References

(n.d.). *http://www.matweb.com*.

Mechanics of Materials, M. (2016). Lab 3: Bending Lab. *Boston University, ME 305*.

8 Appendix

Bending Lab Data Collection Worksheet

Section 1: Cantilever Beam Bending From Fixed Central Collar to Hole	ALUMINUM	BRASS	STEEL
1. Level the MegaZoid: Check that apparatus is level, if not adjust feet			
2. Measure the distance between the Central Collar and the hole	L = 9 in	L = 9 in	L = 9 in
3. With paperclip in hole move caliper to touch without exerting force on beam			
4. Zero the gauge dial on the caliper			
5. Apply 20g mass to the paperclip	Mass = 20g	Mass = 20g	Mass = 20g
6. Move caliper to touch paperclip without exerting force on beam			
7. Measure deflection	D_{20} = 0.096 in	D_{20} = 0.065 in	D_{20} = 0.031 in
8. Zero the gauge dial on the caliper			
9. Apply 20g mass to the paperclip for 40g total mass	Mass = 40g	Mass = 40g	Mass = 40g
10. Move caliper to touch paperclip without exerting force on beam			
11. Measure additonal deflection	D_{40} = 0.147 in	D_{40} = 0.089 in	D_{40} = 0.040 in
12. Zero the gauge dial on the caliper			
13. Apply 20g mass to the paperclip for 60g total mass	Mass = 60g	Mass = 60g	Mass = 60g
14. Move caliper to touch paperclip without exerting force on beam			
15. Measure additonal deflection	D_{60} = 0.107 in	D_{60} = 0.092 in	D_{60} = 0.035 in
16. Zero the gauge dial on the caliper			
17. Apply 20g mass to the paperclip for 80g total mass	Mass = 80g	Mass = 80g	Mass = 80g
18. Move caliper to touch paperclip without exerting force on beam			
19. Measure additonal deflection	D_{80} = 0.090 in	D_{80} = 0.073 in	D_{80} = 0.038 in

Note: The total deflection is the sum of all previous deflections

Section 2.1: Three Point Bending 6"	ALUMINUM	BRASS	STEEL
1. Move U-Bolt Slider to 12" Position on Red Ruler			
2. Move the Gauge Mount Slider to 6" Position on Red Ruler			
3. Center sample between U-Cutout Sidewall and U-Bolt Slider support			
4. Hang load wire hook beneath Gauge Mount at 6" Position on Red Ruler			
5. Move caliper to touch load wire hook without exerting force on beam			
6. Zero the gauge dial on the caliper			
7. Apply 50g mass to the load wire hook	Mass = 50g	Mass = 50g	Mass = 50g
8. Move caliper to touch load wire hook without exerting force on beam*			
9. Measure deflection	D_{50} = 0.046 in	D_{50} = 0.026 in	D_{50} = 0.011 in
10. Zero the gauge dial on the caliper			
11. Apply 50g mass to the load wire hook for 100g total mass	Mass = 100g	Mass = 100g	Mass = 100g
12. Move caliper to touch load wire hook without exerting force on beam*			
13. Measure additonal deflection	D_{100} = 0.041 in	D_{100} = 0.027 in	D_{100} = 0.090 in
14. Zero the gauge dial on the caliper			
15. Apply 50g mass to the load wire hook for 150g total mass	Mass = 150g	Mass = 150g	Mass = 150g
16. Move caliper to touch load wire hook without exerting force on beam*			
17. Measure additonal deflection	D_{150} = 0.046 in	D_{150} = 0.025 in	D_{150} = 0.012 in
18. Zero the gauge dial on the caliper			
19. Apply 50g mass to the load wire hook for 200g total mass	Mass = 200g	Mass = 200g	Mass = 200g
20. Move caliper to touch load wire hook without exerting force on beam*			
21. Measure additonal deflection	D_{200} = 0.040 in	D_{200} = 0.030 in	D_{200} = 0.090 in

*Watch for number of full rotations of gauge dial while moving down to measure displacement

