

Refrigerator Example

- inside compartment acts as cold body and ambient air surrounding refrigerator is hot body
- energy Q_{in} passes to circulating refrigerant from the food inside comp.
→ for heat xfer to occur, refrigerant temp is necessarily below that of refrigerator contents
- energy Q_{out} passes from refrigerant to surrounding air
→ for heat xfer to occur, temp of circulating refrigerant must necessarily be above that of surrounding air
- to achieve these effects, a work input is required
- for a fridge, W_{input} is provided in form of electricity

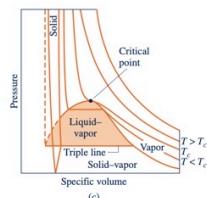
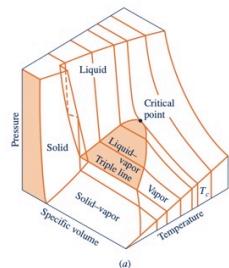
Phase - refers to quantity of matter that is homogeneous throughout in both chemical composition and physical structure
ex. all matter is solid, or all liquid, or all vapor
→ a system can contain one or more phases

* 2 phases coexist during the changes in phase called vaporization, melting, and sublimation

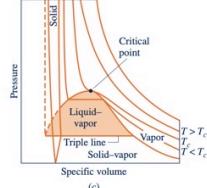
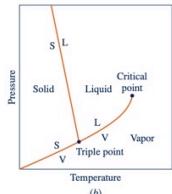
Pure substance - one that is uniform and invariantable in chemical composition, can exist in more than one phase but its chemical composition must be same in each phase
ex. if liquid H_2O and H_2O vapor form a system w/two phases, the system can be regarded as a pure substance because each phase has the same composition

* PURE, SIMPLE COMPRESSIBLE SUBSTANCES

→ look at relations among these properties w/pressure, specific volume, and temperature



← note the appearance of constant temp lines ... ISOTHERMS

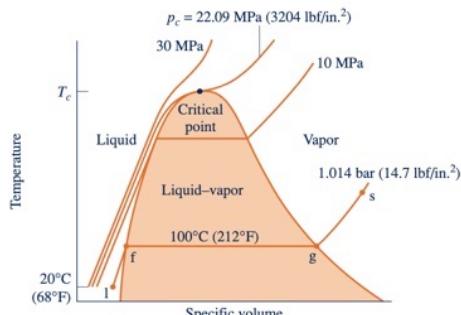


- within 2-phase regions, the state can be fixed by specific volume and either temperature or pressure
 - liquid-vapor, solid-liquid, and solid-vapor
- 2-phases can coexist during changes in phase such as vaporization, melting, and sublimation
- within the 2-phase region, pressure & temp are not independent (one cannot be changed w/out changing the other)
 - ∴ the state cannot be fixed by temp and press alone → the state can be fixed by τ and either press or temp
- 3 phases can exist in equilibrium along the triple line.
- * a state @ which a phase change begins or ends is the saturation state
- * vapor dome - composed of 2-phase liquid-vapor states
- * critical point - where saturated liquid and saturated vapor lines meet
- * critical temp (T_c) of pure substance is max temperature at which liquid and vapor phases can coexist in equilibrium
 - critical pressure (P_c)
 - critical specific volume (v_c)

Pr-diagram

- for any specified temp less than the critical temp, press remains constant as the 2-phase liquid-vapor region is traversed
- for temps greater than or equal to critical temp, press ↓ continuously at fixed temperature as $v \uparrow$ and there is no passage across the two-phase liquid-vapor region
- the critical isotherm passes thru a point of inflection at the critical pt and the slope is zero there

T-v-diagram



- for pressures less than the critical press such as 10 MPa isobar, pressure remains constant w/temp as two-phase region is traversed
- for pressures greater than or equal to crit press (such as 30 MPa), temp ↑ continuously at fixed pressure as $v \uparrow$ and there is no passage across the two-phase liquid-vapor region

* note the appearance of constant pressure lines
150 kbps

From previous Tr-diagram for H₂O

→ Pt. (1): 1 kg H₂O in closed system at 20 °C in piston-cylinder... the H₂O is slowly heated while its press is kept constant and uniform throughout at 1.014 bar (see position (a))

→ Pt. (f): as system heated at constant press, temp ↑ considerably while r ↑ slightly. This is sat. liq. state corresponding to specified pressure
* these states are referred to as COMPRESSED

