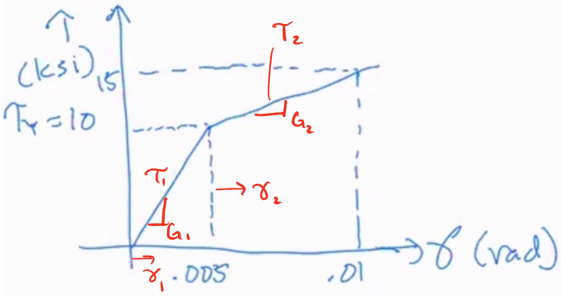
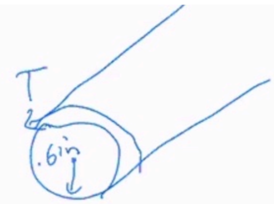
# Materials – Lecture #17

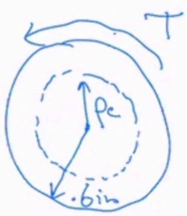
## TORSION REVIEW

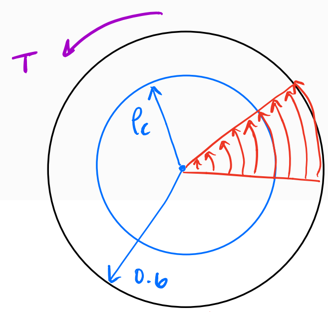
– First problem




Question!
– What is T to create an elastic core with radius $\rho_c = 0.5$ in




strain is linear and continuous

### Stress-strain Relationship

$$\frac{\tau_1}{\gamma} = \frac{10(10^3)}{0.005} \rightarrow \tau_1 = \underbrace{2(10^6)}_{G_1}\gamma$$

$$\frac{\tau_2 - 10(10^3)}{\gamma - .005} = \frac{15(10^3) - 10(10^3)}{.01 - .005} \Rightarrow \tau_2 = 1(10^6)\gamma + 5(10^3)$$

Find relationship between $\gamma$ and $\rho_c$ because $T = 2\pi\int_0^c \tau\rho^2 d\rho$

$$\gamma_{max} = \frac{0.6\text{in}}{0.5\text{in}}(.005)$$

strain in cross-section is still linear

$\gamma_y$ at $\rho_c = 0.5$ in $\Rightarrow$ .005 rad (?)

$\gamma_{max} =$ .006 radians
(at outer surface)

$$\gamma = \frac{\rho}{c}\gamma_{max} \rightarrow \gamma = \frac{\rho}{0.6\text{in}}(.006) \Rightarrow \gamma = .01\rho$$

$$\tau_1 = 2(10^6)\gamma = 2(10^6)(.01\rho) \Rightarrow \tau_1 = 20(10^3)\rho$$

$$\tau_2 = 1(10^6)\gamma + 5(10^3) = 1(10^6)(.01\rho) + 5(10^3) \Rightarrow \tau_2 = 10(10^3)\rho + 5(10^3)$$

$$T = 2\pi \int_0^c \tau \rho^2 \, d\rho$$

$$= 2\pi \int_0^{0.5} \overbrace{(20)(10^3)\rho}^{\tau_1} \rho^2 \, d\rho + 2\pi \int_{0.5}^{0.6} \overbrace{\left[(10)(10^3)\rho + 5(10^3)\right]}^{\tau_2} \rho^2 \, d\rho$$

$$T = 3970 \text{ lb in}$$

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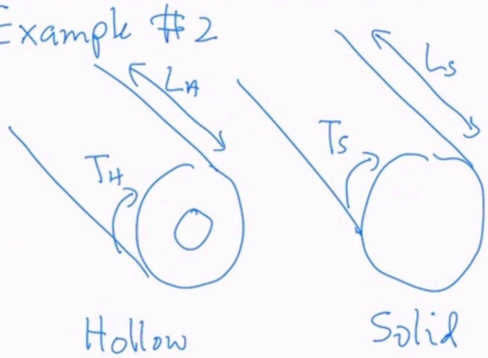
## Example #2