Note: The total deflection is the sum of all previous deflections

Section 2.2: Three Point Bending 7.5"	ALUMINUM	BRASS	STEEL
1. Move U-Bolt Slider to 15" Position on Red Ruler			
2. Move the Gauge Mount Slider to 7.5" Position on Red Ruler			
3. Center sample between U-Cutout Sidewall and U-Bolt Slider support			
4. Hang load wire hook beneath Gauge Mount at 7.5" Position on Red Ruler			
5. Move caliper to touch load wire hook without exerting force on beam			
6. Zero the gauge dial on the caliper			
7. Apply 50g mass to the load wire hook	Mass = 50g	Mass = 50g	Mass = 50g
8. Move caliper to touch load wire hook without exerting force on beam*			
9. Measure deflection	D_{50} = 0.069 in	D_{50} = 0.039 in	D_{50} = 0.025 in
10. Zero the gauge dial on the caliper			
11. Apply 50g mass to the load wire hook for 100g total mass	Mass = 100g	Mass = 100g	Mass = 100g
12. Move caliper to touch load wire hook without exerting force on beam*			
13. Measure additonal deflection	D_{100} = 0.066 in	D_{100} = 0.045 in	D_{100} = 0.020 in
14. Zero the gauge dial on the caliper			
15. Apply 50g mass to the load wire hook for 150g total mass	Mass = 150g	Mass = 150g	Mass = 150g
16. Move caliper to touch load wire hook without exerting force on beam*			
17. Measure additonal deflection	D_{150} = 0.056 in	D_{150} = 0.047 in	D_{150} = 0.026 in
18. Zero the gauge dial on the caliper			
19. Apply 50g mass to the load wire hook for 200g total mass	Mass = 200g	Mass = 200g	Mass = 200g
20. Move caliper to touch load wire hook without exerting force on beam*			
21. Measure additonal deflection	D_{200} = 0.063 in	D_{200} = 0.049 in	D_{200} = 0.023 in

*Watch for number of full rotations of gauge dial while moving down to measure displacement

Note: The total deflection is the sum of all previous deflections

Section 2.3A: Three Point Bending 9"	ALUMINUM	BRASS	STEEL
1. Move U-Bolt Slider to 18" Position on Red Ruler			
2. Move the Gauge Mount Slider to 9" Position on Red Ruler			
3. Center sample between U-Cutout Sidewall and U-Bolt Slider support			
4. Hang load wire hook beneath Gauge Mount at 9" Position on Red Ruler			
5. Move caliper to touch load wire hook without exerting force on beam			
6. Zero the gauge dial on the caliper			
7. Apply 50g mass to the load wire hook	Mass = 50g	Mass = 50g	Mass = 50g
8. Move caliper to touch load wire hook without exerting force on beam*			
9. Measure deflection	D_{50} = 0.148 in	D_{50} = 0.075 in	D_{50} = 0.039 in
10. Zero the gauge dial on the caliper			
11. Apply 50g mass to the load wire hook for 100g total mass	Mass = 100g	Mass = 100g	Mass = 100g
12. Move caliper to touch load wire hook without exerting force on beam*			
13. Measure additonal deflection	D_{100} = 0.123 in	D_{100} = 0.068 in	D_{100} = 0.038 in
14. Zero the gauge dial on the caliper			
15. Apply 50g mass to the load wire hook for 150g total mass	Mass = 150g	Mass = 150g	Mass = 150g
16. Move caliper to touch load wire hook without exerting force on beam*			
17. Measure additonal deflection	D_{150} = 0.120 in	D_{150} = 0.069 in	D_{150} = 0.039 in
18. Zero the gauge dial on the caliper			
19. Apply 50g mass to the load wire hook for 200g total mass	Mass = 200g	Mass = 200g	Mass = 200g
20. Move caliper to touch load wire hook without exerting force on beam*			
21. Measure additonal deflection	D_{200} = 0.106 in	D_{200} = 0.070 in	D_{200} = 0.038 in

*Watch for number of full rotations of gauge dial while moving down to measure displacement