↑ LIQUID STATE ↑

↓ now two phase liq-vapor ↓

→ then addnl heat xfer @ fixed press results in formation of vapor w/out any change in temp but w/considerable ↑ in r as shown in (b) where the system now consists of 2-phase liquid-vapor mixture

→ if system heated further until last bit of liquid has vaporized, it is brought to the sat. vapor state (Pt.(g))

↳ this intervening 2-phase liquid-vapor mixture states can be distinguished from one another by the quality (an intensive property)

$$x = \frac{m_{vap}}{m_{liq} + m_{vap}}$$

x = 0 at saturated liquid

x = 1.0 at saturated vapor

↓ VAPOR STATES ↓

° superheated vapor - state in which system is at a temperature greater than the sat. temperature corresponding to the given pressure

→ further heating at fixed pressure results in ↑ in both temp and r

ex. now say same process but at 10 MPa

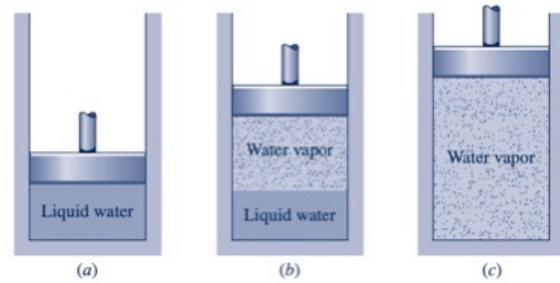
- vapor would form at higher T now bc sat p is higher

- less of an ↑ in r as evidenced from narrowing vapor dome

ex. now say heated @ crit pressure or higher

- there would be no phase change from liquid to vapor; at all states there's only one phase

∴ vaporization & condensation can only occur when p < crit-p



USING SATURATION TABLES

$$H = H_L + H_V$$

← process to find specific vol of 2-phase liq-vap mixture

$$\text{divide by mass: } \bar{v} = \frac{\bar{V}}{m} = \frac{H_L}{m} + \frac{H_V}{m}$$

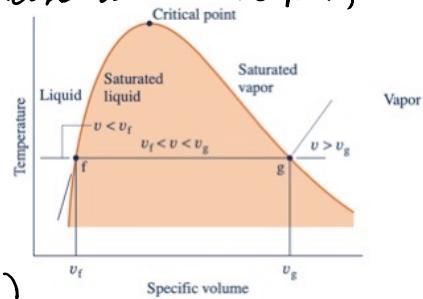
since liquid phase is sat. liq and vapor phase is sat. vapor,
 $H_{\text{liq}} = m_L v_f$ and $H_{\text{vap}} = m_V v_g$ so

$$\bar{v} = \left(\frac{m_L}{m}\right) v_f + \left(\frac{m_V}{m}\right) v_g$$

knowing that

$$x = \frac{m_V}{m} \text{ and } \frac{m_L}{m} = 1 - x \text{ then}$$

$$\bar{v} = (1-x) v_f + x v_g = v_f + x(v_g - v_f)$$



ENTHALPY

Enthalpy - sum of internal energy U and product of pressure P and volume V

$$H = U + PV \quad \text{since } U, P, V \text{ are all properties, this combo is also a property}$$

enthalpy can be expressed per unit mass basis:

$$h = u + PV$$

enthalpy can be expressed per mole basis:

$$\bar{h} = \bar{u} + \bar{P}\bar{V}$$

→ units are same as those for internal energy

- the specific internal energy for a two-phase liquid-vapor mix

$$u = (1-x) u_f + x u_g = u_f + x(u_g - u_f)$$

- the specific enthalpy for a two-phase liquid-vapor mix

$$h = (1-x) h_f + x h_g = h_f + x(h_g - h_f)$$

In applications where changes in KE & gravitational potential energy between end states can be ignored (for processes between end states denoted 1 & 2)

$$\Delta U_{\text{between end states}} = Q - W$$

↓ xfer of energy by heat between sys. & sur.
 ↓ xfer of energy by work between system and surroundings

