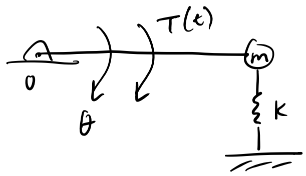
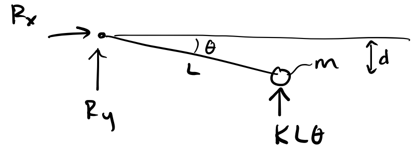
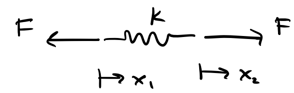
Vibrations - Lecture 6
2021-02-11







$\sin\theta = \frac{d}{L} \approx \theta$

$d = L\theta$

$F = k\delta$
$= k(x_2 - x_1)$

$\circlearrowleft$ $\Sigma M_0 = J_0 \ddot{\theta}$

$-kL^2\theta + T = J_0\ddot{\theta}$

$J_0\ddot{\theta} + kL^2\theta = T$

$mL^2\ddot{\theta} + kL^2\theta = T$

$Ax = b$ aka $[A]\{x\} = \{b\}$

if the determinant of $A = \emptyset$ then there are zero or infinite # of solns to this

if $|A| = 0$ and $\{b\} = \{0\}$, then there are ∞ # solns

$[A]\{x\} = \{b\} = 0$ (homogeneous bc of the 0)

$\{x\} = 0$ => the trivial soln ... since there is one soln then the theorem above means that we have infinite

1D system ex. = $5x = 2$ $\quad x = \frac{2}{5}$
$\quad [A] \uparrow \quad \uparrow \{b\}$

$5x = 0$
determinant is 5
then one soln, $x = 0$

$$\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \Rightarrow \begin{aligned} x_1 &= -2x_2 \\ -4x_1 - 4x_2 &= 0 \end{aligned}$$

ex. let's say $x_2 = 1$ then $x_1 = -2$ => $\begin{Bmatrix} -2 \\ 1 \end{Bmatrix}$

$$\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \begin{Bmatrix} -2 \\ 1 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

now pretend we multiply both sides by 5 ... still be correct

$$\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \underbrace{5 \begin{Bmatrix} -2 \\ 1 \end{Bmatrix}}_{c\{x\}} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

then if $\{x\}$ is a soln, $5\{x\}$ is also a solution and that's part of the nature of a homogeneous system

$\alpha [A] \{x\} = \alpha \{b\} = \alpha \{0\}$

rewrite as $\rightarrow$ $[A] \underbrace{\alpha \{x\}}_{\text{soln}} = \{0\}$

any linear homogeneous system, if $\{x\}$ is a solution then $\alpha\{x\}$ is also a solution

how does this apply to vibrations?

$$\Big[[K] - \omega^2 [M]\Big]\{\bar{X}\} = \{0\}$$

if we want multiple $\bar{X}$ then determinant of $[[K]-\omega^2[M]]$ must be zero

$\{\bar{X}\} = 0$

for more solutions to exist $|\underset{N \times N}{[K]} - \omega^2 \underset{N \times N}{[M]}| = 0$

polynomial of highest power $(\omega^2)^N = 0$

$N=2$ $\quad (\ )(\omega^2)^2 + (\ )(\omega^2) + (\ ) = 0$

$N=3$ $\quad (\ )(\omega^2)^3 + (\ )(\omega^2)^2 + (\ )(\omega^2) + (\ ) = 0$

one often finds N values of ω^2

roots() command in MATLAB allows us to find roots of polynomials