Note: The total deflection is the sum of all previous deflections

Section 2.3B: Three Point Bending 9" - Deflection Along Length at 100g	ALUMINUM	BRASS	STEEL
1. Unload the mass, zero the gauge dial on the caliper on surface of beam			
2. Apply 100g mass to the load wire hook	Mass = 100g	Mass = 100g	Mass = 100g
3. Measure deflection at 1" on Red Ruler	D_1 0.015 in	D_1 0.010 in	D_1 0.007 in
4. Measure deflection at 2" on Red Ruler	D_2 0.035 in	D_2 0.020 in	D_2 0.017 in
5. Measure deflection at 3" on Red Ruler	D_3 0.081 in	D_3 0.040 in	D_3 0.025 in
6. Measure deflection at 4" on Red Ruler	D_4 0.125 in	D_4 0.064 in	D_4 0.037 in
7. Measure deflection at 5" on Red Ruler	D_5 0.173 in	D_5 0.083 in	D_5 0.055 in
8. Measure deflection at 6" on Red Ruler	D_6 0.221 in	D_6 0.108 in	D_6 0.063 in
9. Measure deflection at 7" on Red Ruler	D_7 0.251 in	D_7 0.123 in	D_7 0.073 in
10. Measure deflection at 8" on Red Ruler	D_8 0.266 in	D_8 0.137 in	D_8 0.079 in
11. Measure deflection at 9" on Red Ruler**	D_9 0.276 in	D_9 0.141 in	D_9 0.080 in

**Do not include thickness of load wire hook when measuring displacement

Section 2.3C: Three Point Bending 9" - Deflection Along Length at 200g	ALUMINUM	BRASS	STEEL
1. Unload the mass, zero the gauge dial on the caliper on surface of beam			
2. Apply 200g mass to the load wire hook	Mass = 200g	Mass = 200g	Mass = 200g
3. Measure deflection at 1" on Red Ruler	D_1 0.030 in	D_1 0.080 in	D_1 0.010 in
4. Measure deflection at 2" on Red Ruler	D_2 0.081 in	D_2 0.042 in	D_2 0.029 in
5. Measure deflection at 3" on Red Ruler	D_3 0.161 in	D_3 0.085 in	D_3 0.045 in
6. Measure deflection at 4" on Red Ruler	D_4 0.240 in	D_4 0.135 in	D_4 0.069 in
7. Measure deflection at 5" on Red Ruler	D_5 0.328 in	D_5 0.175 in	D_5 0.078 in
8. Measure deflection at 6" on Red Ruler	D_6 0.404 in	D_6 0.210 in	D_6 0.122 in
9. Measure deflection at 7" on Red Ruler	D_7 0.458 in	D_7 0.232 in	D_7 0.140 in
10. Measure deflection at 8" on Red Ruler	D_8 0.492 in	D_8 0.269 in	D_8 0.154 in
11. Measure deflection at 9" on Red Ruler**	D_9 0.500 in	D_9 0.279 in	D_9 0.156 in

**Do not include thickness of load wire hook when measuring displacement

Bending Analysis

Section 3.1 Validation Measures

```
% Import data
lab3data = xlsread("materialslab3.xlsx");
cant_load = lab3data(1:4,1); % grams
cant_al = lab3data(1:4,2);
cant_br = lab3data(1:4,3);
cant_st = lab3data(1:4,4);
tp_load = lab3data(1:4,5); % grams
tp_al_6 = lab3data(1:4,6);
tp_br_6 = lab3data(1:4,7);
tp_st_6 = lab3data(1:4,8);
tp_al_75 = lab3data(1:4,9);
tp_br_75 = lab3data(1:4,10);
tp_st_75 = lab3data(1:4,11);
tp_al_9 = lab3data(1:4,12);
tp_br_9 = lab3data(1:4,13);
tp_st_9 = lab3data(1:4,14);
def_len = lab3data(:,15);
def_al_100 = lab3data(:,16);
def_br_100 = lab3data(:,17);
def_st_100 = lab3data(:,18);
def_al_200 = lab3data(:,19);
def_br_200 = lab3data(:,20);
def_st_200 = lab3data(:,21);
% all lengths in inches

% Convert grams to lb-force
cant_load = cant_load * 0.00220462;
tp_load = tp_load * 0.00220462;

% Make deflection vectors as cummulative sums for Cantilever & Three Pt
% Bending
cant_al = cumsum(cant_al);
cant_br = cumsum(cant_br);
cant_st = cumsum(cant_st);
tp_al_6 = cumsum(tp_al_6);
tp_br_6 = cumsum(tp_br_6);
tp_st_6 = cumsum(tp_st_6);
tp_al_75 = cumsum(tp_al_75);
tp_br_75 = cumsum(tp_br_75);
tp_st_75 = cumsum(tp_st_75);
tp_al_9 = cumsum(tp_al_9);
tp_br_9 = cumsum(tp_br_9);
tp_st_9 = cumsum(tp_st_9);
al_tp_mat = [tp_al_6, tp_al_75, tp_al_9];
br_tp_mat = [tp_br_6, tp_br_75, tp_br_9];
st_tp_mat = [tp_st_6, tp_st_75, tp_st_9];
```

