# Materials- lecture #19

Hibbeler: Ch. 7: Transverse Shear
→ shear/moment diagrams from ch. 6
→ normal bending stresses from ch. 6
→ haven't found/discussed shear stresses yet

Final Project posted next week

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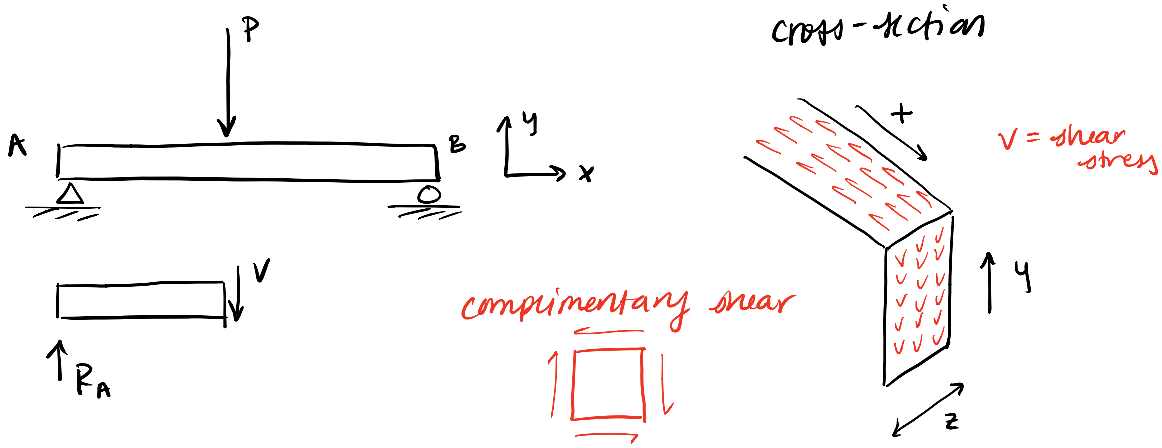
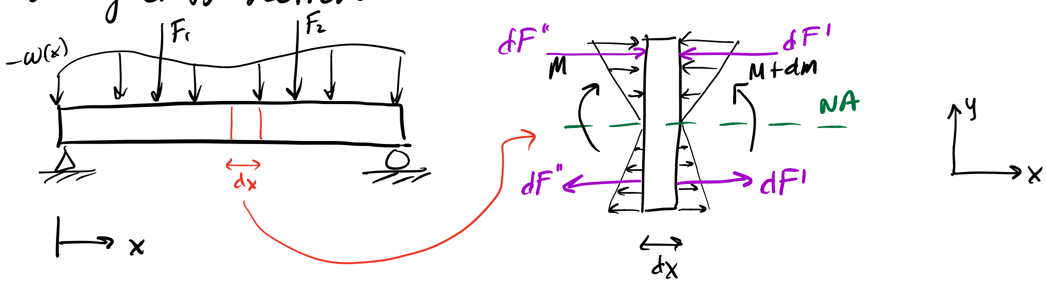
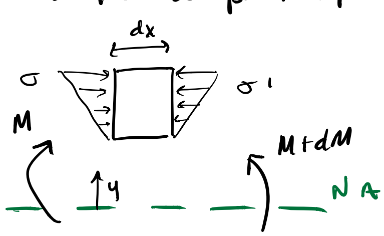
TRANSVERSE SHEAR




→ use complimentary shear to derive shear stress distribution along cross-section

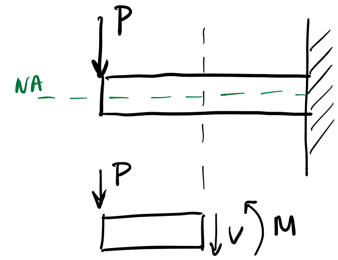


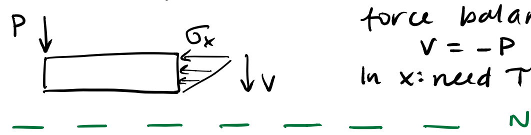

① forces above NA balance forces below (equilibrium)
Take a part of section above NA




