An Analysis of Reinforcement Methods for a Water Holding Tank

Leah Gaeta
ME 305: Mechanics of Materials
ltgaeta@bu.edu

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Contents

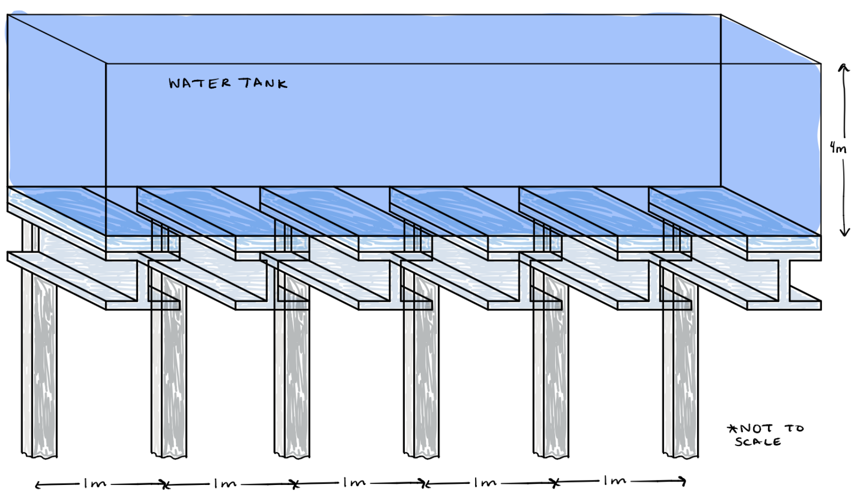


Figure 1: Side view schematic of the overall water holding tank. Note that this is not to scale.

1 Introduction

In today's engineering applications involving the use of or construction with various materials, one often needs to know the strength of such materials when exposed to certain loads and temperatures. Detailed in this report is the investigation into the best reinforcement for a water holding tank structure. A schematic of the overall structure can be seen in Figure 1 on the Table of Contents page. There are six columns with plates on two opposing sides and six wide-flange beams, or I-beams, holding a gate and a tank of water above the gate, and all structural materials are made of A36 Steel. Additionally, the structure is exposed to temperatures ranging from 5 - 40 degrees Celsius.

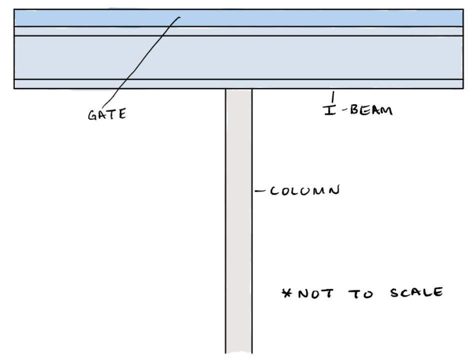


Figure 2: Front view of the column, I-beam, and gate.

The initial structure composed of only the columns, gate, and water tank holding a height of 2 meters of water. The reinforcements to be analyzed are tasked with supporting an additional 2 meters of water (for a total of 4m) without exceeding a maximum allowed normal stress σ_y of 250 MPa and maximum shear stress τ_y of 150 MPa.

The focus of this analysis will be on one column with one I-beam for simplification due to the underlying assumption that the findings can be applied to the other five identical components. Visuals of the single column and I-beam can be seen in Figures 2 and 3, which display the front and side views, respectively. Important assumptions made throughout the analysis will be specified in the proceeding sections, which follow the order of the analysis conducted. In addition to satisfying the previously mentioned requirements, the choice of I-beam and column plate thickness was driven by the motivation to minimize costs of material, in which the costs are directly proportional to the amount of material used. Finally, the MATLAB program used for this analysis can be found in the Appendix and the .mlx with all markdown and visualizations can be requested via email.

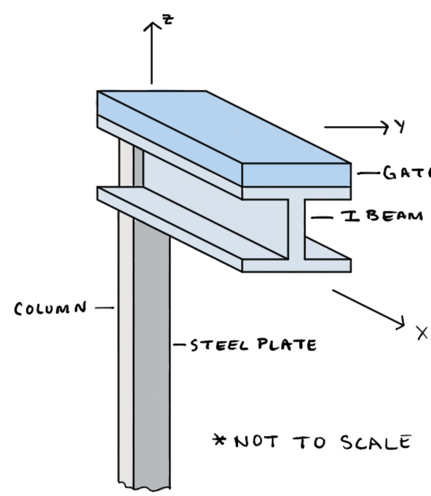


Figure 3: Side view with x-, y-, and z-axes.

2 Thermal Analysis

When exposed to different temperatures, A36 steel will either expand or contract by an amount that is proportional to the structure length, temperature, and thermal coefficient of expansion. This relationship is expressed by Eq. 1 below, in which α is the linear coefficient of thermal expansion, ΔT is the change in temperature, L is the length of the structure, and δ is the change in length. For A36 Steel, the coefficient of thermal expansion is $\alpha = 11.7 \cdot 10^{-6}$. (http://www.matweb.com, n.d.) Additionally, it is important to note the assumption that this material is homogeneous and isotropic; a requirement for application of Eq. 1. (Hibbeler, 2017)

$$\delta = \alpha \Delta T L \tag{1}$$

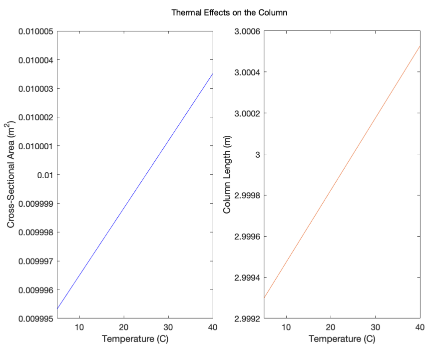


Figure 4: Thermal effects on the column length and cross-sectional area.

Thermal analysis was first completed on the column as the reinforcing I-beam had not yet been chosen. Given the length of the column ($L = 3m$), α, and the temperature range of 5 to 40 degrees Celsius, the change in dimensions were calculated. At 5 degrees, the column is shortest at 2.9993 m, while its longest length ($L = 3.0005m$) is found at 40 degrees. (The baseline length of 3 m was measured at 25 degrees Celsius.) Given that the change in length δ is so small, the effects on the cross-section of the column are negligible and the cross-section was practically 0.1000x0.1000 m^2 throughout, which can be observed in Figure 4. Thermal analysis was extended into the buckling analysis of the column in the following section.

3 Buckling Analysis of the Column

The buckling of a column, or other long and slender structures, is the lateral deflection that can occur due to compressive loading. For a column that is loaded axially, as is the case for this scenario, has a critical load P_{cr} which is the maximum force that the column can sustain when it is about to buckle. (Hibbeler, 2017) The critical load is defined below in Eq. 2 in which E is the modulus of elasticity ($= 200GPa$ for A36 steel), I is the least moment of inertia for the cross-section, K is the effective-length factor, and L is the length of the column. Since the column is comprised of a square cross-section (before adding the additional reinforcement plates) as seen in Figure 5, there is only one moment of inertia I which can be calculated from Eq. 3 in which b is the base and h is the height of the cross-section.

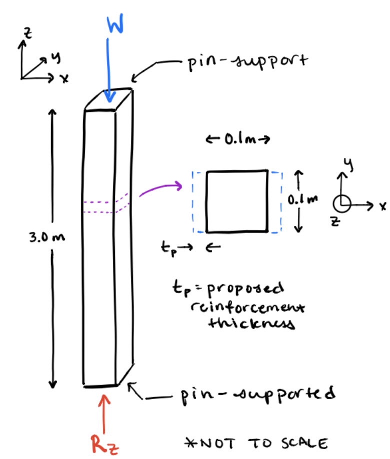


Figure 5: The column acted upon by a compressive load and its cross-section.

From Eq. 2 one can immediately see the dilemma of the thermal stress on the column. The critical load will increase with increasing cross-sectional area while it will decrease with increasing length. The critical load was therefore analyzed for all temperature scenarios and plotted to visually understand optimal axially-loaded column dimensions. This can be observed in Figure 6 which displays the critical loading and critical stress changes over the temperature range. The smallest critical load was found to be 1863.4 kN at 5 degrees and the critical stress stayed consistently below the 250 MPa threshold (marked by the dash-dot line) at

182.77 kPa. The critical stress, σ_{cr}, was also important to analyze so that the design met the normal stress requirements previously stated, and is defined above in Eq. 4 in which r is the smallest radius of gyration from the least moment of inertia I ($r = \sqrt{I/A}$, A is cross-sectional area).