Example #2




$L_s = L_H$

$T_s = T_H$

- Both are same material
- Both have same mass

$$J_H = \frac{\pi}{2}(c_o^4 - c_i^4)$$

$$J_s = \frac{\pi}{2} c^4$$

1. $\phi_s < \phi_H$
2. $\phi_s = \phi_H$, both $\neq 0$
3. $\phi_s = \phi_H$, both $= 0$
4. $\phi_s > \phi_H$

$$\phi = \frac{TL}{JG}$$

$T, L, G$ are the same, $J$ is different

let's say $\frac{TL}{G} = k$ then $\phi = \frac{k}{J}$

$$\phi_H = \frac{k}{(\frac{\pi}{2})(c_o^4 - c_i^4)} \quad \text{vs.} \quad \phi_s = \frac{k}{(\frac{\pi}{2})c^4} \qquad \frac{1}{c_o^4 - c_i^4} \quad \text{vs.} \quad \frac{1}{c^4}$$

$m = m_H = m_s \quad \therefore \rho = \rho_H = \rho_s$

what he did

$$m_s = \rho \pi r_s^2 L \qquad m_H = \pi \rho (r_{oH}^2 - r_{iH}^2) L \quad \Rightarrow m_s = m_H$$

$$\rho \pi r_s^2 L = \pi \rho (r_{oH}^2 - r_{iH}^2) L \quad \rightarrow \underbrace{r_s^2 = r_{oH}^2 - r_{iH}^2}$$

substitute this into J! (for $J_s$)

$$J_s = \frac{\pi}{2}(r_{oH}^4 - 2 r_{oH}^2 r_{iH}^2 + r_{iH}^4)$$

vs.

$$J_H = \frac{\pi}{2}(r_{oH}^4 - r_{iH}^4)$$

$$\underbrace{\cancel{\frac{\pi}{2}}(\cancel{r_{oH}^4} - 2 r_{oH}^2 r_{iH}^2 + r_{iH}^4)}_{J_s} \quad \text{vs.} \quad \underbrace{\cancel{\frac{\pi}{2}}(\cancel{r_{oH}^4} - r_{iH}^4)}_{J_H}$$

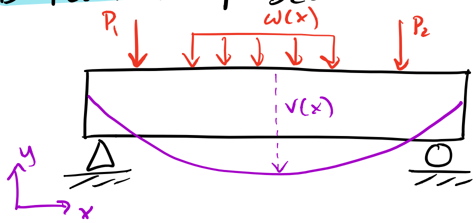
$$-2 r_{oH}^2 r_{iH}^2 + r_{iH}^4 \quad \text{vs} \quad -r_{iH}^4$$

assume that $r_{oH} > r_{iH}$

then $J_H > J_s \quad \therefore \phi_s > \phi_H$

Ch. 12 Hibbeler

## Deflection of Beams

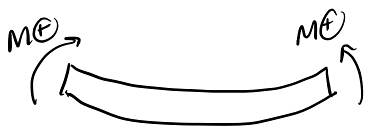



— = Hypothetical deflected shape $v(x)$

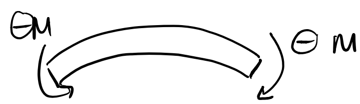
Are these beams going to deflect so much that they become structurally unstable?

- beams are designed to deflect downward
- need to place limits on how much deflection beams can undergo

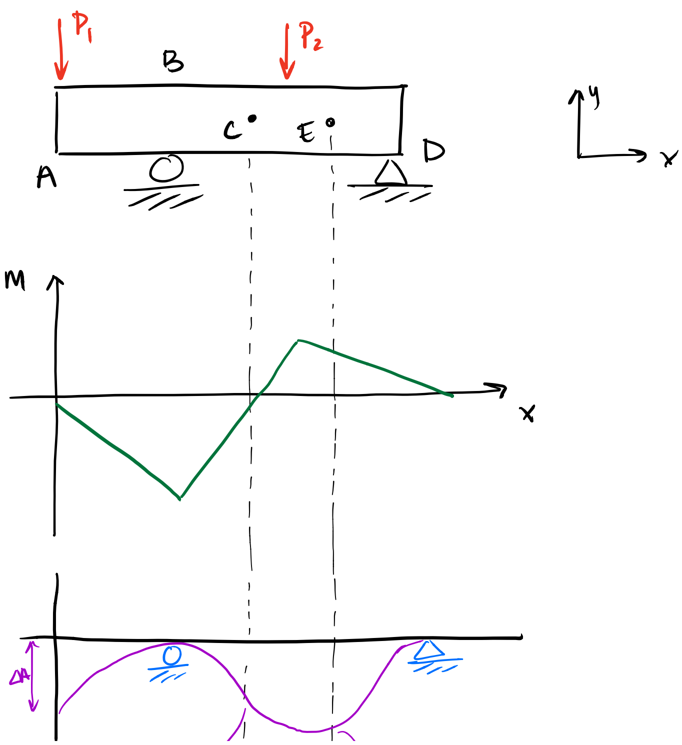
→ Goal: mathematically characterize deflection $v(x)$ as function of internal shear or moment