- equilibrium is not satisfied <u>unless</u> have longitudinal shear stress $\tau$
- assume that $\tau$ is a constant, acts on an area $t\,dx$
  $t$ = thickness into page (z-dir.)
  $dx$ = length

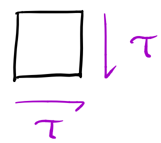
# Another way of thinking about longitudinal shear stress




force balance
$V = -P$
In x: need $\tau$ to balance $\sigma_x$

- then use complimentary shear stress to get vertical shear stress




$$\overset{\oplus}{\leftarrow}\ \sum F_x = 0 = \int_{A'} \sigma' dA' - \int_{A'} \sigma\, dA' - \tau(t\,dx)$$

$dA'$ = area above where $\tau$ is

flexure formula $\sigma = \frac{My}{I}$

$$\Rightarrow \int_{A'} \left(\frac{M+dM}{I}\right) y\, dA' - \int_{A'} \left(\frac{M}{I}\right) y\, dA' - \tau(t\,dx) = 0$$

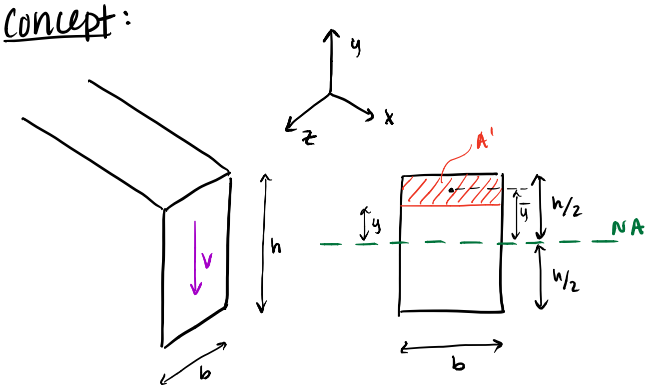
$\Rightarrow$ cancel out moments

$$\left(\frac{dM}{I}\right)\int_{A'} y\, dA' = \tau(t\,dx) \qquad \tau = \frac{1}{It}\overbrace{\left(\frac{dM}{dx}\right)}^{=V}\int_{A'} y\, dA'$$

$$\boxed{\tau = \frac{VQ}{It}}$$

$Q = \int_{A'} y\, dA'$ (first moment of the area about NA)

## Concept:




V = resultant internal shear force

I = cross-sectional moment of inertia

t = thickness of cross-section where want to calculate $\tau$

$Q = \int_{A'} y\, dA' = \bar{y}' A'$
→ $\bar{y}'$ = distance from NA to centroid of A'
→ A' = area above or below where calc. shear stress $\tau$

$Q = \bar{y}' A'$ $\qquad A' = b\left(\frac{h}{2} - y\right)$

$\bar{y}' = \bar{y} = \left(y + \frac{1}{2}\left(\frac{h}{2} - y\right)\right)$

$Q = \frac{b}{2}\left(\frac{h^2}{4} - y^2\right)$

→ apply shear formula $\qquad I = \frac{1}{12}bh^3 \quad t = b$

$\tau = \frac{VQ}{It}$

$$\tau = \frac{V(b/2)(h^2/4 - y^2)}{(1/12)bh^3\, b} = \boxed{\frac{6V}{bh^3}\left(\frac{h^2}{4} - y^2\right) = \tau}$$

this tells us...

