Report V

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0. Distribution set-up

$$P(D^+ = d^+, D^- = d^-) = P(Poisson(W^+) = d^+, Poisson(W^-) = d^-)$$
$$P(W^+ > x) = (\frac{x}{b})^{-\alpha}, \qquad x > b$$
$$P(W^- > x) = (\frac{x}{c})^{-\beta}, \qquad x > c$$
$$(\alpha, \beta > 1)$$
$$W^+ = a(W^-)^d$$

1. Relations between degree correlation and rank correlation

In this session, we want to investigate the relations between degree correlation and rank correlation: whether does one increases the other one increases?

Here, **degree correlation** refers to the theoretical correlation that can be computed by fixed distribution.

sample degree correlation refers to the pearson's correlation of sample in-degree sequence and out-degree sequence.

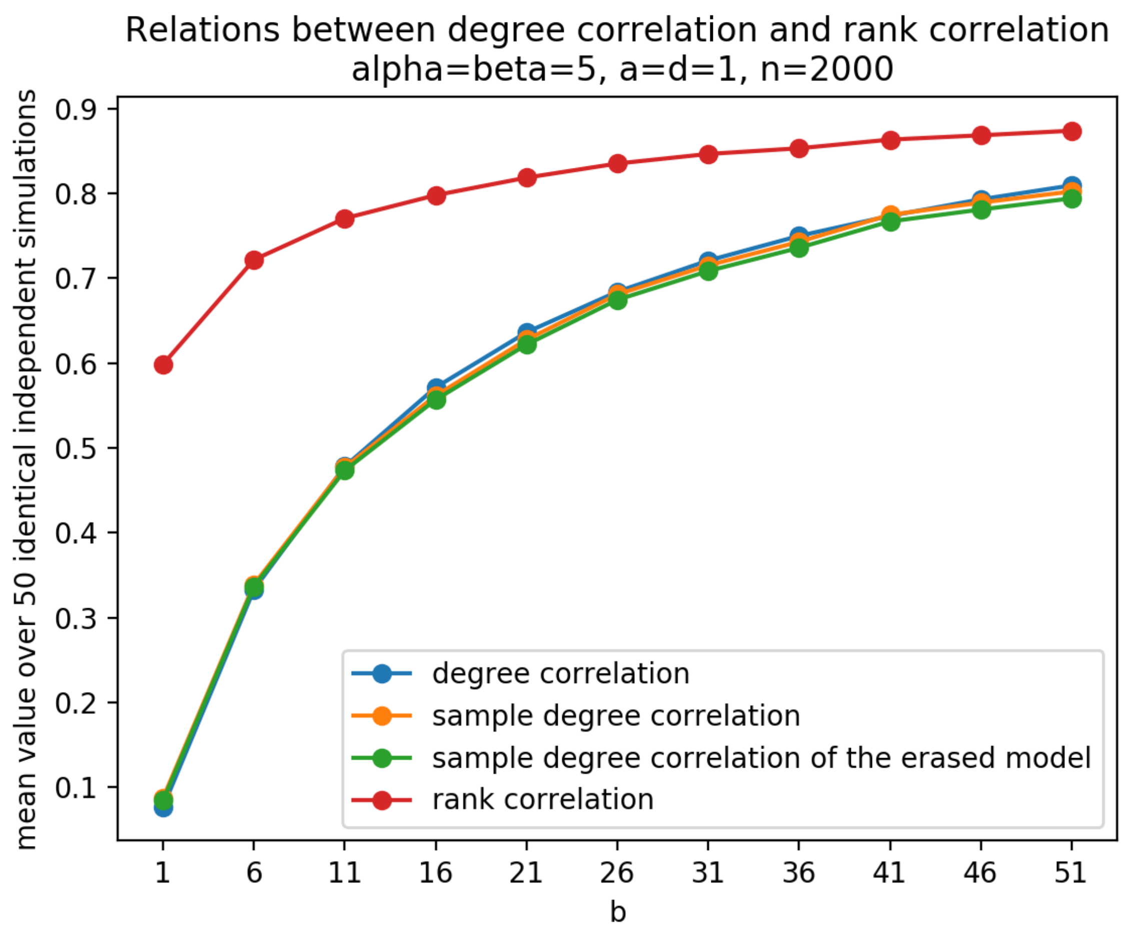
sample degree correlation of the erased model refers to the perason's correlation of graph's in-degree sequence and out-degree sequence of the erased model(after erased parallel edges and self loops).

rank correlation refers to the spearson's rank correlation of page rank and betweenness centrality.