- positive internal moments
- concave up




- negative internal moments
- concave down

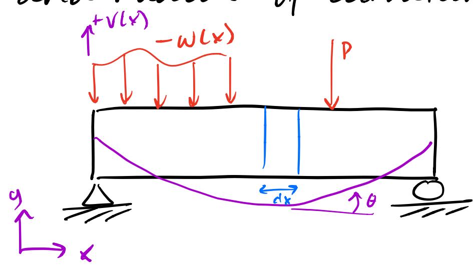


elastic curve = deflection pt.

inflection pt
where curve changes concavity
moment = 0

slope of "elastic curve" = 0

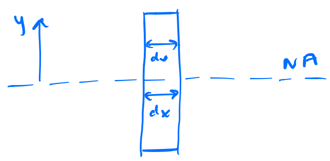
→ Develop relationship between internal moment in beam and radius of curvature $\rho$ of elastic curve



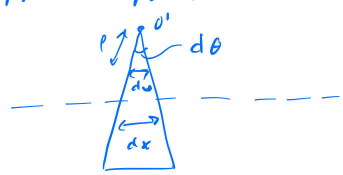

$\theta$ = slope of elastic curve

$v(x)$ = deflection of beam (positive up)

Before Deformation




after deformation




we want to find strain in ds: $\epsilon = \dfrac{ds' - ds}{ds}$

$ds = dx = \rho\, d\theta$

$ds' = (\rho - y)\, d\theta$

$$\epsilon = \frac{ds' - ds}{ds} = \frac{(\rho - y)\,d\theta - \rho\, d\theta}{\rho\, d\theta}$$

$$\hookrightarrow \frac{1}{\rho} = -\frac{\epsilon}{y}$$

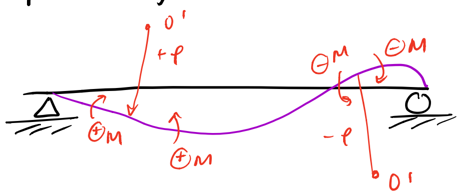
assume satisfies Hooke's Law, so it's a linear elastic material

$\epsilon = \dfrac{\sigma}{E}$ flexure formula $\sigma = -\dfrac{My}{I}$

$\epsilon = \dfrac{-My}{EI}$ then $\dfrac{1}{\rho} = \dfrac{M}{EI}$ $EI$ = "flexural rigidity" $> \phi$

... this tells us that sign of $\rho$ (radius of curvature) depends on sign of moment $M$

graphically, this means




... turn this into an equation of elastic curve deflection: $v(x)$

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{\frac{d^2v}{dx^2}}{\left[1+\left(\frac{dv}{dx}\right)^2\right]^{3/2}} \quad \leftarrow \text{from calculus}$$

Define $dv/dx = \theta$ = slope of elastic curve

→ assume $dv/dx$ is small → then if square small number, $d^2v/dx^2 \approx 0$

so then $\dfrac{d^2v}{dx^2} = \dfrac{M}{EI}$

take derivative of both sides

$$\frac{d}{dx}\left(EI\frac{d^2v}{dx^2}\right) = \frac{dM}{dx} = V(x) \quad \leftarrow \text{shear}$$

$$\frac{d^2}{dx^2}\left(EI\frac{d^2v}{dx^2}\right) = \frac{d^2M}{dx^2} = \frac{dV}{dx} = w(x) \quad \leftarrow \text{distributed load}$$

assume EI flexural rigidity is constant then

$$\boxed{\begin{aligned} EI\frac{d^4v}{dx^4} &= w(x) \\ EI\frac{d^3v}{dx^3} &= V(x) \\ EI\frac{d^2v}{dx^2} &= M(x) \end{aligned}}$$

