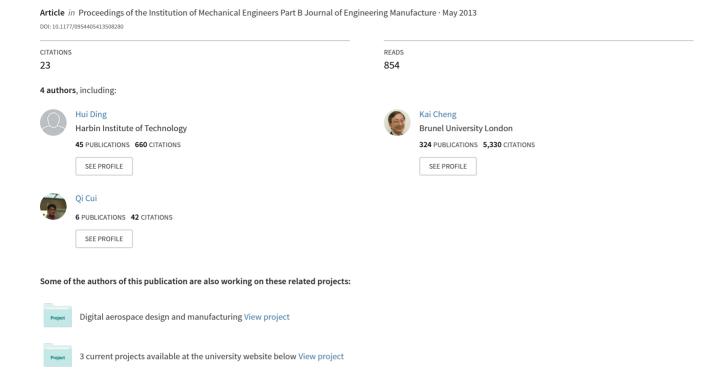
## An investigation on quantitative analysis of energy consumption and carbon footprint in the grinding process





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#### **Abstract**

The research presented in this article aims at developing an innovative integrated modelling methodology to quantify the energy consumption and equivalent carbon footprint in the grinding process. This article categorizes the energy consumption in four compositional parts and identifies the associated key factors affecting the energy consumption and equivalent carbon dioxide emission during the grinding process. Considering energy consumption (E), resource utilization (R), waste generation (W) and their collective effect on equivalent carbon dioxide (C) emission (ERWC), quantitative analysis modelling of the entire grinding process is developed against the roughing, finishing and spark-out stages of the process. The modelling and simulation analysis are carried out with the MATLAB environment, supported by the evaluation and validation through well-designed grinding trials.

#### **Keywords**

Energy consumption, ERWC modelling, low-carbon manufacturing, machining carbon footprint, grinding

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### Introduction

Low-carbon manufacturing or energy-/resource-efficient manufacturing is becoming increasingly important for modern manufacturing industry, while quantitative analysis of energy consumption and carbon dioxide emission in manufacturing processes is essential for scientific understanding and industrial implementation of energy-/resource-efficient manufacturing. Grinding is a widely used precision machining method, but the process consumes more energy and lubricant and generates more waste than other processes such as turning and milling. The thermal prone grinding zone usually requires a large amount of flooded fluid, but its reuse and disposal impose severe environmental impact. Furthermore, regular dressing of grinding wheels is an inevitable consumption process of energy and resources. Therefore, the investigation on energy consumption and carbon footprint in the grinding process is of high industrial and scientific significance, although it is carried out as an exemplar and the methodology is applicable to other machining processes.

Based on the machining processing sequence, Dahmus and Gutowski<sup>1</sup> categorize the power consumption of

machine tools into three different modes, that is, idle mode, run-time mode and production mode. Numerous studies on energy have been carried out based on these three modes, for instance, Li and Kara² developed a reliable method to predict the total energy consumption of a selected machine tool performing a turning operation. Dietmair and Verl³ established the energy consumption behaviour of machines and plants based on a statistical discrete event formulation to exploit the minimization of energy consumption at any given machine or production system. Tridech and Cheng⁴ concluded that decreasing the idle time and downtime help minimize energy consumption. Behrendt et al.⁵ developed an energy consumption monitoring procedure for

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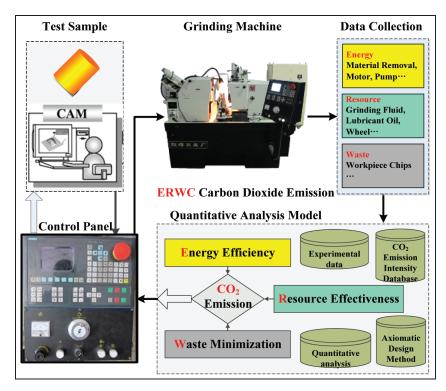


Figure 1. Quantitative analysis of energy consumption and equivalent  $CO_2$  emission in the grinding process. CAM: computer-aided manufacturing.

machine tools, which is useful for companies to establish some standard practices. He et al.<sup>6</sup> analysed the correlation between numerical control (NC) codes and energy-consuming components of machine tools and proposed a practical method for estimating the energy consumption of NC machining. Ball et al.<sup>7</sup> developed a tool to support the development of a zero carbon manufacturing system in a very broad scope. However, an essential challenge is a lack of a generic quantitative analysis method on energy consumption in the machining process, which can be comprehensively applicable to the machining operations and the associated machine tools.