WHEN SYSTEM HAS ONE OR TWO PHASES

case (1): system consisting initially and finally of a single phase of water (vap or liq)

$$m(u_2 - u_1) = Q - W$$

→ when the initial & final temps T_1, T_2 and press p_1, p_2 are known, the internal energies u_1 and u_2 can be obtained from steam tables

case (2): system consisting initially of H_2O vapor and finally as 2-phase mix of $m_1 H_2O$ and $m_2 H_2O$ vap

$$U_1 = m_1 u_1 \quad \text{and} \quad U_2 = (U_1 + U_v) \\ = m_1 u_f + m_2 u_g$$

case (3): system consisting initially of 2 separate masses of H_2O vapor that mix to form a total mass of H_2O vapor

$$U_1 = m' u(T', p') + m'' u(T'', p'')$$

$$U_2 = (m' + m'') u(T_2, p_2) \Rightarrow = m u(T_2, p_2)$$

SPECIFIC HEATS

$$C_V = \left(\frac{\partial u}{\partial T} \right)_V \quad C_P = \left(\frac{\partial h}{\partial T} \right)_P \quad \text{where the subscripts denote the variables held fixed during differentiation}$$

Specific Heats, C_V and C_P (these are INTENSIVE PROPERTIES), defined for pure, simple compressible substances

$$k = \frac{C_P}{C_V} = \frac{\text{specific heat ratio}}{\downarrow \text{SI Units} \quad \downarrow \text{English Units}} \quad \frac{\text{KJ}}{\text{Kg} \cdot \text{K}} \text{ or } \frac{\text{KJ}}{\text{kmol}} \quad \frac{\text{Btu}}{\text{lb} \cdot \text{R}} \text{ or } \frac{\text{Btu}}{\text{lbmol} \cdot \text{R}}$$

In general, C_V is a function of V and T (or p and T) while C_P depends on both p and T (or V and T) ... $C_P \propto 1/T$

INCOMPRESSIBLE SUBSTANCE MODEL

• for liquids or solids, ν (density) assumed to be constant & assumed to vary only w/temp ... a substance idealized this way is called incompressible

↳ then specific heat C_V also a function of temp alone

$$C_V(T) = \frac{du}{dT} \quad (\text{incompressible})$$

• although ν constant and internal energy depends on T only, enthalpy varies w/both p and T according to:

$$h(T, p) = u(T) + \nu p v \quad (\text{incompressible})$$

• therefore, for substances modeled as incompressible, $C_V = C_P$

$$u_2 - u_1 = \int_{T_1}^{T_2} c(T) dT \quad h_2 - h_1 = u_2 - u_1 + \nu(p_2 - p_1) = \int_{T_1}^{T_2} c(T) dT + \nu(p_2 - p_1)$$

If c taken as constant, then: $u_2 - u_1 = c(T_2 - T_1)$ this term is sometimes dropped if small

UNIVERSAL GAS CONSTANT, \bar{R}

$$\bar{R} = \left\{ \begin{array}{l} 8.314 \text{ J/Kmol} \cdot \text{K} \\ 1.986 \text{ cal/mol} \cdot {}^\circ\text{R} \\ 1545 \text{ ft} \cdot \text{lbf/mol} \cdot {}^\circ\text{R} \end{array} \right.$$

COMPRESSIBILITY FACTOR Z

compressibility factor $Z = \frac{pV}{RT} \quad \bar{V} = \bar{M}V \text{ where } \bar{M} = \text{atomic weight}$

$$\text{then } Z = \frac{pV}{RT} \quad \text{where } R = \frac{\bar{R}}{\bar{M}}$$

plotting: Z is plotted versus a dimensionless reduced pressure p_r and a reduced temp T_r defined as:

$$p_r = \frac{p}{p_c} \quad \text{where } p_c \text{ and } T_c \text{ denote the crit press \& temp}$$

$$T_r = \frac{T}{T_c}$$

... results in a generalized compressibility chart of form $Z = f(p_r, T_r)$

IDEAL GAS MODEL (IGM)

- in states where the pressure p is small relative to critical p_c (low p_r) and/or the temp T is large relative to critical temp T_c (high T_r) the compressibility factor $Z = pV/RT$ is approximately 1.

$$\boxed{pV = MRT} \rightarrow p\bar{V} = \bar{R}T \quad \text{or } \bar{V} = \frac{T}{n} \text{ then } pV = nRT$$

$$pV = RT \uparrow \frac{M}{n} = r$$

- for any gas whose equation of state is given exactly by $pV = RT$, the u depends on temp T only
 \rightarrow the specific ethalpy h of gas described by $pV = RT$ also depends on T only

$$\left. \begin{array}{l} pV = RT \\ u = u(T) \\ h = h(T) = u(T) + RT \end{array} \right\} \text{IGM}$$