We make simulations under 2 set-ups: alpha = beta and alpha != beta . In each set-up, we fix the calue of alpha and beta, and let other parameter(a or b) change.

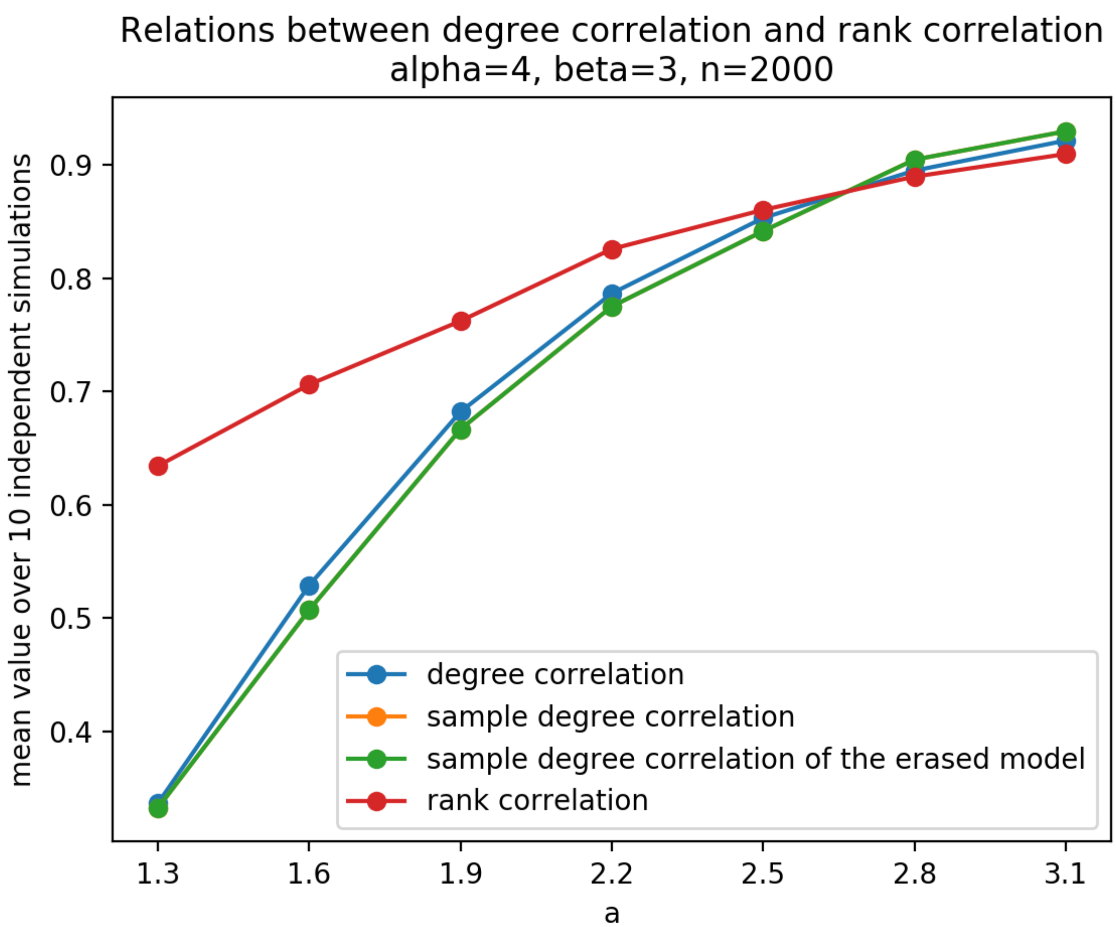
(1) Alpha = beta = 5, a = d = 1. We change the value of b.