This article presents an innovative integrated methodology for quantitative analysis of energy consumption in the grinding process covering four parts material removal energy consumption, basic energy consumption, frequency-converted energy consumption and response energy consumption – and consequently the equivalent carbon footprint of the process. The research has taken the centreless grinding process, as shown in Figure 1, as the application exemplar to evaluate and validate the methodology through well-designed grinding trials. The modelling and simulation of the energy consumption focus on a generic ERWC approach to quantitatively analyse the collective energy consumption (E), resource utilization (R) and waste generation (W) and the resultantly equivalent carbon dioxide emission (C) in the grinding process.

### **Energy consumption in the grinding process**

### Energy consumption in material removal

According to Wang et al., Wang and Ding, 2hu<sup>10</sup> and Zhou, 1 tangential and normal grinding forces are

$$F_{t} = 1000kb\omega^{-2\varepsilon} d_{\varrho}^{\varepsilon/2} [a_{p}(t)]^{1-\varepsilon/2} V_{W}^{1-\varepsilon} V_{S}^{\varepsilon-1}$$
 (1)

$$F_n = \frac{4000}{\pi} kb \tan \gamma d_e^{\epsilon/2} \omega^{-2\epsilon} [a_p(t)]^{1-\epsilon/2} V_W^{1-\epsilon} V_S^{\epsilon-1}$$
 (2)

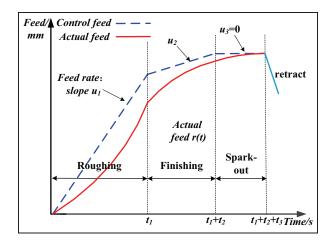
In the grinding process, the machine stiffness is a key factor to influence  $a_p(t)$ , grinding force, surface quality and even carbon footprint. It is very important to take stiffness into consideration in building the real-time ERWC model. Due to small ' $\varepsilon$ ' number,  $1 - \varepsilon/2 \approx 1$ . The normal grinding force can be written as

$$F_n \approx k_c a_p(t) \tag{3}$$

where the grinding system equivalent stiffness coefficient is  $k_e$ , grinding machine stiffness coefficient is  $k_m$ , workpiece stiffness coefficient is  $k_w$  and grinding wheel and workpiece contact stiffness coefficient is  $k_a$ . Then, the grinding system stiffness equation can be calculated as

$$\frac{1}{k_e} = \frac{1}{k_m} + \frac{1}{k_w} + \frac{1}{k_a} \tag{4}$$

Assume that x' is the control feed rate, r' is the actual feed rate, s' is the wear rate of the grinding wheel and



**Figure 2.** A typical grinding process cycle with roughing, finishing and spark-out stages.

**Table 1.** Power consumption data measured on the centreless grinding machine.

| Pumps and motors          | Power consumption (W) |  |  |
|---------------------------|-----------------------|--|--|
| Wheel dresser motor       | 455.85                |  |  |
| Electrical cabinet        | 211.88                |  |  |
| Lubricant pump            | 110.28                |  |  |
| Grinding fluid pump       | 110.28                |  |  |
| U-axis servo motor        | 15.33                 |  |  |
| X-axis servo motor        | 5.99                  |  |  |
| Guide wheel dresser motor | 55.63                 |  |  |
|                           |                       |  |  |

 $\delta'$  is the wear rate of the workpiece. Then, the grinding system can be expressed using the following differential equation

$$x' - r' - s' - \delta' = 0 ag{5}$$

Suppose

$$1 + \frac{d_W}{d_S G} = \zeta, \ \frac{k_c}{k_e n_W} = \tau \text{ and by } s'$$
$$= \frac{d_W}{d_S G} r', \ \delta = \frac{F_n}{k_s} = \frac{k_c a_p(t)}{k_s} = \frac{k_c r'}{k_s n_W}$$

then equation (5) can be expressed as

$$\tau r'' + \zeta r' = u \tag{6}$$

here u is the control feed rate of the guide wheel frame, where during the roughing, finishing and spark-out stages, u is  $u_1$ ,  $u_2$  (= constant) and  $u_3$  (= 0) as shown in Figure 2.

