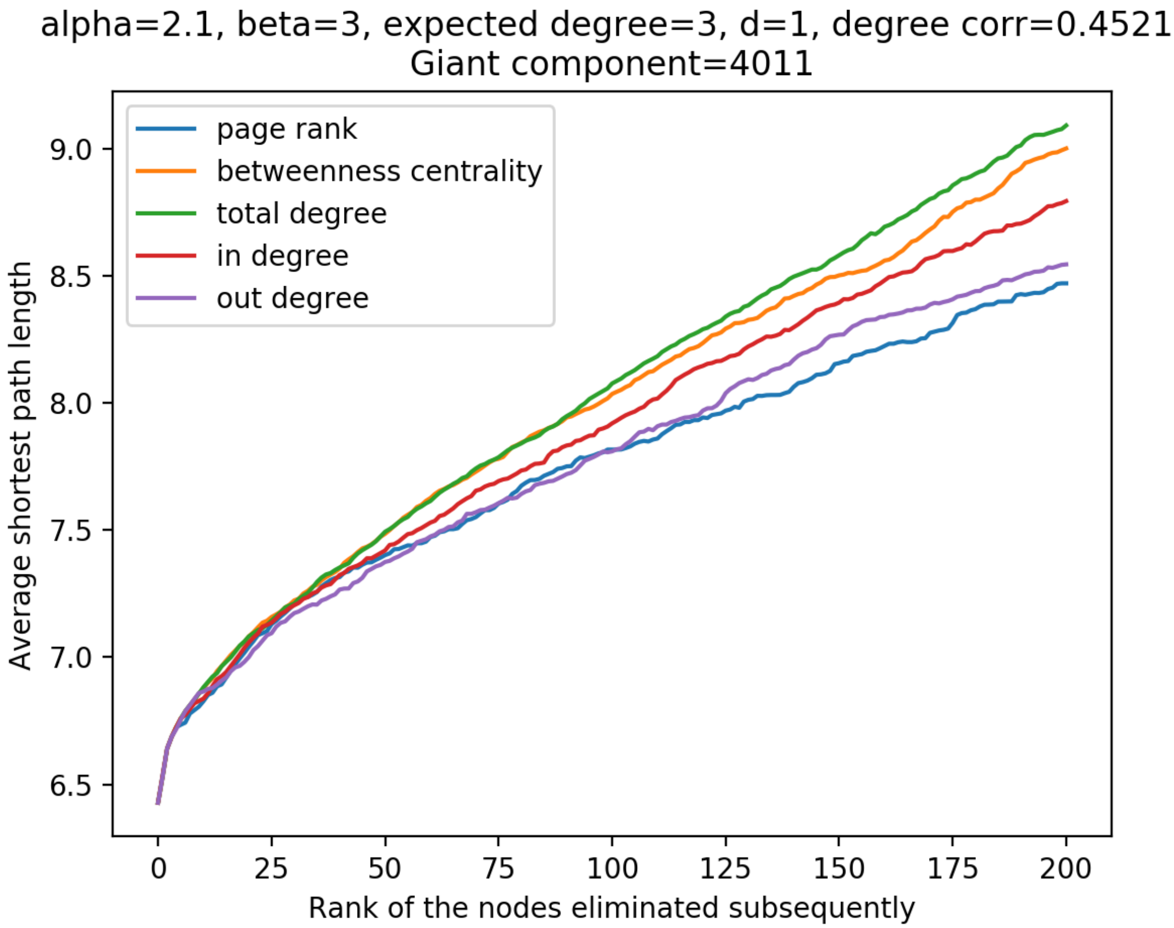