* to verify that a gas can be modeled as an ideal gas, the states of interest can be located on a compressibility chart to determine how well $Z=1$ is satisfied

For a gas obeying IGM, u depends only on T , then specific heat c_v is also a function of T alone

$$c_v(T) = \frac{du}{dT} \text{ (ideal gas)} \Rightarrow du = c_v(T)dT$$

then the change in specific internal energy is

$$u(T_2) - u(T_1) = \int_{T_1}^{T_2} c_v(T)dT \text{ (ideal gas)}$$

For a gas obeying IGM, h depends only on T so specific heat C_p is also a function of T alone

$$C_p(T) = \frac{dh}{dT} \text{ (ideal gas)} \Rightarrow dh = C_p(T)dT$$

then the change in specific enthalpy is

$$h(T_2) - h(T_1) = \int_{T_1}^{T_2} C_p(T)dT \text{ (ideal gas)}$$

Then we can develop an important relationship between ideal gas specific heats by differentiating w.r.t T

$$\frac{dh}{dT} = \frac{du}{dT} + R$$

$$* C_p(T) = C_v(T) + R \text{ (ideal gas)}$$

and on a molar basis: $\bar{C}_p(T) = \bar{C}_v(T) + R$ (ideal gas)

* this shows that specific heats differ by just the gas constant ... so we can calculate one from the other ... $C_p > C_v$ and $\bar{C}_p > \bar{C}_v$

$$K = \frac{C_p(T)}{C_v(T)} \text{ (ideal gas)} \quad \dots K > 1$$

$$\left\{ \begin{array}{l} C_p(T) = \frac{kR}{k-1} \\ C_v(T) = \frac{R}{k-1} \end{array} \right\} \text{ for ideal gas}$$

APPLYING ENERGY BALANCE USING IDEAL GAS TABLES, CONSTANT SPECIFIC HEATS

$$h(T) = \int_0^T C_p(T)dT \quad \dots \quad u = h - RT$$

when specific heats taken as constants

$$\left\{ \begin{array}{l} u(T_2) - u(T_1) = C_v(T_2 - T_1) \\ h(T_2) - h(T_1) = C_p(T_2 - T_1) \end{array} \right\}$$

POLYTROPIC PROCESS RELATIONS

- quasi-equilibrium process described by $pV^n = C$ or $pV^{-n} = C$
- for a polytropic process between two states

$$P_1 V_1^{-n} = P_2 V_2^{-n} \text{ or } \frac{P_2}{P_1} = \left(\frac{V_1}{V_2} \right)^n$$

when $n=0$, process is isobaric and when $n=\pm\infty$ process is isometric

$$\int_1^2 p dT = \frac{P_2 V_2 - P_1 V_1}{1-n} \quad (n \neq 1) \quad \text{or} \quad \int_1^2 p dT = -P_1 V_1 \ln \left(\frac{V_2}{V_1} \right) \quad (n=1)$$

these apply for any gas or liquid undergoing polytropic process

When additional idealization of ideal gas behavior is appropriate further relations can be derived ...

$$\left. \begin{array}{l} \frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^{(n-1)/n} = \left(\frac{v_1}{v_2} \right)^{n-1} \\ \int_1^2 p dV = \frac{m R (T_2 - T_1)}{1-n} \quad \text{for } n \neq 1 \\ \int_1^2 p dV = m R T_1 \ln \left(\frac{v_2}{v_1} \right) \quad \text{for } n=1 \end{array} \right\} \text{for ideal gas}$$

for an ideal gas, the case $n=1$ corresponds to an isothermal process

Analyzing Polytropic Processes of Air as an Ideal Gas

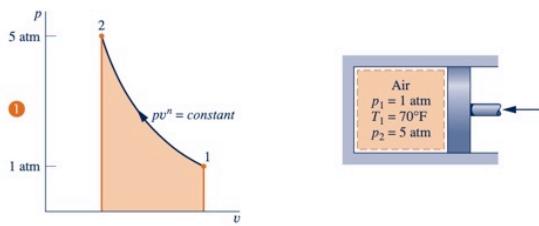
Air undergoes a polytropic compression in a piston-cylinder assembly from $p_1 = 1 \text{ atm}$, $T_1 = 70^\circ\text{F}$ to $p_2 = 5 \text{ atm}$. Employing the ideal gas model with constant specific heat ratio k , determine the work and heat transfer per unit mass, in Btu/lb, if (a) $n = 1.3$, (b) $n = k$. Evaluate k at T_1 .

