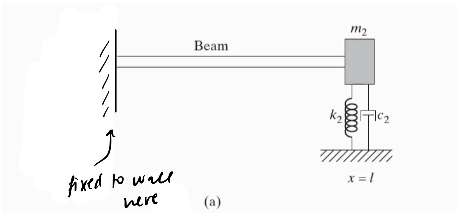
NEED TO REFERENCE THIS LECTURE IN ORDER TO COMPLETE ASSIGNMENT #7. DO NOT USE BOOK!

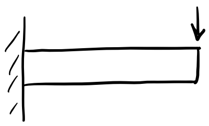
Vibrations - Lecture 20
Thur., 2021-04-08



Eq. 8.97 + 8.98 for RHS of beam
Eq. 8.96 (clamped) for LHS of beam

* do not include zero natural frequency if it is one ... ω_1, ω_2, + ω_3 must all be nonzero.

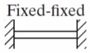
c = 0 because if c is nonzero then searching for complex numbers and makes this more complicated
(he'll change this in case given .mat is still showing c as nonzero



$k_{eq} = \frac{3EI}{l^3}$ ← the equivalent stiffness, aka stiffness of the beam

Chapter 8 solutions posted.*

Ex. Longitudinal vibration of rod/beam



Fixed-fixed $\quad u(0,t) = 0 \quad u(l,t) = 0 \quad \sin\frac{\omega l}{c} = 0 \quad U_n(x) = C_n \cos\frac{n\pi x}{l} \quad \omega_n = \frac{n\pi c}{l}; \quad n = 1,2,3,\ldots$

$u(x,t) = U(x)T(t) = \left(\underset{\sim}{A}\cos\frac{\omega x}{c} + \underset{\sim}{B}\sin\frac{\omega x}{c}\right) * \left(C\cos\omega t + D\sin\omega t\right)$

$u(x=0,\, t) = 0$
$U(x=0)\cancel{T(t)} = 0$

$u(x=l,\, t) = 0$
$U(x=l)\cancel{T(t)} = 0$ cancel these out...

$U(x) = A\cos\frac{\omega x}{c} + B\sin\frac{\omega x}{c}$ ... Find ω_n

$U(0) = A = 0$
$U(x) = B\sin\left(\frac{\omega x}{c}\right)$
$U(x=l) = B\sin\underbrace{\left(\frac{\omega l}{c}\right)}_{n\pi} = 0$

$n = 0, 1, 2, 3, \ldots$
↑ this is a soln but it's a trivial soln.

$\frac{\omega_n l}{c} = n\pi$

Prof. McDaniel is proposing a new method: don't use any algebra!

$U(x=0) = A + 0 \cdot B = 0$
$U(x=l) = A \cdot \cos\left(\frac{\omega l}{c}\right) + B \cdot \sin\left(\frac{\omega l}{c}\right) = 0$

↑ take these two equations and put in matrix form

$$\begin{bmatrix} 1 & 0 \\ \cos\left(\frac{\omega l}{c}\right) & \sin\left(\frac{\omega l}{c}\right) \end{bmatrix} \begin{Bmatrix} A \\ B \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

*remember, if the determinant is zero, then there's either no solutions or infinite number of solutions

↳ how do we know det() = 0? well, we hope so, so we set it equal to zero :)

bar_fixed.m
b = width
l = length
A = x-sectional area
I = ? need that on this assignment! ($bh^3(\frac{1}{12})$?)
take α and divide by l
↳ why?

$\underline{\alpha = \omega/c}$

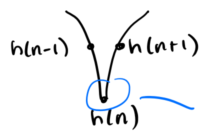
$\frac{\omega l}{c} = n\pi$ ... the linspace() should span the first few modes

D = matrix we made
d(n) = det(D)

why log10(abs(

↳ bc log of 0 is $-\infty$! ... so can see these points

second for loop: why start at 2? (creates empty index to start with)

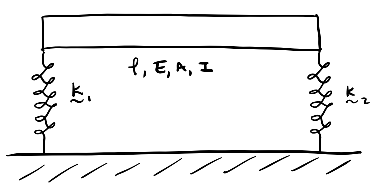


checks to see if in between those two pts!

↳ then able to find red o's (values we want) in plot!

→ should give you ans as being in radians per second

Problem in class: 8.37 (matlab code for this: problem_8_37.m)



Derive the frequency equation for the transverse vibration of a uniform beam resting on springs at both ends. Springs can deflect vertically only, and the beam is horizontal in the equilibrium position.

→ need to work two limits!
K = very stiff! → π, 2π, 3π, 4π
K = 0, nothing → 4.73, 7.85, 10.9956, 14.137

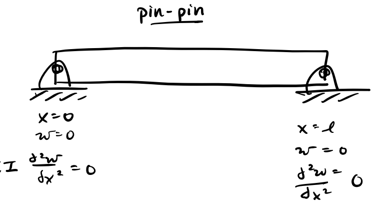
*he rewrote the program over the break to make simpler to understand

going to use: $W(x) = C_1 \cos\beta x + C_2 \sin\beta x + C_3 \cosh\beta x + C_4 \sinh\beta x$

$$M(x,t) = EI(x)\frac{d^2 w}{dx^2}$$

$$V(x,t) = \frac{d}{dx}\left(EI(x)\frac{d^2 w}{dx^2}\right) = EI\frac{d^3 w}{dx^3} \quad \text{(if uniform beam)}$$

pin-pin



$$w(x,t) = W(x)\,T(t)$$
$$w(x=0,t) = W(0)\,\cancel{T(t)} = 0$$

$$W(0) = \left.\frac{d^2W}{dx^2}\right|_{x=0} = W(\ell) = \frac{d^2W}{dx^2} = 0$$

① ② ③ ④ ← four conditions (4x4 matrix)

$W(x) = C_1 \cos\beta x + C_2 \sin\beta x + C_3 \cosh\beta x + C_4 \sinh\beta x$ ← our assumed soln

$W'(x) = C_1(-\beta)\sin\beta x + C_2(\beta)\cos\beta x + C_3(\beta)\sinh\beta x + C_4(\beta)\cosh\beta x$

$W''(x) = C_1(-\beta^2)\cos\beta x + C_2(-\beta^2)\sin\beta x + C_3\beta^2\cosh\beta x + C_4\beta^2\sinh\beta x$

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ -\beta^2 & 0 & \beta^2 & 0 \\ \cos\beta\ell & \sin\beta\ell & \cosh\beta\ell & \sinh\beta\ell \\ -\beta^2\cos\beta\ell & -\beta^2\sin\beta\ell & \beta^2\cosh\beta\ell & \beta^2\sinh\beta\ell \end{bmatrix} \begin{Bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \begin{array}{l} \leftarrow W(0) \\ \leftarrow d^2W/dx^2|_{x=0} \\ \leftarrow W(\ell) \\ \leftarrow d^2W/dx^2|_{x=\ell} \end{array}$$

beam_pinned.m ←

↳ need to convert from β to omega $\quad \omega = \beta^2\sqrt{\frac{EI}{\rho A}} = (\beta\ell)^2\sqrt{\frac{EI}{\rho A \ell^4}}$