Vibrations – Lecture 7
Thur. 2021 – 02 – 18

Expressions for potential + kinetic energy in any size systems:

$$V = \frac{1}{2}\sum_{i=1}^{n}\left(\sum_{j=1}^{n} k_{ij}x_j\right)x_i = \frac{1}{2}\vec{x}^T[k]\vec{x}$$

POTENTIAL ENERGY

a scalar

$$T = \frac{1}{2}\sum_{i=1}^{n}$$

vector of complex numbers in MATLAB
- if you have a complex vector and transpose it gives conjugate.
→ need to do if just want pure transpose : X.'

ex. $X = \begin{Bmatrix} 1+2i \\ 2+4i \end{Bmatrix}$ $X' = \begin{bmatrix} 1-2i & 2-4i \end{bmatrix}$ $X.' = \begin{bmatrix} 1+2i & 2+4i \end{bmatrix}$

<u>Assignment #2 Notes</u>
- F(t) only acts on m_3
- he gives us displacements but not force on mass
- use the hint!

  conservation of power statement

  $$P_{in} - P_{out} = \dot{E}$$

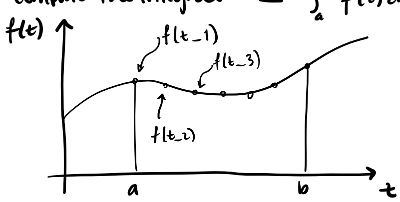
  P_{in}: instantaneous power flowing in from outside

  P_{out}: power out, working on environment (or heat)

  ... if dashpot turns mechanical energy into heat then P_{out}
- there's a .mat file given that contains all of the numerical values
- one problem w/multiple parts
  - he's grading the To Find parts (check the sizes!)
  - 1st bullet from natural frequencies
  - next to bullets unrelated to natural frequency
  - multiple ways to approach the 3rd bullet pt.

in general, for vibrations, making things stiffer doesn't necessarily reduce vibrations (for instance, it could move natural frequencies towards excitation frequency)

compute the integral $I = \int_a^b f(t)\,dt$

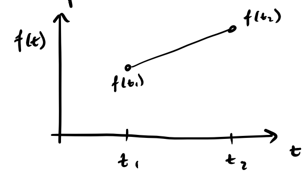


t	f(t)
0	2
0.1	8
0.2	16
⋮	⋮

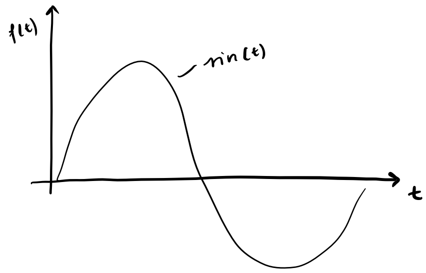
trapezoid method will be very effective for assignment

simplest way to get integral in MATLAB (when don't have f(t) given) : trapezoid $I = \sum_{n=1}^{N} A_n$ add up all of the areas differentiation for MATLAB

$\frac{df}{dx}$



$$\frac{df}{dx} = \frac{\Delta f}{\Delta x} = \frac{f(t_2) - f(t_1)}{t_2 - t_1}$$



if integrate a sine function from 0 to 2π get zero
if integrate a cosine funct. from 0 to 2π get zero

use trapz() in MATLAB ... need to multiply trapz by time spacing (or else it automatically assumes 1)

aka: delta * trapz(f)
delta * trapz (f(1:end/2))

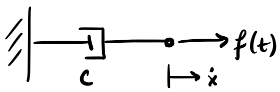
this gets 1st 1/2 of vector

for derivative, use diff() command

↳ plot(t(1:end-1), diff(f)./diff(t))

⊛ can look at sample script given w/exotic function that he made to show how to use these techniques

Reminder about Dashpots



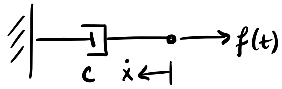
$f = c\dot{x}$

What power is being deliverd to the dashpot?

power
$P = f\dot{x}$
$P = c\dot{x}^2$

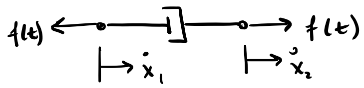
energy $E = \int_{t_1}^{t_2} P\,dt$

What about this?



$P = -f\dot{x}$

What about?



$f = c\dot{\delta}$

$\dot{\delta} = \dot{x}_2 - \dot{x}_1$ ... the rate of extension

$P = c(\dot{x}_2 - \dot{x}_1)^2$

Equation on the handout:

$$\ddot{T} + \omega^2 T = 0$$

$$T(t) = A\cos\omega t + B\sin\omega t$$

frequency

$$\omega = 2\pi f$$

rad/sec

Hz
cps
cyc/sec

$$|[K] - \omega^2[M]| = 0$$

characteristic polynomial

$$P(\omega^2) = 0$$

$[K]$ & $[M]$ N×N matrices then N values of ω^2

$\omega_1^2, \omega_2^2, \ldots, \omega_N^2$ $\qquad \omega_n^2 \quad n = 1, 2, \ldots, N$

in the assignment, have N×N as 3x3

without the square, we call these natural frequencies $\omega_1, \omega_2, \ldots \omega_N$

in MATLAB:

```
[~, D] = eig(K,M);
```

$$D = \begin{matrix} 1 & 0 \\ 0 & 3 \end{matrix}$$

ω_1^2

ω_2^2 — note: if get a negative # then made a mistake

```
omega_squared = diag(D)

omega = sqrt(omega_squared)
```

assignment wants units of Hertz though:

```
f = omega/(2*pi)
```