Administrative Stuff:

Vibrations – Lecture 13
2021-03-11

- Assignment #4 due Tuesday
- some new code he wrote that goes into theory that we'll use for future assignments

*new zip file on BB of codes:
mdof_undamped_force.m

A#4

Part 1: just need to write forcing routine, use the code provided
→ there's no distinction between x_h and x_p though
→ can't use unforced code because we don't know initial conditions for x_h and x_p
therefore none of the codes given can give us x_h or x_p

Part 2 & 3: $P(t) = \dot{E}(t)$
→ sanity checks!
→ these must be computed separately

~ Lecture ~

Book doesn't cover free vibration of damped systems $[m]\ddot{\vec{x}} + [c]\dot{\vec{x}} + [k]\vec{x} = \vec{0}$ ... can we just make $\vec{F} = \vec{0}$ and use code for that? Something to think about

My question to look up:
What is "state space"?

Read pages 160+ on (2.6 Free Vibration w/ Viscous Damping) for background

6.15 Forced Vibration of Viscously Damped Systems
→ mdof_damped_free_vibration
- as of now this will only return x, not $\dot{x}$ or $\ddot{x}$
- can follow sequence as done in the text (he did not verify that there's no errors but there are likely none)

⇒ $x = \underline{X}\, q$ at the end
- this is a completely analytic integration

→ test code: mdof_damped_free_vibration_test.m

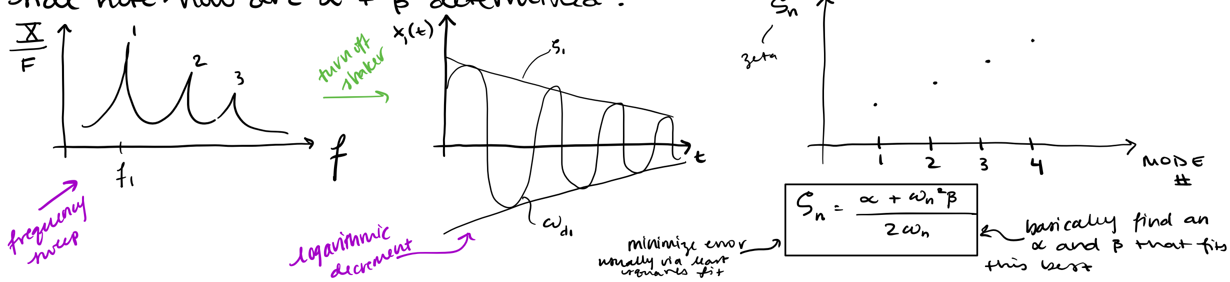
$M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $K = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 4 \end{bmatrix}$ $\alpha = 0.001$, $\beta = 0.001$

[C] = alpha*M + beta*K

plot(t, x) ... get plot w/ 3 lines

- we'll find it's going to be very similar to the undamped free vibration
plot this then compare

side note: how are α + β determined?



minimize error usually via least squares fit

$$\zeta_n = \frac{\alpha + \omega_n^2 \beta}{2\omega_n}$$

basically find an α and β that fits this best

Damped + Forced Modal Equation

$$\ddot{q}_i + 2\zeta_i \omega_i \dot{q}_i + \omega_i^2 q_i = Q_i$$

$$\dot{y} = Ay + Bu$$

$$y = \begin{Bmatrix} q \\ \dot{q} \end{Bmatrix}$$

[A] & [B] are state-space matrices

mdof_damped_force

$$\begin{Bmatrix} \dot{q} \\ \ddot{q} \end{Bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega_i^2 & -2\zeta_i\omega_i \end{bmatrix} \begin{Bmatrix} q \\ \dot{q} \end{Bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} 0 \\ Q \end{Bmatrix}$$

$$\ddot{q}_i = -2\zeta_i\omega_i\dot{q}_i - \omega_i^2 q_i + Q_i$$

can see this in the code
- ↳ make state-space matrices
- ↳ then implement initial conditions

mdof_damped_forced_test
- uses 2x2 system
- low alpha + beta
- initial conditions all zero
- using cosine forcing (applied to 2^{nd} mass)
  - ↳ then code knows to evaluate this as a character vector ⇒ force = 'force_cosine'
  - ↳ mdof_damped_forced_ode45.m
- this is underdamped
  - ↳ red is forced
  - ↳ blue is xient

  from plot

* for our assignment, need to make a forcing function