Assignment #4 is posted and we'll talk a lot about it today.

Vibrations - Lecture 12
2021-03-09

- → expects us to work the problem in many ways to verify answer (check if all three ways work and get 100% for instance)
- → always look at the intermediate answers
- → use the scripts in the zip file
- → use modular programming and local functions

A little recap:

Forced vibration of an undamped system:

(1) $[m]\ddot{\vec{x}} + [k]\vec{x} = \vec{F}(t)$ with initial conditions $\vec{x}_0$, $\dot{\vec{x}}_0$

(2) $\vec{x}(t) = \vec{x}_p(t) + \vec{x}_h(t)$

the total solution (no subscript h, p)

particular (3) $[m]\ddot{\vec{x}}_p + [k]\vec{x}_p = F(t)$

homogeneous (4) $[m]\ddot{\vec{x}}_h + [k]\vec{x}_h = \vec{0}$

substitute (2) → (1)

$$[m](\ddot{\vec{x}}_p + \ddot{\vec{x}}_h) + [k](\vec{x}_p + \vec{x}_h) = \vec{F}(t)$$ ← equivalent to (3) + (4)

the initial conditions

$$\vec{x}(0) = \vec{x}_0$$
$$\vec{x}_p(0) + \vec{x}_h(0) = \vec{x}_0$$
$$\left.\begin{aligned}\vec{x}_h(0) &= \vec{x}_0 - \vec{x}_p(0)\\ \dot{\vec{x}}_h(0) &= \dot{\vec{x}}_0 - \dot{\vec{x}}_p(0)\end{aligned}\right\}$$

$$m\ddot{x}_p + kx_p = \underbrace{\sin\omega t}_{F(t)}$$

$x_p = A\sin\omega t$

$$(-\omega^2 m + k)A\sin\omega t = \sin\omega t$$

$$A = \frac{1}{-\omega^2 m + k}$$

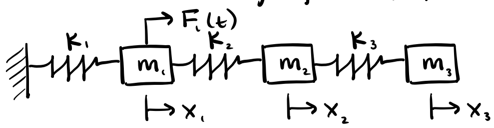
$$x_p^{(1)} = \frac{\sin\omega t}{-\omega^2 m + k}$$

$$x_p(0) \neq x_0$$
$$\dot{x}_p(0) \neq \dot{x}_0$$

why do we need homogeneous + particular?
- → because need the unknown constants to adjust
- → the initial conditions

ASSIGNMENT #4 INFORMATION

We call this a chain system
↳ here we have a highly simplified elongated bar for instance



* the initial conditions given are for the total solution (not just the particular or homogeneous)

Force : $F_1(t) = \sin(\omega t)$

For 1, 2, and 3, the forcing frequency ω is 1 rad/s

codes to use: mdof_undamped_forced (task 1)
undamped_free_vibration (task 2) ... but we don't know the initial conditions so we'll need to figure this out

For task 4: now omega is not fixed, find frequency at which mass 2 has ...

even though forcing mass 1, look for possibility that there's a frequency @ which mass 2 doesn't move (may not in this problem, but interesting phenomenon)
↳ basically if see a magical frequency @ which mass 2 has zero disp, don't worry

$\vec{X}_h(t)$ = sum of eigenvectors

The codes he's given us execute the theory of 6.14
→ read Ch. 6.14 because may see examples that could be useful for our coding functions

Example of using the Q's:
simple linear system

$$5x_1 - 7x_2 = 8$$
$$6x_1 + 9x_2 = 10$$

define $q_1 = 5x_1 - 7x_2$
$q_2 = 6x_1 + 9x_2$

Find q_1 and q_2. Then, $q_1 = 8$
$q_2 = 10$

we use q's because they make things simpler... they essentially uncouple
↳ then once find q's, can find the x's

Section 6.15... start to now also look at effects of damping

$$\underbrace{[m]\ddot{\vec{x}}}_{\text{=ma, force needed to accelerate mass}} + \underbrace{[c]\dot{\vec{x}}}_{\text{damping force}} + \underbrace{[k]\vec{x}}_{\text{force needed to displace spring}} = \vec{F} \text{ for } n \text{ DoF}$$

← example of viscous damping

viscous damping model is what we see in this class so far
↳ the damping matrix times velocity vector
↳ what's missing in this model is the "history"
→ different materials may have different velocity history that impacts future
→ we just see the velocity now in this model

Positive Definite Matrix

[c]

→ means that $\dot{\vec{x}}^T [c] \dot{\vec{x}} > 0$ for any $\vec{x}$
basically always dissipating energy
power always going into the dashpot

$\vec{x}^T [k] x \geq 0$ always have ⊕ potential energy stored in springs, likewise for masses $\ddot{\vec{x}}^T [m] \ddot{\vec{x}} > 0$

we always know the least about the c matrix
↳ most analysts will estimate a lot of things in it, kind of based on experience

looking at:

$[c] = \alpha[m] + \beta[k]$ proportional damping ... "important if true"
↳ aka "Rayleigh Damping"
↳ seems to be popular model in FEA programs

if this is not true ... need to take brute force numerical approach to
$[m]\ddot{\vec{x}} + [c]\dot{\vec{x}} + [k]\vec{x} = \vec{F}(t)$

in the case of proportional damping:

$[m]\ddot{\vec{x}} + (\alpha[m] + \beta[k])\dot{\vec{x}} + [k]\vec{x} = \vec{F}$ for n DoF

→ then look at eigenvalue problem
→ then eigenvector expansion
→ substitution & premultiplication
→ generalized forces
→ uncoupled equations for generalized coordinates

$\Big\}$ all from the summary sheets

ζ_i = modal loss factor