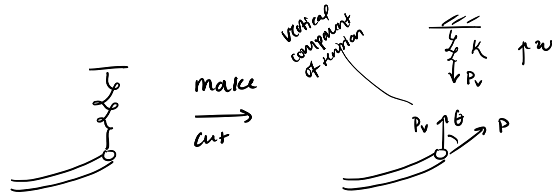
For assignment #6:

Vibrations - Lecture 17
Tue. 2021-03-30

- $\underset{\sim}{k}$ is kappa in assignment
- boundary condition is tricky:



$P_v = -k w \quad (x = l, t)$

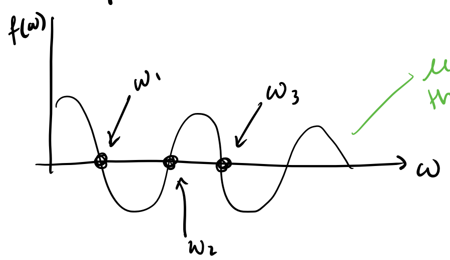
- testing understanding of math that we did

FIXED-FIXED: $\sin(KL) = 0$ $\quad KL = n\pi$ (solved by inspection)

↳ $K = \frac{n\pi}{L}$ $\quad \omega_n = \frac{n\pi c}{L}$ $\quad$ } from before

↳ can't use for this assignment since not fixed-fixed
→ we'll find $f(\omega) = 0$
∘ may not be able to solve by inspection or analytically

- assignment is 50/50 analysis & MATLAB
- need to plot to solve numerically



let's say it's this random function

⇒ may need to zoom in on plot and individually pick points

- note that if had ends fixed to ground then $\omega_n = \frac{n\pi c}{L}$ ... but won't work here!
- MUST LOOK AT BOOK

The travelling wave solution aka D'alambert's

$$c^2 \frac{\partial^2 w}{\partial x^2} = \frac{\partial^2 w}{\partial t^2}$$

$w = f(x - ct)$ ← is that a solution to the wave equat.?

$$\frac{\partial w}{\partial x} = f'(x-ct) \qquad \frac{\partial w}{\partial t} = -c f'(x-ct)$$

$$\frac{\partial^2 w}{\partial x^2} = f''(x-ct) \qquad \frac{\partial^2 w}{\partial t^2} = c^2 f''(x-ct)$$

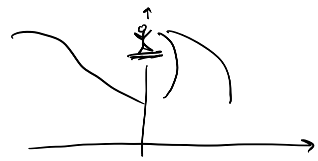
plug these in

$c^2 f''(x-ct) = c^2 f''(x-ct)$ ... so it works!

↳ same thing if $w = f(x+ct)$, too.

Now, how fast does this wave travel?

• if surfer stays @ same height, what's horizontal velocity?



$\frac{d}{dt}\left((x-ct = \text{constant})\right) \rightarrow \dot{x} = c$ (moves right)

$\frac{d}{dt}\left((x+ct = \text{constant})\right) \rightarrow \dot{x} = -c$ (moves left)

waves move one direction @ a speed c

$$\underline{w(x,t) = w_1(x-ct) + w_2(x+ct)}$$

initial conditions: $w(x,t=0) = w_1(x) + w_2(x) = w_0$

↳ apply these to find w_1 and w_2

$\dot{w}(x,t=0) = -c\,w_1' + c\,w_2' = \dot{w}_0$

integrate this equation

$$-c\,w_1 + c\,w_2 = \int_{x_0}^{x} \dot{w}_0(x')\,dx'$$

$$\Downarrow$$

$$w_1(x) = \frac{1}{2}\left[w_0(x) - \frac{1}{c}\int_{x_0}^{x} \dot{w}_0(x')\,dx'\right]$$ ... kind of like taking initial condition and sending to the right

$$w_2(x) = \frac{1}{2}\left[w_0(x) + \frac{1}{c}\int_{x_0}^{x} \dot{w}_0(x')\,dx'\right]$$

