THERMO CH.4 NOTES

Conservation of mass principle

$$\begin{bmatrix}\text{time rate of change}\\ \text{of mass contained within}\\ \text{control volume at time } t\end{bmatrix} = \begin{bmatrix}\text{time rate of flow}\\ \text{of mass in across}\\ \text{inlet } i \text{ at time } t\end{bmatrix} - \begin{bmatrix}\text{time rate of flow}\\ \text{of mass out across}\\ \text{exit } e \text{ at time } t\end{bmatrix}$$

$$\frac{dm_{cv}}{dt} = \underbrace{\dot{m}_i \quad - \quad \dot{m}_e}_{\text{instantaneous mass flow rates}}$$

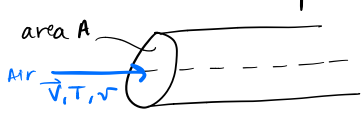
In general (because there may be several locations on the boundary through which mass enters or exits)

$$\frac{dm_{cv}}{dt} = \sum_i \dot{m}_i - \sum_e \dot{m}_e \qquad \text{and} \quad \dot{m} = \int_A \rho \vec{V}_n \, dA$$

One-Dimensional Flow Form of the Mass Rate Balance

→ flow is normal to boundary at locations where mass enters or exits the control volume

→ all intensive properties, including velocity and density are uniform w/position (bulk average values) over each inlet or exit area through which matter flows



$$\dot{m} = \rho A \vec{V} \quad \text{(1D flow)} \qquad A\vec{V} = \text{volumetric flow rate}$$

$$\dot{m} = \frac{A\vec{V}}{v} \quad \text{(1D flow)}$$

$$\frac{dm_{cv}}{dt} = \sum_i \frac{A_i \vec{V}_i}{v_i} - \sum_e \frac{A_e \vec{V}_e}{v_e} \quad \text{(1D flow)}$$

Steady-State Form of the Mass Rate Balance

→ steady-state: all properties are unchanging w/time

→ for a CV @ s-s the identity of the matter within the CV changes continuously but the total amount present at any instant remains constant so $dm_{cv}/dt = 0$

$$\underset{\text{(mass rate in)}}{\sum_i \dot{m}_i} = \underset{\text{(mass rate out)}}{\sum_e \dot{m}_e}$$

→ the equality of total incoming and outgoing rates of mass flow does not necessarily imply that a CV is at s-s.

- total amount of mass within CV at any instant would be constant, other properties such as temp and press might be varying w/time

→ when a CV is at s-s, <u>every</u> property is independent of time

mass flux: $\rho \vec{V}_n$, time rate of mass flow per unit area

$$m_{cv}(t) = \int_V \rho \, dV \Rightarrow \frac{d}{dt}\int_V \rho \, dV = \sum_i \left(\int_A \rho \vec{V}_n \, dA\right)_i - \sum_e \left(\int_A \rho \vec{V}_n \, dA\right)_e$$

Steady-State Application: mass rate balance

- for cv @ s-s, the conditions of mass w/in cv and at boundary do not vary w/time
- mass flow rates constant w/time
- volumetric flow rate at exit does not necessarily equal sum of volumetric flow rates at inlets

Time-Dependent (Transient) Application

- many devices undergo periods of operation during which the state changes with time
  - for ex. start up + shut down of motors
  - containers being filled/emptied

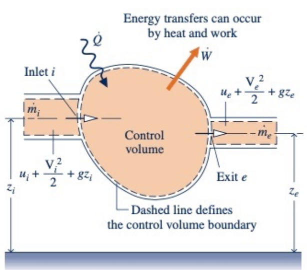
→ the steady-state model is not appropriate when analyzing time-dependent (transient cases)

Conservation of Energy for a Control Volume

- energy is an extensive property (like mass) so it too can be transferred into or out of a control volume as a result of mass crossing the boundary

Conservation of Energy applied to a cv

[time rate of change of the energy contained w/in the cv at time t] = [net rate at which energy is being xferred in by Q at time t] − [net rate at which energy is being xferred out by work at time t] + [net rate of energy transfer into the cv accompanying mass flow]



$$\frac{dE_{cv}}{dt} = \dot{Q} - \dot{W} + \dot{m}_i\left(u_i + \frac{\vec{V}_i^2}{2} + gz_i\right) - \dot{m}_e\left(u_e + \frac{\vec{V}_e^2}{2} + gz_e\right)$$

