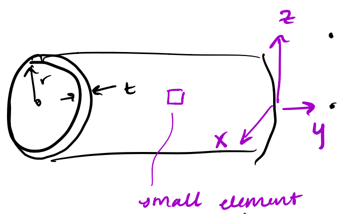
Materials Lecture #24

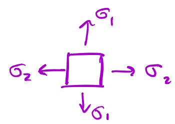
Hibbeler Ch. 8
① pressure vessels
② combined loading

→ Thin Walled Pressure Vessels (basically storage tanks, like H_2O heaters)

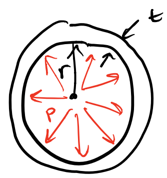


• thin walled if $r/t \geq 10$

• if vessels are "thin-walled," then stresses don't vary through wall thickness t



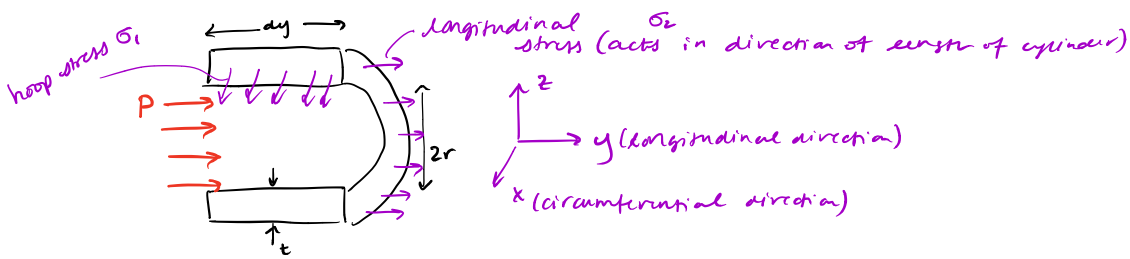
cross section



Internal pressure P pushing on walls, typically caused by fluid

→ have 2 stresses of interest
σ_1 : circumferential stress "hoop stress"
σ_2 : longitudinal, or axial, stress

FBD of cross section



Balance forces:

two bc top | bottom

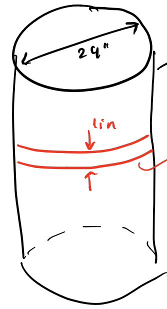
$\overset{+}{\swarrow} \Sigma F_x = 0 : 2\sigma_1 (t\,dy) - p(2r\,dy) \Rightarrow \boxed{\sigma_1 = \frac{Pr}{t}}$ → "hoop stress"

$\overset{+}{\rightarrow} \Sigma F_y = 0 : \sigma_2 (2\pi r t) - P(\pi r^2) \Rightarrow \boxed{\sigma_2 = \frac{Pr}{2t}}$

→ cylindrical pressure vessel: subject to biaxial stress state, ie is normal acting in 2 directions simultaneously

$\Rightarrow$ "uniaxial" stress state

Example Problem on Pressure vessels / thin-walled structures

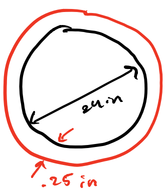


rigid cylinder

A36 steel hoop
↳ inner diameter of 23.99 in valid at $T = 65°F$
$t = 0.25$ in
$w = 1$ in

$\alpha = 6.6 \times 10^{-6}$ /°F

Find: ① T for steel hoop to just slip around the cylinder?
② pressure hoop exerts on cylinder?
③ tensile stress on hoop when T back to 65°F?



① Thermal expansion $\delta_T = \alpha \Delta T L$

$$(24 - 23.99) = (6.6 \times 10^{-6}\ ^1/_F)(T_{new} - 65°)(23.99$$

$$T_{new} = 129.16°F$$

② cool down: $\delta_F = \delta_T$

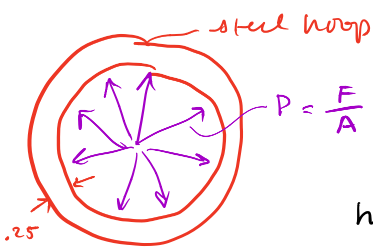
$$\frac{FL}{AE} = \alpha \Delta T L$$

P = normal force acting on cylinder

$$\frac{F(24)}{(wt)(29 \times 10^6)} = (6.6 \times 10^{-6})(129.16 - 65)(24)$$

$$\frac{F(24)}{(1)(.25)(29 \times 10^6)} = (6.6 \times 10^{-6})(129.16 - 65)(24)$$

$$F = 3022.21 \text{ lbs}$$



$$P = \frac{F}{A} = \frac{3022.21}{wt} = \frac{3022.21}{1\ (.25)} = 12088 \text{ psi} = P$$

hoop stress: $\sigma_1 = \frac{pr}{t} = \frac{12088 \text{ psi}(12)}{0.25}$

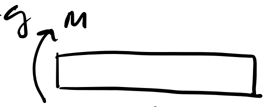
$\sigma_1 = 252$ psi (acts longitudinally, does not expand)

→ State of stress by combined loadings

Forms of axial stress

① 

$\sigma = \frac{P}{A}$

② Bending 

$\sigma = \frac{My}{I}$

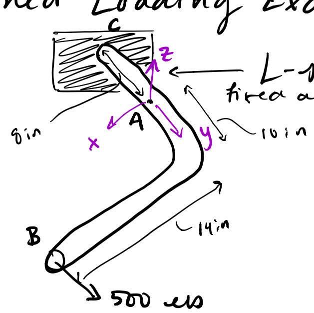
→ didn't do so far: normal stresses arising from both axial load + bending in same problem