← three differential equations

→ this implies that need to integrate $w(x)$ four times to get $v(x)$ and then need four constants of integration

→ need to integrate $M(x)$ two times to get the deflection $v(x)$

need two constants of integration

↳ get this from the boundary conditions of the problem

So we have three different ways to calculate the beam deflection (3 different differential equations)

↳ need to pick most convenient way for problem that doing

↳ generally involves solving

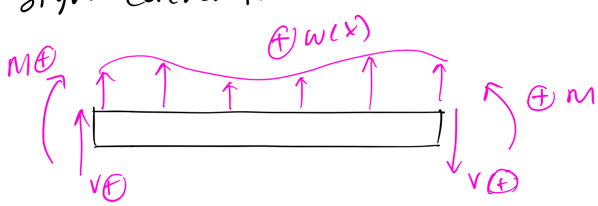
$$EI \frac{d^2v}{dx^2} = M(x)$$

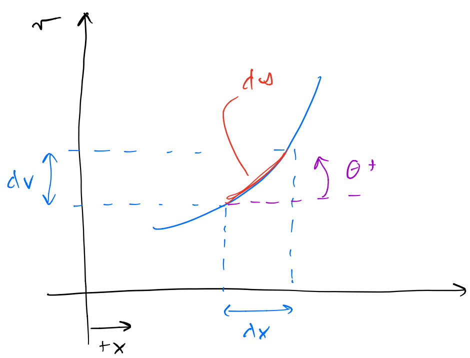
because only need 2 boundary conditions to integrate only twice

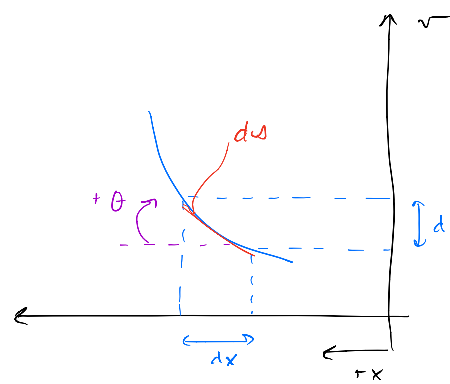
↓

then need to calculate internal moment diagram, i.e. $M(x)$

Sign Convention




for both cases

$$ds = \sqrt{(dx)^2 + (dv)^2}$$

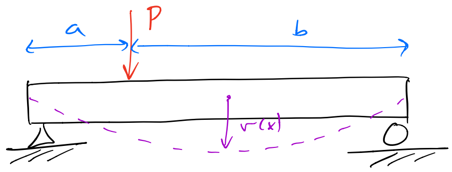
$$= \sqrt{1 + (\sim 0)}\ dx$$

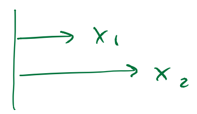
$$ds = dx$$

→ implies that points on elastic curve displaced vertically and not horizontally

→ then elastic curve represents deflection of neutral axis

## Continuity Conditions




where $0 \le x_1 \le a$
$0 \le x_2 \le b$

2 differential equations to solve

$$EI \frac{d^2 v_1}{dx^2} = M_1(x) \text{ and}$$

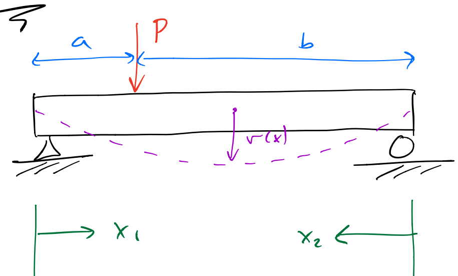
$$EI \frac{d^2 v_2}{dx^2} = M_2(x)$$

↑ we'll connect these DE's using continuity equations

Continuity conditions: ① $v_1(a) = v_2(a)$
② $\theta_1(a) = \theta_2(a)$

remember that $\theta$ = slope

BUT!




where $0 \le x_1 \le a$
$0 \le x_2 \le b$

Continuity conditions:

① $v_1(a) = v_2(b)$
② $\theta_1(a) = -\theta_2(b)$

slopes go in different directions bc $\theta_1$ is ⊕ ccw and $\theta_2$ is ⊕ cw