From equation (6), we can find out that the grinding depth per revolution depending on grinding time t in the roughing, finishing and spark-out stages can be calculated as follows

Roughing stage:

$$a_p(t) = \frac{r'(t)}{n_w} = \frac{u_1}{\xi n_w} (1 - e^{-\frac{\xi}{\tau}t})$$

Finishing stage:

$$a_p(t) = \frac{1}{\xi n_W} \left[ u_2 - u_1 e^{-\frac{\xi}{\tau}t} - (u_2 - u_1) e^{-\frac{\xi(t - t_1)}{\tau}} \right]$$

Spark-out stage:

$$a_p(t) = -\frac{1}{Fn_W} \left[ u_1 - u_2 e^{\frac{\xi}{\tau}(t_1 + t_2)} + (u_2 - u_1) e^{\frac{\xi}{\tau}t_1} \right] e^{-\frac{\xi}{\tau}t_1}$$

Substituting  $a_p(t)$  into equation (1), tangential grinding force  $F_t$  can be obtained. Then, energy consumption caused by grinding force can be calculated as

$$E_C = \int_{0}^{t_1 + t_2 + t_3} F_t \times (V_S \pm V_W) dt$$
 (7)

where the signs correspond to the climb-cut and downcut, respectively.

### Basic energy consumption

Some basic energy consumption is needed to maintain the normal operation of the grinding machine, which includes the energy consumption of coolant pump, lubricant pump and electric cabinet. This part of power consumption can be measured by using a Fluke 1735 Power Logger. Table 1 shows the measurement results of the centreless grinding machine.

### Frequency-converted energy consumption

Besides basic energy, energy consumption of grinding wheel drive motor and guide wheel drive motor in noload are two other indispensable elements to make the grinding machine function well. These two adjustable frequency motors can change the rotational speed by control knobs. According to Zhang, 12 the power of these two motors can be expressed as a quadratic function of rotational speed. Then, the frequency-converted power of the grinding wheel motor  $P_D$  can be calculated as

$$\begin{cases}
P_S = P_{0S} + k_{1S}n_S + k_{2S}n_S^2 \\
P_D = P_{0D} + k_{1D}n_D + k_{2D}n_D^2
\end{cases}$$
(8)

 $P_{0S}$ ,  $k_{1S}$ ,  $k_{2S}$ ,  $P_{0D}$ ,  $k_{1D}$ ,  $k_{2D}$  are the constants that need to be verified by experiments and  $n_S$  and  $n_D$  are the rotational speeds of the grinding wheel and the guide wheel, respectively. So, the frequency-converted energy consumption is calculated as

$$E_{FC} = \int_{0}^{t_1 + t_2 + t_3} (P_S + P_D)dt = (P_S + P_D)(t_1 + t_2 + t_3)$$
(9)

### Response energy consumption

Because of the workpiece being grinded, the grinding machine needs more energy besides the material removal, basic and frequency-converted energy. We Ding et al. 953

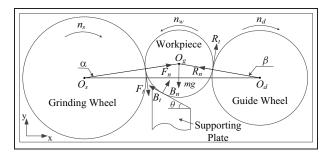
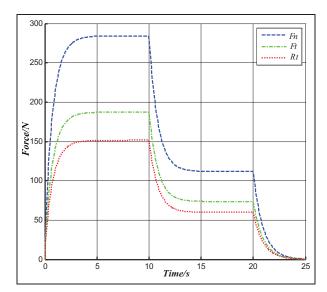


Figure 3. Workpiece force diagram.



**Figure 4.** Curves of the three force components in the grinding process.

call it as response energy. It includes additional guide wheel drive motor power load caused by bearing friction and tangential friction force between the workpiece and the guide wheel. Because of the bearing's small size and fine lubrication conditions, we emphasize the latter one.