SOLUTION

Known: Air undergoes a polytropic compression process from a given initial state to a specified final pressure.

Find: Determine the work and heat transfer, each in Btu/lb.

Schematic and Given Data:



Engineering Model:

1. The air is a closed system.
2. The air behaves as an ideal gas with constant specific heat ratio k evaluated at the initial temperature.
3. The compression is polytropic and the piston is the only work mode.
4. There is no change in kinetic or potential energy.

Fig. E3.12

$$W = \int_1^2 p dV \Rightarrow \frac{W}{m} = \frac{R(T_2 - T_1)}{1-n} \quad \text{Energy Balance: } \frac{Q}{m} = \frac{W}{m} + (u_2 - u_1)$$

when specific heat ratio K constant then C_V constant

$$\frac{Q}{m} = \frac{W}{m} + C_V(T_2 - T_1)$$

[for $n = 1.3$]

$$T_2 = T_1 \left(\frac{P_2}{P_1} \right)^{(n-1)/n} = 530^\circ\text{R} \left(\frac{5}{1} \right)^{3/1.3} = 768^\circ\text{R} (302^\circ\text{F})$$

$$\frac{W}{m} = \frac{R(T_2 - T_1)}{1-n} = \left(\frac{1.986 \text{ Btu}}{29.97 \text{ lb} \cdot \text{R}} \right) \left(\frac{768^\circ\text{R} - 530^\circ\text{R}}{1 - 1.3} \right) = -54.39 \text{ Btu/lb}$$

$$C_V = \frac{R}{K-1} = \frac{(1.986 / 29.97) \text{ Btu/lb} \cdot \text{R}}{(1.401 - 1)} = 0.171 \frac{\text{Btu}}{\text{lb} \cdot \text{R}}$$

$$\Rightarrow \frac{Q}{m} = -54.39 \frac{\text{Btu}}{\text{lb}} + (0.171 \frac{\text{Btu}}{\text{lb} \cdot \text{R}})(768^\circ\text{R} - 530^\circ\text{R}) = -13.69 \text{ Btu/lb}$$

[for $n = K$]

$$\frac{Q}{m} = \frac{R(T_2 - T_1)}{1-K} + \frac{R(T_2 - T_1)}{K-1} = 0$$

Because NO HEAT TRANSFER OCCURS IN THE POLYTROPIC PROCESS OF AN IDEAL GAS FOR WHICH $n = K$

TYPICAL IDEAL GASES :

air, O_2 , N_2 , CO_2 , CO , H_2

WORK the work W done by, or on, a system evaluated in terms of macroscopically observable forces and displacements is

$$W = \int_{x_1}^{x_2} F \cdot d\mathbf{x}$$

$W > 0 \dots$ work done by the system $W < 0 \dots$ work done on the system

$$W = \int_{t_1}^{t_2} \dot{W} dt = \int_{t_1}^{t_2} \dot{F} \cdot \dot{\mathbf{v}} dt$$

POWER rate of energy transfer by work, denoted as \dot{W}

$$\dot{W} = \dot{F} \cdot \dot{\mathbf{v}}$$

WORK IS NOT A PROPERTY. THE VALUE OF WORK DEPENDS ON THE NATURE OF THE PROCESS BETWEEN THE END STATES

Polytropic Process quasi-equilibrium process $pV^n = C$ or $pr^n = C$
Extension of a Solid Bar Power transmitted By a Shaft Electric Power

$$W = - \int_{x_1}^{x_2} \sigma A dx \quad \dot{W} = F_t V = (\tau/R)(R\omega) = \tau \omega \quad \dot{W} = - \dot{E}_i \\ SW = - \dot{E} dz$$

$Q > 0$: heat transfer into the system $\dot{Q} = \int_{t_1}^{t_2} \dot{S} Q$

$Q < 0$: heat transfer from the system $\dot{Q} = \int_{t_1}^{t_2} \dot{S} \dot{Q} dt$