b	1	6	11	16	21	26	31	36	41
expected degree	1.25	7.5	13.75	20.0	26.25	32.5	38.75	45.0	51.25
degree correlation	0.08	0.33	0.48	0.57	0.64	0.68	0.72	0.75	0.77
rank correlation	0.599	0.721	0.771	0.797	0.818	0.835	0.846	0.853	0.863



(2) alpha = 4, beta = 3, d = 0.75. We change the value of a.

a	1.3	1.6	1.9	2.2	2.5	2.8	3.1
b	2.01	4.60	9.15	16.45	27.43	43.17	64.86
c	1.78	4.09	8.14	14.62	24.39	38.37	57.65
expected degree	2.67	6.14	12.20	21.94	36.58	57.56	86.48
degree correlation	0.34	0.53	0.68	0.79	0.85	0.90	0.92
rank correlation	0.63	0.71	0.76	0.83	0.86	0.89	0.91



2. Power law index of page rank and betweenness centrality

In this part, we want to find out the power law index of page rank and betweenness centrality.

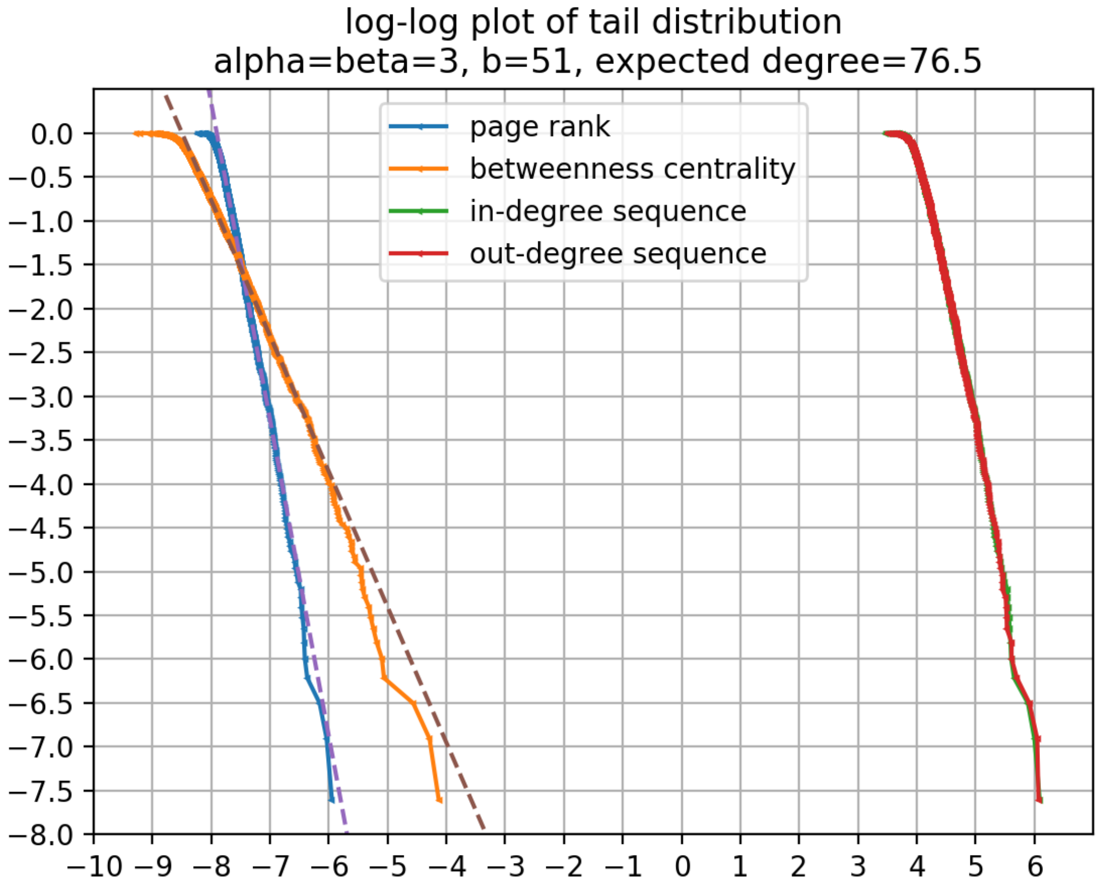
The simulation results show:

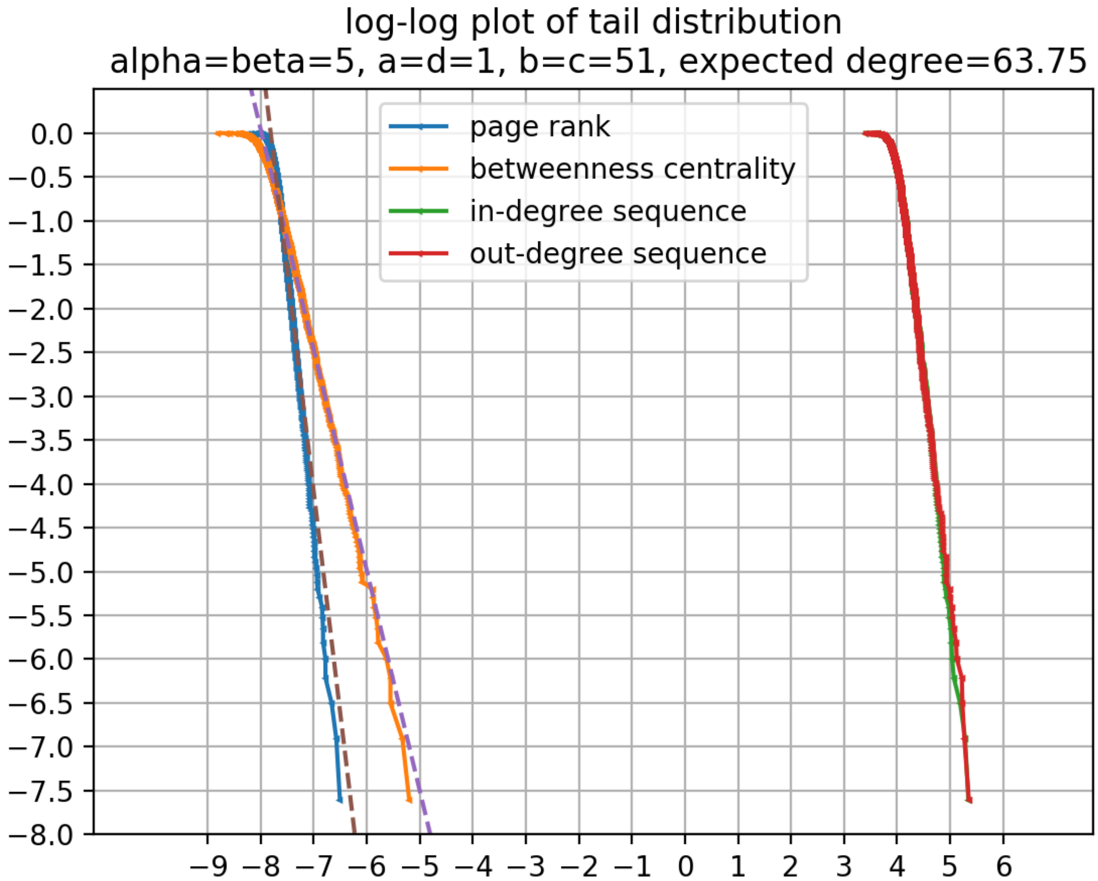
(1)As has been proved, the index of page rank is equal to alpha when alpha = beta.