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} \tag{2}$$

$$I = \frac{1}{12} bh^3 \tag{3}$$

$$\sigma_{cr} = \frac{\pi^2 E}{(KL/r)^2} \tag{4}$$

The loading, P, experienced by the column was calculated by way of Eq. 5, in which ρ is the density of water equal to 1000 $\frac{kg}{m^3}$, A is the cross-sectional area, h is the height of the water (4 m), and g is the acceleration due to gravity. The loading was calculated to be 392.4 N which is substantially smaller than the any of the critical loads calculated for the temperature range and the column was therefore determined to suffice supporting the load of the water. It can therefore also be assumed that the column could support the 2 m of water before the reinforcements were added. Another important point to note is that the water was treated as a distributed load and converted into a point force acting axially on the column. Water as a distributed load will be discussed further in the later sections.

$$\rho \cdot Ahg = P \tag{5}$$

In addition, the column was analyzed with different supporting conditions. First, it was analyzed as a fixed-fixed support, corresponding to $k = 0.5$, then as a fixed-pinned with $k = 0.7$, and finally as a pin-pin support of $k = 1.0$. It was determined that while either of these supports could withstand the weight of the water, only the pin-pin supported column could satisfy the critical stress requirement and the column was therefore determined to be supported by pinned ends. Finally, since steel plates were required in the reinforcement, a very small thickness of 0.001 m was chosen to minimize costs. The thermal effects on the buckling of the column with reinforcements can be seen in Figure 7. Finally, since the thermal effects are so small the remainder of the analysis was conducted with the assumption that length and cross-section of the I-beam and gate will be essentially equal to their measured values at 25 degrees Celsius.

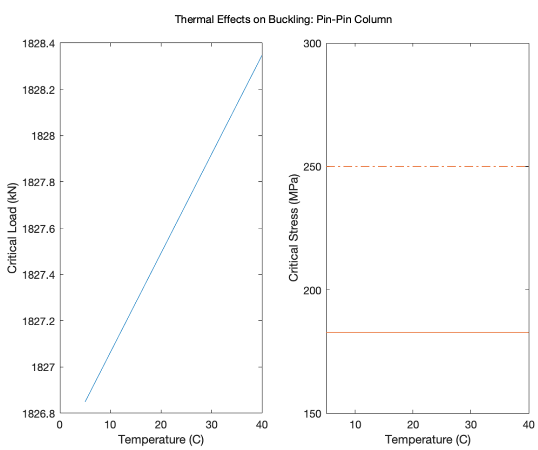


Figure 6: Thermal effects on column buckling.

4 Shear Stress Analysis of the Beam

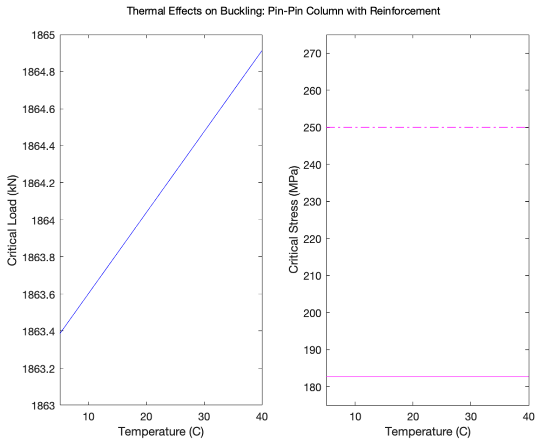


Figure 7: Thermal effects on the column with reinforcements.

With the column reinforcements chosen, the remainder of the study was focused on the selection of the I-beam. The shear stress was analyzed next due to better understand how the shear stress was distributed over the cross-section of an I-beam at 25 degrees C. The shear stress τ is defined below in Eq. 6 in which V is the shear force, I is the moment of inertia, t is the width of the cross-section where τ is measured, and $Q = \bar{z}'A'$ (or the distance from the neutral axis to the centroid of area A' multiplied by A'). I and the neutral axis NA were calculated with functions created within MATLAB which can be seen in the Appendix. Additionally, Figure 10 in the following section shows a visual corresponding to the different areas of the I-beam that were used to create the function.

$$\tau = \frac{VQ}{It} \tag{6}$$

For the purpose of this analysis, the gate was assumed to be strongly bonded to the I-beam, which therefore increased the cross-sectional areas of all of the I-beams provided to choose from. The maximum shear stress was then analyzed for all of the I-beams using Eq. 6. The maximum shear stress was found to be 8.818 kPa which is well below the 150 MPa limit. Therefore, the smallest I-beam cross-sectional area was chosen for the remainder of the analysis to see if it would satisfy the other requirements. This I-beam is a 'W150x14' in which 150 is the depth of 150 mm and 14 is the mass per length (kg/m); the maximum shear stress experienced over the cross-section of this particular beam was found to be 7.523 kPa and its NA at 0.0504 m from the top of the gate/I-beam, or 149.6 mm from the bottom, which is within the top flange of the I-beam.

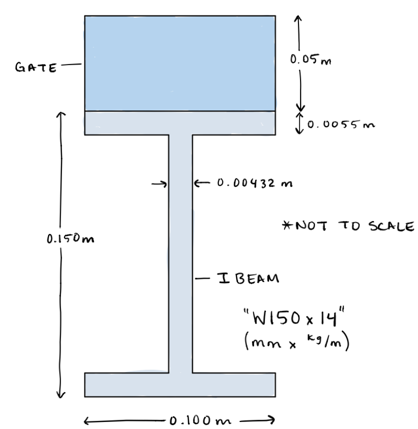


Figure 8: I-beam reinforcement of gate cross-section.

5 Bending & Normal Stress Analysis of the Beam

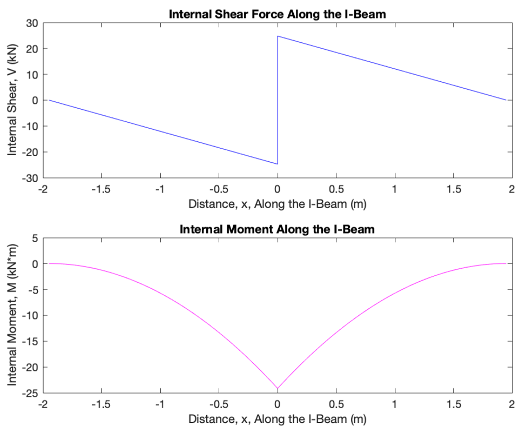


Figure 9: Shear & Moment diagrams through the length of the beam. Note that $x = 0$ signifies the center of the I-beam.

In addition to finding the shear stress distribution over the cross-section of the chosen I-beam, the normal stress was also analyzed. Though there is no axial loading on the beam, the internal moment created by the loading on the beam causes a normal stress throughout the length of the beam. In order to conduct this analysis, an important assumption was made: the beam can be treated as two cantilever beams with the fixed ends at the location of the column. This was assumption was made based on the fact that the beam has two free ends and could then be split into two cantilevers with loading symmetry on both. The cantilevers were analyzed based on the method of sections with the water as a distributed and the assumption that the gate is strongly bonded to the I-beam. The force of the water was found in a similar manner as with the column; by taking the volume of water over the I-beam geometry and multiplying by the density of water and the acceleration due to gravity. Figure 12 in the following section shows the free body diagram of the right side of the beam (with the bonded gate) treated as a cantilever.

$$\sigma_{max} = \frac{Mc}{I} \tag{7}$$

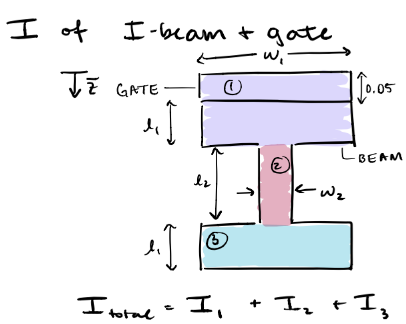


Figure 10: Visual of areas used to calculate I. I and the neutral axis NA were calculated using these sections and variables for each I-beam listed with functions created in MATLAB.