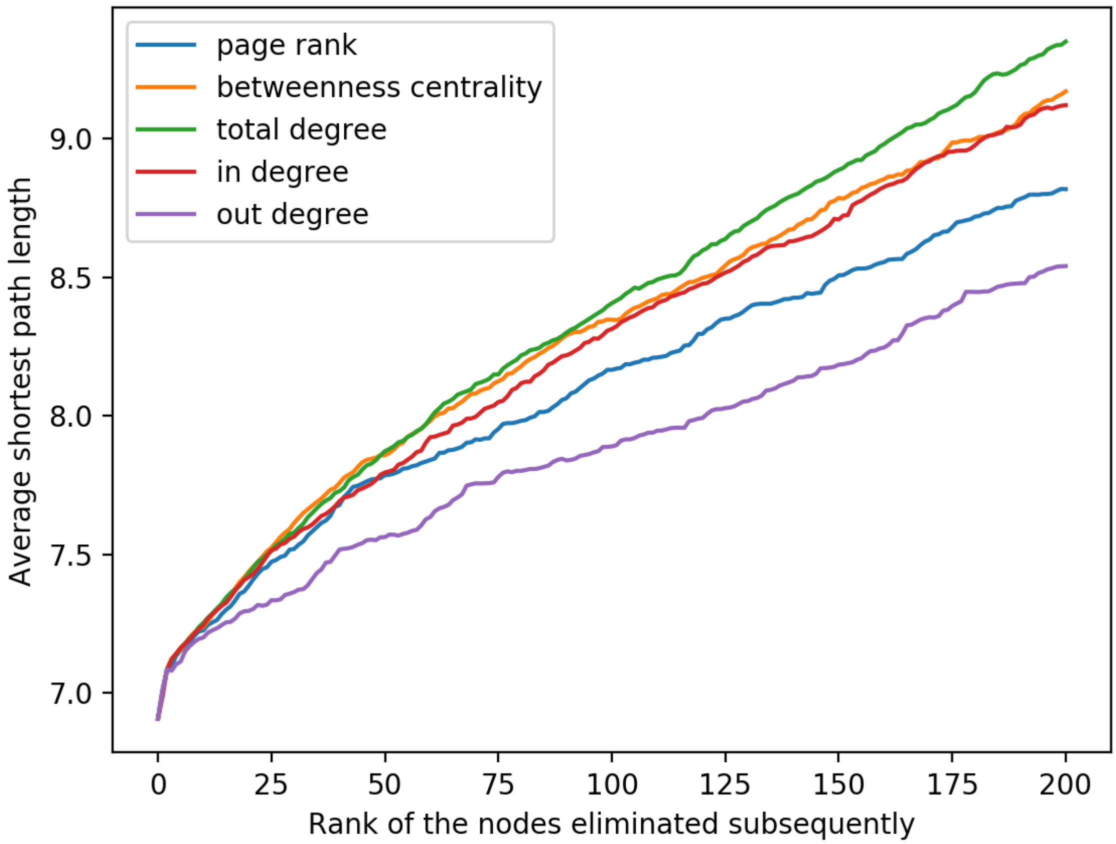
