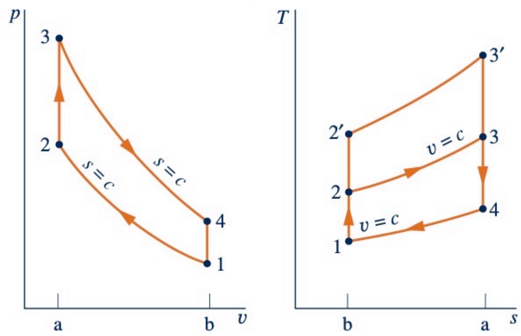
Air Standard Otto Cycle

- an ideal cycle that assumes heat addition occurs instantaneously while piston is at TDC
- consists of four internally reversible processes in series



1→2: isentropic compression of air as piston moves from BDC to TDC

2→3: constant $\forall$ heat xfer to air from external source while piston @ TDC (ignition of fuel/air mixture)

3→4: isentropic expansion (power stroke

4→1: completes cycle by constant $\forall$ process in which heat is rejected from air while piston is @ BDC

Since air-standard Otto cycle composed of internally reversible processes, areas on T-s, p-v diagrams can be interpreted as heat and work

On T-s: 2-3-a-b-2 = heat added (per unit mass)
1-4-a-b-1 = heat rejected (per unit mass)

On Pv: 1-2-a-b-1 = work input (per unit mass) during compres
3-4-b-a-3 = work done (per unit mass) in expansion proc.

Enclosed area of each figure = net work output, net heat added

1→2, 3→4: work, no Q xfer

2→3, 4→1: Q xfer, no work

$$\frac{W_{12}}{m} = u_2 - u_1 \quad \text{(input)} \qquad \frac{W_{34}}{m} = u_3 - u_4 \qquad \frac{Q_{23}}{m} = u_3 - u_2 \qquad \frac{Q_{41}}{m} = u_4 - u_1 \quad \text{(heat rejected)}$$

net work of cycle: $$\frac{W_{cycle}}{m} = \frac{W_{34}}{m} - \frac{W_{12}}{m} = (u_3 - u_4) - (u_2 - u_1)$$

which can also be interpreted as <u>net</u> heat added: $$\frac{W_{cycle}}{m} = \frac{Q_{23}}{m} - \frac{Q_{41}}{m} = (u_3 - u_2) - (u_4 - u_1)$$

$$\eta = \frac{\text{net work}}{\text{heat added}} = \frac{(u_3 - u_2) - (u_4 - u_1)}{u_3 - u_2} = 1 - \frac{u_4 - u_1}{u_3 - u_2}$$

These following formulas apply for the <u>isentropic</u> processes 1→2, 3→4

$$v_{r2} = v_{r1}\left(\frac{\forall_2}{\forall_1}\right) = \frac{v_{r1}}{r} \qquad v_{r4} = v_{r3}\left(\frac{\forall_4}{\forall_3}\right) = r\, v_{r3}$$

Since $V_3 = V_2$, $V_4 = V_1$ then $r = V_1/V_2 = V_4/V_3$

✱ When the Otto cycle is analyzed on a cold air-standard basis the following expressions can be used for the isentropic processes $1 \to 2$, $3 \to 4$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{k-1} = r^{k-1} \qquad \frac{T_4}{T_3} = \left(\frac{V_3}{V_4}\right)^{k-1} = \frac{1}{r^{k-1}} \qquad k = c_p/c_v$$

Otto cycle $\eta_{th} \uparrow$ as compression ratio $r \uparrow$ (also w/ cold)

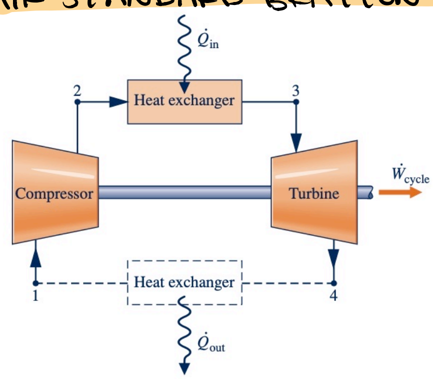
$\to$ for cold: for constant c_v: $\eta = 1 - \dfrac{c_v(T_4 - T_1)}{c_v(T_3 - T_2)}$