The shear and moment diagrams of the loading over the entire length of the beam can be seen in Figure 9. One can see that at the free ends of the beam there is no internal shear or moments experienced. By observation of the internal moment diagram, one can decipher that the maximum normal stress in the member will therefore occur at the center of the beam. This is further supported by Eq. 7, which is also known as the flexure formula. Here, σ_{max} is the maximum normal stress occurring on a cross-section of the I-beam. The maximum normal stress will occur at the point of the cross-section that is furthest from the neutral axis. In addition, M is the internal moment found from the method of sections, c is the perpendicular distance from the point where the normal stress acts to the NA, and I is the moment of inertia. The equations used for finding the distribution of water, internal shear force, and internal moment for the right end of the beam are shown in Eq. 8, 9, and 10 respectively. The 0.324 m is the flange width of the 'W150x14' I-beam chosen. Over a beam length of 1.949 m (subtracting the width of the column in the x-direction), the force is 24792 N which can be treated as a point force at the center of this right side.

$$w(x) = (1000\frac{kg}{m^3})(.324m)(4m)(9.81\frac{m}{s^2} = 12713.76\frac{N}{m} \tag{8}$$

$$v(x) = \int w(x)dx = 24792N - 12713.76x \tag{9}$$

$$M(x) = \int v(x)dx = -\frac{12713.76}{2}x^2 + 24792x - 24172 \tag{10}$$

Using the flexure formula defined in Eq. 7, the maximum normal stress over the length of the beam was plotted. This can be seen in Figure 9, which shows that the greatest normal stress occurs at the center where the internal moment is at its highest magnitude. In addition, of most importance is that this maximum normal stress found to be 81.432 MPa is well below the required limit of 250 MPa.

6 Deflection Analysis of the Beam

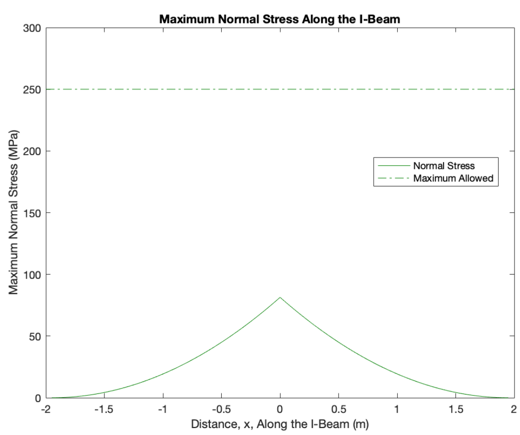
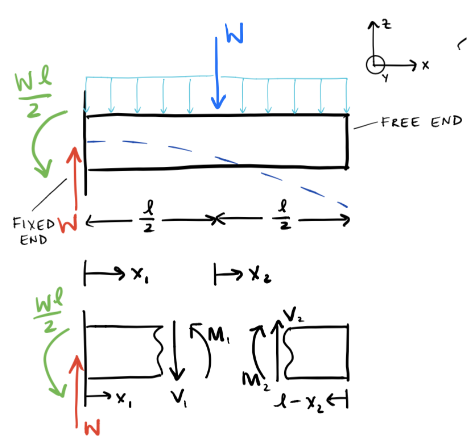


Figure 11: The normal stress experienced at the cross-section of the I-beam along over its entire length.

The final requirement that must be satisfied for this structure is that the deflection of the gate be no more than 20 mm, or 0.020 m. Assuming a constant flexural rigidity EI along the length of the beam, the deflection v of the beam can be found by integrating Eq. 11 twice in which $M(x)$ is the internal moment, as mentioned previously. For this particular scenario, the right side of the beam was analyzed and then symmetry applied to

Figure 12: Right side of the the I-beam free body diagram using method of sections



the left. Boundary conditions of $v_1 = 0$ and $\frac{dv_1}{dx_1} = 0$ at $x_1 = 0$ were applied to the fixed end and continuity conditions of $\frac{dv_1}{dx_1} = \frac{dv_2}{dx_2}$ at $x_1 = x_2 = L/2$ in the middle result in Eq. 12 and 13, which were used to calculate the deflection. A visual to aid these equations for this right side beam treated as a cantilever can be seen in Figure 12.

$$M(x) = EI\frac{d^2v}{dx^2} \tag{11}$$

$$v_1 = \frac{1}{12EI}P(2x_1^3 - 3x_1^2) \tag{12}$$

$$v_2 = \frac{1}{48EI}Pl^2(-6x_2 + l) \tag{13}$$

In these equations, P is used to represent the load so as not to be confused with W (though w is used in the corresponding figure) and l represents the length of the beam. Using these equations, the maximum deflection of the I-beam chosen was found to be at the free end as expected, and equal to 8.51x10$^-$7 mm, which is well below the required maximum of 20 mm. The deflection along the beam is plotted in Figure 13 as the red dash-dot line. The blue line corresponds to what the deflection of the gate would be without any reinforcement. Though this deflection of the gate is still small (found to be 7.46x10$^-$6), the plot shows just how strong the reinforcement is in holding the gate up. Though this plot shows only the beam to the right of the column, symmetry can be applied to the left end.

7 Conclusion

In conclusion, the reinforcement system shown to be able to handle the increased accommodations for the water tank include using reinforcing plates of 0.001 m thickness on the columns, and a 'W150x14' I-beam to be strongly bonded to the gate. These were chosen because they satisfied the requirements listed and were small enough to decrease costs. In the future, it would be more applicable to apply a factor of safety for this structure as well as do a more thorough analysis that includes the other five identical column and I-beam structures.

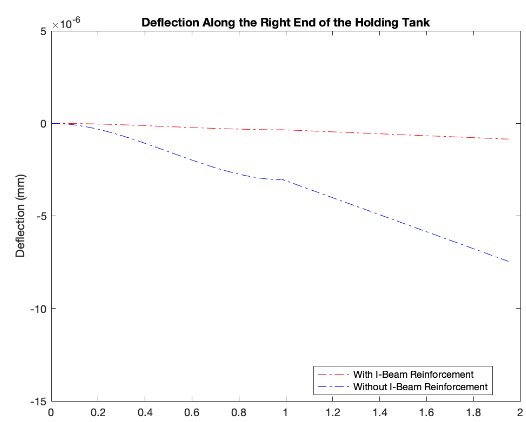


Figure 13: The deflection of the right end of the I-beam. Via symmetry, the deflection will be the same for the left end.

References

Hibbeler, R. C. (2017). Chapter 4, 6, 7, 12, 13. *Mechanics of Materials*, *10thEdition*.

http://www.matweb.com. (n.d.).

8 Appendix

Mechanics of Materials Final Project

```matlab
% Load I-Beam Data
[ibeam_data ibeam_text] = xlsread("Wide Flange Beams SI.xlsx");
[row col] = size(ibeam_data);
[row_text col_text] = size(ibeam_text);

% Create dataframe
ibeam_df(row) = struct('Designation', ' ', 'Area', 0, 'Depth', 0, 'WThick', 0, ...
    'FWidth', 0, 'FThick', 0, 'YY_I', 0, 'YY_S', 0, 'YY_r', 0, ...
    'ZZ_I', 0, 'ZZ_S', 0, 'ZZ_r', 0, 'I_final', 0);
ibeam_text(1:(row_text - row), :) = [];
for i = row:-1:1
    ibeam_df(i).Designation = ibeam_text(i, 1);
    ibeam_df(i).Area = ibeam_data(i,1)/(10^6);
    ibeam_df(i).Depth = ibeam_data(i,2)/(10^3);
    ibeam_df(i).WThick = ibeam_data(i,3)/(10^3);
    ibeam_df(i).FWidth = ibeam_data(i,4)/(10^3);
    ibeam_df(i).FThick = ibeam_data(i,5)/10^3;
    ibeam_df(i).YY_I = ibeam_data(i,6)/(10^6);
    ibeam_df(i).YY_S = ibeam_data(i,7)/(10^6);
    ibeam_df(i).YY_r = ibeam_data(i,8)/(10^3);
    ibeam_df(i).ZZ_I = ibeam_data(i,9)/(10^6);
    ibeam_df(i).ZZ_S = ibeam_data(i,10)/(10^6);
    ibeam_df(i).ZZ_r = ibeam_data(i,11)/(10^3);
    ibeam_df(i).I_final = 0;
end

% Define Constants
E = 200*10^9; % Young's Modulus A36 Steel (Pa)
sigma_y = 250*10^6; % Max Allowable Normal Stress A36 Steel (Pa)
tau_y = 150*10^6; % Max Allowable Shear Stress A36 Steel (Pa)
rho_water = 1000; % Density Water (kg/m^3)
rho_steel = 7800; % Density A36 Steel (kg/m^3)
g = 9.81; % Acceleration due to gravity (m/s^2)
alpha = 11.7*10^(-6); % Coefficient of Thermal Expansion A36 Steel (1/C)

% Define Dimensions, Cartesian Basis (m)
col_x = 0.1;
col_y = 0.1;
col_z = 3;

gate_x = 4;
gate_z = 0.05;

ht_water_i = 2;
ht_water_f = 4;

temp = 5:40; % Exposed Temp Range (C)
temp_i = 25; % Initial Temp, all measurements taken here

% Variables to adjust
col_t = 0.001; % Thickness of Plates to add to columns (m)
%k = 0.5; % Fixed-Fixed Column
%k = 0.7; % Fixed-Pinned Column
k = 1.0; % Pinned-Pinned Column

% Calculate new I Values for Ibeams based on gate thickness
I_final = zeros(1,row);
for i = row:-1:1
    ibeam_df(i).I_final = calc_Ivalue(ibeam_df(i).FThick, ibeam_df(i).WThick, ibeam_df(i).Depth, ibeam_df(i).FWidth, gate_z);
end
ibeam_df
```

