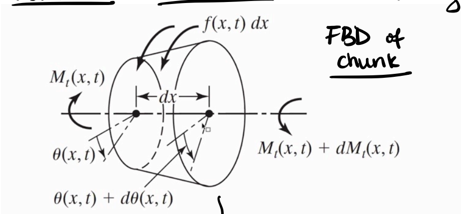
Presentation on his Drug Sub Research

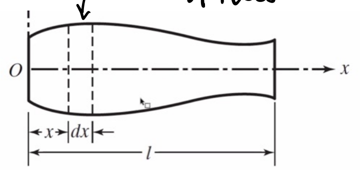
Vibrations - Lecture 18
Thur. 2021-04-01

Tortional Vibration - a twisting of a shaft

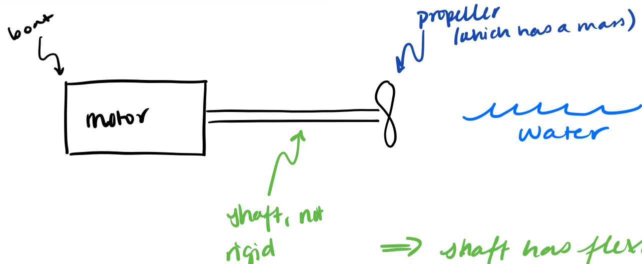


FBD of chunk ← left end will rotate differently than right end, creating a vibration → lead to wave equation again

chunk of this

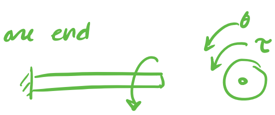


θ = rotation angle ← this is the vibration



⇒ shaft has flexibility to it

fix one end



$M_t(x,t) = GJ(x)\frac{\partial\theta}{\partial x}(x,t)$ ← the twisting moment is proportional to the torsional deflection by G (shear modulus) and J is the polar moment of inertia of the cross section in the case of a circular section

(J = area moment of inertia)

if the mass polar moment of inertia of shaft per unit length is I_0,

GJ = tortional stiffness

the inertia torque acting on an element of length dx becomes

$$I_0\,dx\frac{\partial^2\theta}{\partial t^2}$$

$f(x,t)$ = external torque acts on shaft

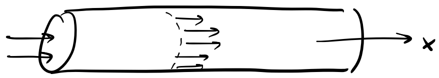
if an external torque $f(x,t)$ acts on a shaft of unit length ...

$$\frac{\partial}{\partial x}\left[GJ(x)\frac{\partial\theta}{\partial x}(x,t)\right] + f(x,t) = I_0(x)\frac{\partial^2\theta}{\partial t^2}(x,t)$$

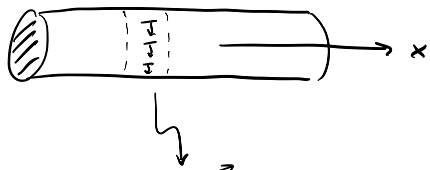
uniform shaft, ignore the force ...

$$GJ\frac{\partial^2\theta}{\partial x^2}(x,t) + f(x,t) = I_0\frac{\partial^2\theta}{\partial t^2}(x,t) \xrightarrow[\text{vibration}]{\text{free}} \overbrace{c^2\frac{\partial^2\theta}{\partial x^2}(x,t) = \frac{\partial^2\theta}{\partial t^2}(x,t)}^{\text{wave equation}}$$

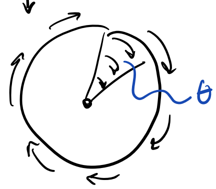
sound speed $c = \sqrt{\frac{GJ}{I_0}}$



longitudinal waves (last lecture)



get twisting (this lecture)



greatest motion at outer circumference

* if shaft has a uniform cross section $I_0 = \rho J$ then sound speed $c = \sqrt{\frac{G}{\rho}}$