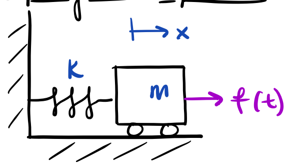
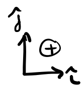
Spring Mass System $\Rightarrow$ Newton's 2nd Law

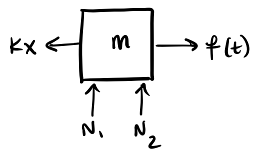


$m\ddot{x} + kx = f(t)$ How do we derive this?

VERY INTRODUCTORY MATERIAL



break this up...



$$\Sigma F_i = m\ddot{x}$$
$$f(t) - kx = m\ddot{x}$$
$$m\ddot{x} + kx = f(t) \quad \text{... standard form of an ODE}$$

$\rightarrow$ if given $x(0)$, $\dot{x}(0)$, m, and k, can then find $x(t)$

The Free Vibration Problem : $f(t) = 0$

$$m\ddot{x} + kx = 0$$
$$x(0) = x_0$$
$$\dot{x}(0) = \dot{x}_0$$

divide by m...

$$\ddot{x} + \omega_n^2 x = 0$$
$$\omega_n = \sqrt{\frac{k}{m}} \quad \text{(the natural frequency)}$$

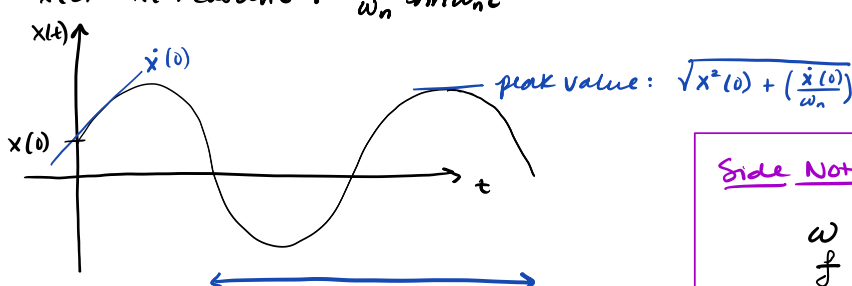
$$x(t) = A\cos\omega_n t + B\sin\omega_n t$$
$$x(0) = A\underbrace{\cos\omega_n(0)}_{=1} + B\underbrace{\sin\omega_n(0)}_{0}$$

$$\therefore\ x(0) = A$$

$$\dot{x}(t) = A(-\omega_n)\sin\omega_n t + B(\omega_n)\cos\omega_n t$$
$$\dot{x}(0) = A(-\omega_n)\underbrace{\sin\omega_n(0)}_{0} + B(\omega_n)\underbrace{\cos\omega_n(0)}_{1}$$

$$\therefore\ \dot{x}(0) = B\omega_n$$
$$B = \dot{x}(0)/\omega_n$$

$$x(t) = x(0)\cos\omega_n t + \frac{\dot{x}(0)}{\omega_n}\sin\omega_n t$$



natural period $T_n = \frac{2\pi}{\omega_n}$

$$D\cos(\omega_n t - \phi) \Rightarrow D = \sqrt{x^2(0) + \left(\frac{\dot{x}(0)}{\omega_n}\right)^2}$$

Side Note :

ω	FREQUENCY	(rad/sec)
f	FREQUENCY	(Hz = cps)

$$\omega = 2\pi f$$

$$\frac{rad}{sec} \times \frac{cycles}{2\pi\ rad}$$

ex. $6\ \frac{rad}{s} \times \frac{sec}{2\pi\ rad}$

~ Supplemental Textbook Notes ~

for simplicity of analysis, continuous systems are often approximated as multidegree-of-freedom systems

there's one equation of motion for each degree of freedom
→ if generalized coordinates used, there's one generalized coordinate for each DoF

there are n natural frequencies, each associated w/ its own mode shape, for a system having n degrees of freedom
→ method of determining the natural frequencies from the characteristic equation obtained by equating the determinant to zero also applies to these systems
→ however, as # DoF ↑, soln to characteristic equation becomes more complex
→ mode shapes exhibit orthogonality, which can be used for the solution of undamped forced-vibration problems using a procedure known as modal analysis

modeling of continuous systems as multidegree-of-freedom systems

- replace the distributed mass or inertia of the system by a finite number of lumped masses or rigid bodies
  - lumped masses assumed to be connected by massless elastic + damping members
  - linear or angular coordinates are used to describe the motion of the lumped mass or rigid bodies

↳ these are called lumped-parameter or lumped-mass or discrete-mass systems

→ the minimum number of coordinates necessary to describe the motion of the lumped masses and rigid bodies defines the # of DoF of the system
  - larger the # of lumped masses used in the model, higher the accuracy of the resulting analysis