ibeam_df = 1×55 struct

Fields	Designation	Area	Depth	WThick	FWidth	FThick	YY_I	YY_S	YY_r	ZZ_I	ZZ_S	ZZ_r
1	1×1 cell	0.0198	0.6110	0.0127	0.3240	0.0190	0.0013	0.0042	0.2550	1.0800e-04	6.6700e-04	0.0739
2	1×1 cell	0.0179	0.6170	0.0131	0.2300	0.0222	0.0011	0.0036	0.2500	4.5100e-05	3.9200e-04	0.0502
3	1×1 cell	0.0159	0.6120	0.0119	0.2290	0.0196	9.8500e-04	0.0032	0.2490	3.9300e-05	3.4300e-04	0.0497
4	1×1 cell	0.0144	0.6080	0.0112	0.2280	0.0173	8.7500e-04	0.0029	0.2470	3.4300e-05	3.0100e-04	0.0488
5	1×1 cell	0.0129	0.6030	0.0105	0.2280	0.0149	7.6400e-04	0.0025	0.2430	2.9500e-05	2.5900e-04	0.0478
6	1×1 cell	0.0118	0.6030	0.0109	0.1790	0.0150	6.4600e-04	0.0021	0.2340	1.4400e-05	1.6100e-04	0.0349
7	1×1 cell	0.0105	0.5990	0.0100	0.1780	0.0128	5.6000e-04	0.0019	0.2310	1.2100e-05	1.3600e-04	0.0339
8	1×1 cell	0.0123	0.4660	0.0114	0.1930	0.0190	4.4500e-04	0.0019	0.1900	2.2800e-05	2.3600e-04	0.0431
9	1×1 cell	0.0114	0.4630	0.0105	0.1920	0.0177	4.1000e-04	0.0018	0.1900	2.0900e-05	2.1800e-04	0.0428
10	1×1 cell	0.0104	0.4600	0.0099	0.1910	0.0160	3.7000e-04	0.0016	0.1890	1.8600e-05	1.9500e-04	0.0423

Fields	Designation	Area	Depth	WThick	FWidth	FThick	YY_I	YY_S	YY_r	ZZ_I	ZZ_S	ZZ_r
11	1×1 cell	0.0095	0.4570	0.0090	0.1900	0.0145	3.3300e-04	0.0015	0.1880	1.6600e-05	1.7500e-04	0.0419
12	1×1 cell	0.0087	0.4590	0.0091	0.1540	0.0154	2.9700e-04	0.0013	0.1840	9.4100e-06	1.2200e-04	0.0328
13	1×1 cell	0.0076	0.4550	0.0080	0.1530	0.0133	2.5500e-04	0.0011	0.1830	7.9600e-06	1.0400e-04	0.0324
14	1×1 cell	0.0066	0.4500	0.0076	0.1520	0.0108	2.1200e-04	9.4200e-04	0.1790	6.3400e-06	8.3400e-05	0.0309
15	1×1 cell	0.0108	0.4170	0.0109	0.1810	0.0182	3.1500e-04	0.0015	0.1710	1.8000e-05	1.9900e-04	0.0408
16	1×1 cell	0.0095	0.4130	0.0097	0.1800	0.0160	2.7500e-04	0.0013	0.1700	1.5600e-05	1.7300e-04	0.0405
17	1×1 cell	0.0086	0.4100	0.0088	0.1790	0.0144	2.4500e-04	0.0012	0.1690	1.3800e-05	1.5400e-04	0.0402
18	1×1 cell	0.0068	0.4030	0.0075	0.1770	0.0109	1.8600e-04	9.2300e-04	0.1650	1.0100e-05	1.1400e-04	0.0385
19	1×1 cell	0.0059	0.4030	0.0070	0.1400	0.0112	1.5600e-04	7.7400e-04	0.1630	5.1400e-06	7.3400e-05	0.0295
20	1×1 cell	0.0050	0.3990	0.0063	0.1400	0.0088	1.2600e-04	6.3200e-04	0.1590	4.0200e-06	5.7400e-05	0.0285
21	1×1 cell	0.0101	0.3540	0.0094	0.2050	0.0168	2.2700e-04	0.0013	0.1500	2.4200e-05	2.3600e-04	0.0489
22	1×1 cell	0.0081	0.3470	0.0077	0.2030	0.0135	1.7900e-04	0.0010	0.1480	1.8800e-05	1.8500e-04	0.0480
23	1×1 cell	0.0072	0.3580	0.0079	0.1720	0.0131	1.6000e-04	8.9400e-04	0.1490	1.1100e-05	1.2900e-04	0.0393
24	1×1 cell	0.0065	0.3550	0.0072	0.1710	0.0116	1.4100e-04	7.9400e-04	0.1480	9.6800e-06	1.1300e-04	0.0387
25	1×1 cell	0.0057	0.3520	0.0069	0.1710	0.0098	1.2100e-04	6.8800e-04	0.1460	8.1600e-06	9.5400e-05	0.0378
26	1×1 cell	0.0050	0.3530	0.0065	0.1280	0.0107	1.0200e-04	5.7800e-04	0.1430	3.7500e-06	5.8600e-05	0.0275
27	1×1 cell	0.0042	0.3490	0.0058	0.1270	0.0085	8.2900e-05	4.7500e-04	0.1410	2.9100e-06	4.5800e-05	0.0264
28	1×1 cell	0.0165	0.3180	0.0131	0.3080	0.0206	3.0800e-04	0.0019	0.1370	1.0000e-04	6.4900e-04	0.0778
29	1×1 cell	0.0095	0.3100	0.0094	0.2050	0.0163	1.6500e-04	0.0011	0.1320	2.3400e-05	2.2800e-04	0.0497
30	1×1 cell	0.0085	0.3060	0.0085	0.2040	0.0146	1.4500e-04	9.4800e-04	0.1300	2.0700e-05	2.0300e-04	0.0493
31	1×1 cell	0.0049	0.3100	0.0058	0.1650	0.0097	8.4800e-05	5.4700e-04	0.1310	7.2300e-06	8.7600e-05	0.0383
32	1×1 cell	0.0042	0.3130	0.0066	0.1020	0.0108	6.5000e-05	4.1500e-04	0.1250	1.9200e-06	3.7600e-05	0.0214
33	1×1 cell	0.0030	0.3050	0.0056	0.1010	0.0067	4.2800e-05	2.8100e-04	0.1190	1.1600e-06	2.3000e-05	0.0195
34	1×1 cell	0.0027	0.3030	0.0051	0.1010	0.0057	3.7000e-05	2.4400e-04	0.1170	9.8600e-07	1.9500e-05	0.0192
35	1×1 cell	0.0190	0.2820	0.0173	0.2630	0.0284	2.5900e-04	0.0018	0.1170	8.6200e-05	6.5600e-04	0.0674
36	1×1 cell	0.0102	0.2560	0.0094	0.2550	0.0156	1.2600e-04	9.8400e-04	0.1110	4.3100e-05	3.3800e-04	0.0650
37	1×1 cell	0.0086	0.2570	0.0089	0.2040	0.0157	1.0400e-04	8.0900e-04	0.1100	2.2200e-05	2.1800e-04	0.0509
38	1×1 cell	0.0074	0.2520	0.0080	0.2030	0.0135	8.7300e-05	6.9300e-04	0.1090	1.8800e-05	1.8500e-04	0.0504
39	1×1 cell	0.0057	0.2660	0.0076	0.1480	0.0130	7.1100e-05	5.3500e-04	0.1120	7.0300e-06	9.5000e-05	0.0351
40	1×1 cell	0.0036	0.2600	0.0063	0.1020	0.0100	3.9900e-05	3.0700e-04	0.1050	1.7800e-06	3.4900e-05	0.0222
41	1×1 cell	0.0029	0.2540	0.0058	0.1020	0.0069	2.8800e-05	2.2700e-04	0.1010	1.2200e-06	2.3900e-05	0.0207
42	1×1 cell	0.0023	0.2510	0.0048	0.1010	0.0053	2.2500e-05	1.7900e-04	0.0993	9.1900e-07	1.8200e-05	0.0201
43	1×1 cell	0.0127	0.2290	0.0145	0.2100	0.0237	1.1300e-04	9.8700e-04	0.0943	3.6600e-05	3.4900e-04	0.0537
44	1×1 cell	0.0110	0.2220	0.0130	0.2090	0.0206	9.4700e-05	8.5300e-04	0.0928	3.1400e-05	3.0000e-04	0.0534
45	1×1 cell	0.0091	0.2160	0.0102	0.2060	0.0174	7.6600e-05	7.0900e-04	0.0917	2.5400e-05	2.4700e-04	0.0528
46	1×1 cell	0.0076	0.2100	0.0091	0.2050	0.0142	6.1200e-05	5.8300e-04	0.0899	2.0400e-05	1.9900e-04	0.0519
47	1×1 cell	0.0059	0.2030	0.0072	0.2030	0.0110	4.5500e-05	4.4800e-04	0.0879	1.5300e-05	1.5100e-04	0.0510
48	1×1 cell	0.0046	0.2010	0.0062	0.1650	0.0102	3.4400e-05	3.4200e-04	0.0868	7.6400e-06	9.2600e-05	0.0409
49	1×1 cell	0.0029	0.2060	0.0062	0.1020	0.0080	2.0000e-05	1.9400e-04	0.0836	1.4200e-06	2.7800e-05	0.0223
50	1×1 cell	0.0047	0.1620	0.0081	0.1540	0.0116	2.2200e-05	2.7400e-04	0.0685	7.0700e-06	9.1800e-05	0.0387
51	1×1 cell	0.0038	0.1570	0.0066	0.1530	0.0093	1.7100e-05	2.1800e-04	0.0672	5.5400e-06	7.2400e-05	0.0382
52	1×1 cell	0.0029	0.1520	0.0058	0.1520	0.0066	1.2100e-05	1.5900e-04	0.0650	3.8700e-06	5.0900e-05	0.0368
53	1×1 cell	0.0031	0.1600	0.0066	0.1020	0.0103	1.3400e-05	1.6800e-04	0.0662	1.8300e-06	3.5900e-05	0.0245
54	1×1 cell	0.0023	0.1530	0.0058	0.1020	0.0071	9.1900e-06	1.2000e-04	0.0633	1.2600e-06	2.4700e-05	0.0235
55	1×1 cell	0.0017	0.1500	0.0043	0.1000	0.0055	6.8400e-06	9.1200e-05	0.0629	9.1200e-07	1.8200e-05	0.0230

```
% Calculate Initial & Final Weights of Water and Gate
% Over column
Wt_water_col_i = col_x*col_y*ht_water_i*rho_water*g;
Wt_steel_col = col_x*col_y*gate_z*rho_steel*g;
Wt_col_i = (Wt_water_col_i + Wt_steel_col)/1000; % kN
Wt_water_col_f = col_x*col_y*ht_water_f*rho_water*g;
Wt_col_f = (Wt_water_col_f + Wt_steel_col)/1000; % kN
```