→ shear stress distribution over cross section is parabolic ($y^2$)

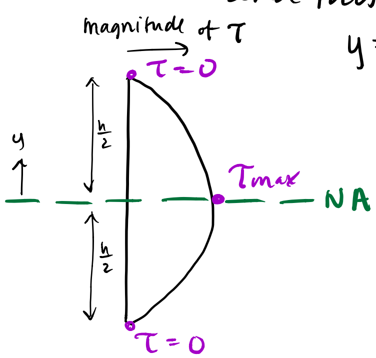
→ since $A = bh$, $\tau = \frac{6V}{Ah^2}\left(\frac{h^2}{4} - y^2\right)$

and if substitute $y = 0$,

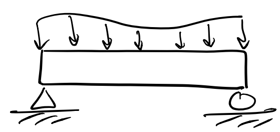
$$\boxed{y = 0 \rightarrow \tau = 1.5\frac{V}{A} = \tau_{max}}$$

and this is going to be maximum shear stress

$y = \pm\frac{h}{2} \rightarrow \tau = 0$



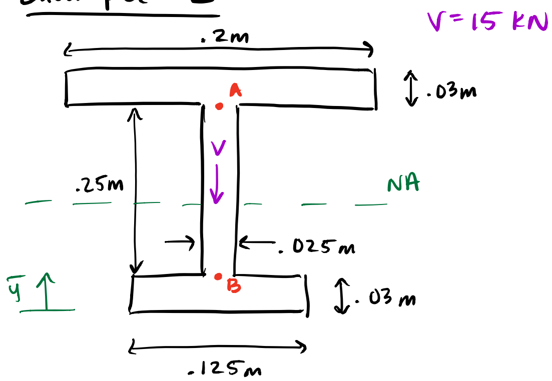

– why (physically) is $\tau = 0$ at the surfaces?



→ all applied forces are vertical!

– no horizontal (x-direction) loads on surface → complementary shear on surface = 0

Example #1:




① Find $\tau_A$
② Find $\tau_B$

Steps:
① find neutral axis
② find $I$
③ find $Q_A$, $Q_B$

①

$$\bar{y} = \frac{(.125)(.03)(.015) + (.025)(.25)(.155) + (.03)(.2)(.295)}{(.125)(.03) + (.025)(.25) + (.03)(.2)}$$

$\bar{y} = .175\text{m}$ up from bottom

②

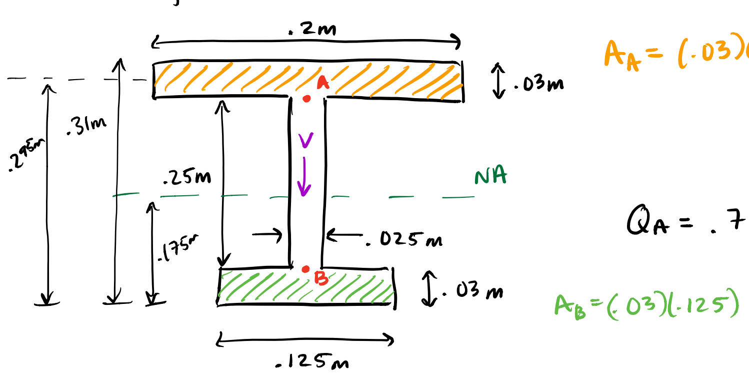
$$I = \frac{1}{12}(.125)(.03)^3 + (.125)(.03)(.175-.015)^2 + \frac{1}{12}(.025)(.25)^3 + (.025)(.25)(.175-.155)^2 + \frac{1}{12}(.2)(.03)^3 + (.2)(.03)(.295-.175)^2$$

$$I = .218(10^{-3})\ m^4$$

③

$Q_A = \bar{y}\, A_A$ $\quad$ $A_A$ = area above pt A = $(.2)(.03)$

$\bar{y}$ = dist from NA to centroid of A = $(.295 - .175)$




$Q_A = .72(10^{-3})m^3$

$Q_B = \bar{y}\, A_B$ $\quad$ $A_B = (.03)(.125)$ $\quad$ $\bar{y} = \left(.175 - \frac{.03}{2}\right)$

$Q_B = .6(10^{-3})\ m^3$

$$\tau_A = \frac{VQ_A}{It} = \frac{(15\times10^3\ N)(.72\times10^{-3}\ m^3)}{(.218\times10^{-3}m^4)(.025\ m)} = 1.99\ \text{MPa}$$

$$\tau_B = \frac{VQ_B}{It} = \frac{(15\times10^3\ N)(.6\times10^{-3}\ m^3)}{(.218\times10^{-3}m^4)(.025\ m)} = 1.65\ \text{MPa}$$

→ What is correct thickness at A & B?