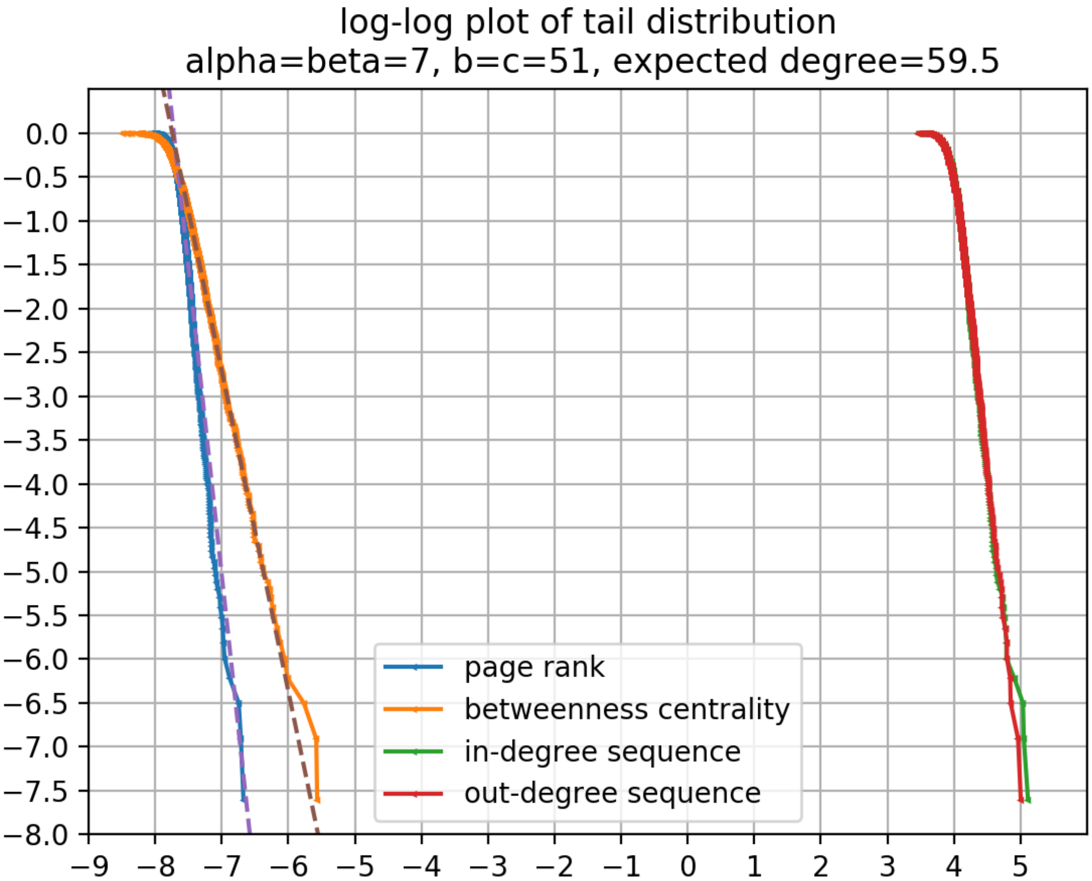
(2) In general cases, the index of page rank and betweenness centrality is determined by alpha and beta. More specifically, if we fix **alpha** and **beta**, and let other parameter changes (**a** or **d**) change, the log-log plots of page rank and betweenness centrality are parallel to each other. (So we leave a few blanks in the table because the plots have same slope.)

(3) The slope of betweenness centrality's plot is smaller than the slope of page rank's plot (in absolute value).