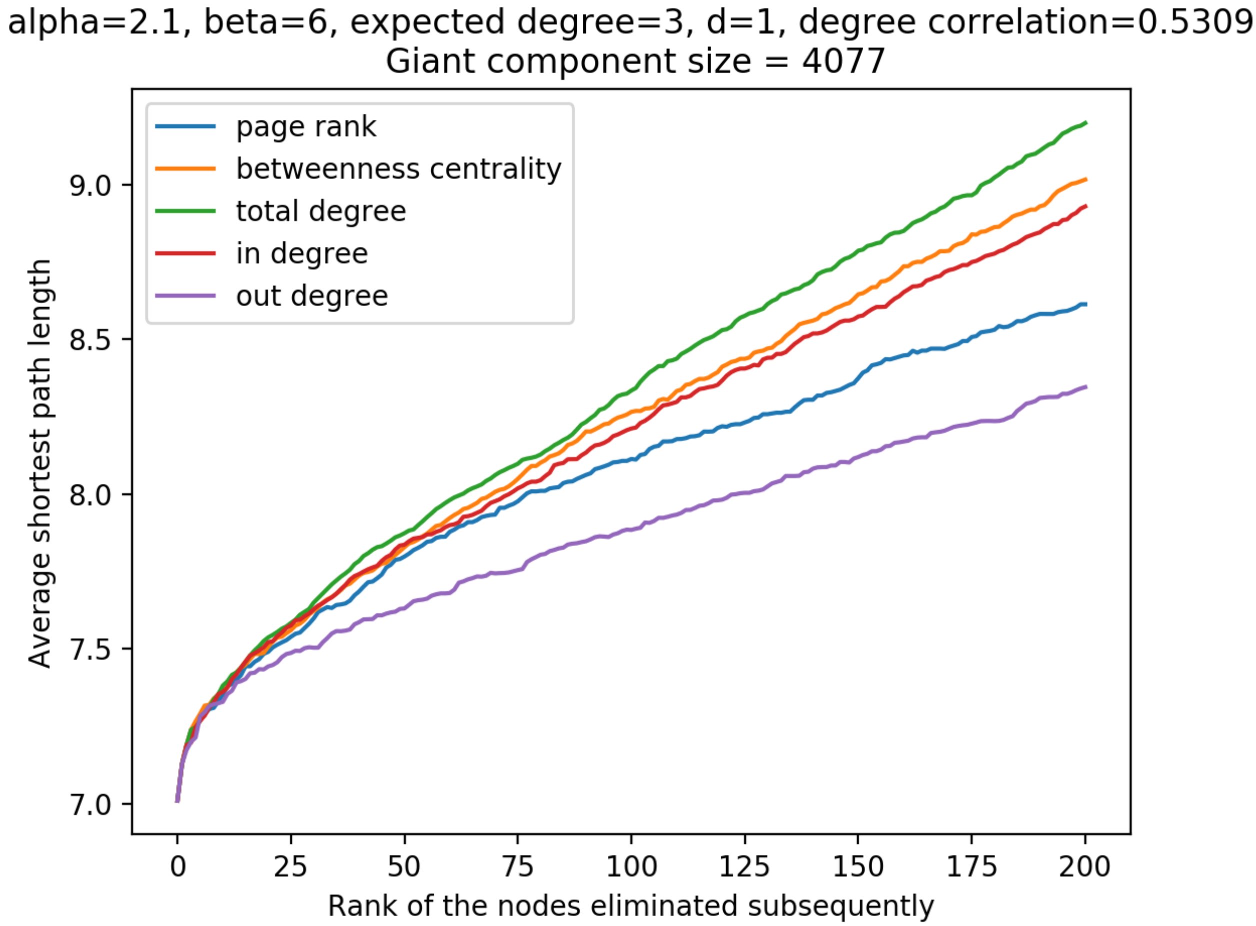
Changes as beta increases