Section 3.2 Initial Display of Results

```
% Generate Plot 1 with all raw data
plot(cant_al, cant_load, 'd', 'MarkerFaceColor','r', 'MarkerEdgeColor', 'r') % Aluminum
xlabel('Deflection (inches)')
ylabel('Loading (lb)')
title('Load vs. Deflection')
axis([0 0.5 0 0.5])
hold on
plot(tp_al_6, tp_load, 'o', 'MarkerFaceColor','r', 'MarkerEdgeColor', 'r')
hold on
plot(tp_al_75, tp_load, 's', 'MarkerFaceColor','r', 'MarkerEdgeColor', 'r')
hold on
plot(tp_al_9, tp_load, '^', 'MarkerFaceColor','r', 'MarkerEdgeColor', 'r')
hold on
plot(cant_br, cant_load, 'd', 'MarkerFaceColor','b', 'MarkerEdgeColor', 'b') % Brass
hold on
plot(tp_br_6, tp_load, 'o', 'MarkerFaceColor','b', 'MarkerEdgeColor', 'b')
hold on
plot(tp_br_75, tp_load, 's', 'MarkerFaceColor','b', 'MarkerEdgeColor', 'b')
hold on
plot(tp_br_9, tp_load, '^', 'MarkerFaceColor','b', 'MarkerEdgeColor', 'b')
hold on
plot(cant_st, cant_load, 'd', 'MarkerFaceColor', [0 0.5 0], 'MarkerEdgeColor', [0 0.5 0]) % Steel
hold on
plot(tp_st_6, tp_load, 'o', 'MarkerFaceColor', [0 0.5 0], 'MarkerEdgeColor', [0 0.5 0])
hold on
plot(tp_st_75, tp_load, 's', 'MarkerFaceColor', [0 0.5 0], 'MarkerEdgeColor', [0 0.5 0])
hold on
plot(tp_st_9, tp_load, '^', 'MarkerFaceColor', [0 0.5 0], 'MarkerEdgeColor', [0 0.5 0])
legend('Al Cantilever', 'Al 3-Pt 6 in.', 'Al 3-Pt 7.5 in.', 'Al 3-Pt 9 in.', 'Br Cantilever', 'Br 3-Pt 6
in.', 'Br 3-Pt 7.5 in.', 'Br 3-Pt 9 in.', 'St Cantilever', 'St 3-Pt 6 in.', 'St 3-Pt 7.5 in.', 'St 3-Pt 9
in.', 'location', 'eastoutside')
hold off
```

