THERMO FINAL EXAM REVIEW

Isentropic Processes → constant entropy

Ideal gas model: $0 = s^\circ(T_2) - s^\circ(T_1) - R\ln\left(\frac{p_2}{p_1}\right)$

4 Property values: T_1, T_2, P_1, P_2

$$p_2 = p_1 \exp\left[\frac{s^\circ(T_2) - s^\circ(T_1)}{R}\right]$$

air modelled as an ideal gas, 2 states having same specific entropy

$$\frac{p_2}{p_1} = \frac{\exp[s^\circ(T_2)/R]}{\exp[s^\circ(T_1)/R]} \Rightarrow \frac{p_2}{p_1} = \frac{p_{r2}}{p_{r1}} \quad (s_1 = s_2\text{, air only})$$

* relative pressure *

where $p_{r1} = p_r(T_1)$, $p_{r2} = p_r(T_2)$

$v = \frac{RT}{P}$ → take ratio of specific volumes

$$\frac{v_2}{v_1} = \left(\frac{RT_2}{P_2}\right)\left(\frac{P_1}{RT_1}\right) \Leftrightarrow \left[\frac{RT_2}{P_r(T_2)}\right]\left[\frac{P_r(T_1)}{RT_1}\right]$$

for 2 states having same specific entropy

$$\Updownarrow \qquad \Updownarrow$$

$$v_r(T_2) \qquad 1/v_r(T_1)$$

* relative volume *

$$\frac{v_2}{v_1} = \frac{v_{r2}}{v_{r1}} \quad \text{for } s_1 = s_2 \text{ air only}$$

Isentropic Processes of ideal gas when Specific Heats Constant

$$0 = c_p \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1} \qquad 0 = c_v \ln\frac{T_2}{T_1} + R\ln\frac{v_2}{v_1}$$

→ ideal gas relations $c_p = \frac{kR}{k-1}$ $\quad c_v = \frac{R}{k-1}$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} \quad \text{for } s_1 = s_2\text{, constant } k$$

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{k-1}$$

$$\Leftrightarrow \quad \frac{P_2}{P_1} = \left(\frac{v_1}{v_2}\right)^{k} \quad \text{for } s_1 = s_2\text{, constant } k$$

The polytropic process $pv^k = C$ of an ideal gas w/ constant specific heat ratio k is an isentropic process.

⇒ from $pv^n = C$, for ideal gas $n=1$ == isothermal, for any fluid $n=0$ == isobaric

Isentropic Efficiencies – comparison between actual performance of a device and performance that would be achieved under idealized circumstances for the same inlet state and same exit pressure.

ISENTROPIC TURBINE EFFICIENCY

$\frac{\dot{W}_{cv}}{\dot{m}} = h_1 - h_2$

- heat xfer w/ surroundings ignored
- KE/PE effects ignored
- since state 1 fixed, sp. enthalpy known h_1

$W\uparrow$ as $h_2\downarrow$ ⇒ max value of turbine work corresponds to smallest allowed value for the specific enthalpy at exit

$\frac{\dot{\sigma}_{cv}}{\dot{m}} = s_2 - s_1 \geq 0$ bc entropy production $\frac{\dot{\sigma}_{cv}}{\dot{m}}$ can't be ⊖, states with $s_2 < s_1$ are not accessible in an adiabatic expansion

⇒ the only states that actually can be attained adiabatically are those with $s_2 > s_1$

⇒ smallest allowed value for h_2 corresponds to state 2s and then the maximum value for turbine work is $\left(\frac{\dot{W}_{cv}}{\dot{m}}\right)_s = h_1 - h_{2s}$

→ in actual expansion thru turbine $h_2 > h_{2s}$ and thus less work than max developed

$$\eta_t = \frac{\dot{W}_{cv}/\dot{m}}{(\dot{W}_{cv}/\dot{m})_s} = \frac{h_1 - h_2}{h_1 - h_{2s}}$$ typically 0.7 - 0.9

ISENTROPIC NOZZLE EFFICIENCIES

— ratio of actual specific KE of gas leaving nozzle to KE at exit that would be achieved in an isentropic expansion

$$\eta_{nozzle} = \frac{V_2^2/2}{(V_2^2/2)_s}$$ 95% + ... well designed, nearly free of internal reversibilities