(for one inlet, one exit control volume w/1D flow)

→ if there's no mass flow in or out, respective mass flow rates vanish and equation reduces to rate form of energy balance for closed systems

Evaluating Work for a cv

- since work is done on or by cv where matter flows across the boundary, convenient to separate $\dot{W}$ into two contributions
  - (1) work associated w/fluid pressure as mass is introduced at inlets and removed at exits
  - (2) $\dot{W}_{cv}$, includes all other work effects, such as those associated with rotating shafts, displacement of boundary, and electrical effects

$$\dot{W} = \dot{W}_{cv} + (p_e A_e)\vec{V}_e - (p_i A_i)\vec{V}_i \qquad A\vec{V} = \dot{m} v$$

$$\dot{W} = \dot{W}_{cv} + \underbrace{\dot{m}_e(p_e v_e) - \dot{m}_i(p_i v_i)}_{\text{flow work}}$$

One Dimensional Flow Form of the Control Volume Energy Rate Balance

$$\frac{dE_{cv}}{dt} = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_i\left(u_i + p_i v_i + \frac{\vec{V}_i^2}{2} + gz_i\right) - \dot{m}_e\left(u_e + P_e v_e + \frac{\vec{V}_e^2}{2} + gz_e\right)$$

$$h = u + pv$$

$$\frac{dE_{cv}}{dt} = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_i\left(h_i + \frac{\vec{V}_i^2}{2} + gz_i\right) - \dot{m}_e\left(h_e + \frac{\vec{V}_e^2}{2} + gz_e\right)$$

Accounting Balance for Energy of Control Volume

→ mechanisms of energy xfer are heat and work (like closed systems) and energy that accompanies the mass entering/exiting

$$\frac{dE_{cv}}{dt} = \dot{Q}_{cv} - \dot{W}_{cv} + \sum_i \dot{m}_i\left(h_i + \frac{\vec{V}_i^2}{2} + gz_i\right) - \sum_e \dot{m}_e\left(h_e + \frac{\vec{V}_e^2}{2} + gz_e\right)$$

→ change in energy of a C∀ over a time period can be obtained by integration of the energy rate balance wrt time

$$E_{cv}(t) = \int_{\forall} \rho e \, d\forall = \int_{\forall} \rho\left(u + \frac{\vec{V}^2}{2} + gz\right) d\forall$$

$$= \dot{Q}_{cv} - \dot{W}_{cv} + \sum_i \left[\int_A \left(h + \frac{\vec{V}^2}{2} + gz\right)\rho \vec{V}_n \, dA\right]_i - \sum_e \left[\int_A \left(h + \frac{\vec{V}^2}{2} + gz\right)\rho \vec{V}_n \, dA\right]_e$$

Analyzing Control Volumes at Steady State

→ steady-state forms do not apply to the transient start up or shut down periods of operation of devices, only to periods of steady operation

For C∀ at s-s, conditions of mass w/in and at boundary do not vary w/time. Mass flow rates and rates of energy transfer by heat and work are also constant w/time

$$\frac{dm_{cv}}{dt} = 0 \Rightarrow \underbrace{\sum_i \dot{m}_i}_{\text{(mass rate in)}} = \underbrace{\sum_e \dot{m}_e}_{\text{(mass rate out}}$$

$$\frac{dE_{cv}}{dt} = 0 \Rightarrow 0 = \dot{Q}_{cv} - \dot{W}_{cv} + \sum_i \dot{m}_i\left(h_i + \frac{\vec{V}_i^2}{2} + gz_i\right) - \sum_e \dot{m}_e\left(h_e + \frac{\vec{V}_e^2}{2} + gz_e\right)$$

alternatively . . .

$$\underbrace{\dot{Q}_{cv} + \sum_i \dot{m}_i\left(h_i + \frac{\vec{V}_i^2}{2} + gz_i\right)}_{\text{energy rate in}} = \underbrace{\dot{W}_{cv} + \sum_e \dot{m}_e\left(h_e + \frac{\vec{V}_e^2}{2} + gz_e\right)}_{\text{energy rate out}}$$