In the grinding process, the workpiece has relevant immobility under the action of seven forces: workpiece's own gravity and three pairs of pressure and friction from the grinding wheel, guide wheel and supporting plate to workpiece. They are marked in Figure 3.

Then, the force balance equations are as follows

$$\begin{cases} \sum x = 0 \\ \sum y = 0 \end{cases} \Rightarrow \begin{cases} F_n \cos \alpha + R_t \sin \beta + B_n \sin \theta = \\ R_n \cos \beta + B_t \cos \theta - F_t \sin \alpha \\ F_n \sin \alpha + R_n \sin \beta + B_n \cos \theta = \\ mg + F_t \cos \alpha - B_t \sin \theta - R_t \cos \beta \end{cases}$$
(10)

The tangential grinding force  $F_t$ , normal grinding force  $F_n$  and tangential static friction force between the

workpiece and the guide wheel  $R_t$  are shown in Figure 4.

So, the response energy consumption is calculated as

$$E_{response} = \int_{0}^{t_1 + t_2 + t_3} R_t \times V_d dt$$
 (11)

Suppose grinding efficiency coefficient is  $\eta$ , the energy consumption of grinding process E is as follows

$$E = E_{base} + E_{FC} + \frac{E_C}{\eta} + E_{response}$$
 (12)

Carbon dioxide emission and energy consumption of grinding machine are calculated as

$$C_E = \frac{EC_e}{3.6 \times 10^6} \tag{13}$$

# Equivalent carbon dioxide emission resulted from the process resource usage and waste disposal

Equivalent carbon dioxide emission from the grinding wheel usage

The grinding ratio G (G-ratio) is defined as the ratio of the volume of accumulated metal removal to the accumulated lost volume of the grinding wheel, and it is expressed as

$$G = \frac{\Delta V_W}{\Delta V_S} \tag{14}$$

Then, the accumulated lost volume of wheel is expressed as

$$\Delta V_S = \frac{\pi r(t) l d_W}{G} \tag{15}$$

The accumulated lost volume of wheel dressing is expressed as

$$V_D = N_D B a_d \pi \frac{(d_{S \max} + d_{S \min})}{2} \tag{16}$$

The equivalent carbon dioxide emission of wheel dressing is expressed as

$$C_{GWDR} = \frac{C_e}{3.6 \times 10^6} \times P_{GWDR} \times t_{GWDR}$$
$$= \frac{C_e \times P_{GWDR}}{3.6 \times 10^3} \times \frac{B\pi (d_{S\max} + d_{S\min})/2}{s_d V_S}$$
(17)

The equivalent carbon dioxide emission from the grinding wheel usage is calculated as

$$C_{GW} = \frac{\Delta V_S}{V_A - V_D} \times [(C_{GWP} + C_{GWD}) \times G_{GW} + N_D \times C_{GWDR}]$$
(18)

Table 2. Equivalent carbon dioxide emission intensities.

| Equivalent carbon dioxide emission intensity | Quantitative value |  |  |
|--|--------------------|--|--|
| $C_e$ (g-CO <sub>2</sub> /kW h)              | 381                |  |  |
| C <sub>GWP</sub> (g-CO <sub>2</sub> /kg)     | 33,747.8           |  |  |
| $C_{GWD}$ (g- $CO_2/kg$ )                    | 13.46              |  |  |
| $C_{GFP}$ (g- $CO_2/L$ )                     | 977.6              |  |  |
| $C_{GFD}$ (g- $CO_2/L$ )                     | 2.9                |  |  |
| $C_{LOP}$ (g- $CO_2/L$ )                     | 469                |  |  |
| Cwcp (g-CO2/kg)                              | 63.4               |  |  |

### Equivalent carbon dioxide emission from the grinding fluid usage

Grinding fluid is an indispensable resource in the grinding process. Grinding fluid can not only cool down and lubricate the grinding zone but also swash wear debris. As we all know, grinding is a precision machining method. To ensure grinding precision, grinding fluid should be entirely replaced regularly. The equivalent carbon dioxide emission from the grinding fluid usage is calculated as

$$C_{GF} = \frac{T_{GFU}}{T_{GFR}} \times (C_{GFP} + C_{GFD}) \times Q_{GFI}$$
 (19)