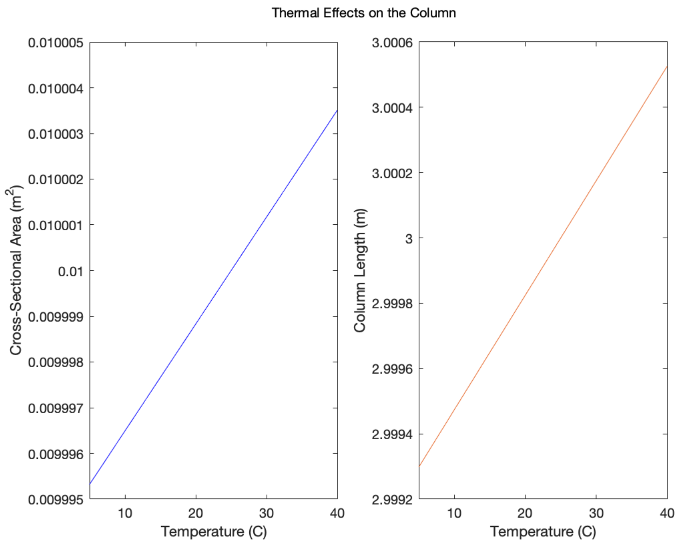
Thermal Analysis

```
% Column Initial Analysis
buckling_initial(temp(end)-temp(1)+1) = struct('Temp', 0, 'X', 0, 'Y', 0, 'Z', 0, 'Crit_Load', 0, 'Crit_Stress', 0, 'Max_All_L
for i = length(temp):-1:1
    buckling_initial(i).Temp = temp(i);
    buckling_initial(i).X = thermal_exp(alpha, (temp(i)-temp_i), col_x) + col_x;
    buckling_initial(i).Y = thermal_exp(alpha, (temp(i)-temp_i), col_y) + col_y;
    buckling_initial(i).Z = thermal_exp(alpha, (temp(i)-temp_i), col_z) + col_z;
    [buckling_initial(i).Crit_Load, buckling_initial(i).Crit_Stress, buckling_initial(i).Max_All_Load] = calc_buckling(bucklin
        buckling_initial(i).Y, buckling_initial(i).Z, E, k, 2);
end

buckling_initial
```

```
buckling_initial = 1×36 struct
```

Fields	Temp	X	Y	Z	Crit_Load	Crit_Stress	Max_All_Load
1	5	0.1000	0.1000	2.9993	1.8268e+03	182.7705	913.4246
2	6	0.1000	0.1000	2.9993	1.8269e+03	182.7705	913.4460
3	7	0.1000	0.1000	2.9994	1.8269e+03	182.7705	913.4674
4	8	0.1000	0.1000	2.9994	1.8270e+03	182.7705	913.4888
5	9	0.1000	0.1000	2.9994	1.8270e+03	182.7705	913.5101
6	10	0.1000	0.1000	2.9995	1.8271e+03	182.7705	913.5315
7	11	0.1000	0.1000	2.9995	1.8271e+03	182.7705	913.5529
8	12	0.1000	0.1000	2.9995	1.8271e+03	182.7705	913.5743
9	13	0.1000	0.1000	2.9996	1.8272e+03	182.7705	913.5957
10	14	0.1000	0.1000	2.9996	1.8272e+03	182.7705	913.6170
11	15	0.1000	0.1000	2.9996	1.8273e+03	182.7705	913.6384
12	16	0.1000	0.1000	2.9997	1.8273e+03	182.7705	913.6598
13	17	0.1000	0.1000	2.9997	1.8274e+03	182.7705	913.6812
14	18	0.1000	0.1000	2.9998	1.8274e+03	182.7705	913.7026
15	19	0.1000	0.1000	2.9998	1.8274e+03	182.7705	913.7240
16	20	0.1000	0.1000	2.9998	1.8275e+03	182.7705	913.7453
17	21	0.1000	0.1000	2.9999	1.8275e+03	182.7705	913.7667
18	22	0.1000	0.1000	2.9999	1.8276e+03	182.7705	913.7881
19	23	0.1000	0.1000	2.9999	1.8276e+03	182.7705	913.8095
20	24	0.1000	0.1000	3.0000	1.8277e+03	182.7705	913.8309
21	25	0.1000	0.1000	3	1.8277e+03	182.7705	913.8523
22	26	0.1000	0.1000	3.0000	1.8277e+03	182.7705	913.8736
23	27	0.1000	0.1000	3.0001	1.8278e+03	182.7705	913.8950
24	28	0.1000	0.1000	3.0001	1.8278e+03	182.7705	913.9164
25	29	0.1000	0.1000	3.0001	1.8279e+03	182.7705	913.9378
26	30	0.1000	0.1000	3.0002	1.8279e+03	182.7705	913.9592
27	31	0.1000	0.1000	3.0002	1.8280e+03	182.7705	913.9806
28	32	0.1000	0.1000	3.0002	1.8280e+03	182.7705	914.0020
29	33	0.1000	0.1000	3.0003	1.8280e+03	182.7705	914.0233
30	34	0.1000	0.1000	3.0003	1.8281e+03	182.7705	914.0447
31	35	0.1000	0.1000	3.0004	1.8281e+03	182.7705	914.0661
32	36	0.1000	0.1000	3.0004	1.8282e+03	182.7705	914.0875
33	37	0.1000	0.1000	3.0004	1.8282e+03	182.7705	914.1089
34	38	0.1000	0.1000	3.0005	1.8283e+03	182.7705	914.1303
35	39	0.1000	0.1000	3.0005	1.8283e+03	182.7705	914.1517
36	40	0.1000	0.1000	3.0005	1.8283e+03	182.7705	914.1730

```
figure
%yyaxis left
subplot(1,2,1)
plot([buckling_initial.Temp], [buckling_initial.X].*[buckling_initial.Y], 'b-')
annotation('textbox', [0 0.9 1 0.1], ...
    'String', 'Thermal Effects on the Column', ...
    'EdgeColor', 'none', ...
    'HorizontalAlignment', 'center')
xlabel('Temperature (C)')
ylabel('Cross-Sectional Area (m^2)')
axis([5 40 9.995*10^(-3) 10.005*10^(-3)])
%yyaxis right
subplot(1,2,2)
plot([buckling_initial.Temp], [buckling_initial.Z], '-', 'Color', [0.91 0.41 0.17])
xlabel('Temperature (C)')
ylabel('Column Length (m)')
axis([5 40 2.9992 3.0006])
```