ISENTROPIC COMPRESSOR & PUMP EFFICIENCIES

work input per unit mass flowing through compressor $\left(-\frac{\dot{W}_{cv}}{\dot{m}}\right) = h_2 - h_1$

— neg. heat xfer w/surr
— neg. KE/PE effects

→ value of work input depends on specific enthalpy at exit h_2

→ magnitude of work input ↓ as h_2 ↓

→ minimum work input corresponds to smallest allowed value for the specific enthalpy at the compressor exit

∴ smallest allowed enthalpy at exit state would be achieved in an isentropic compression from the specific inlet state to the specified exit pressure

$$\left(-\frac{\dot{W}_{cv}}{\dot{m}}\right)_s = h_{2s} - h_1$$ ⇐ minimum work input

⇒ in actual compression, $h_2 > h_{2s}$ thus more work than minimum required

$$\eta_c = \frac{(-\dot{W}_{cv}/\dot{m})_s}{(-\dot{W}_{cv}/\dot{m})} = \frac{h_{2s} - h_1}{h_2 - h_1}$$ typically 75% to 85%

η_p defined similarly

Isothermal, internally reversible $0 = \frac{\dot{Q}_{cv}}{T} + \dot{m}(s_1 - s_2) + \cancel{\dot{\sigma}_{cv}}^{\,0}$

$$\frac{\dot{Q}_{cv}}{\dot{m}} = T(s_2 - s_1) \iff \left(\frac{\dot{Q}_{cv}}{\dot{m}}\right)_{\substack{int\\rev}} = \int_1^2 T\,ds$$ [Area under T-s curve]

$$\left(\frac{\dot{W}_{cv}}{\dot{m}}\right)_{\substack{int\\rev}} = \int_1^2 T\,ds + (h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right) + g(z_1 - z_2)$$

$$\Rightarrow T\,ds = dh - v\,dp \Rightarrow \int_1^2 T\,ds = (h_2 - h_1) - \int_1^2 v\,dp$$

$$\Rightarrow \left(\frac{\dot{W}_{cv}}{\dot{m}}\right)_{\substack{int\\rev}} = \underbrace{-\int_1^2 v\,dp + \left(\frac{V_1^2 - V_2^2}{2}\right) + g(z_1 - z_2)}_{\text{applicable to turbines, compressors, pumps}}$$ [magnitude of integral is area shaded behind curve on PV diagram]

$$-\int_1^2 v\,dp + \left(\frac{V_1^2 - V_2^2}{2}\right) + g(z_1 - z_2) \rightarrow \text{if no KE/PE} \Rightarrow \left(\frac{\dot{W}_{cv}}{\dot{m}}\right)_{\substack{int\\rev}} = -\int_1^2 v\,dp$$

shows that work value is related to magnitude of v of gas or liquid as flows from inlet to exit

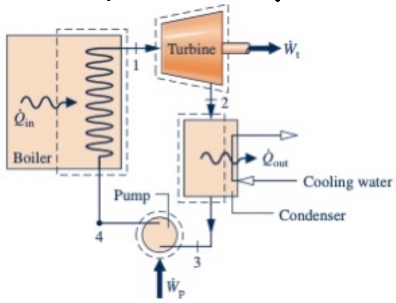
if v constant, as in many applications w/liquids

$$\left(\frac{\dot{W}_{cv}}{\dot{m}}\right)_{\substack{int\\rev}} = -v(p_2 - p_1) \qquad v = C, \Delta KE, \Delta PE = 0$$

*ex: for the same pressure rise, pump requires smaller work input per unit of mass flowing than compressor bc liquid v is much smaller than that of vapor (also correct for actual pumps & compressors where irreversibilities are present during operation

THE RANKINE CYCLE VAPOR POWER PLANTS

- 1st law of TD requires that net work developed by a system undergoing a power cycle must equal the net energy added by heat xfer to the system
- 2nd law of TD requires that the thermal efficiency of a power cycle must be less than 100%



*4 principal components: turbine, condenser, pump, boiler

*most large scale plants use water as the working fluid

*each component regarded as operating at steady-state for our purposes

① TURBINE vapor from the boiler @ state 1 w/ its elevated temp & press, expands thru turbine to produce work, then is discharged to condenser at state 2 w/ relatively low pressure

$$\frac{\dot{W}_t}{\dot{m}} = h_1 - h_2$$

② CONDENSER here there's heat xfer from working fluid to cooling H_2O flowing in separate stream. The working fluid condenses and temp of cooling water ↑

$$\frac{\dot{Q}_{out}}{\dot{m}} = h_2 - h_3$$

... energy xfer by heat from working fluid to cooling H_2O

③ PUMP liquid leaving condenser is pumped to into high press boiler $\frac{\dot{W}_p}{\dot{m}} = h_4 - h_3$ ... $\dot{W}_p/\dot{m}$ is work input