→ value of heat transfer depends on the details of a process and not just the end states HEAT IS NOT A PROPERTY

CONDUCTION $\dot{Q}_x = -KA \frac{dT}{dx}$ $\dot{Q}_x = -KA \left[\frac{T_2 - T_1}{L} \right]$

⇒ sign consequence of energy transfer in direction of decreasing temperature

\dot{Q}_e rate at which energy is emitted from a surface of area A is quantified macroscopically by a modified form of the $0 \leq \epsilon \leq 1.0$

STEFAN-BOLTZMANN LAW $\sigma = 5.67(10^{-8}) \frac{W}{m^2 K^4} = 0.1714(10^8) \frac{Btu}{hr \cdot ft^2 \cdot F^4}$

$$\dot{Q}_e = \epsilon \sigma A T_b^4 \quad \dot{Q}_e = \epsilon \sigma A [T_b^4 - T_s^4]$$

CONVECTION rate of energy transfer from the surface to the air $\dot{Q}_c = hA(T_b - T_f)$

$$dE = \dot{S}Q - SW \quad dE = \text{differential of energy (property)}$$

$\dot{S}Q, SW$ = differentials (not properties)

$$\frac{dE}{dt} = \dot{Q} - \dot{W} \quad \begin{bmatrix} \text{time rate form of} \\ \text{the energy balance} \end{bmatrix} \quad \frac{dE}{dt} = \frac{dKE}{dt} + \frac{dPE}{dt} + \frac{dU}{dt}$$

$\begin{bmatrix} \text{time rate of change} \\ \text{of energy contained} \\ \text{within system at time } t \end{bmatrix} = \begin{bmatrix} \text{net rate at which} \\ \text{energy being transferred} \\ \text{in by heat xfer at } t \end{bmatrix} - \begin{bmatrix} \text{net rate at which} \\ \text{energy being transferred} \\ \text{out by work at time } t \end{bmatrix}$
 then ...

$$\frac{dKE}{dt} + \frac{dPE}{dt} + \frac{dU}{dt} = \dot{Q} - \dot{W}$$

POWER CYCLE $n = \frac{W_{cycle}}{Q_{in}}$ (power cycle) or $n = \frac{Q_{in} - Q_{out}}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}}$
 • deliver a net work xfer of energy

to surroundings

$$W_{cycle} = Q_{out} - Q_{in} \quad (\text{refrigeration \& heat pump cycles})$$

$$\beta = \frac{Q_{in}}{W_{cycle}} \text{ or } \frac{Q_{in}}{Q_{out} - Q_{in}} \quad (\text{refrigeration})$$

$$\gamma = \frac{Q_{out}}{W_{cycle}} \text{ or } \frac{Q_{out}}{Q_{out} - Q_{in}} \quad (\text{heat pump})$$

heat xfer
into system
from hot body heat xfer
out to
cold body

$$x = \frac{m_{\text{vapor}}}{m_{\text{mix}} + m_{\text{vap}}} \quad H = H_f + H_v \quad \bar{v} = \frac{\bar{v}_f}{m} = \frac{H_f}{m} + \frac{H_v}{m}$$

$$H_{\text{mix}} = m_e v_f \text{ and } v_{\text{vap}} = m_v \bar{v}_g \quad \underbrace{x - \frac{m_v}{m}}_{\text{and }} \quad \underbrace{\frac{m_e}{m} = 1-x}_{\text{then}}$$

$$\bar{v} = \left(\frac{m_e}{m}\right) \bar{v}_f + \left(\frac{m_{\text{vap}}}{m}\right) \bar{v}_g \quad \bar{v} = (1-x) \bar{v}_f + x \bar{v}_g = \bar{v}_f + x(\bar{v}_g - \bar{v}_f)$$

$$H = U + pV \quad h = u + p\bar{v} \quad \bar{h} = \bar{u} + p\bar{v}$$

$$u = (1-x) u_f + x u_g = u_f + x(u_g - u_f)$$

$$h = (1-x) h_f + x h_g = h_f + x(h_g - h_f)$$

SPECIFIC HEATS

$$C_v = \left(\frac{\partial u}{\partial T}\right)_v \quad C_p = \left(\frac{\partial h}{\partial T}\right)_p \quad k = \frac{C_p}{C_v} = \text{specific heat ratio}$$

$$h(T, p) = u(T) + p\bar{v} \text{ (incompressible)}$$

• therefore, for substances modeled as incompressible, $C_v = C_p$

$$u_2 - u_1 = \int_{T_1}^{T_2} c(T) dT \quad h_2 - h_1 = u_2 - u_1 + v(p_2 - p_1) = \int_{T_1}^{T_2} c(T) dT + v(p_2 - p_1)$$