$\Rightarrow \eta = 1 - \dfrac{T_1}{T_2}\left(\dfrac{T_4/T_1 - 1}{T_3/T_2 - 1}\right) \Rightarrow \eta = 1 - \dfrac{T_1}{T_2}$ bc $T_4/T_1 = T_3/T_2$

$$\eta = 1 - \frac{1}{r^{k-1}}$$ for cold air-standard basis

✱ advantageous for internal combustion engines to have high compression ratios

AIR STANDARD BRAYTON CYCLE – ideal gas



- air drawn into compressor @ 1 from surroundings and later returned to surroundings @ 4 w/ temperature > ambient temp
- assuming turbine operates adiabatically w/ negligible KE & PE effects

$$\frac{\dot{W}_t}{\dot{m}} = h_3 - h_4 \qquad \frac{\dot{W}_c}{\dot{m}} = h_2 - h_1 \qquad \frac{\dot{Q}_{in}}{\dot{m}} = h_3 - h_2$$

$\dfrac{\dot{Q}_{out}}{\dot{m}} = h_4 - h_1 \Rightarrow \eta = \dfrac{\dot{W}_t/\dot{m} - \dot{W}_c/\dot{m}}{\dot{Q}_{in}/\dot{m}} = \dfrac{(h_3 - h_4) - (h_2 - h_1)}{h_3 - h_2}$

back work ratio: $bwr = \dfrac{\dot{W}_c/\dot{m}}{\dot{W}_t/\dot{m}} = \dfrac{h_2 - h_1}{h_3 - h_4}$ ~ 40-80%

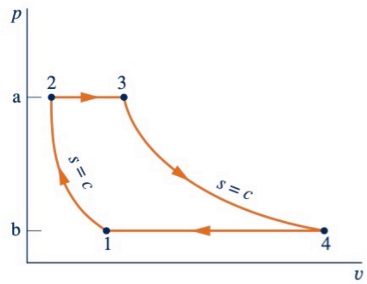
$\to$ relatively large portion of work developed turbine required to drive compressor, hence why bwr higher than vapor power pe.

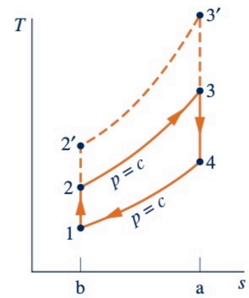
$\to$ take sp. heat as constant, then COLD AIR-STANDARD ANALYSIS

IDEAL AIR-STANDARD BRAYTON CYCLE

- ignore irreversibilities
- air flows @ constant pressure through heat exchangers
- stray heat xfer ignored

✱ turbine & compressor are isentropic





Ts: 2-3-a-b-2 = heat added
1-4-a-b-1 = heat rejected
pv: 1-2-a-b-1 = compressor W
3-4-b-a-3 = turbine W

Enclosed areas = net work output, net heat added

for ideal Brayton, isentropic processes:

$p_{r2} = p_{r1} \frac{P_2}{P_1}$ $\quad p_{r4} = p_{r3} \frac{P_4}{P_3} = p_{r3} \frac{P_1}{P_2}$ $\quad$ compressor ratio $= \frac{P_2}{P_1}$ $\quad$ TABLE A22

$P_4/P_3 = P_1/P_2$ since air flows thru heat exchangers @ const. press

for ideal Brayton, cold air-standard:
sp. heats taken constant

$$T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} \qquad T_4 = T_3 \left(\frac{P_4}{P_3}\right)^{\frac{k-1}{k}} = T_3 \left(\frac{P_1}{P_2}\right)^{\frac{k-1}{k}} \qquad k = \frac{c_p}{c_v}$$

(*) η_{th} increases w/ increasing pressure ratio across compressor for air-standard and cold air-standard

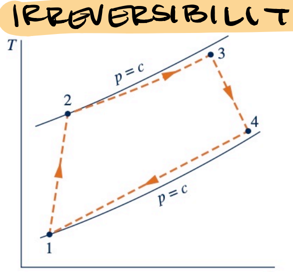
for constant c_p & k:

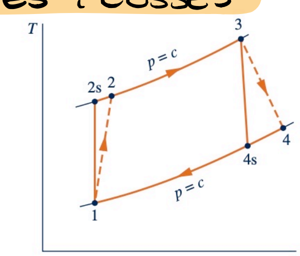
$$\eta = \frac{c_p(T_3 - T_4) - c_p(T_2 - T_1)}{c_p(T_3 - T_2)} = 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)} = 1 - \frac{T_1}{T_2}\left(\frac{T_4/T_1 - 1}{T_3/T_2 - 1}\right)$$

and since $T_4/T_1 = T_3/T_2$: $\eta = 1 - \frac{T_1}{T_2}$

for cold air-stand: $\eta = 1 - \frac{1}{(P_2/P_1)^{(k-1)/k}}$

IRREVERSIBILITIES & LOSSES





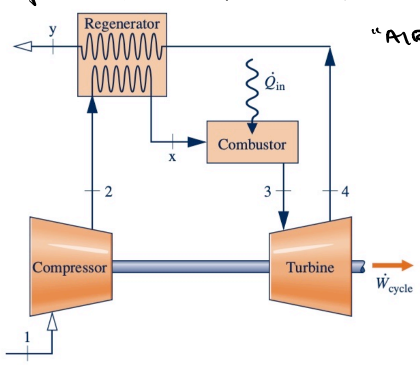
- frictional effects w/in comp & turbine causing ↑ in specific entropy
- stray heat xfer

- work developed by turbine ↓, work input to comp ↑

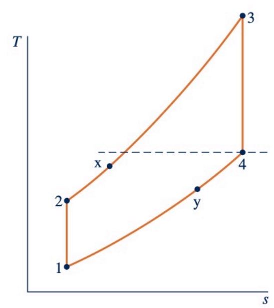
$$\eta_t = \frac{\dot{W}_t/\dot{m}}{(\dot{W}_t/\dot{m})_s} = \frac{h_3 - h_4}{h_3 - h_{4s}} \qquad \eta_c = \frac{(\dot{W}_c/\dot{m})_s}{(\dot{W}_c/\dot{m})} = \frac{h_{2s} - h_1}{h_2 - h_1}$$

REGENERATIVE GAS TURBINES

Regenerator: allows air exiting compressor to be pre-heated before entering combustor, thereby reducing amount of fuel that must be burned



"AIR STANDARD BRAYTON"



- regenerator is a counter-flow heat exchanger thru which hot turbine exhaust gas and cooler air leaving compressor pass in opposite directions
- turbine exhaust gas cooled from 4 to y, air exiting comp heated from 2 to x

heat added per unit mass: $\frac{\dot{Q}_{in}}{\dot{m}} = h_3 - h_x$

the net work is not altered by addition of regenerator

... thus, since heat added reduced, η_{th} increases

η_{reg} = regenerator effectiveness: ratio of actual enthalpy increase of air flowing thru comp side of regenerator to maximum theoretical enthalpy increase [p. 538-539]

$$\eta_{reg} = \frac{h_x - h_2}{h_4 - h_2}$$

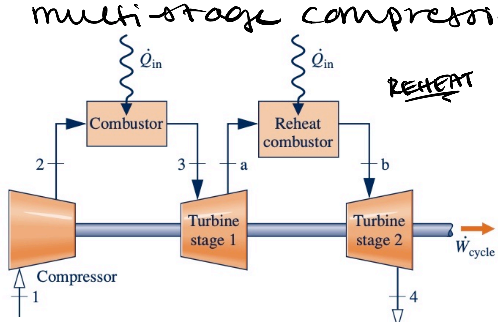
→ as heat xfer approaches reversibility, $h_x \rightarrow h_4$ and $\eta_{reg} \rightarrow 100\%$

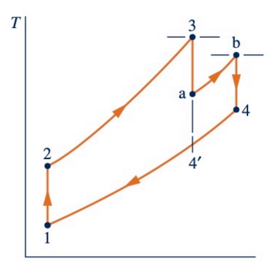
... in practice, typically 60-80% and thus T_x of air exiting comp typically well below turbine exhaust T

