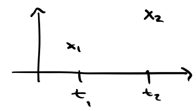
Announcement about Assignment #2:

- using diff() to get force vector
- $\frac{x_2 - x_1}{t_2 - x_1}$ ... this applies for derivative at $t=1$

  which means will miss last one

  put 1 in the 1000^{th} element of $F(t)$

  ⇒ so only 1^{st} thru 999^{th} element of $F(t)$ will be graded
- can load the given mat file (do not rename this given file!)



Undamped Free Vibrations

ω^2 = eigenvalue $\vec{X}$ = eigenvector (aka mode shape)

MATLAB will sort ω^2 from smallest to largest on diagonal

$$\begin{matrix} \omega_1^2 \\ \omega_2^2 \\ \vdots \\ \omega_n^2 \end{matrix} \Big\downarrow \text{largest}$$

$([K] - \omega_1^2 [M])\vec{X}_1 = \vec{0}$ find $\vec{X}_1$

$([K] - \omega_2^2 [M])\vec{X}_2 = \vec{0}$ find $\vec{X}_2$

$(\omega_1^2, \vec{X}_1)$ first eigenpair

take 1^{st} eigenvector $(\vec{X}_1)_1$ and set $= 1$ (first element $= 1$)

require this triple product: $\vec{X}_1^T [M] \vec{X}_1 = 1$ (looks like kinetic energy - the ½)

```
[V, D] = eig(K, M);
diag(D)
    ans =
```

ω_1^2 ← fundamental frequency (always the smallest)

ω_2^2

f in assignment 2 must be in units of Hz

i.e. f = omega / (2 * pi)

V is the eigenvectors, 1^{st} column in MATLAB is the first eigenvector

i.e. X_1 = V(:, 1)

X_2 = V(:, 2)

(K - omega(1)^2 * M) * X_1 should be zero! (or close)

(K - omega(2)^2 * M) * X_2 should be zero! (or close)

det(K - omega(1)^2 * M)

ans = 0

X_1.'* M * X_1 should be 1 and X_2.'* M * X_2 should be 1, etc

4 x_1.' * M * x_2
ans =
0

Book tells us that $\vec{X}^{(j)T}[m]\vec{X}^{(i)} = \begin{cases} 0 & i \neq j \\ 1 & i = j \end{cases}$ ... this is the orthogonality proof

$\left([K] - \omega_i^2[m]\right)\vec{X}^{(i)} = \vec{0}$

$\left([K] - \omega_j^2[m]\right)\vec{X}^{(j)} = \vec{0}$

premultiply these by $\vec{X}^{(j)T}$ & $\vec{X}^{(i)T}$

$\vec{X}^{(j)T}\left[\left([K] - \omega_i^2[m]\right)\vec{X}^{(i)} = \vec{0}\right]$ ①

$\vec{X}^{(i)T}\left[\left([K] - \omega_j^2[m]\right)\vec{X}^{(j)} = \vec{0}\right]$ ②

$\vec{X}^{(j)T}[K]\vec{X}^{(i)} - \omega_i^2\vec{X}^{(j)T}[m]\vec{X}^{(i)} = 0$ ③

$\vec{X}^{(i)T}[K]\vec{X}^{(j)} - \omega_j^2\vec{X}^{(i)T}[m]\vec{X}^{(j)} = 0$ ④

side note:
$(ABC)^T = C^T B^T A^T$

rewrite eq. ③ with xposing every term:

$\vec{X}^{(i)T}[K]^T\vec{X}^{(j)} - \omega_i^2\vec{X}^{(i)T}[m]^T\vec{X}^{(j)} = 0$ ⑤

take ④ − ⑤ and get:

$-(\omega_j^2 - \omega_i^2)\vec{X}^{(i)}[m]\vec{X}^{(j)} = 0$

↳ if $\omega_i \neq \omega_j$ then $\vec{X}^{(i)}[m]\vec{X}^{(j)} = 0$

back to equation 1: wrote this ...

$\vec{X}_j^T\left[\left([K] - \omega_i^2[m]\right)\vec{X}^{(i)} = \vec{0}\right]$

↑ premultiply eq.

$i \neq j$

$\vec{X}^{(j)T}[K]\vec{X}^{(i)} = 0$

orthonormality, scale to 1 for mass matrix: $\vec{X}^{(i)T}[m]\vec{X}^{(i)} = 1$

then what is $\vec{X}^{(i)T}[K]\vec{X}^{(i)} = ?$

$\vec{X}^{(i)T}\left[\left([K] - \omega_i^2[m]\right)X\right.$

Expansion Theorem

$$\vec{x} = \sum_{i=1}^{n} c_i \vec{\underline{X}}^{(i)}$$

$$= \begin{bmatrix} \vec{\underline{X}}^{(1)} & \vec{\underline{X}}^{(2)} & \dots & \vec{\underline{X}}^{(n)} \end{bmatrix} \begin{Bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{Bmatrix}$$

$$[\underline{X}] \qquad\qquad \vec{c}$$

$$[\underline{X}]\vec{c} = \vec{x}$$

$\vec{c} = [\underline{X}]^{-1}\vec{x}$ ... since $\underline{X}$ has linearly independent columns it's invertible

how do we use math orthogonality?

$$\vec{\underline{X}}^{(j)T}[m]\left(\vec{x} = \sum_{i=1}^{n} c_i \vec{\underline{X}}^{(i)}\right)$$

$$\vec{\underline{X}}^{(j)T}[m]\vec{x} = \sum_{i=1}^{n} c_i \vec{\underline{X}}^{(j)T}[m]\vec{\underline{X}}^{(i)}$$

let's say $j=4$ ... get ϕ until $i=4$ when $\vec{\underline{X}}^{(j)T}[m]\vec{\underline{X}}^{(i)} = 1$

$$\underbrace{\vec{\underline{X}}^{(j)T}[m]\vec{\underline{X}}^{(i)}}_{\begin{matrix} 0 & i \neq j \\ 1 & i = j \end{matrix}}$$

How do we relate this to vibrations?

$$\vec{x} = \sum_{i=1}^{n} \vec{\underline{X}}^{(i)} A_i \cos(\omega_i t + \phi_i)$$

is the most general solution to $[m]\ddot{\vec{x}} + [k]\vec{x} = 0$

proof $i=1$

$$\vec{x}_1 = \vec{\underline{X}}^{(1)} A_{(1)} \cos(\omega_1 t + \phi_1)$$

$$\ddot{\vec{x}}_1 = (-\omega_1)^2 \vec{\underline{X}}^{(1)} A_1 \cos(\omega_1 t + \phi_1)$$

$$= -\omega_1^2 \vec{x}_1$$

substitute into EOM

$$\left(([m](-\omega_1)^2 + [k]\right)\vec{x}_1 = \vec{0}$$

$$(-\omega_1^2[m]$$