Section 3.3 Testing Linearity of Response

```
% Aluminum - generate 2x4 of P0 and k values for each trial
bls_al = ones(2,4);

bls_al(:,1) = regress(cant_load, [ones(4, 1) cant_al]); % cantilever
for i = 1:3
    bls_al(:,i+1) = regress(tp_load, [ones(4,1) al_tp_mat(:,i)]); % three pt bending
end
bls_al = flipud(bls_al); % makes k values row 1, P0 values row 2

% Brass - generate 2x4 of P0 and k values for each trial
bls_br = ones(2,4);

bls_br(:,1) = regress(cant_load, [ones(4, 1) cant_br]); % cantilever
for i = 1:3
    bls_br(:,i+1) = regress(tp_load, [ones(4,1) br_tp_mat(:,i)]); % three pt bending
end
bls_br = flipud(bls_br); % makes k values row 1, P0 values row 2

% Steel - generate 2x4 of P0 and k values for each trial
bls_st = ones(2,4);

bls_st(:,1) = regress(cant_load, [ones(4, 1) cant_st]); % cantilever
for i = 1:3
    bls_st(:,i+1) = regress(tp_load, [ones(4,1) st_tp_mat(:,i)]); % three pt bending
end
bls_st = flipud(bls_st); % makes k values row 1, P0 values row 2

% Generate Figure 2 Plot with trendlines
xtrend = linspace(0,0.5,100); % x trendline
trendeq = 'F = %.4f + %.4f%s';
plot(cant_al, cant_load, 'd', 'MarkerFaceColor','r', 'MarkerEdgeColor', 'r') % Aluminum
xlabel('Deflection (inches)')
ylabel('Loading (lb)')
title('Load vs. Deflection')
axis([0 0.5 0 0.5])
hold on
plot(xtrend, polyval(bls_al(:,1),xtrend), 'r-')
hold on
plot(tp_al_6, tp_load, 'o', 'MarkerFaceColor','r', 'MarkerEdgeColor', 'r')
hold on
plot(xtrend, polyval(bls_al(:,2),xtrend), 'r-')
hold on
plot(tp_al_75, tp_load, 's', 'MarkerFaceColor','r', 'MarkerEdgeColor', 'r')
hold on
plot(xtrend, polyval(bls_al(:,3),xtrend), 'r-')
hold on
plot(tp_al_9, tp_load, '^', 'MarkerFaceColor','r', 'MarkerEdgeColor', 'r')
hold on
plot(xtrend, polyval(bls_al(:,4),xtrend), 'r-')
hold on
plot(cant_br, cant_load, 'd', 'MarkerFaceColor','b', 'MarkerEdgeColor', 'b') % Brass
hold on
plot(xtrend, polyval(bls_br(:,1),xtrend), 'b-')
hold on
plot(tp_br_6, tp_load, 'o', 'MarkerFaceColor','b', 'MarkerEdgeColor', 'b')
hold on
plot(xtrend, polyval(bls_br(:,2),xtrend), 'b-')
hold on
plot(tp_br_75, tp_load, 's', 'MarkerFaceColor','b', 'MarkerEdgeColor', 'b')
hold on
plot(xtrend, polyval(bls_br(:,3),xtrend), 'b-')
hold on
plot(tp_br_9, tp_load, '^', 'MarkerFaceColor','b', 'MarkerEdgeColor', 'b')
hold on
plot(xtrend, polyval(bls_br(:,4),xtrend), 'b-')
hold on
plot(cant_st, cant_load, 'd', 'MarkerFaceColor', [0 0.5 0], 'MarkerEdgeColor', [0 0.5 0]) % Steel
hold on
plot(xtrend, polyval(bls_st(:,1),xtrend), '-', 'color', [0 0.5 0])
hold on
plot(tp_st_6, tp_load, 'o', 'MarkerFaceColor', [0 0.5 0], 'MarkerEdgeColor', [0 0.5 0])
hold on
plot(xtrend, polyval(bls_st(:,2),xtrend), '-', 'color', [0 0.5 0])
hold on
plot(tp_st_75, tp_load, 's', 'MarkerFaceColor', [0 0.5 0], 'MarkerEdgeColor', [0 0.5 0])
hold on
plot(xtrend, polyval(bls_st(:,3),xtrend), '-', 'color', [0 0.5 0])
hold on
plot(tp_st_9, tp_load, '^', 'MarkerFaceColor', [0 0.5 0], 'MarkerEdgeColor', [0 0.5 0])
hold on
plot(xtrend, polyval(bls_st(:,4),xtrend), '-','color', [0 0.5 0])
legend('Al Cantilever', sprintf(trendeq, bls_al(2,1), bls_al(1,1), '\delta'), 'Al 3-Pt 6 in.',
sprintf(trendeq, bls_al(2,2), bls_al(1,2), '\delta'), ...
    'Al 3-Pt 7.5 in.', sprintf(trendeq, bls_al(2,3), bls_al(1,3), '\delta'), 'Al 3-Pt 9 in.',
sprintf(trendeq, bls_al(2,4), bls_al(1,4), '\delta'), ...
    'Br Cantilever', sprintf(trendeq, bls_br(2,1), bls_br(1,1), '\delta'), 'Br 3-Pt 6 in.', sprintf(trendeq,
bls_br(2,2), bls_br(1,2), '\delta'), ...
    'Br 3-Pt 7.5 in.', sprintf(trendeq, bls_br(2,3), bls_br(1,3), '\delta'), 'Br 3-Pt 9 in.',
sprintf(trendeq, bls_br(2,4), bls_br(1,4), '\delta'), ...
    'St Cantilever', sprintf(trendeq, bls_st(2,1), bls_st(1,1), '\delta'), 'St 3-Pt 6 in.', sprintf(trendeq,
bls_st(2,2), bls_st(1,2), '\delta'), ...
    'St 3-Pt 7.5 in.', sprintf(trendeq, bls_st(2,3), bls_st(1,3), '\delta'), 'St 3-Pt 9 in.',
sprintf(trendeq, bls_st(2,4), bls_st(1,4), '\delta'), ...
    'location', 'eastoutside')
hold off

% Compute the Offset values - order is Cantilever, 6 in, 7.5 in, 9 in 3Pt
al_offset = [-bls_al(2,1)/bls_al(1,1) -bls_al(2,2)/bls_al(1,2) -bls_al(2,3)/bls_al(1,3) -bls_al(2,4)/
bls_al(1,4)];
br_offset = [-bls_br(2,1)/bls_br(1,1) -bls_br(2,2)/bls_br(1,2) -bls_br(2,3)/bls_br(1,3) -bls_br(2,4)/
bls_br(1,4)];
st_offset = [-bls_st(2,1)/bls_st(1,1) -bls_st(2,2)/bls_st(1,2) -bls_st(2,3)/bls_st(1,3) -bls_st(2,4)/
bls_st(1,4)];
```

