Most common vibration calculation

Vibrations - Lec 26
2021-04-29

"Frequency Sweep"

$$[m]\ddot{\vec{x}} + [c]\dot{\vec{x}} + [k]\vec{x} = \vec{F}(t)$$

proportional damping

Steady-State Vibration Particular Soln

$$\vec{x}(t) = \vec{X} \exp(j\omega t)$$

real part

i

complex amplitude

$$\vec{X} = \vec{X}_r + i\,\vec{X}_i$$

$$\vec{x}(t) = \mathrm{Re}\left\{(\vec{X}_r + j\,X_i)(\cos\omega t + j\sin\omega t)\right\}$$

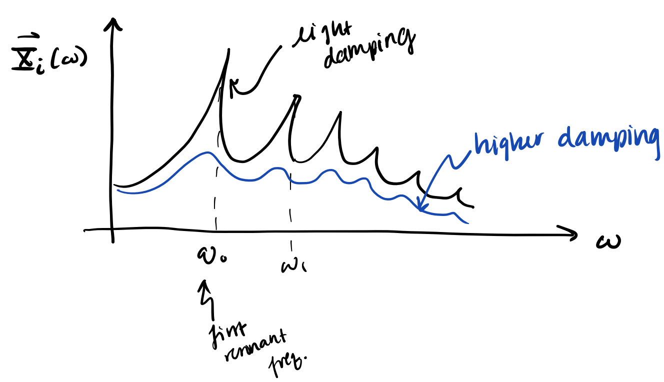
$$= \vec{X}_r \cos\omega t - \vec{X}_i \sin\omega t$$

$$\vec{F}(t) = \vec{F}_0 \exp(j\omega t)$$

complex-valued force vector

- frequency sweep:
  given $\vec{F}_0$ & $a \le \omega \le b$
  $[m], [c], [k]$

  find $\vec{X}(\omega)$



- substitute time dependencies

$$\left((j\omega)^2[M] + (j\omega)[c] + [k]\right)\vec{X} = \vec{F}_0$$

$$[D(\omega)]\,\vec{X} = \vec{F}_0$$

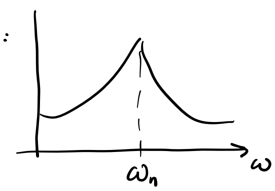
we call this the dynamic stiffness

$$\vec{X}(\omega) = [D(\omega)]^{-1}\vec{F_0}$$

- why peaks in FRF = frequency response function ?

$$[c] = [0]$$
$$([k] - \omega^2[m])\vec{X} = \vec{F_0}$$

in One DoF:



$$X = \frac{F_0}{k - \omega^2 m}$$

- associated eigenvalue problem

$$([k] - \omega_n^2[m])\vec{\Phi}_n = \vec{0}$$

$$\vec{X} = \sum_{n=1}^{N} c_n \vec{\Phi}_n = [\Phi]\{c\}$$

$$[\vec{\Phi}_1 \ \vec{\Phi}_2 \ \dots \ \vec{\Phi}_n]$$ ... eigenvalue matrix

$$\vec{X} = [\Phi]\vec{c}$$

$$[\Phi]^T([k] - \omega^2[m])[\Phi]\vec{c} = [\Phi]^T\vec{F_0}$$

$$[\Phi]^T[k][\Phi] = diag(\omega_n^2)$$
$$[\Phi]^T[m][\Phi] = [I] \quad N \times N$$

$$c_n = \frac{([\Phi]^T\vec{F_0})_n}{\omega_n^2 - \omega^2}$$

putting it all together into one final answer:

$$\vec{X}(\omega) = \sum_{n=1}^{N} \underbrace{\frac{\vec{\Phi}_n^{\,T}\vec{F_0}}{\omega_n^2 - \omega^2}}_{\text{scalar \#, call this the MODAL AMPLITUDE}} \vec{\Phi}_n$$

eigenvector