④ BOILER working fluid completes cycle as liquid leaving pump @ 4 (the "boiler feedwater") is heated to saturation & evaporated in boiler

$$\frac{\dot{Q}_{in}}{\dot{m}} = h_1 - h_4$$

... $\dot{Q}_{in}/\dot{m}$ = energy xfer by heat from energy source into working fluid per unit mass

THERMAL EFFICIENCY gauges extent to which energy input to working fluid passing thru boiler is converted to <u>net</u> work output
→ net work output = net heat input

$$\eta = \frac{\dot{W}_t/\dot{m} - \dot{W}_p/\dot{m}}{\dot{Q}_{in}/\dot{m}} = \frac{(h_1-h_2)-(h_4-h_3)}{h_1-h_4}$$

$$\eta = \frac{\dot{Q}_{in}/\dot{m} - \dot{Q}_{out}/\dot{m}}{\dot{Q}_{in}/\dot{m}} = 1 - \frac{\dot{Q}_{out}/\dot{m}}{\dot{Q}_{in}/\dot{m}} = 1 - \frac{(h_2-h_3)}{(h_1-h_4)}$$

HEAT RATE amount of energy added by heat xfer to cycle to produce a unit of net work output
→ heat rate is 1/α to η

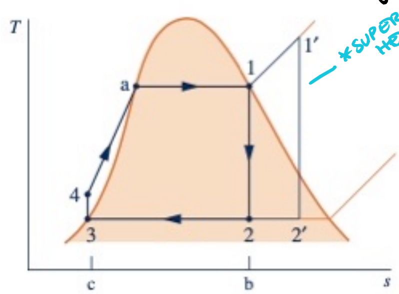
BACK WORK RATIO (BWR) ratio of pump work input to work developed by turbine

$$bwr = \frac{\dot{W}_p/\dot{m}}{\dot{W}_t/\dot{m}} = \frac{(h_4-h_3)}{(h_1-h_2)}$$

→ typically quite low since change in specific enthalpy for expansion of vapor through turbine is normally many times greater than the increase in enthalpy for the liquid passing through the pump

IDEAL RANKINE CYCLE

- w/out irreversibilities, no friction in boiler/condenser so working fluid would flow thru these components @ constant press
- processes thru turbine and pump are isentropic
- ideal Rankine cycle consists of internally reversible processes



SUPERHEAT

1′–2′–3–4–1′ = * example of * superheating

1→2 isentropic expansion of working fluid through turbine from sat. vapor at state 1 to the condenser press

2→3 heat xfer <u>from</u> working fluid as it flows @ constant press thru condenser exiting as sat. liq. at state 3

3→4 isentropic compression in pump to state 4 in compressed liquid region

4→1 heat xfer <u>to</u> working fluid as it flows at constant press through boiler to complete cycle

Area 1–b–c–4–a–1 heat xfer to working fluid passing thru boiler

Area 2–b–c–3–2 heat xfer from working fluid passing thru condenser

Enclosed Area 1–2–3–4–a–1 the net heat input per unit mass, or equivalently, net work output per unit mass

$$\left(\frac{\dot{W}_p}{\dot{m}}\right)_{\substack{int\\rev}} = v_3(p_4-p_3)$$ → isentropic, int. rev., adiabatic

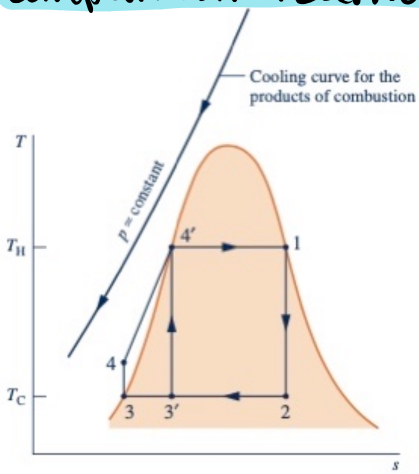
* η_{th} ↑ ave temp @ which energy added by heat xfer ↑ and/or ave temp @ which energy rejected by heat xfer ↓