$w = w_1 + w_2$

$w = w_1(x-ct) + w_2(x+ct)$

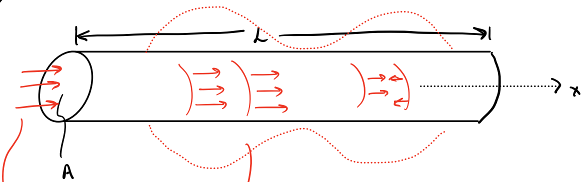
$$\underbrace{-\frac{1}{c}\int_{x_0}^{x-ct}} \quad + \quad \frac{1}{c}\int_{x_0}^{x+ct}$$

$$+\frac{1}{c}\int_{x-ct}^{x_0}(\quad) \; + \; \frac{1}{c}\int_{x_0}^{x+ct}(\quad)$$

$$\frac{1}{c}\int_{x-ct}^{x+ct}(\quad)$$

final answer: $w(x,t) = \frac{1}{2}\left[w_0(x-ct) + w_0(x+ct)\right] + \frac{1}{2c}\int_{x-ct}^{x+ct} \dot{w}(x')\,dx'$

A Bar:



$k_{eq} = \frac{EA}{L}$

center is not bending or flexing, note!

Poisson effect ν: without this, would never hear bar vibrating

cut out a slice dx thick ← Rao, Fig. 8.6

$$(P + dP) + \underbrace{f\,dx}_{\text{possible distributed load}} - P = \rho \overbrace{A\,dx}^{\text{volume}} \underbrace{\frac{\partial^2 u}{\partial t^2}}_{\text{acceleration}}$$

now ρ is mass per unit volume, density

setting $f=0$

$$\underbrace{c^2 \frac{\partial^2 u}{\partial x^2}(x,t) = \frac{\partial^2 u}{\partial t^2}(x,t)}_{u = \text{axial displacement}} \qquad \underbrace{c = \sqrt{\frac{E}{\rho}}}_{\text{sound speed}}$$

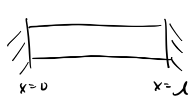
} very similar for the string

the book doesn't tell us: $u(x,t) = \sum_n \underbrace{U_n(x)}_{\text{mode shape}} \underbrace{T_n(t)}_{C\cos\omega t + D\sin\omega t \text{ evaluated @ } \omega_n}$

→ the free-free bar: $\omega = \frac{n\pi c}{l}$
DEMO:

$$= \left(\underset{\sim}{A}\cos\frac{\omega x}{c} + \underset{\sim}{B}\sin\frac{\omega x}{c}\right) + \left(C\cos\omega t + D\sin\omega t\right)$$

FIXED – FIXED BAR



$$\left.\begin{aligned} u(x=0,t) &= 0 \\ u(x=l,t) &= 0 \end{aligned}\right\} \text{B.C.}$$

$$u(x=0,t) = \underbrace{U(x=0)}_{\text{then this } U(x=0)=0 \ (1)} \underbrace{T(t)}_{\text{if this was zero, trivial soln}} = 0$$

$$u(x=l,t) = \underbrace{U(x=l)}_{U(x=l)=0 \ (2)} T(t) = 0$$

$$U(x) = \underset{\sim}{A}\cos\left(\frac{\omega x}{c}\right) + \underset{\sim}{B}\sin\left(\frac{\omega x}{c}\right)$$

⇓

$U(0) = \underset{\sim}{A} = 0$

$U(l) = \underset{\sim}{B}\sin\frac{\omega l}{c} = 0$ ...if $\underset{\sim}{B}=0$ then trivial so sin func. is zero $\Rightarrow \frac{\omega_n l}{c} = n\pi$ → $\underline{\frac{n\pi c}{l} = \omega_n}$ $n = 1, 2, 3, \ldots$

then $U_n(x) = \underset{\sim}{B}_n \sin\left(\frac{\omega_n x}{c}\right)$

$$\underset{\sim}{B}_n \sin\left(\frac{n\pi c}{l}\frac{x}{c}\right) = \underset{\sim}{B}_n \sin\left(\frac{n\pi x}{l}\right)$$