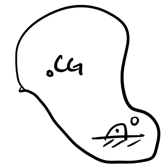
New assignment details:
- make all scripts buid_assignmentno.m

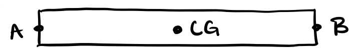
Vibrations - Lec4
2021-02-04

Can only use following equation for CG or fixed pt.

$$\Sigma M_0 = J_0 \ddot{\theta}$$



In Rao:



we'll see

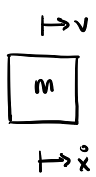
$\Sigma M_A = (\qquad) + (\quad)$

ΣM_B

*need to use: $\underset{\sim}{\Sigma M_A} = J_G(\alpha \hat{k}) + \underset{\sim}{r_{G/A}} \times (m\underset{\sim}{a_G})$

see handout on BB for quick study of planar motion
(in image G is center of gravity and α is angular acceleration) ... $\alpha \hat{k} \Leftrightarrow \ddot{\theta}$

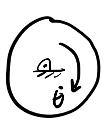
Rao 6.5 Energy



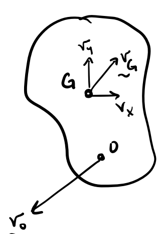
$T = \frac{1}{2} m v^2$

$v = \dot{x}$

$T = \frac{1}{2} m \dot{x}^2$



$T = \frac{1}{2} J_0 \dot{\theta}^2$

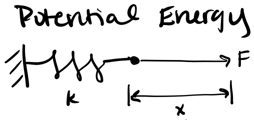


$T = \frac{1}{2} m |\underset{\sim}{V_G}|^2$

$T = \frac{1}{2} m \sqrt{V_x^2 + V_y^2}$

CHASLE'S THEOREM:

Potential Energy



(x is final distance

$\mapsto y$ (y is intermediate distance)

$U = \frac{1}{2} k x^2$

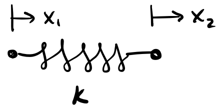
F is a function of y

$U = \int_0^x F(y)\, dy$

plug in Hooke's Law: ky

$U = \int_0^x k y\, dy = \frac{1}{2} k y^2 \Big|_0^x$

now, if have spring and both ends can move:
What's the potential energy?



$$U = \frac{1}{2}k(x_2 - x_1)^2$$

$$U = \frac{1}{2}k(x_2^2 - 2x_1x_2 + x_1^2)$$

now pretend that given a vector $\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix}$ and we're told that potential energy of system is:

$$U = \frac{1}{2}\begin{bmatrix} x_1 & x_2 \end{bmatrix}\begin{bmatrix} k \end{bmatrix}\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix}$$

k matrix is 4x4, symmetric

$$\begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix}$$

$$U = \frac{1}{2}\begin{bmatrix} x_1 & x_2 \end{bmatrix}\begin{Bmatrix} k_{11}x_1 + k_{12}x_2 \\ k_{21}x_1 + k_{22}x_2 \end{Bmatrix}$$

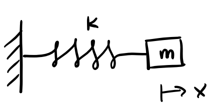
$$U = \frac{1}{2}\left(k_{11}x_1^2 + k_{12}x_1x_2 + k_{21}x_1x_2 + k_{22}x_2^2\right)$$

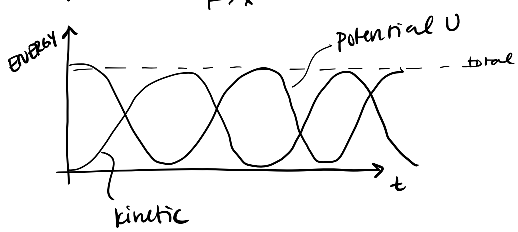
$$= \frac{1}{2}\left(k_{11}x_1^2 + (k_{12} + k_{21})x_1x_2 + k_{22}x_2^2\right) \iff \frac{1}{2}k\left(x_2^2 - 2x_1x_2 + x_1^2\right)$$

$$\therefore\ k_{11} = k,\quad k_{22} = k$$
$$k_{12} + k_{21} = -2k \quad\Rightarrow\quad [k] = \begin{bmatrix} k & -k \\ -k & k \end{bmatrix}$$
$$k_{12} = -k$$
$$k_{21} = -k$$

This form is generally written as $U = \frac{1}{2}\underbrace{\underset{1\times n}{\{x\}^T}\ \underset{n\times n}{[k]}\ \underset{n\times 1}{\{x\}}}_{\text{aka a "triple product"?}}$ ... this is a scalar

with n-DoF system, potential energy is scalar





as this thing vibrates through, there's a dance between potential + kinetic energy

$$T + U = \text{constant}$$

$U = \frac{1}{2}\{x\}^T[k]\{x\}$ inner product of x wrt k $= \frac{1}{2}\sum_m \sum_n k_{mn}x_m x_n$

$T = \frac{1}{2}\{\dot{x}\}^T[m]\{\dot{x}\}$ inner product of $\dot{x}$ wrt m $= \frac{1}{2}\sum_m \sum_n m_{mn}\dot{x}_m \dot{x}_n$

these are called quadratic sums

initial potential energy : $U_0 = \frac{1}{2}kx_0^2$
initial kinetic energy: $T_0 = \frac{1}{2}mv_0^2$

$$\underbrace{U_0 + T_0 = E_0}_{\text{initial total energy}} = E(t)$$ ~ say $E(t)$ because the energy is same

Conservation of Energy:
the time rate of change of the energy of the system is the net Power flowing into the system

$$\dot{T} + \dot{U} = P_{in} - P_{dis}$$

dissipate

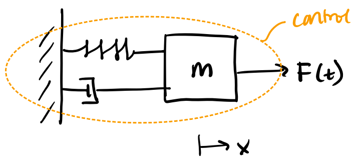
chain rule ($\frac{1}{2}mv^2 = T$)

$\dot{T} = m\dot{x}\ddot{x}$
$\dot{U} = kx\dot{x}$

if assume no dampots, etc then $\Sigma P = 0$

↳ plug these in: $m\cancel{\dot{x}}\ddot{x} + kx\cancel{\dot{x}} = 0$
$m\ddot{x} + kx = 0$ ... equation of motion!

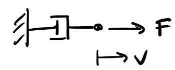
now add dashpot & force:



$\dot{T} = m\dot{x}\ddot{x}$
$\dot{U} = kx\dot{x}$

$P_{in} = F\dot{x}$ power is force · velocity

the hooke's law equivalent of dashpot:



$P_{dis} = Fv = cv^2$

$F = cv$

$$m\dot{x}\ddot{x} + kx\dot{x} = F\dot{x} - c\dot{x}^2$$
$$m\ddot{x} + c\dot{x} + kx = F$$ the equation of motion for the damped system