Materials – Lecture #21 4/16/20

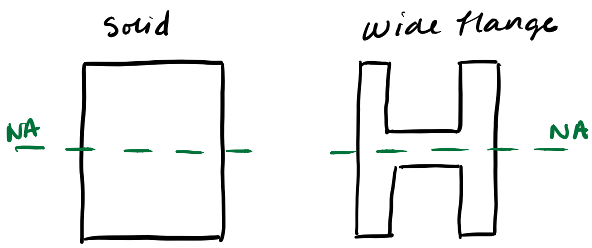
Continuation of buckling: (fixed/fixed column)

$$P_{CR} = \frac{\pi^2 E I}{L^2} \qquad v = c_1 \sin\left(\frac{n\pi x}{L}\right)$$

P_{CR} depends on:
- stiffness E
- moment of inertia I (column dimensions/geometry)
- inversely to L
- tells us that the ability to carry load increases w/ I, so idea is to move area as far from from centroid as possible

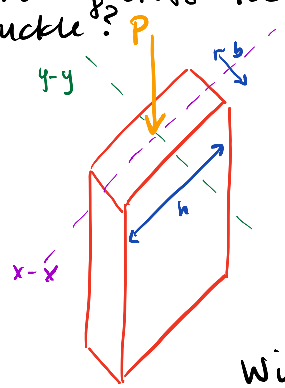
↳ parallel axis theorem: $I = \Sigma \bar{I} + A d^2$ → more mass away from neutral axis, ↑ I

also tells us that...
→ hollow tubes or columns are more economical than solid ones
→ wide flange/built up cross-sections better than solid or rectangular ones



* want to shift mass away from NA to resist buckling

What if cross-section isn't square? → which way will column buckle?



$$I_{x-x} = \frac{1}{12} h b^3 \qquad I_{y-y} = \frac{1}{12} b h^3$$

assuming $h > b$, then $I_{y-y} > I_{x-x}$

↳ column buckle about "weaker" axis, i.e. buckle around I_{x-x}

Will column buckle or yield?

consider $I = A r^2$, r = "radius of gyration"

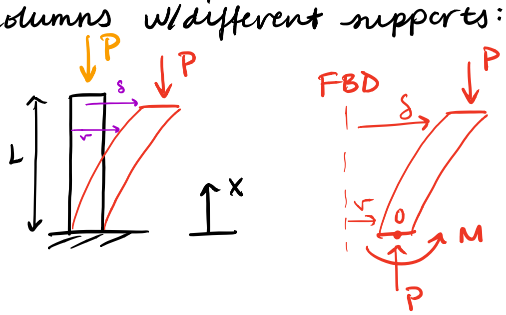
critical buckling load: $P_{CR} = \dfrac{\pi^2 E I}{L^2} = \dfrac{\pi^2 E (A r^2)}{L^2}$

So, $\dfrac{P_{CR}}{A} = \dfrac{\pi^2 E}{(L/R)^2}$ => critical stress $\sigma_{CR} = \dfrac{\pi^2 E}{(L/R)^2}$

1. check if $\sigma_{cr} < \sigma_y$
→ if yes, column fails by buckling
→ if no, column yields

$\frac{L}{R}$ = "slenderness ratio" → classify columns as long or short

Columns w/different supports: fixed-free column



$\circlearrowleft + \sum M_0 = 0 = M - P\delta + Pv$

$M = P(\delta - v)$

plug into beam deflection ODE: $EI^2 \frac{d^2v}{dx^2} = P(\delta - v)$

$$\frac{d^2v}{dx^2} + \frac{P}{EI} v = \underbrace{\frac{P}{EI}\delta}$$

different from fixed-fixed
"non-homogeneous"

deflection : $v = c_1 \sin\left(\sqrt{\frac{P}{EI}}x\right) + c_2 \cos\left(\sqrt{\frac{P}{EI}}x\right) + \delta$

→ need boundary conditions to solve for c_1 and c_2

① $x=0,\ v=0 \rightarrow 0 = c_1(0) + c_2(1) + \delta$

$$c_2 = -\delta$$

② $x=0,\ \frac{dv}{dx}=0$ (slope also equal to zero bc cantilevered)

$$\frac{dv}{dx} = c_1\sqrt{\frac{P}{EI}}\cos\left(\sqrt{\frac{P}{EI}}x\right) - c_2\sqrt{\frac{P}{EI}}\sin\left(\sqrt{\frac{P}{EI}}x\right)$$

$$0 = c_1(1) - c_2(0) \Rightarrow c_1 = 0$$

$$v = \delta\left[1 - \cos\left(\sqrt{\frac{P}{EI}}x\right)\right]$$

→ also know that at $x=L,\ v=\delta$

so $\delta = \delta\left[1 - \cos\left(\sqrt{\frac{P}{EI}}L\right)\right]$

$\delta = \delta - \delta\cos\left(\sqrt{\frac{P}{EI}}L\right) \Rightarrow \delta\cos\left(\sqrt{\frac{P}{EI}}L\right) = 0$

→ at $\delta = 0$, but that's a trivial solution
instead,

$\cos\left(\sqrt{\frac{P}{EI}}L\right) = 0 \Rightarrow \sqrt{\frac{P}{EI}}L = \frac{n\pi}{2}$, $n = 1, 3, 5, \ldots$

solve for P,

$$\frac{P}{EI} = \frac{n^2\pi^2}{4L^2} \Rightarrow \underline{P = \frac{n^2\pi^2 EI}{4L^2}}$$

so the smallest load is at $n=1$

→ tells us that the critical buckling load

$$P_{CR} = \frac{\pi^2 EI}{4L^2} \rightarrow \text{for fixed-free column}$$

$P_{CR} = \frac{1}{4}$ of fixed-fixed column

so it's significantly easier to buckle a fixed-free column than a fixed-fixed column

↳ we can generalize this using the concept of an effective length (L_e)

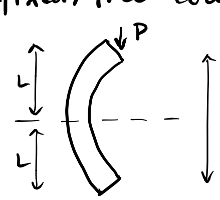
$$P_{CR} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2 EI}{(L_e)^2}$$

$$\sigma_{CR} = \frac{\pi^2 E}{(KL/R)^2}$$

What does L_e mean physically?