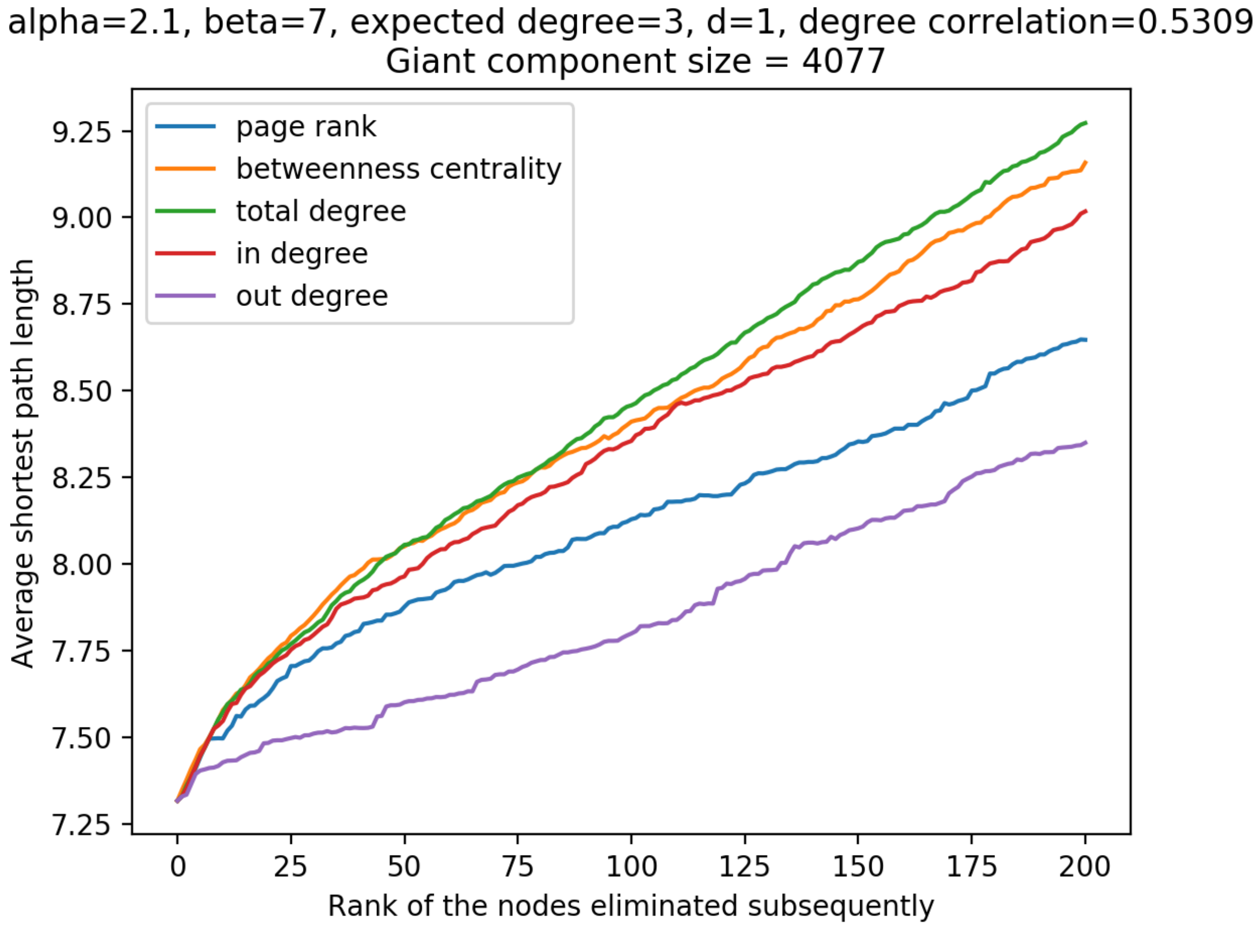


alpha=2.1, beta=5, expected degree=3, d=1, degree correlation=0.5122
Giant component size = 4066