If c taken as constant, then: $u_2 - u_1 = c(T_2 - T_1)$
this term is sometimes dropped if small

UNIVERSAL GAS CONSTANT, \bar{R}

$$\bar{R} = \begin{cases} 8.314 \text{ J/Kmol} \cdot \text{K} \\ 1.986 \text{ cal/Kmol} \cdot {}^\circ\text{R} \\ 1545 \text{ ft-lbf/lbmol} \cdot {}^\circ\text{R} \end{cases} \quad Z = \frac{p\bar{v}}{RT} \quad \bar{v} = \frac{M}{R} \quad \text{where } M = \text{atomic weight}$$

$$Z = \frac{p\bar{v}}{RT} \quad \text{where } R = \frac{p_c}{M} \quad p_c = \frac{P}{P_c} \quad T_c = \frac{T}{T_c}$$

IDEAL GAS MODEL (IGM)

$$z = p\bar{v}/RT \approx \text{approximately 1.}$$

$$pH = MRT \rightarrow p\bar{v} = \bar{R}T \quad C_v(T) = \frac{du}{dT} \quad du = C_v(T) dT$$

$$p\bar{v} = RT \quad \uparrow \frac{M}{m} = r \quad \text{or } \bar{v} = \frac{M}{n} \quad \text{then } pH = n\bar{R}T \quad C_p(T) = \frac{dh}{dT} \quad dh = C_p(T) dT$$

$$\left. \begin{array}{l} p\bar{v} = RT \\ u = u(T) \\ h = h(T) = u(T) + RT \end{array} \right\} \text{IGM}$$

then the change in specific enthalpy is

$$h(T_2) - h(T_1) = \int_{T_1}^{T_2} C_p(T) dT \dots k > 1$$

$$\frac{dh}{dT} = \frac{du}{dT} + R \quad K = \frac{C_p(T)}{C_v(T)}$$

APPLYING ENERGY BALANCE USING IDEAL GAS TABLES, CONSTANT SPECIFIC HEATS

$$\bar{C}_p(T) = \bar{C}_v(T) + \bar{R}$$

$$* C_p(T) = C_v(T) + R$$

$$\left. \begin{array}{l} C_p(T) = \frac{kR}{k-1} \\ C_v(T) = \frac{R}{k-1} \end{array} \right\} \left. \begin{array}{l} (u(T_2) - u(T_1)) = C_v(T_2 - T_1) \\ (h(T_2) - h(T_1)) = C_p(T_2 - T_1) \end{array} \right\}$$

$$h(T) = \int_0^T C_p(T) dT \quad \dots \quad u = h - RT$$

POLYTROPIC PROCESS RELATIONS

- quasi-equilibrium process described by $pV^n = C$ or $pV^{-n} = C$
- for a polytropic process between two states

$$P_1 V_1^{-n} = P_2 V_2^{-n} \quad \text{or} \quad \frac{P_2}{P_1} = \left(\frac{V_1}{V_2} \right)^n$$

when $n=0$, process is isobaric and when $n=\pm\infty$ process is isometric

$$\int_1^2 p dV = \frac{P_2 V_2 - P_1 V_1}{1-n} \quad \text{or} \quad \int_1^2 p dV = -P_1 V_1 \ln \left(\frac{V_2}{V_1} \right) \quad (n=1)$$

these apply for any gas or liquid undergoing polytropic process

When additional idealization of ideal gas behavior is appropriate further relations can be derived ...

$$\left\{ \begin{array}{l} \frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^{(n-1)/n} = \left(\frac{V_1}{V_2} \right)^{n-1} \\ \int_1^2 p dV = \frac{m R (T_2 - T_1)}{1-n} \quad \text{for } n \neq 1 \\ \int_1^2 p dV = m R T_1 \ln \left(\frac{V_2}{V_1} \right) \quad \text{for } n=1 \end{array} \right\} \quad \text{for ideal gas}$$

for an ideal gas, the case $n=1$ corresponds to an isothermal process