Effects of Boiler and Condenser Pressures on the Rankine Cycle

↑ boiler pressure, ↑ η_{th}
↓ condenser press, ↑ η_{th}

* including a condenser in which steam side is operated at press below atmospheric, turbine then has lower-press region in which to discharge, resulting in significant ↑ in net work and η_{th}

Comparison w/ Carnot Cycle



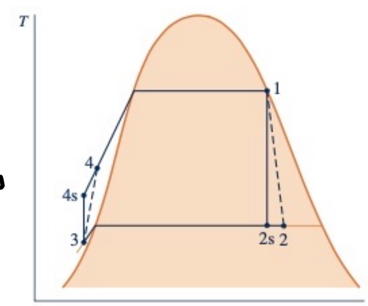
Rankine: 1-2-3-4-4'-1
Carnot: 1-2-3'-4'-1

$(\eta_{th})_{Rankine} < (\eta_{th})_{Carnot}$

but shortcoming of Carnot is that pump has to pump 2-phase mixture and it's better to condense vapor completely as in Rankine

Principal Irreversibilities and Losses

I. Internal Effects



- Turbine – actual adiabatic expansion thru turbine is accompanied by an ↑ in entropy
  - → the work developed is <u>less</u> than corresponding isentropic expansion 1→2s

$$\eta_t = \frac{(\dot{W}_t/\dot{m})}{(\dot{W}_t/\dot{m})_s} = \frac{h_1 - h_2}{h_1 - h_{2s}}$$ ← actual work / ← isentropic work

⇒ net result: ↓ power plant power output + η_{th}

- Pump – work input to pump required to overcome irreversibilities reduces net power output of plant
  - → actual pumping accompanied by ↑ entropy
  - → ∴ W_{input} actual > W_{input} isentropic 3→4s

$$\eta_p = \frac{(\dot{W}_p/\dot{m})_s}{(\dot{W}_p/\dot{m})} = \frac{h_{4s} - h_3}{h_4 - h_3}$$ ← actual and larger in magnitude than numerator

⇒ can also say $\eta_p = \frac{(\dot{W}_p/\dot{m})_s}{(\dot{W}_p/\dot{m})} = \frac{v_3(p_4 - p_3)}{h_4 - h_3}$

→ because pump work is much less than turbine work, irreversibilities in pump have smaller effect on η_{th} than turbine

II. External Effects

- energy discharged by heat xfer to cooling H_2O as working fluid condenses

IMPROVING PERFORMANCE — RANKINE

Remember:

increase in boiler press or decrease in condenser press may result in reduction of steam quality at exit of turbine
→ if quality becomes too low, erode turbine blades and cause ↓ in η_t (superheat & reheat avoid this)

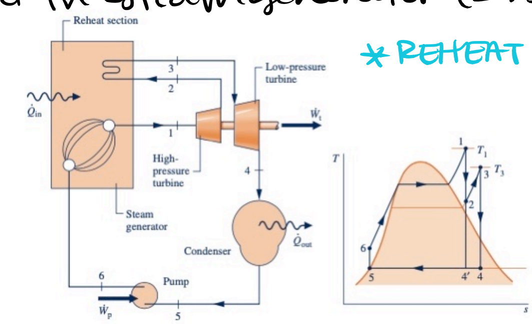
SUPERHEAT energy added by heat xfer to steam bringing it to a superheated vapor condition at turbine inlet
Steam generator = boiler + superheater

↑ η_{th} ↑ x of steam

REHEAT steam doesn't expand to condenser pressure in a single stage; instead, steam expands thru first stage turbine (1→2) to some press between steam generator and condenser pressures; steam then reheated in steam generator (2→3) and there's no pressure drop
→ after reheating, steam expands in 2nd stage turbine to condenser pressure (3→4)

x ↑

* REHEAT *

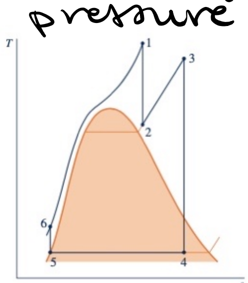


SUPERCRITICAL

temp of steam entering turbine is restricted by metallurgical limitations imposed by materials used to fabricate the superheater, reheater, and turbine

- high press in steam generator also requires piping that can withstand great stresses at elevated temperatures