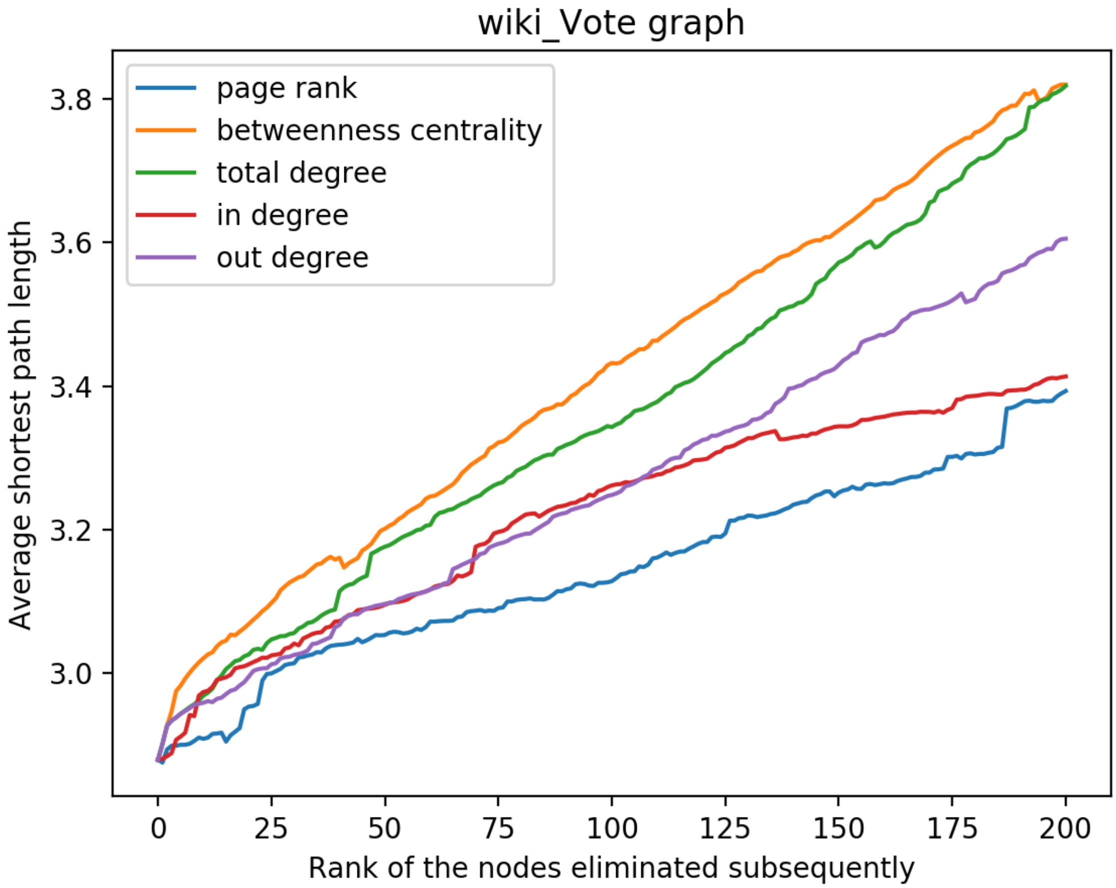
alpha=2.1, beta=6, expected degree=3, d=1, degree correlation=0.5309
Giant component size = 4077
page rank
betweenness centrality
total degree
in degree
out degree
Average shortest path length
9.0
8.5
8.0
7.5
7.0
0
25
50
75
100
125
150
175
200
Rank of the nodes eliminated subsequently