Section 3.4 Testing Dependence of Bending Stiffness on Length and Boundary Conditions

```
% Input E, L, and I parameters
d = 0.125; % diameter in inches
L_cant = 9; % length of cantilever beam in inches
L_3pt6 = 12; % length of 3pt trial measured at 6 inch mark
L_3pt75 = 15; % length of 3pt trial measured at 7.5 inch mark
L_3pt9 = 18; % length of 3pt trial measured at 9 inch mark
E_al = 10200000; % Young's Modulus in lbf/in^2
E_br = 14100000; % Young's Modulus in lbf/in^2
E_st = 28000000; % Young's Modulus in lbf/in^2
I = pi*(d^4)/64; % inches^4

% Compute k_bending(tilda)
al_kb = [bls_al(1,1)/3 bls_al(1,2)/48 bls_al(1,3)/48 bls_al(1,4)/48];
br_kb = [bls_br(1,1)/3 bls_br(1,2)/48 bls_br(1,3)/48 bls_br(1,4)/48];
st_kb = [bls_st(1,1)/3 bls_st(1,2)/48 bls_st(1,3)/48 bls_st(1,4)/48];

% Calculate theoretical Euler-Bernoulli values
al_eb = (1/(E_al*I))^(1/3);
br_eb = (1/(E_br*I))^(1/3);
st_eb = (1/(E_st*I))^(1/3);

% Calculate actual Euler-Bernoulli values using same order as offset
al_eb_actual = [((1/al_kb(1))^(1/3))*(1/L_cant) ((1/al_kb(2))^(1/3))*(1/L_3pt6) ((1/al_kb(3))^(1/3))*(1/
L_3pt75) ((1/al_kb(4))^(1/3))*(1/L_3pt9)];
br_eb_actual = [((1/br_kb(1))^(1/3))*(1/L_cant) ((1/br_kb(2))^(1/3))*(1/L_3pt6) ((1/br_kb(3))^(1/3))*(1/
L_3pt75) ((1/br_kb(4))^(1/3))*(1/L_3pt9)];
st_eb_actual = [((1/st_kb(1))^(1/3))*(1/L_cant) ((1/st_kb(2))^(1/3))*(1/L_3pt6) ((1/st_kb(3))^(1/3))*(1/
L_3pt75) ((1/st_kb(4))^(1/3))*(1/L_3pt9)];

% Calculate percent difference from theoretical using same order as offset
al_perdiff = ones(1,4);
for i = 1:length(al_perdiff)
    al_perdiff(1,i) = ((al_eb_actual(1,i) - al_eb)/al_eb)*100;
end

br_perdiff = ones(1,4);
for i = 1:length(br_perdiff)
    br_perdiff(1,i) = ((br_eb_actual(1,i) - br_eb)/br_eb)*100;
end

st_perdiff = ones(1,4);
for i = 1:length(st_perdiff)
    st_perdiff(1,i) = ((st_eb_actual(1,i) - st_eb)/st_eb)*100;
end

% Calculate average with percent difference underneath - order is Al, Br, St
Ave_eb = [mean(al_eb_actual) mean(br_eb_actual) mean(st_eb_actual); ((mean(al_eb_actual) - al_eb)/al_eb)*100
((mean(br_eb_actual) - br_eb)/br_eb)*100 ((mean(st_eb_actual) - st_eb)/st_eb)*100];
```

