MATH 271, Homework 10

Due November 20^{nd}

Problem 1. For each description below, construct a single 3×3 transformation matrix.

(a) A rotation by $\frac{\pi}{2}$ in the yz-plane.

(b) A rotation by $\frac{\pi}{2}$ in the xy-plane, an interchange of the x and y coordinates.

(c) A matrix which undoes everything in part (b).

Problem 2. Consider the system of linear equations

$$\begin{aligned} 2x - y - 2z &= 4 \\ -x + 3y + 4z &= -2 \\ 2x + y - 2z &= 8. \end{aligned}$$

(a) Write this system of equations as a matrix/vector equation

$$[A]\vec{\boldsymbol{x}} = \vec{\boldsymbol{y}}.$$

(b) Solve the system of equations by finding the inverse of the matrix $[A]$.

Problem 3. Consider the matrix

$$[A] = \begin{pmatrix} 1 & 3 & 0 \\ 3 & 1 & 0 \\ 0 & 4 & 1 \end{pmatrix}$$

(a) Compute the eigenvalues and eigenvectors for the matrix $[A]$.

(b) Show that $\det([A]) = \lambda_1\lambda_2\lambda_3$ and $\operatorname{tr}([A]) = \lambda_1 + \lambda_2 + \lambda_3$ where λ_1, λ_2, and λ_3 are the eigenvalues you found in (a).

(c) Argue why the eigenvalues of $[A]^{-1}$ must be $\frac{1}{\lambda_1}$, $\frac{1}{\lambda_2}$, and $\frac{1}{\lambda_3}$.

Problem 4. Consider the matrix

$$[A] = \begin{pmatrix} 1 & 1 \\ 0 & 2 \end{pmatrix}.$$

(a) Find the eigenvalues and eigenvectors for this matrix.

(b) Construct the matrix $[P]$ such that

$$[\Lambda] = [P]^{-1}[A][P]$$

from the eigenvectors you found.

(c) Find $[P]^{-1}$ and compute

$$[\Lambda] = [P]^{-1}[A][P].$$

Is this $[\Lambda]$ diagonal?

Problem 5. Let $A\colon \mathbb{C}^n \to \mathbb{C}^n$ be hermitian. We prove the two theorems in the text.

(a) Show that all eigenvalues of A are real.

(b) Show that eigenvectors corresponding to different eigenvalues are orthogonal with the hermitian inner product.