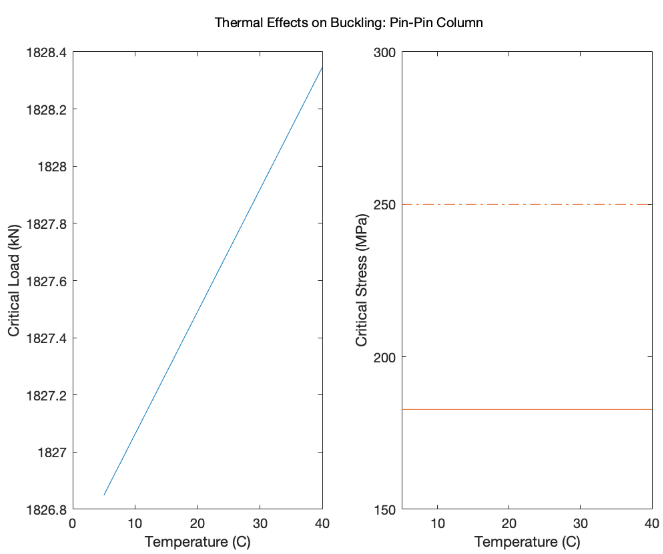


The dliemna here is that we can see that critical load will increase with increasing cross-sectional area, but it will decrease with increasing length. We therefore must analyze the critical load for all scenarios and plot to visually understand the optimal axially-loaded column dimensions for the design.

```
% Gate Thermal Analysis
```

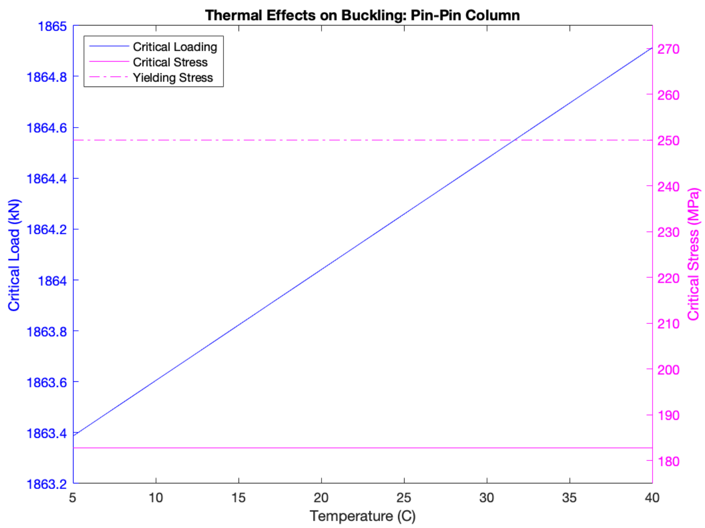
Buckling Analysis

```
% Plot initial conditions for buckling - safety factor of 2

figure
%yyaxis left
subplot(1,2,1)
plot([buckling_initial.Temp], [buckling_initial.Crit_Load])
annotation('textbox', [0 0.9 1 0.1], ...
    'String', 'Thermal Effects on Buckling: Pin-Pin Column', ...
    'EdgeColor', 'none', ...
    'HorizontalAlignment', 'center')
xlabel('Temperature (C)')
ylabel('Critical Load (kN)')
%yyaxis right
subplot(1,2,2)
plot([buckling_initial.Temp], [buckling_initial.Crit_Stress], '-', 'color', [0.91 0.41 0.17])
xlabel('Temperature (C)')
ylabel('Critical Stress (MPa)')
hold on
plot([buckling_initial.Temp], (ones(1,length(temp)).*sigma_y)./(10^6), '-.', 'color', [0.91 0.41 0.17])
axis([5 40 150 300])
%legend('Critical Loading', 'Critical Stress', 'Yielding Stress', 'Location', 'northwest')
hold off
```

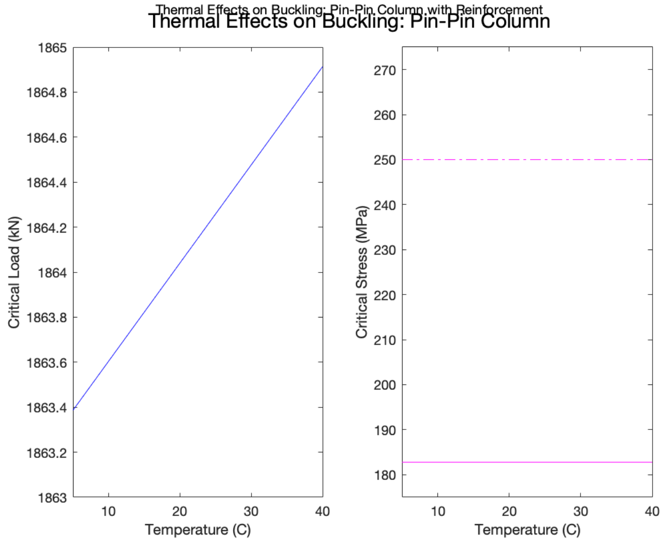


After several trials of different k-values, in order for this column to buckle before yielding (which we want), the k-value must be 1.0. This means it is a pin-pin supported column. There is no other way for the column to be supported (several trials were run for the various k-value that were commented in and out). We always want the critical stress to be less than the yielding stress throughout all temperatures.

```
% Find final conditions for buckling
% Column Dimensions - Final
column_final(temp(end)-temp(1)+1) = struct('Temp', 0, 'X', 0, 'Y', 0, 'Z', 0, 'Crit_Load', 0, 'Crit_Stress', 0, 'Max_All_Load'
for i = length(temp):-1:1
    column_final(i).Temp = temp(i);
    column_final(i).X = thermal_exp(alpha, (temp(i)-temp_i), (col_x+2*col_t)) + (col_x+2*col_t);
    column_final(i).Y = thermal_exp(alpha, (temp(i)-temp_i), col_y) + col_y;
    column_final(i).Z = thermal_exp(alpha, (temp(i)-temp_i), col_z) + col_z;
    [column_final(i).Crit_Load, column_final(i).Crit_Stress, column_final(i).Max_All_Load] = calc_buckling(column_final(i).X,
        column_final(i).Y, column_final(i).Z, E, k, 2);
end

figure
colororder({'b','m'})
yyaxis left
plot([column_final.Temp], [column_final.Crit_Load])
title('Thermal Effects on Buckling: Pin-Pin Column')
xlabel('Temperature (C)')
ylabel('Critical Load (kN)')
yyaxis right
plot([column_final.Temp], [column_final.Crit_Stress])
ylabel('Critical Stress (MPa)')
hold on
plot([column_final.Temp], (ones(1,length(temp)).*sigma_y)./(10^6), '-.')
axis([5 40 175 275])
legend('Critical Loading', 'Critical Stress', 'Yielding Stress', 'Location', 'northwest')
hold off
```




```
figure
subplot(1,2,1)
plot([column_final.Temp], [column_final.Crit_Load], 'b-')
annotation('textbox', [0 0.9 1 0.1], ...
    'String', 'Thermal Effects on Buckling: Pin-Pin Column with Reinforcement', ...
    'EdgeColor', 'none', ...
    'HorizontalAlignment', 'center')
sgtitle('Thermal Effects on Buckling: Pin-Pin Column')
xlabel('Temperature (C)')
ylabel('Critical Load (kN)')
axis([5 40 1863 1865])
subplot(1,2,2)
plot([column_final.Temp], [column_final.Crit_Stress], 'm-')
xlabel('Temperature (C)')
ylabel('Critical Stress (MPa)')
hold on
plot([column_final.Temp], (ones(1,length(temp)).*sigma_y)./(10^6), 'm-.')
axis([5 40 175 275])
hold off
```




```
column_final
```