### Equivalent carbon dioxide emission from the lubricant oil usage

Many parts of the grinding machine system should be lubricated regularly. We make an equivalent to spindle lubrication. The equivalent carbon dioxide emission from lubrication oil usage is calculated as

$$C_{LO} = \frac{T_{LOU}}{T_{LOS}} \times C_{LOP} \times Q_{LOS}$$
 (20)

### Equivalent carbon dioxide emission from the grinding chips disposal

The accumulated metal removal volume  $\Delta V_W$  is stated in aforementioned equations (14) and (15). So, carbon dioxide emission from the grinding chips disposal can be calculated as

$$C_{WC} = \Delta V_W \times \rho_W \times C_{WCD} \tag{21}$$

Based on the study by Narita et al., <sup>13</sup> carbon dioxide emission intensities mentioned above are listed in Table 2.

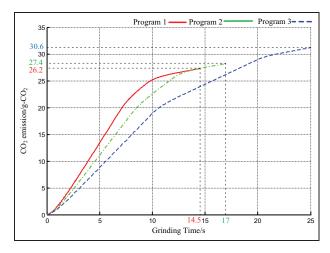
# ERWC carbon dioxide emission quantitative analysis model and a case study

From the above analysis, the time-varying equivalent carbon dioxide emission quantitative analysis model in the grinding process can be represented as follows

Table 3. Grinding NC program parameters.

|           | u <sub>I</sub> /mm | t <sub>I</sub> /s | u <sub>2</sub> /mm | t <sub>2</sub> /s |
|-----------|--------------------|-------------------|--------------------|-------------------|
| Program I | 0.025              | 7                 | 0.01               | 2.5               |
| Program 2 | 0.020              | 8                 | 0.01               | 4                 |
| Program 3 | 0.015              | 10                | 0.005              | 10                |

NC: numerical control.



**Figure 5.** Equivalent carbon dioxide emission curves during grinding processes.

$$C_{Grinding}(t) = \vec{E}(t)\vec{C}_{Energy} + \vec{R}(t)\vec{C}_{Resouce} + \vec{W}(t)\vec{C}_{Waste}$$
(22)

We have some 20-mm-diameter cylindrical work-pieces, which are made of 45 gauge steel. MK1080 centreless grinding machine is chosen and the grinding depth is 0.2 mm. Table 3 shows three NC programs used for comparison, and the parameters are empirical data from the grinding machine operator.

The time-varying equivalent carbon emission curves of the three NC programs were shown in Figure 5. For program 1, due to its greater feed rate, it let out more carbon dioxide at the beginning. However, its final emission was the least one because of consuming least time

Comparing Figures 2 and 5, we can also find out that equivalent carbon emission curves had strong relevance to feed rate. In general, bigger feed rate meant faster carbon emission rate. Compared with other machining processes, the result is consistent with the conclusion that the energy requirement decreases as the material removal rate increases, as described by Gutowski et al.<sup>14</sup> and Rajemi et al.<sup>15</sup>

Figure 6(a) shows the equivalent  $CO_2$  emission of the four kinds of grinding energy consumptions mentioned before. We can find that the material removal energy accounts for the largest proportion, while response energy the smallest one. The percentage of material removal energy consumption is 56%. The

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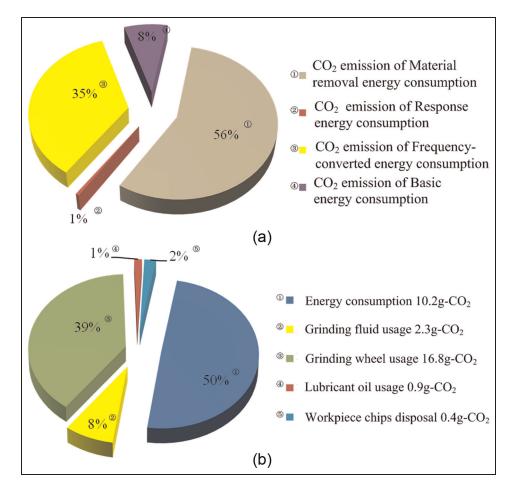