→ for mass rate balance, $\dot{m}_1 = \dot{m}_2$

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \frac{(\vec{V}_1^2 - \vec{V}_2^2)}{2} + g(z_1 - z_2)\right] \quad \text{or}$$

$$0 = \frac{\dot{Q}_{cv}}{\dot{m}} - \frac{\dot{W}_{cv}}{\dot{m}} + (h_1 - h_2) + \frac{(\vec{V}_1^2 - \vec{V}_2^2)}{2} + g(z_1 - z_2)$$

General Modeling Considerations for CV at Steady State

→ assume s-s operation
→ assume flow is 1D at exit/entrance
→ if $\dot{Q}_{cv} = \emptyset$ (either bc it is or is very small compared to other terms)
- outer surface of CV is well insulated
- outer surface area too small for there to be effective heat xfer
- temp diff between CV and its surroundings is so small that the heat xfer can be ignored
- gas or liquid passes through CV so quickly that there's not enough time for significant heat xfer to occur

→ if $\dot{W}_{cv} = \emptyset$
- no rotating shafts
- no displacements of boundary
- no electrical effects

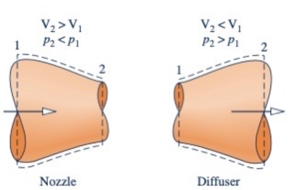
* steady-state can be assumed in cases of periodic time observations

Steady-State assumption can be applied to control volumes enclosing devices if these are satisfied:
(1) no net change in total energy and total mass w/in CV
(2) time-averaged mass flow rates, heat xfer rates, work rates, and properties of substances crossing all remain constant

Nozzles + Diffusers

nozzle — flow passage of varying cross-sectional area in which velocity of gas or liquid increases in direction of flow

diffuser — gas or liquid decelerates in direction of flow

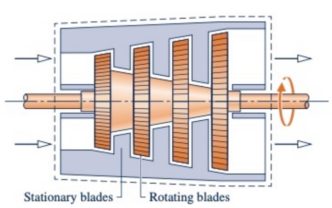


modeling considerations:
- only work is flow work at inlet/outlet of CV so W_{cv} drops out
- ΔPE is negligible
- often Q_{cv} can be neglected too bc avoidable (stray heat xfer)

$$0 = \dot{Q}_{cv} + \dot{m}\left(h_1 - h_2 + \frac{\vec{V}_1^2 - \vec{V}_2^2}{2}\right) \qquad 0 = (h_1 - h_2) + \frac{\vec{V}_1^2 - \vec{V}_2^2}{2}$$

Turbines

turbine — device in which power is developed as a result of a gas or liquid passing through a set of blades attached to a shaft free to rotate



→ widely used for power generation in vapor power plants, gas turbine power plants, aircraft engine

→ superheated steam or a gas enters the turbine and expands to a lower pressure as power is made

→ generator converts shaft power to electricity

turbine modeling considerations

- KE, PE negligible

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}(h_1 - h_2)$$

note that any $\dot{Q}_{cv}$ would be small in magnitude and ⊖ since xfer would be from turbine to surroundings

- often heat transfer is stray/unavoidable and small enough to be neglected

$$\dot{W}_{cv} = \dot{m}(h_1 - h_2)$$

Compressors + Pumps

- devices in which work is done on the substance flowing through them in order to change the state of the substance, typically to ↑ the pressure and/or elevation

compressor – when substance is a gas

pump – when substance is a liquid

modeling considerations

- heat xfer w/surroundings is frequently a secondary effect

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}(h_1 - h_2) \Rightarrow \dot{W}_{cv} = \dot{m}(h_1 - h_2)$$

- value of $\dot{W}_{cv}$ is negative because power input is required

Heat Exchangers

- common type of heat exchanger is a mixing chamber in which hot and cold streams are mixed directly
- another common type is one in which a gas or liquid is separated from another gas or liquid by a wall through which energy is conducted

modeling considerations

- only work is flow work so $\dot{W}_{cv}$ drops out of energy rate balance
- KE & PE can usually be ignored

$$0 = \dot{Q}_{cv} + \sum_i \dot{m}_i h_i - \sum_e \dot{m}_e h_e$$

- heat xfer w/surroundings is often small enough to be neglected so the $\dot{Q}_{cv}$ term can drop out to just enthalpy terms

