Assignment #4 due today
Assignment #5 now up

Vibrations – Lec 14
2021-03-16

$\cos(t)$
$\omega = 1$ rad/s } interpret t as nondimensional time

$t = \omega \tau$ ← τ time variable

$\omega = 1$ rad/s $\quad 0 \le \tau \le \tau_{max} \quad 0 \le t \le \tau_{max}$
$\omega = 2$ rad/s $\quad 0 \le \tau \le \tau_{max} \quad 0 \le t \le 2 * \tau_{max}$

what could have also been done if we didn't change force sine to add a parameter

$$T = \frac{1}{2} \underbrace{\dot{\vec{x}}^T}_{1\times3} \underbrace{[M]}_{3\times3} \underbrace{\dot{\vec{x}}}_{3\times1}$$

w/plot my code takes ~1 min

assignment #5 — UNFORCED SYSTEMS

- make sure proportionally damp as in $\alpha[M] + \beta[K] = [C]$ (?)
- mass matrix is diagonal
- C matrix is diagonal (dashpots are stretched relative to ground) } 1st part (→ C proportional to M?)
- C matrix is proportional to K matrix here ← 2nd part

hint: $\begin{bmatrix} c_1 & 0 & 0 \\ 0 & c_2 & 0 \\ 0 & 0 & c_3 \end{bmatrix} = C \qquad M = \begin{bmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_3 \end{bmatrix}$

$C = \alpha M$

... so if $c_1 = 3M_1$, $c_2 = 3M_2$, $c_3 = 3M_3$ $\therefore$ $\alpha = 3$, $\beta = 0$

- given 2 systems, given usual stuff
- find M, C, K + displacement $\bar{x}(t)$ ← part A + B

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B =

  struct with fields:

          m: [3×1 double]
          k: [3×1 double]
          c: [3×1 double]
        x_0: [3×1 double]
    x_dot_0: [3×1 double]
          t: [1×1000 double]

>> B.k

ans =

     3
     4
     5

>> B.c

ans =

    1.5000
    2.0000
    2.5000
```

*nothing too tricky w/ this assignment

$[M]\ddot{\vec{x}} + [C]\dot{\vec{x}} + [K]\vec{x} = \vec{F}(t)$ $\qquad$ $\vec{x}_0$ & $\dot{\vec{x}}_0$

Find $\vec{x}(t)$.

$[C]$ general

may not be $\alpha[M] + \beta[K]$ $\qquad$ ... use state space matrices

$$[S]\dot{\vec{y}} - [R]\vec{y} = \begin{Bmatrix} \vec{0} \\ \vec{F}(t) \end{Bmatrix}$$

$$\vec{y} = \begin{Bmatrix} \vec{x} \\ \dot{\vec{x}} \end{Bmatrix}$$

$$\begin{bmatrix} [I] & [0] \\ [0] & [M] \end{bmatrix} \begin{Bmatrix} \dot{\vec{x}} \\ \ddot{\vec{x}} \end{Bmatrix} - \begin{bmatrix} [0] & [I] \\ -[K] & -[C] \end{bmatrix} \begin{Bmatrix} \vec{x} \\ \dot{\vec{x}} \end{Bmatrix} = \begin{Bmatrix} \vec{0} \\ \vec{F}(t) \end{Bmatrix}$$

this row comes from eq of motion

not symmetric ... need to premult. first row by $-[K]$

now we have symmetric matrices

$$\underbrace{\begin{bmatrix} -[K] & [0] \\ [0] & [M] \end{bmatrix}}_{[S]} \begin{Bmatrix} \dot{\vec{x}} \\ \ddot{\vec{x}} \end{Bmatrix} - \underbrace{\begin{bmatrix} [0] & -[K] \\ -[K] & -[C] \end{bmatrix}}_{[R]} \begin{Bmatrix} \vec{x} \\ \dot{\vec{x}} \end{Bmatrix} = \begin{Bmatrix} \vec{0} \\ \vec{F}(t) \end{Bmatrix}$$

→ write more compactly:

$$[S]\dot{\vec{y}} - [R]\vec{y} = \begin{Bmatrix} \vec{0} \\ \vec{F}(t) \end{Bmatrix}$$

↓

pre multiply by inverse of S, $[S]^{-1}$

↓

$$\dot{\vec{y}} - [S]^{-1}[R]\vec{y} = [S]^{-1}\begin{Bmatrix} \vec{0} \\ \vec{F}(t) \end{Bmatrix}$$

↓

$$\dot{\vec{y}} = \underbrace{[S]^{-1}[R]}_{[A]}\vec{y} + \underbrace{[S]^{-1}}_{[B]}\underbrace{\begin{Bmatrix} \vec{0} \\ \vec{F}(t) \end{Bmatrix}}_{\vec{u}}$$

use state_space_matrices.m

↳ gives you the state space matrices

mdof_damped_forced_ss.m ← ss means state space

↑ uses mdof_damped_forced_ss_ode45.m

then check with mdof_damped_forced_test.m