Section 3.5 Testing Predictions of the Deflection Curve for Three-Point Bending

```
% Calculate deflection due to load
al_delta_j = def_al_200 - def_al_100;
br_delta_j = def_br_200 - def_br_100;
st_delta_j = def_st_200 - def_st_100;

% Calculate maximum deflection of beam
L = L_3pt9;
x = [1:length(def_len)];

v = ones(1,length(x));
for i = 1:length(v)
    v(i) = (1/(L^3))*((3*L^2 - 4*x(i)^2)*x(i) +4*((abs(x(i)-L/2))^3 + (x(i) - L/2)^3));
end
v = v';
al_delta_max = sum(v'*al_delta_j)/sum(v'*v);
br_delta_max = sum(v'*br_delta_j)/sum(v'*v);
st_delta_max = sum(v'*st_delta_j)/sum(v'*v);

% Calculate delta_delta & error bars & plot
g = 5/32; % gauge width in inches

% Aluminum
al_theta_max = 3 * al_delta_max/L;
al_delta_delta = al_theta_max * g; % magnitude of error
al_error = ones(9,1)*al_delta_delta;

figure(3), errorbar(x, al_delta_j, al_error, al_error, 'ko') % plot data
hold('on')
figure(3), plot(x, al_delta_max*v, 'red') % add theoretical curve
title('Aluminum Deflection Curve')
xlabel('Point of Measurement (inches)')
ylabel('Amount of Deflection (inches)')
axis([0 10 0 0.25])
legend('Measured Deflection', 'Predicted Deflection', 'Location', 'southeast')
hold('off')

% Brass
br_theta_max = 3 * br_delta_max/L;
br_delta_delta = br_theta_max * g; % magnitude of error
br_error = ones(9,1)*br_delta_delta;

figure(3), errorbar(x, br_delta_j, br_error, br_error, 'ko') % plot data
hold('on')
figure(3), plot(x, br_delta_max*v, 'blue') % add theoretical curve
title('Brass Deflection Curve')
xlabel('Point of Measurement (inches)')
ylabel('Amount of Deflection (inches)')
axis([0 10 0 0.15])
legend('Measured Deflection', 'Predicted Deflection', 'Location', 'southeast')
hold('off')

% Steel
st_theta_max = 3 * st_delta_max/L;
st_delta_delta = st_theta_max * g; % magnitude of error
st_error = ones(9,1)*st_delta_delta;

figure(3), errorbar(x, st_delta_j, st_error, st_error, 'ko') % plot data
hold('on')
figure(3), plot(x, st_delta_max*v, 'color', [0 0.5 0]) % add theoretical curve
title('Steel Deflection Curve')
xlabel('Point of Measurement (inches)')
ylabel('Amount of Deflection (inches)')
axis([0 10 0 0.08])
legend('Measured Deflection', 'Predicted Deflection', 'Location', 'southeast')
hold('off')
```