* this process now allows vapor power plants to operate w/ steam generator pressures exceeding the critical pressure of water



Process 6→1: steam generation occurs at press above critical press (no phase change occurs, no boiler is used)

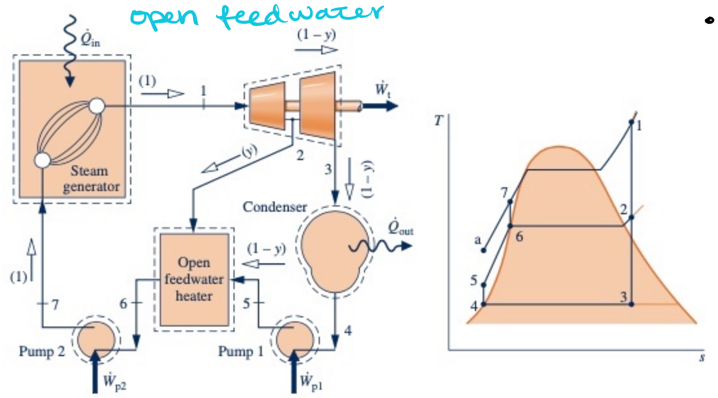
- H_2O flowing thru tubes is gradually heated from liquid to vapor w/out bubbling assoc w/ boil (heating provided by combustion of coal w/ air)

Improving Performance — Regenerative Vapor Power Cycle

↑ η_{th} ... ave temp of heat addition ↑ thereby ↑ efficiency

Open Feedwater Heaters

- type of direct-contact heat exchanger in which streams @ different temperatures mix to form a stream at an intermediate temp open feedwater



- working fluid passes isentropically thru the turbine stages and pumps, and flow thru the steam generator, condenser, and feedwater heater takes place w/no pressure drop in any of these components

1→2: steam enters 1st stage turbine @ 1 and expands to 2 where fraction of total flow is extracted (bleed) into an open feedwater heater operating at the extraction pressure p_2

2→3: rest of steam expands thru 2nd stage turbine to 3.

3→4: this portion of flow is condensed to sat. liq @ 4

4→5: then pumped to the extraction pressure and introduced into feedwater heater @ 5

5→6: single mixed stream exits feedwater heater @ 6
{ 2→6 (6 is sat. liq.)

6→7: liquid then pumped to steam generator pressure and enters 7

7→1: working fluid heated

* operating conditions are such that the reduction in heat added more than offsets the decrease in net work developed, resulting in increased η_{th} in regenerative powerplants)

→ looking @ 2 turbines here: (as the cv)

$$\dot{m}_2 + \dot{m}_3 = \dot{m}_1$$

$\dot{m}_1$ = rate entering turbine, $\dot{m}_2$ = rate extracted, $\dot{m}_3$ = rate exits turbine 2

$$\frac{\dot{m}_2}{\dot{m}_1} + \frac{\dot{m}_3}{\dot{m}_1} = 1$$

$y = \dot{m}_2/\dot{m}_1$ ⟹ then the fraction of the total flow passing thru 2nd stage turbine (see paratheses) $= \frac{\dot{m}_3}{\dot{m}_1} = 1 - y$

$0 = y h_2 + (1-y) h_5 - h_6$ [cons. of mass & energy)

$y = \frac{h_6 - h_5}{h_2 - h_5}$ ... can find fraction y when 2, 5, & 6 fixed

total turb. work : $\frac{\dot{W}_t}{\dot{m}_1} = (h_1 - h_2) + (1-y)(h_2 - h_3)$

total pump work : $\frac{\dot{W}_p}{\dot{m}_1} = (h_7 - h_6) + (1-y)(h_5 - h_4)$

energy added by heat xfer to working fluid passing thru steam gen. : $\frac{\dot{Q}_{in}}{\dot{m}_1} = h_1 - h_7$

energy rejected by heat xfer to cooling H_2O : $\frac{\dot{Q}_{out}}{\dot{m}_1} = (1-y)(h_3 - h_4)$

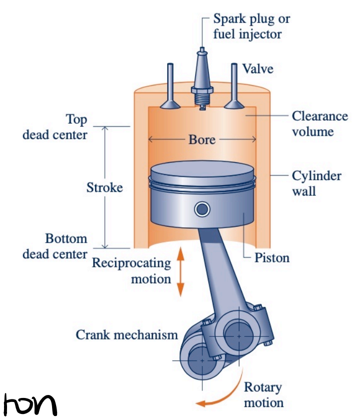
Closed Feedwater Heaters

- shell-and-tube-type recuperators in which the feedwater temp ↑ as extracted steam condenses on outside of tubes carrying feedwater
- since the 2 streams don't mix, they can be @ diff press.

