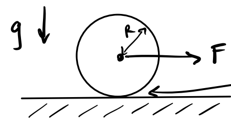
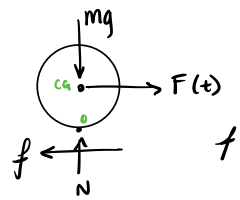
Disc example



no slipping

=> FBD =>



need to draw at instant in time be not static ... we don't know if moving to left or right

$f = \mu N$

don't necessarily need to know μ in order to solve this problem

μ_k - kinetic

μ_s - static

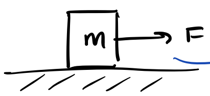
pick pt. O for summing moments (must be a fixed pt)

$\Sigma M = J\ddot{\theta}$

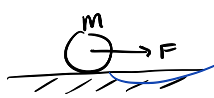
$\Downarrow$

$\Sigma M_0 = J_0 \ddot{\theta}$

fixed doesn't necessarily mean $\emptyset$ acceleration, but we can say $\emptyset$ velocity ... pt is not moving relative to the ground and ground is not moving either ... why we're allowed to sum moments about O as long as we take J_0

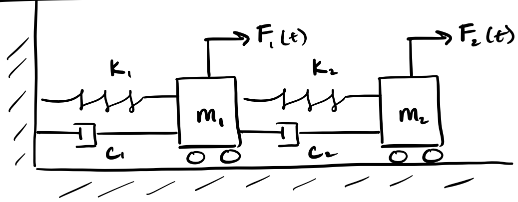


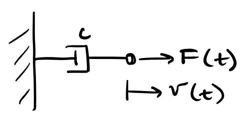
if same mass as disc, this block will accelerate more than the disc



if there's no slipping, it will roll & translate with more inertia

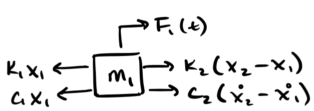
dashpots



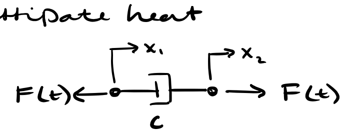


$F(t) = c\,v(t)$
$= c(\dot{x}_2 - \dot{x}_1)$

dashpots cannot store energy, only dissipate heat



$x_2 > x_1$
$\dot{x}_2 > \dot{x}_1$

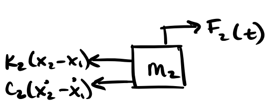


↑ subtle assumption: why make it? have to draw it in motion and not in place ... if hadn't made this assump. couldn't know whether to write $(x_2 - x_1)k_2$ or $k_2(x_1 - x_2)$, etc.

$\Sigma F = m_1 \ddot{x}_1$

$F_1(t) + k_2(x_2 - x_1) + c_2(\dot{x}_2 - \dot{x}_1) - k_1 x_1 - c_1 \dot{x}_1 = m_1 \ddot{x}_1$

$m_1 \ddot{x}_1 + (k_1 + k_2)x_1 + (c_1 + c_2)\dot{x}_1 - k_2 x_2 - c_2 \dot{x}_2 = F_1(t)$ (1)



note that equation (1) only has $F_1(t)$ and not $F_2(t)$ which will be how we order our matrices for assign.

$\Sigma F = m_2 \ddot{x}_2$

$F_2(t) - k_2(x_2 - x_1) - c_2(\dot{x}_2 - \dot{x}_1) = m_2 \ddot{x}_2$

$m_2 \ddot{x}_2 + k_2 x_2 + c_2 \dot{x}_2 - k_2 x_1 - c_2 \dot{x}_1 = F_2(t)$ (2)

$\begin{Bmatrix} F_1(t) \\ F_2(t) \end{Bmatrix}$

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} c_1+c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix}\begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix} + \begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} F_1(t) \\ F_2(t) \end{Bmatrix}$$

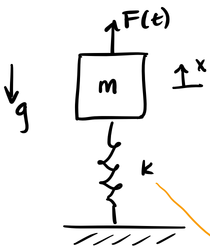
M_1 — the mass matrix

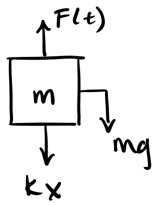
C_1 (dashpot matrix) — the damping matrix

K_1 — the stiffness matrix

gravity

bottom line assumption we can ignore gravity





Hooke's Law

the vertical displacement of the mass is measured from a reference, spring force is zero when relative displacement of mass is zero

X must be zero when spring is unstretched!

$$\Sigma F = m\ddot{x}$$
$$F(t) - kx - mg = m\ddot{x}$$
$$m\ddot{x} + \underbrace{kx + mg}_{k\left(x + \frac{mg}{k}\right)} = F(t)$$

$$\ddot{y} = \ddot{x}$$

$$m\ddot{y} + ky = F(t)$$ ← no gravity!

$$y = x + \frac{mg}{k}$$

$$\boxed{x = y - \frac{mg}{k}}$$

y: the vibration of the mass (the vibration part ... if not vibrating then $y = 0$)

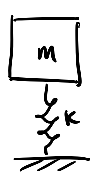
$\frac{mg}{k}$: δ_{st} (the static sag of the system)

*weight of mass will crush the spring

y is deflection from static equilibrium

if static:

block goes down



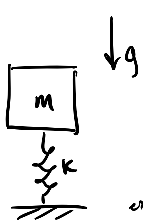
δ_{st} (delta static

$$\Rightarrow \delta_{st} = \frac{mg}{k}$$

(static deflection due to gravity

=> so should I use x or y?
... if measure a displacement from static equilibrium, don't need the mg term

main takeaway!



why do engineers worry about this system? (a low frequency system)

$$\delta_{st} = \frac{mg}{k}, \quad k = \frac{mg}{\delta_{st}} \Rightarrow \omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{mg}{\delta_{st} m}} = \sqrt{\frac{g}{\delta_{st}}}$$

ex. $\omega_n = 1$ $\quad \omega_n = 2\pi f_n \quad f_n = \frac{1}{2\pi} \quad \delta_{st} = \frac{g}{\omega_n^2} = \frac{9.81 \text{ m/s}^2}{(1 \text{ rad/s})^2} = 9.81 \text{ m}$

we want frequency to be really low, but static displacement will always be ~10m

so take $\omega_n = 10$ rad/s

$f_n = \frac{10}{2\pi}$ Hz $\quad \ldots \delta_{st} = \frac{9.81 \text{ m/s}^2}{(\omega_n \text{ rad/s})^2} = 0.1 \text{ m}$

if build low freq. resonator, going to have massive disp.