REGENERATIVE GAS TURBINES WITH REHEAT & INTERCOOLING

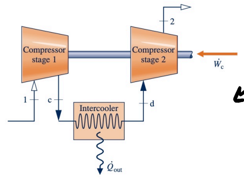
- increase net work w/ multistage expansion (reheat) and multi-stage compression (intercooling)

REHEAT



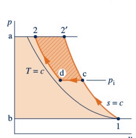


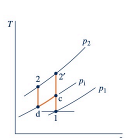
- after expansion from 3→a in 1st turbine, gas reheated @ constant press from a to b
- expansion then completed in 2nd turbine b to 4
- net work ↑



"INTERCOOLING"

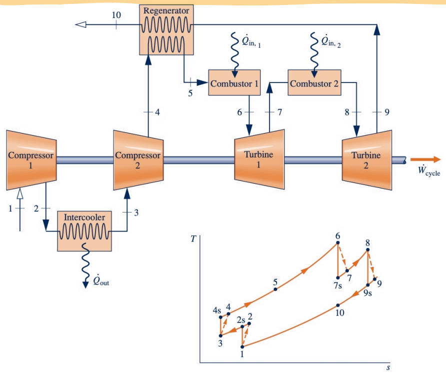
- 1→c: isentropic compression from 1 to c where pressure is p_i
- c→d: constant pressure cooling from temp T_c to T_d
- d→2: isentropic compression to state 2





↑ net work developed by reducing compression work

REHEAT AND INTERCOOLING



net work output ↑
potential for regeneration enhanced

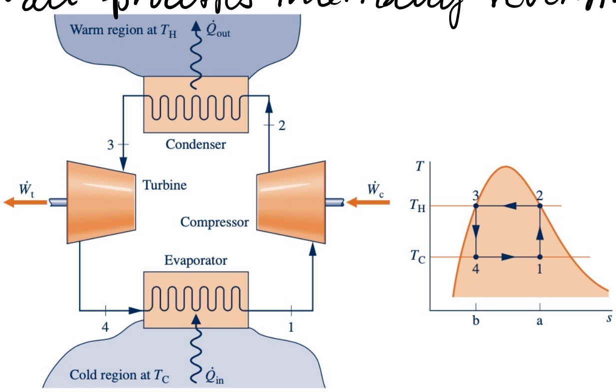
← reheat, intercooling, and regeneration = substantial improvement in performance

VAPOR REFRIGERATION SYSTEMS

- purpose: maintain cold region @ temp below temp of surround.

CARNOT REFRIGERATION CYCLE

- reversing Carnot vapor power cycle
- cycle executed by refrigerant circulating steadily thru series of components
- all processes internally reversible



*refrigerant enters evaporator as 2-phase liquid vapor mixture at state 4

– in evaporator, some of refrigerant changes phase from liquid to vapor as result of heat xfer from region at temp T_c to refrigerant

– temp & press of refrigerant remain constant from 4→1 where it is 2-phase liquid-vapor mixture to state 2 where it is saturated vapor and the temp of refrigerant increases from T_c to T_H and pressure also increases

– refrigerant passes from compressor into condenser where it changes phase from sat. vapor to saturated liq as a result of heat xfer to region at temp T_H

– temp & pressure constant 2→3

– refrigerant returns to state at the inlet of evaporator by expanding adiabatically through a turbine; in this process from 3→4 temp ↓ from T_H to T_c and press ↓

areas on T–s can be interpreted as heat xfers, enclosed area 1-2-3-4-1 = <u>net</u> heat xfer <u>from</u> refrigerant

net heat xfer from refrigerant = <u>net</u> work done <u>on</u> refrigerant

net work = diff between comp work input and turbine work output

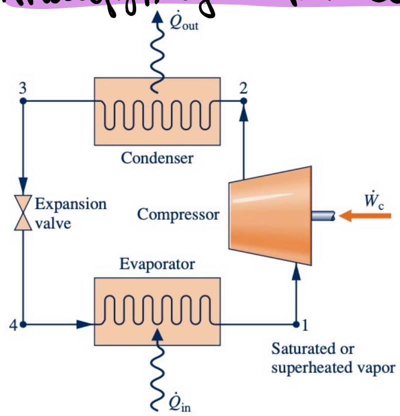
$$\beta_{max} = \frac{\dot{Q}_{in}/\dot{m}}{\dot{W}_c/\dot{m} - \dot{W}_t/\dot{m}} = \frac{\text{area } 1-a-b-4-1}{\text{area } 1-2-3-4-1} = \frac{T_c(s_a - s_b)}{(T_H - T_c)(s_a - s_b)}$$