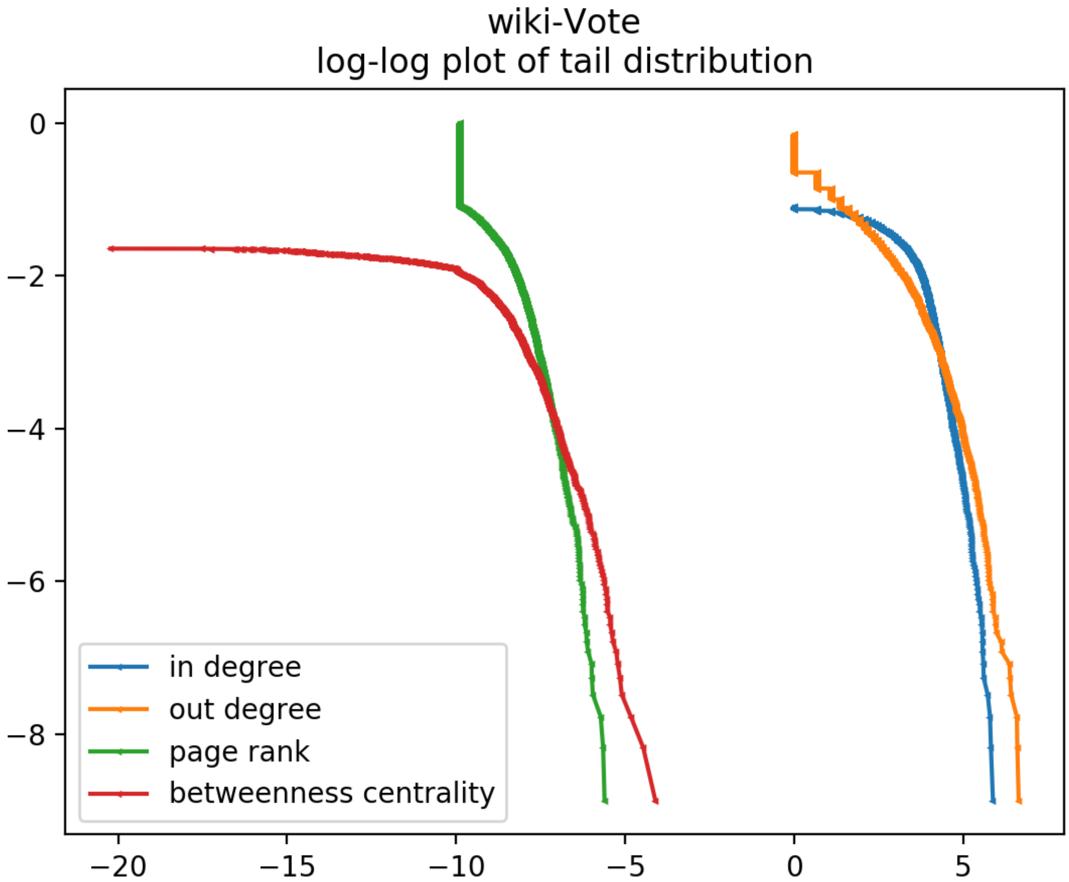


WIKI VOTE

Pearson's correlation: corr, p-value

(in-degree, out-degree)	(0.31763976337716515, 1.5853361463995747e-166)
(total-degree, in-degree)	(0.74782960155018874, 0.0)
(total-degree, out-degree)	(0.86704920856659373, 0.0)
(Page rank, in-degree)	(0.92304846171013577, 0.0)
(page rank, out-degree)	(0.25414821590213482, 2.7315071472270952e-105)
(page rank, total-degree)	(0.66294301163752367, 0.0)
(Page rank, betweeness centrality)	(0.52889191488043785, 0.0)
(betweeness centrality, in-degree)	(0.55171357611680383, 0.0)
(betweeness centrality, out-degree)	(0.57633476842922016, 0.0)
(betweeness centrality, total-degree)	(0.69341012182116557, 0.0)


wiki_Vote graph
page rank
betweenness centrality
total degree
in degree
out degree
Average shortest path length
3.0
3.2
3.4
3.6
3.8
0
25
50
75
100
125
150
175
200
Rank of the nodes eliminated subsequently



Degree correlation

$$
\begin{aligned}
p_{w^-}(x) = PDF(W^-) &= \beta c^{\beta} x^{-\beta-1} \\
\Rightarrow E[(W^-)^s] &= \int_c^{\infty} \beta c^{\beta} x^{-\beta-1} x^s \, dx \\
&= \frac{\beta c^s}{\beta - s} \\
E[(W^-)^{2s}] &= \frac{\beta c^{2s}}{\beta - 2s} \\
E[(W^-)^2] &= \frac{\beta c^2}{\beta - 2} \\
E[W^-] &= \frac{\beta c}{\beta - 1}
\end{aligned}
$$

Since

$$s = \frac{\beta}{\alpha}$$

by setting

$$\alpha, \beta > 2$$

the denominator is not zero.

$$Var(D^+) = E[D^+] - (E[D^+])^2 = E[W^+ + (W^+)^2] - (E[W^+])^2$$
$$E[W^+] = aE[(W^-)^s]$$
$$E[(W^+)^2] = a^2(d^2 + (1-d)^2)E([W^-)^{2s}] + 2a^2d(1-d)(E[(W^-)^s])^2$$

Similarly,

$$Var(D^-) = E[D^-] - (E[D^-])^2 = E[W^- + (W^-)^2] - (E[W^-])^2$$

Then we calculate the covariance of D+ and D-:

$$\begin{aligned} Cov(D^+, D^-) &= E[D^+D^-] - ED^+ED^- \\ E[D^+D^-] &= E_{\hat{W}^-}[E_{W^-}[E[D^+D^-]|W^-]|\hat{W}^-] \\ &= E_{\hat{W}^-}[E_{W^-}[W^+W^-|W^-]|\hat{W}^-] \\ &= E_{\hat{W}^-}[E_{W^-}[(ad(W^-)^s + a(1-d)(\hat{W}^-)^s)(W^-)^s|W^-]|\hat{W}^-] \\ &= adE(W^-)^{s+1} + a(1-d)E(\hat{W}^-)^sEW^- \end{aligned}$$