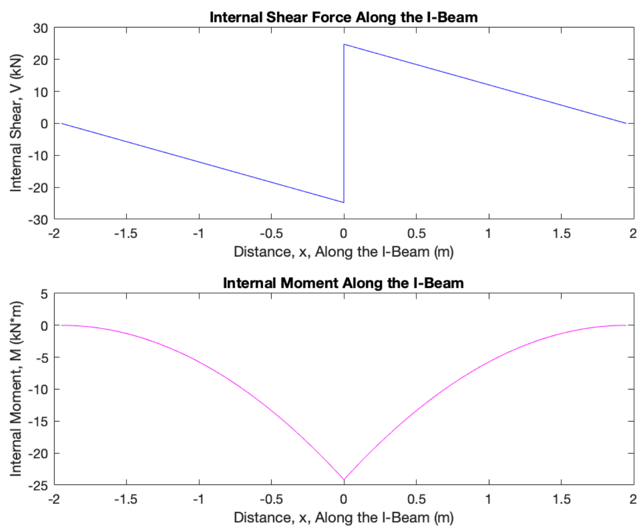
column_final = 1×36 struct

Fields	Temp	X	Y	Z	Crit_Load	Crit_Stress	Max_All_Load
1	5	0.1020	0.1000	2.9993	1.8634e+03	182.7705	931.6931
2	6	0.1020	0.1000	2.9993	1.8634e+03	182.7705	931.7149
3	7	0.1020	0.1000	2.9994	1.8635e+03	182.7705	931.7367
4	8	0.1020	0.1000	2.9994	1.8635e+03	182.7705	931.7585
5	9	0.1020	0.1000	2.9994	1.8636e+03	182.7705	931.7803
6	10	0.1020	0.1000	2.9995	1.8636e+03	182.7705	931.8022
7	11	0.1020	0.1000	2.9995	1.8636e+03	182.7705	931.8240
8	12	0.1020	0.1000	2.9995	1.8637e+03	182.7705	931.8458
9	13	0.1020	0.1000	2.9996	1.8637e+03	182.7705	931.8676
10	14	0.1020	0.1000	2.9996	1.8638e+03	182.7705	931.8894
11	15	0.1020	0.1000	2.9996	1.8638e+03	182.7705	931.9112
12	16	0.1020	0.1000	2.9997	1.8639e+03	182.7705	931.9330
13	17	0.1020	0.1000	2.9997	1.8639e+03	182.7705	931.9548
14	18	0.1020	0.1000	2.9998	1.8640e+03	182.7705	931.9766
15	19	0.1020	0.1000	2.9998	1.8640e+03	182.7705	931.9984
16	20	0.1020	0.1000	2.9998	1.8640e+03	182.7705	932.0202
17	21	0.1020	0.1000	2.9999	1.8641e+03	182.7705	932.0421
18	22	0.1020	0.1000	2.9999	1.8641e+03	182.7705	932.0639
19	23	0.1020	0.1000	2.9999	1.8642e+03	182.7705	932.0857
20	24	0.1020	0.1000	3.0000	1.8642e+03	182.7705	932.1075
21	25	0.1020	0.1000	3	1.8643e+03	182.7705	932.1293
22	26	0.1020	0.1000	3.0000	1.8643e+03	182.7705	932.1511
23	27	0.1020	0.1000	3.0001	1.8643e+03	182.7705	932.1729
24	28	0.1020	0.1000	3.0001	1.8644e+03	182.7705	932.1947
25	29	0.1020	0.1000	3.0001	1.8644e+03	182.7705	932.2166
26	30	0.1020	0.1000	3.0002	1.8645e+03	182.7705	932.2384
27	31	0.1020	0.1000	3.0002	1.8645e+03	182.7705	932.2602
28	32	0.1020	0.1000	3.0002	1.8646e+03	182.7705	932.2820
29	33	0.1020	0.1000	3.0003	1.8646e+03	182.7705	932.3038
30	34	0.1020	0.1000	3.0003	1.8647e+03	182.7705	932.3256
31	35	0.1020	0.1000	3.0004	1.8647e+03	182.7705	932.3474
32	36	0.1020	0.1000	3.0004	1.8647e+03	182.7705	932.3693
33	37	0.1020	0.1000	3.0004	1.8648e+03	182.7705	932.3911
34	38	0.1020	0.1000	3.0005	1.8648e+03	182.7705	932.4129
35	39	0.1020	0.1000	3.0005	1.8649e+03	182.7705	932.4347
36	40	0.1020	0.1000	3.0005	1.8649e+03	182.7705	932.4565

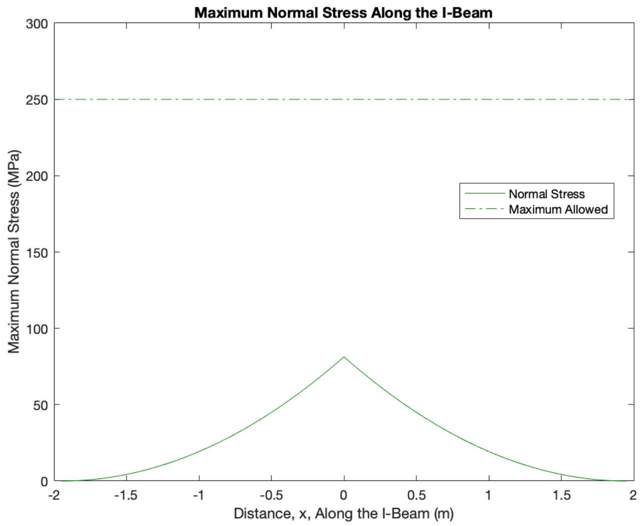
Bending Analysis

```
% Find I-beam with smallest cross-sectional area
small = find([ibeam_df.Area] == min([ibeam_df.Area]));
x_units = 0:0.001:(gate_x/2-col_x/2-col_t);
figure
subplot(2,1,1)
plot([x_units(end:-1:1).*(-1)], (-12.71376.*x_units), 'b-')
```

```
title('Internal Shear Force Along the I-Beam')
xlabel('Distance, x, Along the I-Beam (m)')
ylabel('Internal Shear, V (kN)')
hold on
plot(x_units, (24.792-12.71376.*x_units), 'b-')
hold on
plot(zeros(1,100), linspace(-24.792,24.792,100), 'b-')
hold off
subplot(2,1,2)
plot([x_units(end:-1:1).*(-1)], (-(12.71376/2).*(x_units.^2)), 'm-')
title('Internal Moment Along the I-Beam')
xlabel('Distance, x, Along the I-Beam (m)')
ylabel('Internal Moment, M (kN*m)')
hold on
plot(x_units, (-(12.71376/2).*(x_units.^2)+24.792.*x_units-24.172), 'm-')
hold off
```




```
% Calculate Flexure
% Find Neutral Axis
NA_IBeam = calc_NA(ibeam_df(small).FThick, ibeam_df(small).WThick, ibeam_df(small).Depth, ibeam_df(small).FWidth, gate_z);
c_val = NA_IBeam;
if (ibeam_df(small).Depth - NA_IBeam) > NA_IBeam
    c_val = (ibeam_df(small).Depth - NA_IBeam);
end

M_val = [abs(-(12.71376/2).*(x_units.^2)) abs(-(12.71376/2).*(x_units.^2)+24.792.*x_units-24.172)];
sigma_max = (max_norm_stress(M_val, c_val, ibeam_df(small).I_final))./1000; % To get Mega Pascals
figure
plot([x_units(end:-1:1).*(-1) x_units], sigma_max, '-', 'Color', [0 0.5 0])
title('Maximum Normal Stress Along the I-Beam')
xlabel('Distance, x, Along the I-Beam (m)')
ylabel('Maximum Normal Stress (MPa)')
axis([-2 2 0 300])
hold on
plot(linspace(-2,2,100), ones(1,100).*250, '-.', 'Color', [0 0.5 0])
legend('Normal Stress', 'Maximum Allowed', 'Location',"best")
hold off
```




```
max(sigma_max)
```

```
ans = 81.4316
```

The maximum normal stress found along the I-Beam is 81.43 MPa which is well below the maximum allowed of 250 MPa. Therefore, this I-Beam is still a wise choice.