$$\beta_{max} = \frac{T_c}{T_H - T_c}$$

CARNOT REFRIGERATION

Departures from Carnot Cycle - p. 611-612

Analyzing Vapor-Compression Ref. Systems



- as refrigerant passes thru evaporator, heat xfer from refrigerated space results in vaporization of refrigerant

$$\frac{\dot{Q}_{in}}{\dot{m}} = h_1 - h_4$$

$\dot{Q}_{in}$ = refrig. capacity

- refrigerant leaving evaporator compressed to relatively high press & temp by comp.

$$\frac{\dot{W}_c}{\dot{m}} = h_2 - h_1$$

$\dot{W}_c$ = rate of power input

- refrigerant passes through condenser where refrigerant condenses and there's no heat xfer from ref. to cooler surroundings

$$\frac{\dot{Q}_{out}}{\dot{m}} = h_2 - h_3$$

- refrigerant at state 3 enters expansion valve and expands to evaporator pressure (throttling process)

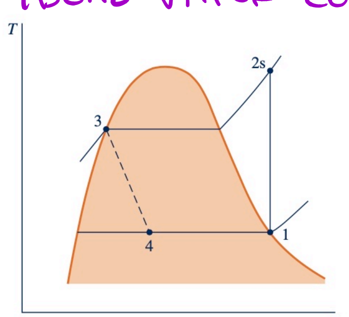
$$h_4 = h_3$$

  - ref. press ↓ in irreversible adiabatic expansion, accompanying ↑ s
  - ref exits valve @ state 4 as 2-phase liq-vapor mixture

* net power input = compressor power since expansion valve involves no power input or output

$$\beta = \frac{\dot{Q}_{in}/\dot{m}}{\dot{W}_c/\dot{m}} = \frac{h_1 - h_4}{h_2 - h_1}$$

IDEAL VAPOR-COMPRESSION SYSTEMS



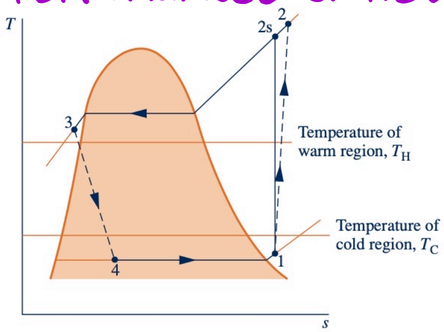
1→2s: isentropic compression of refrigerant from 1 to condenser pressure at state 2s

2s→3: heat xfer from refrigerant as flows at constant P thru condenser, exits as liquid at state 3

3→4: throttling process to 2-phase liq-vap @ 4

4→1: heat xfer to ref. as flows @ c P thru evaporator

PERFORMANCE OF ACTUAL VAPOR COMPRESSION



- heat xfers between refrigerant and warm and cold regions are <u>not</u> accomplished reversibly
  - ref. T in evaporator < T_C
  - ref. T in condenser > T_H
  - * causes $\beta \downarrow$

$1 \to 2$: ↑ s that accompanies adiabatic irreversible compression
⇒ W input of irr. compression > ideal cycle
$\beta \downarrow$

⇒ isentropic comp. efficiency:

$$\eta_c = \frac{(\dot{W}_c/\dot{m})_s}{(\dot{W}_c/\dot{m})} = \frac{h_{2s} - h_1}{h_2 - h_1}$$

2 add'tnl features exhibited by actual:
1. superheated vapor condition at evaporator exit (state 1)
2. subcooling of condenser exit state (state 3)

other sections not read yet:
→ 10.3 selecting refrigerants
→ 10.4 other vapor-comp. applications