→ idea: member subject to several different types of loading simultaneously

→ analyze using principle of superposition to get resultant stress distribution

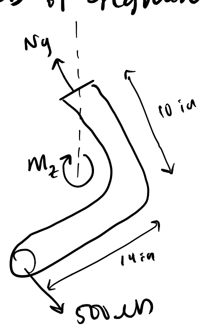
Analysis procedure:
① obtain internal normal/shear bending/torsion forces/moments
② calculate stresses associated with each internal loading
③ use superposition to find resultant stresses

Combined Loading Example #1



solid rod, radius = 0.75 in
Find: state of stress at A

FBD of segment AB



$\overset{+}{\searrow} \Sigma F_y = 0 = 500 - N_y$
$N_y = 500$ lbs

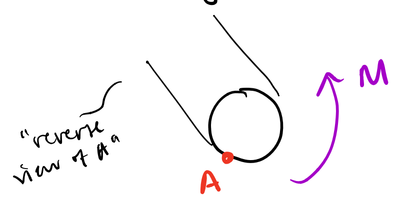
$\overset{+}{\curvearrowleft} \Sigma M_z = 0 = M_z - 500(14)$
$M_z = 7000$ lb·in

Stress Components:

→ normal force: $\sigma_A = \frac{N_y}{A} = \frac{500 \text{ lbs}}{\pi(.75 \text{ in})^2} = 283$ psi $= .283$ ksi
↳ tension

→ normal stress from bending moment:
→ bending stress $= \frac{Mc}{I} = \frac{(7000 \text{ lb in})(.75 \text{ in})}{\frac{\pi}{4}(.75)^4} = 21.13$ ksi $= \sigma_{bending}$
cross section I

Is bending stress ⊕ or ⊖ at A?



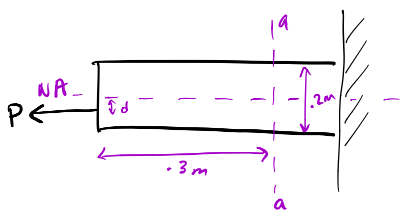
Then A is in tension

bc normal stress from both axial + bending is ⊕

$$\sigma_A = \underbrace{.283 \text{ ksi}}_{\text{axial}} + \underbrace{21.13 \text{ ksi}}_{\text{bending}}$$

$$\boxed{\sigma_A = 21.4 \text{ ksi}}$$

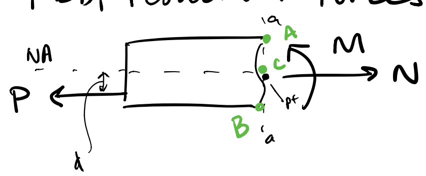
In class Example



$P = 80$ kN   thickness (into screen) $= 0.01$ m

Find: ① normal stress distribution at a-a
② where is NA located along cross section a-a?

FBD, resultant forces at a-a



y, x, ⊕

$\Sigma F_x : N - P = 0 \qquad N = P$

$\Sigma M_{\substack{a-a \\ pt}} : M - Pd = 0 \qquad M = Pd$

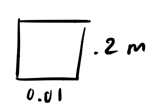
What he did: $\xrightarrow{+} \Sigma F_x = 0 = N - P, \quad N = P = 80 \text{ kN}$

$+\circlearrowleft \Sigma M_c = 0 = M - Pd, \quad M = Pd = (80 \text{ kN})(.05 \text{ m}) = 4 \text{ kN m}$

↑center of x-section

What are resultant normal stresses along a-a?
σ_A, σ_B? Between σ_A and σ_B?

Where is NA at a-a due to combined loading?



$A = (0.01\text{m})(.2\text{m}) = 0.002 \text{ m}^2$

$I = \frac{1}{12} bh^3 = \frac{1}{12}(0.01\text{m})(0.2\text{m})^3 = 6.6667(10^{-6}) \text{ m}^4$

$$\sigma = \frac{80 \text{ kN}}{0.002 \text{m}^2} \pm \frac{(4 \text{ kN m})(0.1\text{m})}{6.6667(10^{-6} \text{m}^4)} = \frac{N}{A} \pm \frac{My}{I}$$

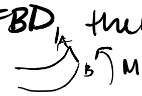
$$40000 \frac{\text{kN}}{\text{m}^2} \pm 60000 \frac{\text{kN}}{\text{m}^2}$$

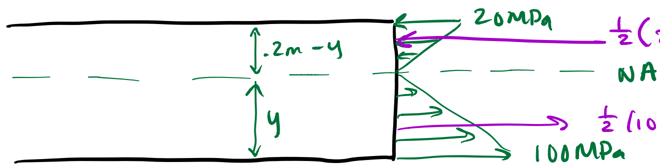
these signs mean bending stress + or − depending on C or T

$\sigma_A = 40000 \text{ kPa} - 60000 \text{ kPa} = -20000 \text{ kPa}$ (compression)

$\sigma_B = 40000 \text{ kPa} + 60000 \text{ kPa} = 100000 \text{ kPa}$ (tension)

* due to how we drew FBD, then A gets compressed while B gets stretched





$\frac{1}{2}(20)(.033)(.01) = .0033 \times 10^6$ N

$\frac{1}{2}(100)(.01)(\frac{1}{6}) = .0833 \times 10^6$ N

$+\circlearrowleft \Sigma M_{NA} = 0$ will get $= 0$

$\frac{.2-y}{y} = \frac{20}{100}$ $\quad 20 - 100y = 20y$

$y = \frac{1}{6} \rightarrow$ NA is 0.167m up from bottom