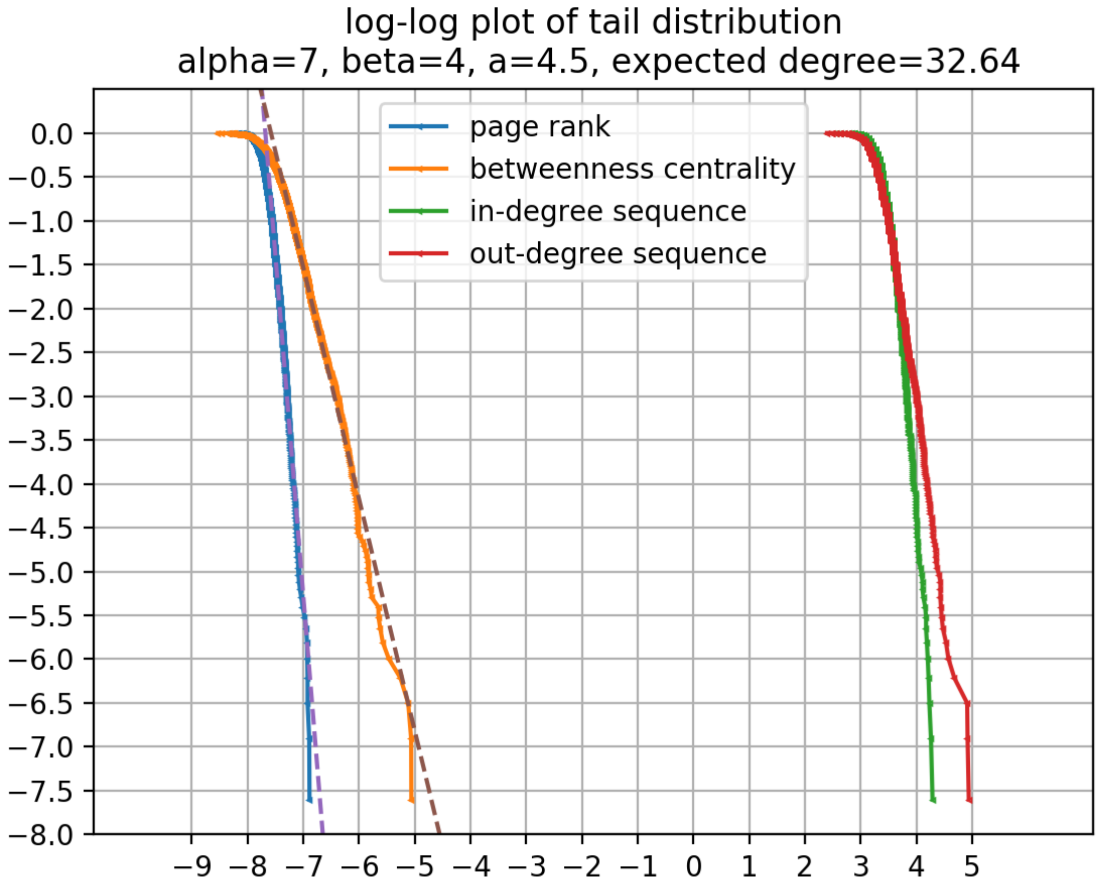
alpha	**3**	**5**	**7**	**7**	**7**	**7**	**4**
beta	3	5	7	4	4	4	3
a	1	1	1	3.5	4.5	5.5	3.1
b	51	51	51	15.57	27.98	44.69	64.86
c	51	51	51	13.62	24.48	39.10	57.65
d	1	1	1	0.57	0.57	0.57	1.4
expected degree	76.5	63.75	59.5	18.16	32.64	55.61	86.48
degree correlation	0.96	0.81	0.63	0.48	0.61	0.70	0.92
rank correlation	0.88	0.88	0.82	0.71	0.78	0.81	0.91
page rank power law index(|slope of the plot|)	3.03	5.04	7.00		7.76		4.49
betweenness centrality power law index(|slope of the plot|)	1.55	2.51	3.65		2.64		1.76


log-log plot of tail distribution
alpha=beta=3, b=51, expected degree=76.5
page rank
betweenness centrality
in-degree sequence
out-degree sequence
0.0
−0.5
−1.0
−1.5
−2.0
−2.5
−3.0
−3.5
−4.0
−4.5
−5.0
−5.5
−6.0
−6.5
−7.0
−7.5
−8.0
−10 −9 −8 −7 −6 −5 −4 −3 −2 −1 0 1 2 3 4 5 6