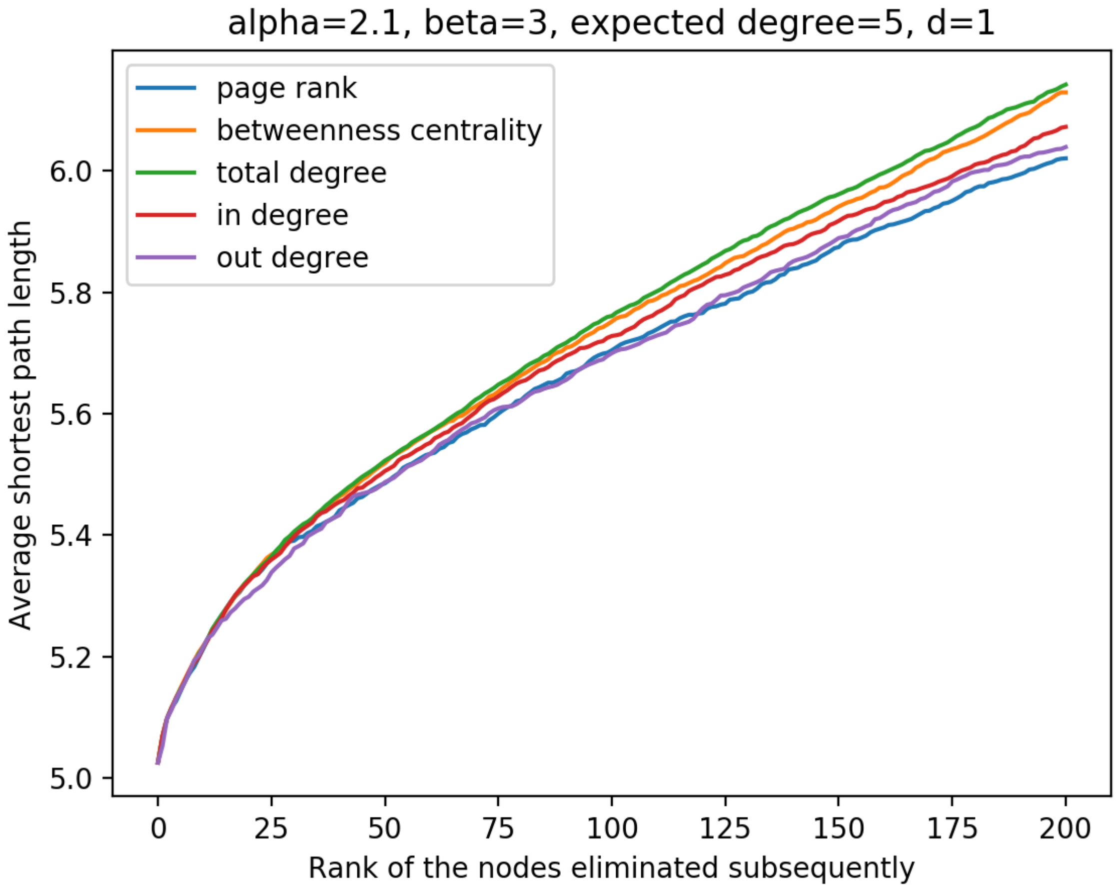
Finally,

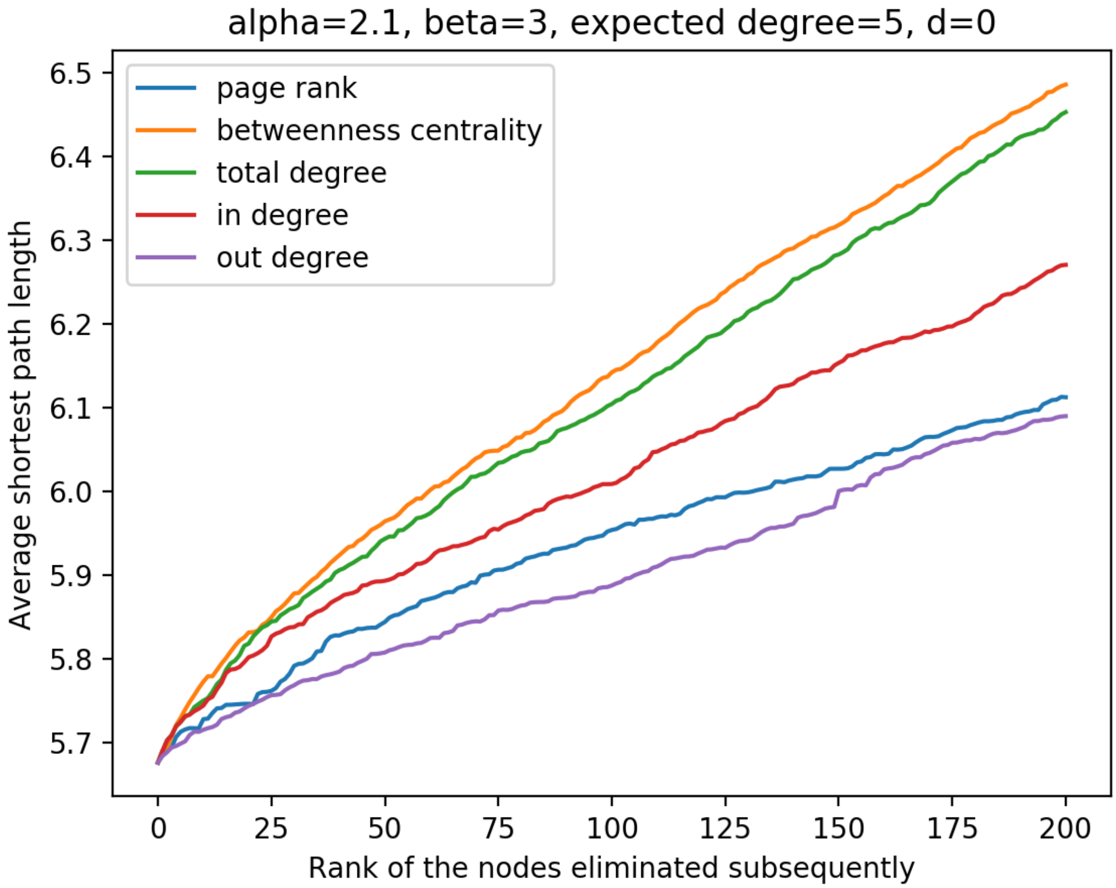
$$\rho = corr(D^+, D^-) = \frac{Cov(D^+, D^-)}{\sqrt{Var(D^+)Var(D^-)}}$$

Relationships between parameters

$$s = \frac{\beta}{\alpha}$$
$$a\frac{\beta c^s}{\beta - s} = \frac{\beta c}{\beta - 1}$$
$$\text{If } s = 1, \text{ then } a = 1$$
$$\text{Else, } a = (\frac{\beta - s}{a(\beta - 1)})^{\frac{1}{s-1}}$$

Other graphs


alpha=2.1, beta=3, expected degree=5, d=1
page rank
betweenness centrality
total degree
in degree
out degree
Average shortest path length
6.0
5.8
5.6
5.4
5.2
5.0
0
25
50
75
100
125
150
175
200
Rank of the nodes eliminated subsequently



alpha=2.1, beta=3, expected degree=5, d=0
page rank
betweenness centrality
total degree
in degree
out degree
Average shortest path length
Rank of the nodes eliminated subsequently
5.7
5.8
5.9
6.0
6.1
6.2
6.3
6.4
6.5
0
25
50
75
100
125
150
175
200