Deflection Analysis

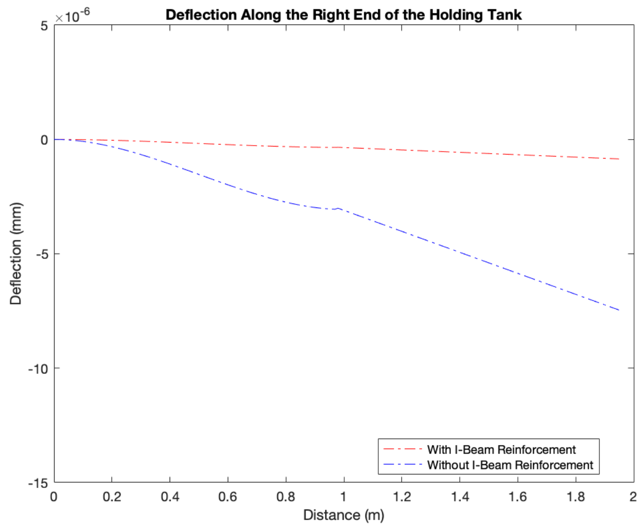
```
% Calculate deflection of Smallest I-Beam
def_ibeam = zeros(1,200);
x_def = linspace(0,(gate_x/2-col_x/2-col_t), 200);
for i = 1:length(def_ibeam)
    def_ibeam(i) = calc_deflect(24792, E, ibeam_df(small).I_final, (gate_x/2-col_x/2-col_t), x_def(i))/1000; % Make into mm
end
max(abs(def_ibeam))
```

```
ans = 8.5104e-07
```

```

% Calculate deflection of gate without support of I-Beam
def_gate = zeros(1, 200);
for i = 1:length(def_gate)
    def_gate(i) = calc_deflect(7651.8, E, (1.041667*10^(-6)), (gate_x/2-col_x/2-col_t), x_def(i))/1000;
end
max(abs(def_gate))
```

```
ans = 7.4567e-06
```

```
figure
plot(x_def, def_ibeam, 'r-.')
xlabel('Distance (m)')
ylabel('Deflection (mm)')
title('Deflection Along the Right End of the Holding Tank')
hold on
plot(x_def, def_gate, 'b-.')
axis([0 2 (-15*10^(-6)) (5*10^(-6))])
legend('With I-Beam Reinforcement', 'Without I-Beam Reinforcement', 'Location',"best")
hold off
```



Transverse Shear Analysis

```
% On the I-beam at 25 degC
shear_vec = [];
for i = 1:row
    shear_vec(i) = shear_stress_ibeam(ibeam_df(i).FThick, ibeam_df(i).WThick, ibeam_df(i).Depth, ibeam_df(i).FWidth, gate_z);
end
max(shear_vec)
```

```
ans = 8.8184e+03
```

```
shear_vec(small)
```

```
ans = 7.5226e+03
```

```
NA_IBeam
```

```
NA_IBeam = 0.0504
```

Based on the shear analysis, the maximum shear stress that will be experienced by any of the I-Beams in the structure is only 8.818 kPa, which is much less than the 150MPa limit.

Therefore, any of the I-Beams will suffice for holding the water. Also, since the thermal effects will be so small in comparison, we can assume that they're negligible here.

Since the goal is to control for cost and assuming that cost is directly proportional to I-Beam size, choose the I-Beam with the smallest cross-sectional area based on max shear.

```
function out = calc_Ivalue(tf, tw, d, bf, th)
% Returns moment of inertia of cross section about neutral axis based on
% reinforcing of gate of certain thickness, measuring from top
% Format of the call: calc_Ivalue()
A1 = bf.*(tf+th);
Z1 = (tf+th)./2;
A2 = tw.*(d-2.*tf);
Z2 = th+tf+(d-2.*tf)./2;
A3 = bf.*tf;
Z3 = th + tf + (d-2.*tf) + (tf./2);
NA = (A1.*Z1 + A2.*Z2 + A3.*Z3)./(A1 + A2 + A3);
I_1 = (1/12).*bf.*((tf+th).^3) + bf.*(tf+th).*((NA - (th+tf)./2).^2);
I_2 = (1/12).*tw.*((d-2.*tf).^3) + tw.*(d-2.*tf).*(th+tf+(d-2.*tf)-NA).^2;
I_3 = (1/12).*bf.*(tf.^3) + bf.*tf.*(th+(d-2.*tf)+(3/2).*tf-NA).^2;
out = I_1 + I_2 + I_3;
end

function out = calc_NA(tf, tw, d, bf, th)
A1 = bf.*(tf+th);
Z1 = (tf+th)./2;
A2 = tw.*(d-2.*tf);
Z2 = th+tf+(d-2.*tf)./2;
A3 = bf.*tf;
Z3 = th + tf + (d-2.*tf) + (tf./2);
out = (A1.*Z1 + A2.*Z2 + A3.*Z3)./(A1 + A2 + A3);
end

function out = thermal_exp(alpha, delT, len)
% Returns thermal expansion in meters of a material
% Format of the call: thermal_exp(alpha_co, deltaT, length)
```

```
out = alpha.*delT.*len;
end

function [P_cr, Sig_cr, P_all] = calc_buckling(dimx, dimy, dimz, YM, k_col, SF)
% Returns column's critical load (kN), critical stress (MPa), and max
% allowable load (kN)
% Format of the call: calc_buckling(cross-section dimension1, cross-section
% dimension2, length of column, young's modulus, k coefficient, safety factor)
A = dimx.*dimy;
I1 = (1/12).*dimx.*(dimy).^3;
I2 = (1/12).*dimy.*(dimx).^3;
if I1 > I2 % Must use smallest I
    I = I2;
else
    I = I1;
end
P_cr = ((pi^2.*YM.*I)./(k_col.*dimz).^2)./1000;
r = (I./A).^(1/2);
Sig_cr = ((pi^2.*YM)./((k_col.*dimz./r).^2))./(10^6);
P_all = P_cr./SF;
end

function out = max_norm_stress(M, c, I)
out = M.*c./I;
end

function out = weight(dim1, dim2, dim3, rho)
% Returns the weight force in kN based on volume and density
% Format of the call: weight(dimension1, dimension2, dimension3,
% density)
out = dim1.*dim2.*dim3.*rho.*9.81./1000;
end

function out = shear_stress_ibeam(tf, tw, d, bf, th)
% Returns the maximum shear of the IBeam
% Format of the call: shear_stress_ibeam()
A1 = bf.*(tf+th);
Z1 = (tf+th)./2;
A2 = tw.*(d-2.*tf);
Z2 = th+tf+(d-2.*tf)./2;
A3 = bf.*tf;
Z3 = th + tf + (d-2.*tf) + (tf./2);
NA = (A1.*Z1 + A2.*Z2 + A3.*Z3)./(A1 + A2 + A3);
I_1 = (1/12).*bf.*((tf+th).^3) + bf.*(tf+th).*((NA - (th+tf)./2).^2);
I_2 = (1/12).*tw.*((d-2.*tf).^3) + tw.*(d-2.*tf).*(th+tf+(d-2.*tf)-NA).^2;
I_3 = (1/12).*bf.*(tf.^3) + bf.*tf.*(th+(d-2.*tf)+(3/2).*tf-NA).^2;
Itot = I_1 + I_2 + I_3;
V = 1000*1.95*4*(bf/1000)*9.81; % Based on water density and dimensions of I-beam (no column)
Q = (NA - (tf+th)./2).*((tf+th).*bf)+(NA - (NA-tf-th)./2).*((NA-th-tf)./2).*(NA-tf-th).*(tw);
out = V.*Q./(Itot.*tw); % Max Shear
end

function out = calc_deflect(F, YM, MI, L, x)
% Calculates the deflection of cantilever with free end on right
if x < (L/2)
    constant = F/(12*YM*MI);
    eq = (2.*x.^3) - (3.*x.^2);
else
    constant = F/(48*YM*MI);
    eq = -6.*x + L;
end
out = constant*eq;
end
```