Throttling Devices

- a significant reduction in pressure can be achieved by introducing a restriction into a line through which a gas or liquid flows, commonly done by means of a partially opened valve or porous plug

modeling considerations

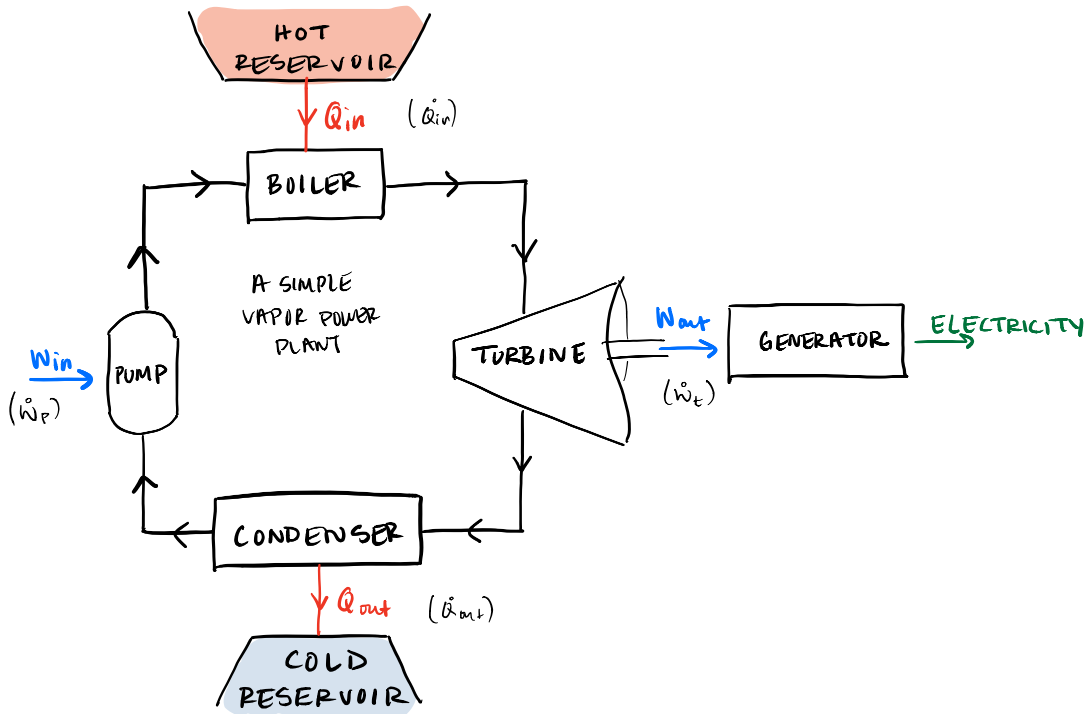
- only work is flow work at locations where mass enters and exits the cv so $\dot{W}_{cv}$ term drops out
- usually no significant heat xfer w/surroundings
- PE is negligible

$$0 = (h_1 - h_2) + \frac{\vec{V}_1^2 - \vec{V}_2^2}{2}$$

- if KE neglected then $h_2 = h_1$ $(P_2 < P_1)$

we call this "idealized" throttling process

System Integration



Transient Analysis

- many devices undergo periods of transient operation during which the state changes w/ time
  for example: startup/shutdown of turbines, compressors, motors

- because property values, work and heat transfer rates, and mass flow rates may vary w/ time during transient operation, the steady-state assumption is not appropriate when analyzing such cases

Mass Balance in Transient Analysis

$$\int_0^t \left(\frac{dm_{cv}}{dt}\right)dt = \int_0^t \left(\sum_i \dot{m}_i\right)dt - \int_0^t \left(\sum_e \dot{m}_e\right)dt = m_{cv}(t) - m_{cv}(0)$$

$\Downarrow$ m_i: amount of mass entering the CV through inlet i, from time 0 to t

$\Downarrow$ m_e: amount of mass exiting CV through exit e, from time 0 to t

mass balance: $m_{cv}(t) - m_{cv}(0) = \sum_i m_i - \sum_e m_e$

↳ meaning, the change in the amount of mass contained in the control volume equals difference between the total incoming and outgoing amounts of mass