L_e represents the distance between points of zero moment

fixed/free column

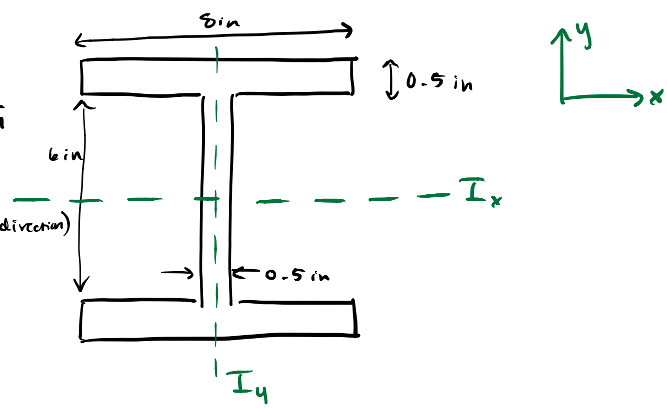


$L_e = 2L$, $K = 2$

Ex. #1

A36 steel:
$E = 29(10^3)$ ksi
$\sigma_y = 36$ ksi

$L = 15$ ft (in z-direction)



Question:
What P_{CR}, and will column buckle or yield?
Which axis will column buckle about?
→ assume that $K = 1$

$$P_{cr} = \frac{\pi^2 E I}{L^2}$$

$$I_x = \frac{1}{12}(8)(7)^3 - \frac{1}{12}(7.5)(6)^3 = 93.67 \text{ in}^4$$

$$I_y = \frac{1}{12}(6)(.5)^3 + 2\left(\frac{1}{12}\right)(.5)(8)^3 = 42.73 \text{ in}^4$$

... so if it buckles, it will buckle about I_y

smaller one

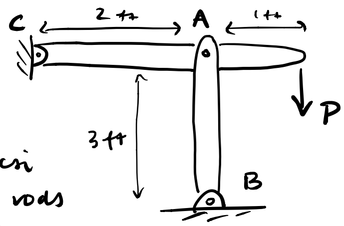
$$P_{CR} = \frac{\pi^2 (29 \times 10^3)(42.73)}{[15(12 \text{ in/ft})]^2} \Rightarrow P_{cr} = 377 \text{ kips}$$

$$\sigma_{CR} = \frac{P_{CR}}{A} = \frac{377 \text{ kips}}{A} \qquad A = (2)(8)(.5) + (6)(.5) = 11 \text{ in}^2$$

$$\sigma_{CR} = \frac{377 \text{ kips}}{11 \text{ in}^2} \Rightarrow \sigma_{CR} = 34.3 \text{ ksi} < \sigma_y = 36 \text{ ksi}$$

... this means that this particular column will buckle before yielding

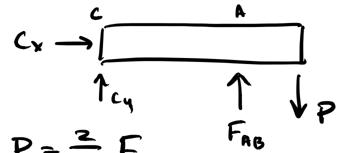
EX: #2



A36 steel
$E = 29 \times 10^3$ ksi
$K = 1$ for all rods
$d_{rods} = 1.25$ in

Find: maximum force P that can be applied such that rod AB does not buckle?

FBD:



$$\circlearrowleft \oplus \; \Sigma M_C = 0 = F_{AB}(2\text{ ft}) - P(3\text{ ft}) \quad \Rightarrow \quad P = \frac{2}{3} F_{AB}$$

Buckling load for rod AB:

$$I = \frac{\pi}{4}(0.625\text{in})^4 = 0.1198422 \text{ in}^4$$

$$A = \pi\,(0.625\text{ in})^2 = 1.22718463 \text{ in}^2$$

$$P_{CR} = \frac{\pi^2 E I}{(KL)^2} = \frac{\pi^2 (29 \times 10^3 \text{ kip/in}^2)(0.1198422 \text{ in}^4)}{[(1)(3\text{ ft})(12\text{ in/ft})]^2} = 26.4669 \text{ kip}$$

$P_{CR} = 26$ kip

... then relate this to force P

$$P = \frac{2}{3} F_{AB} = \frac{2}{3}(26.4669\text{ kip}) \quad \Rightarrow \quad \boxed{P = 17.6446\text{ kip}}$$

... find critical stress

$$\sigma_{CR} = \frac{P_{CR}}{A} = \frac{26.4669\text{ kip}}{1.22718463\text{ in}^2} = 21.567 \text{ ksi}$$

new material next week...
project description below...

Final Project Description (last ~15 min)

Due: May 5, 2020 by 5pm via email

· top of column is steel gate
- can only have steel plate deflect by certain amount
· what is the ... ?

Buckling, bending, shear & normal stresses

reinforcing plate: for column ... square cross-section
want plates to match geometry of column

control for material cost ... same material to reinforce column/etc
... minimize amount of material used

→ need to consider thermal effects

→ we can assume ... plates stuck???

geometry of gate:

fig on left for holding tank : right underneath rubber gasket is gate w/thickness t

gate is bottom of water tank