→ we used web thickness = .025 m

→ why not $t_A = .2$ m?

→ why not $t_B = .125$ m?

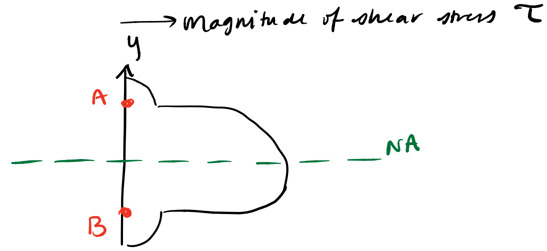
⟹ shear stress distribution is <u>discontinuous</u> at A and B because thickness changes discontinuously

$\tau_{A-} = 1.99$ MPa

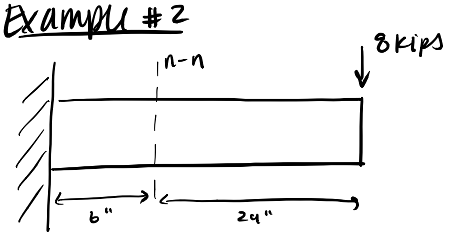
$$\tau_{A+} = \frac{(15\times10^{-3}\,N)(.72\times10^{-3}\,m^3)}{(.210\times10^{-3}\,m^4)(.2\,m)}$$

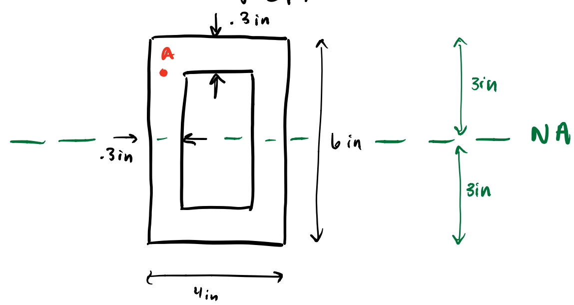
$\tau_{B+} = 1.65$ MPa

$$\tau_{B-} = \frac{(15\times10^{-3}\,N)(.6\times10^{-3}\,m^3)}{(.218\times10^{-3}\,m^4)(.125\,m)}$$



## Example #2



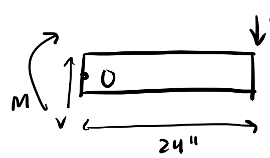




Find:
① $\sigma_{max}$ at n-n
② $\tau_A$ at n-n
③ $\tau_{max}$ at n-n

Quantities to check:
- I
- Moment at n-n
- V at n-n
- $Q_A$

$I$: $\frac{1}{12}(4in)(6in)^3 - \frac{1}{12}(3.4in)(5.4in)^3 = 72 - 44.6148 = 27.39 in^4$

Moment at n-n:




$\oplus\curvearrowleft \Sigma M_0 = 0 = M + 8(24)$ $M = -192$ kip·in

$V = -8$ kip

$\sigma_{max} = ?$ $\sigma_{max} = \frac{Mc}{I} = \frac{(192 \text{ kipin})(3in)}{27.39 in^4} = 21$ ksi

$\bar{y} = (3 in - \frac{.3in}{2}) = 2.85$

$Q_A = (2.85in)(4in)(.3in) = 3.42 in^3$

$\tau_A = \frac{VQ_A}{It} = \frac{(-8 kip)(3.42in^3)}{(27.39in^4)(2)(.3in)} = 1664.84$ psi

Things left for next class
① $\tau_A$
② $\tau_{max}$

See beginning of lecture Tue. April 14th, 2020 to finish this problem...