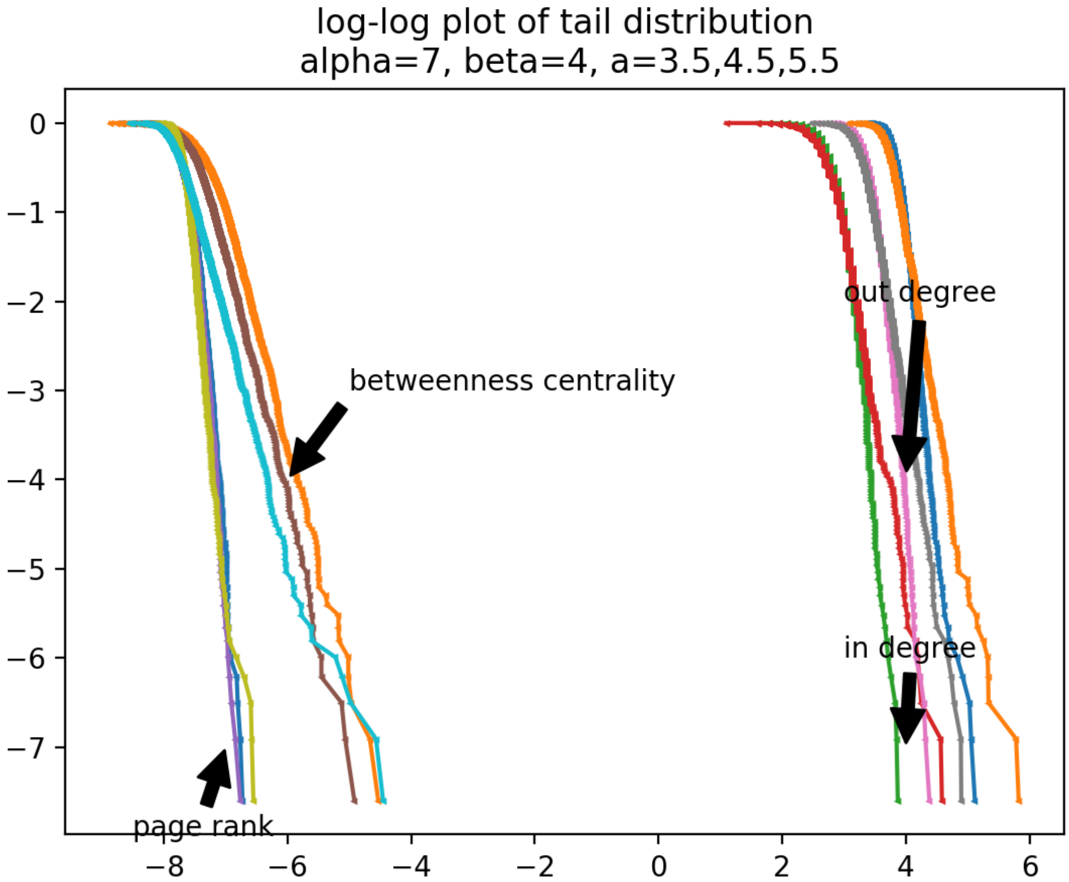

log-log plot of tail distribution
alpha=beta=5, a=d=1, b=c=51, expected degree=63.75
page rank
betweenness centrality
in-degree sequence
out-degree sequence
0.0
−0.5
−1.0
−1.5
−2.0
−2.5
−3.0
−3.5
−4.0
−4.5
−5.0
−5.5
−6.0
−6.5
−7.0
−7.5
−8.0
−9
−8
−7
−6
−5
−4
−3
−2
−1
0
1
2
3
4
5
6