Figure 6. Quantitative analysis results: (a) proportion of energy influencing CO<sub>2</sub> emissions and (b) proportion of factors influencing equivalent CO<sub>2</sub> emissions.

result is not in the range as reported in literatures by Kardonowy, <sup>16</sup> Dahmus and Gutowski<sup>1</sup> and Rajemi et al., <sup>17</sup> who indicated that power distribution for machining lies in the range of 0%–48.1% depending on machine loads. On the other hand, it may indicate that the grinding process consumes more specific energy than turning or milling. Figure 6(b) shows that energy consumption and grinding wheel usage are two main factors resulting in carbon dioxide emission in the grinding process under existing carbon dioxide emission intensities. The result that 50% of carbon dioxide emission is generated by energy, indicates that not only energy consumption, but also resource and waste should be taken into consideration to access carbon dioxide emission in manufacturing.

#### Conclusion

This article presents an ERWC-based modelling approach to quantifying the energy consumption in the centreless grinding process and the associated equivalent carbon dioxide emission. The modelling is evaluated and validated by well-designed grinding trails. Currently, the ERWC models are further implemented in the computer numerical control (CNC) system to

achieve the multi-dimensional process optimization by addressing the quality, productivity, costs and energy consumption simultaneously.

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### **Declaration of conflicting interests**

The authors declare that there is no conflict of interest.

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### Appendix I

| A 1 |         |  |
|-----|---------|--|
| IN  | otation |  |

dressing depth (mm)  $a_d$ grinding depth per revolution (mm) (time  $a_p(t)$ varying)

b grinding width (mm) В grinding wheel width (mm)  $C_{\rho}$ equivalent carbon dioxide emission intensity of electricity (g-CO<sub>2</sub>/kW h) equivalent carbon dioxide emission  $C_{GFD}$ intensity of grinding fluid disposal  $(g-CO_2/L)$ equivalent carbon dioxide emission  $C_{GFP}$ intensity of grinding fluid production  $(g-CO_2/L)$ equivalent carbon dioxide emission  $C_{GWD}$ intensity of the grinding wheel disposal equivalent carbon dioxide emission  $C_{GWP}$ intensity of grinding wheel production  $(g-CO_2/kg)$ carbon dioxide emission intensity of  $C_{LOP}$ lubricant oil production (g-CO<sub>2</sub>/L)  $C_{WCD}$ equivalent carbon dioxide emission intensity of workpiece chips disposal  $(g-CO_2/kg)$ equivalent diameter =  $d_s \times d_w/(d_s + d_w)$  $d_e$  $d_{s}$ diameter of the grinding wheel (mm)  $d_{S \max}$ maximum available grinding wheel diameter (mm) minimum available grinding wheel  $d_{S \min}$ diameter (mm) diameter of the workpiece (mm)  $d_W$ Ggrinding ratio  $G_{GW}$ weight of the grinding wheel (kg) a constant relevant to workpiece material and lubricant conditions length of the workpiece being grinded (mm) rotational speed of the guide wheel (rev/s)  $n_{\scriptscriptstyle W}$ times can be dressed (–)  $N_D$ machine power when dressing (W)  $P_{GWDR}$ initial quantity of grinding fluid (L)  $Q_{GFI}$  $Q_{LOS}$ supplement quantity of lubricant oil (L) *r*(*t*) grinding depth (mm) (time varying) dressing lead (mm/rev)  $S_d$  $\mathsf{T}_{\mathit{GFR}}$ mean interval time between grinding fluid replacement (s)

 $T_{GFU}$ grinding fluid usage time (s)

mean interval time between lubricant oil  $T_{LOS}$ supplies (s)

 $\mathrm{T}_{LOU}$ spindle running time (s)

 $V_d$ linear speed of the guide wheel (m/s)  $V_S$ linear speed of the wheel (m/s)  $V_W$ linear speed of the workpiece (m/s)

average half angle of single abrasive grain  $\gamma$ empirical constant, mostly between 0.2 and 0.5

workpiece chip density (kg/mm<sup>3</sup>)  $\rho_W$ ω

average interval of abrasive grains (mm)