INTERNAL COMBUSTION ENGINES – 2 principal types

1. SPARK-IGNITION ENGINE
   mixture of fuel & air ignited by a spark plug
2. COMPRESSION-IGNITION ENGINE
   air compressed to high enough press & temp that combustion occurs spontaneously when fuel injected

ENGINE TERMINOLOGY



Bore = cylinder diameter
stroke = distance piston moves in one direction

piston is @ TDC when it has moved to a position where cylinder volume is a minimum
→ minimum volume = clearance volume

when piston moves to position of max cylinder volume, piston @ BDC

Displacement volume = volume swept out by piston as it moves from TDC to BDC

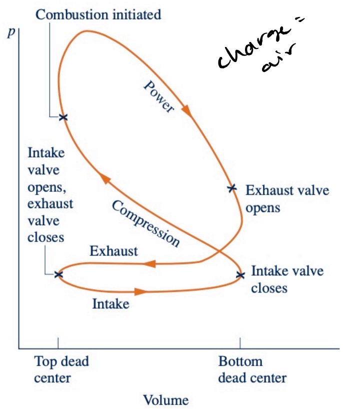
compression ratio, r : $\forall_{BDC} \div \forall_{TDC}$

→ the reciprocating motion of the piston is converted to rotary motion by a crank mechanism

mean effective pressure, mep: theoretical constant pressure that, if it acted on the piston during the power stroke, would produce same net work as actually developed in one cycle

$$mep = \frac{\text{net work for one cycle}}{\text{displacement } \forall}$$

⇒ for 2 engines of equal displacement ∀, one w/ higher mep would produce the greater net work and if the engines run @ same speed, greater power



In a 4-stroke internal combustion engine, the piston executes 4 distinct strokes within the cylinder for every 2 revolutions of the crankshaft.

(1) w/ intake valve open, piston makes intake stroke to draw fresh charge into cylinder
(2) w/ both valves closed, piston undergoes compression stroke, raising temp & press of charge... requires work input from piston to cylinder contents... combustion process then initiated, resulting in high press & high temp gas mixture
(3) Power stroke follows compression stroke during which gas mixture expands and work is done on piston as it returns to BDC
(4) Piston then executes exhaust stroke, burned gases purged from cylinder through open exhaust valve

Air Standard Analysis

- fixed amount of air modeled as IG
- combustion process is replaced by heat xfer from external source
- no exhaust and intake processes as in an actual engine; cycle completed by constant-$\forall$ heat xfer process taking place while piston is at BDC
- all processes are internally reversible

Cold Air-Standard Analysis

- specific heats are assumed constant @ their ambient temperature values

IDEAL GAS MODEL REVIEW

$pv = RT \qquad p\forall = mRT$

$u(T_2) - u(T_1) = \int_{T_1}^{T_2} c_v(T)\,dT$

$h(T_2) - h(T_1) = \int_{T_1}^{T_2} c_p(T)\,dT$

constant sp. heats
Tables A20, 21

variable sp. heats
Tables A22, 23

$u(T_2) - u(T_1) = c_v(T_2 - T_1)$

$h(T_2) - h(T_1) = c_p(T_2 - T_1)$

[AIR ONLY $\frac{P_2}{P_1} = \frac{P_{r2}}{P_{r1}} \quad \frac{v_2}{v_1} = \frac{v_{r2}}{v_{r1}}$]

$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}$

$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{k-1}$

$\frac{P_2}{P_1} = \left(\frac{v_1}{v_2}\right)^{k}$

$k = \frac{c_p}{c_v}$

constant sp. heats

$s(T_2, v_2) - s(T_1, v_1) = c_v \ln\left(\frac{T_2}{T_1}\right) + R \ln\left(\frac{v_2}{v_1}\right)$

$s(T_2, P_2) - s(T_1, P_1) = c_p \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{P_2}{P_1}\right)$

variable sp. heats

$s(T_2, P_2) - s(T_1, P_1) = s^\circ(T_2) - s^\circ(T_1) - R \ln\left(\frac{P_2}{P_1}\right)$