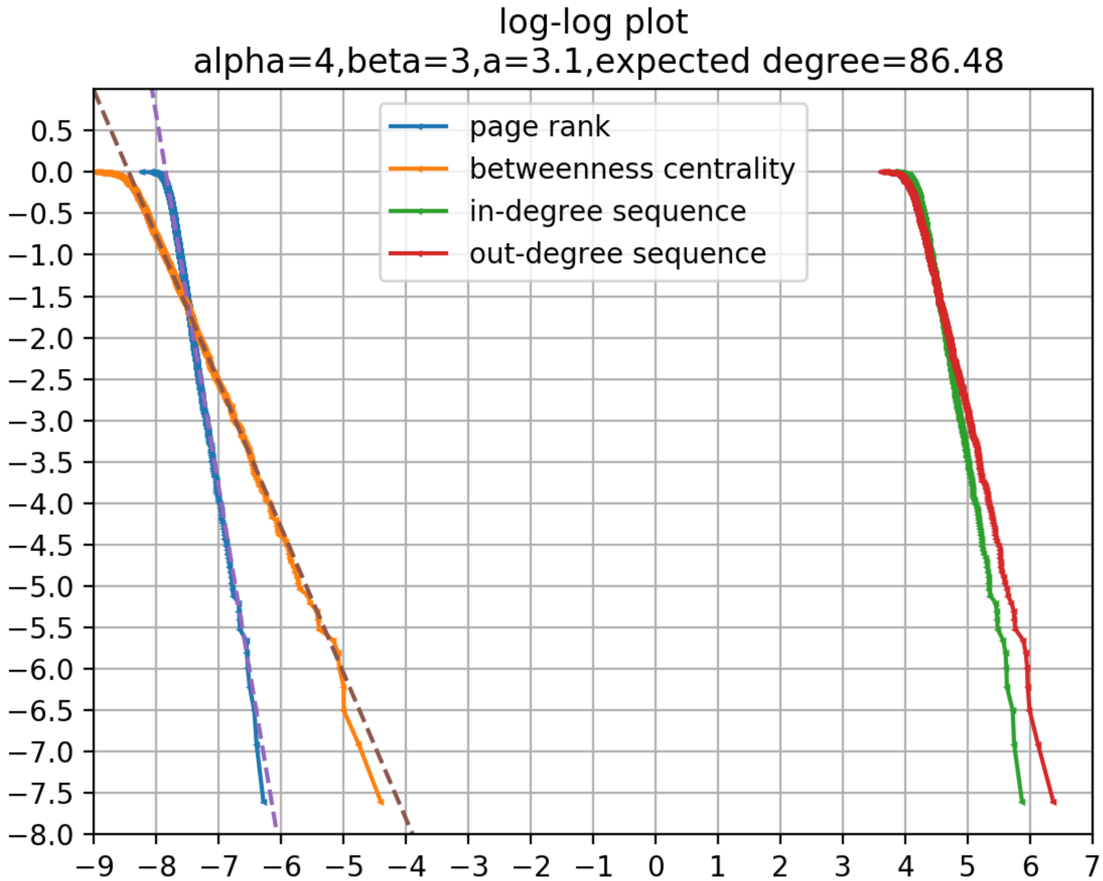
log-log plot of tail distribution
alpha=beta=7, b=c=51, expected degree=59.5
page rank
betweenness centrality
in-degree sequence
out-degree sequence
0.0
−0.5
−1.0
−1.5
−2.0
−2.5
−3.0
−3.5
−4.0
−4.5
−5.0
−5.5
−6.0
−6.5
−7.0
−7.5
−8.0
−9
−8
−7
−6
−5
−4
−3
−2
−1
0
1
2
3
4
5



log-log plot of tail distribution
alpha=7, beta=4, a=4.5, expected degree=32.64
page rank
betweenness centrality
in-degree sequence
out-degree sequence
0.0
−0.5
−1.0
−1.5
−2.0
−2.5
−3.0
−3.5
−4.0
−4.5
−5.0
−5.5
−6.0
−6.5
−7.0
−7.5
−8.0
−9
−8
−7
−6
−5
−4
−3
−2
−1
0
1
2
3
4
5



log-log plot of tail distribution
alpha=7, beta=4, a=3.5,4.5,5.5
betweenness centrality
out degree
in degree
page rank
0
−1
−2
−3
−4
−5
−6
−7
−8
−6
−4
−2
0
2
4
6



3. Degree correlation

We calculate the degree correlation to be:

$$\rho = corr(D^+, D^-) = \frac{Cov(D+, D^-)}{\sqrt{Var(D^+)}\sqrt{Var(D^-))}} = \frac{ED^+D^- - ED^+ED^-}{\sqrt{Var(D^+)}\sqrt{Var(D^-)}}$$
$$= \frac{\frac{\alpha\beta c^{d+1}}{\beta-d-1} - (\frac{b\alpha}{\alpha-1})^2}{\sqrt{Var(D^+)}\sqrt{Var(D^-)}}$$

where

$$Var(D^-) = E(D^-)^2 - (ED^-)^2 = E[W^- + (W^-)^2] - (EW^-)^2$$
$$E(W^-)^2 = \int_c^\infty x^2 \beta c^\beta x^{-\beta-1} dx = \frac{\beta c^2}{\beta - 2}$$
$$EW^- = \frac{\alpha b}{\alpha - 1}$$

However, it is difficult to mathematicallly solve the case when correlation is negative.

So we only test for a few values of the parameters, and we find out that:

When alpha = beta (that is d = 1 and a = 1),

as beta increases, pho (correlation) decreases;

as b increases, pho increases.

We did not obatain the case where correlation is negative, but we could make it very close to zero.