Euclidean Distance Mapping - 2Pass Sequential Algorithm

A 2-Pass sequential Euclidean distance mapping algorithm from the book "Binary Digital Image Processing: A Discrete Approach".

@book{marchand-maillet_binary_2000,

title = {Binary {Digital} {Image} {Processing}: {A} {Discrete} {Approach}},

isbn = {978-0-12-470505-0},

url = {https://books.google.de/books?id=i-lRAAAAMAAJ},

publisher = {Elsevier Science},

author = {Marchand-Maillet, S. and Sharaiha, Y.M.},

year = {2000},

lccn = {99064631}

}

Create a sample binary image

- Create an image of size 8X7, contaning ones (foreground pixels). And two pixels at the center having zero values.

```
close all, clear;
img =logical(ones(8,7));
[m,n] = size(img);
img(ceil(m/2),ceil(n/2)) = 0;
img(ceil(m/2 + 1),ceil(n/2)) = 0;
show_pixels(img)
title("Original IMage")
```

Create and initialize the distance map

- Create a cell structure to store the indipendent x and y distances as arryays [x,y]
- Initialize the distance map using: $DT_D^{(0)} = \begin{cases} (0,0), & \text{if} p \in \Gamma \\ (+\infty,+\infty), & \text{if} p \notin \Gamma \end{cases}$

```
dist = cell(m,n);
dist(~img) = {[0,0]};
dist(img) = {[Inf,Inf]};
```

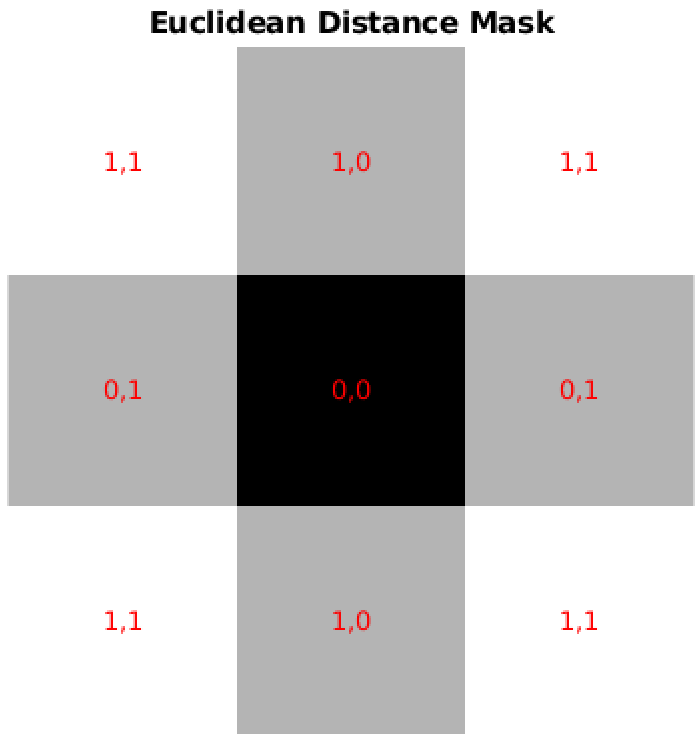
- Create a 3×3 $(8-N)$ distance mask where each value in the matrix $(m_{k,i})_{k,i}$ represents a local distance between a pixel $p = (x_p, y_p)$ and a neighbouring pixel $q = (x_p + k, y_p + l)$. A typical $8-N$ euclidian distance mas looks like this.

1,1	0,1	1,1
1,0	0,0	1,0
1,1	0,1	1,1

```
edm_8 = cell(3,3);
for i = 1:3
    for j = 1:3
        edm_8(i,j) = {[(mod(i,2)),mod(j,2)]};
    end
end
mask = zeros(3,3);
for i = 1:3
    for j = 1:3
        mask(i,j) = norm(edm_8{i,j});
    end
end
```

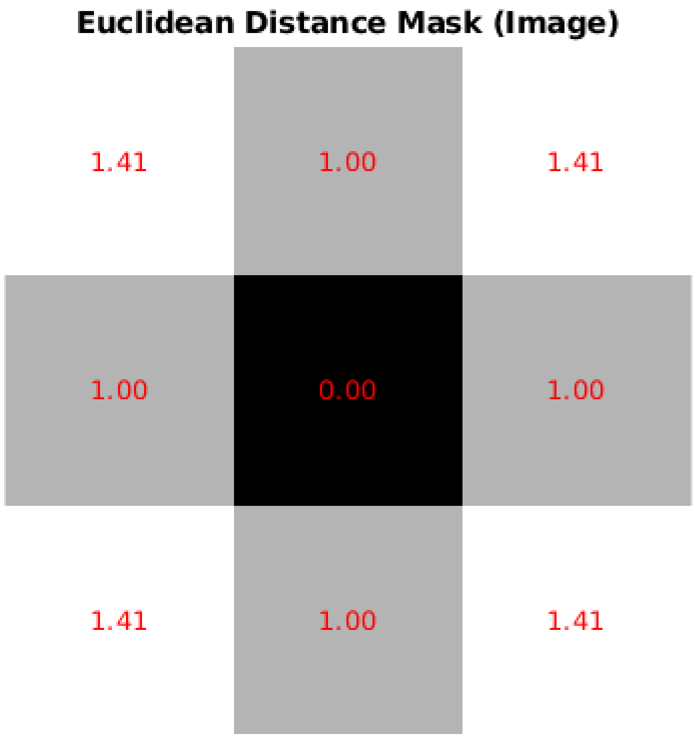
The cell array neighborhood stores the x and y coordinates separately.

```
show_Edist(mask,edm_8)
title("Euclidean Distance Mask")
```



The actual pixel value could be retrieved using the definition of Euclidean distance. $DT_E(p) = ||EDT(p)||_2 = \sqrt{\delta_x(p)^2 + \delta_y(p)^2}$

```
show_pixels(mask)
title("Euclidean Distance Mask (Image)")
```



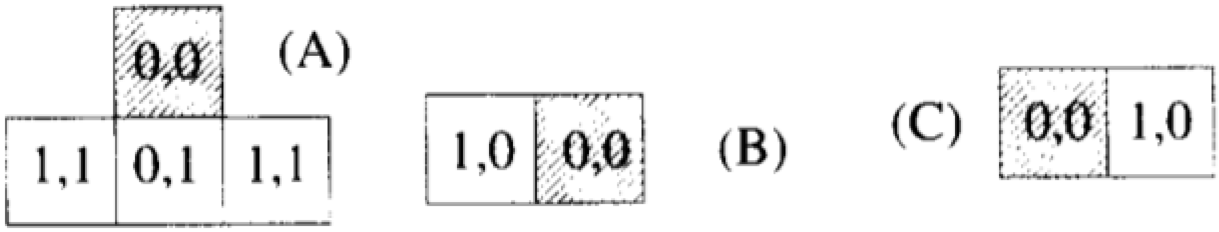
Updating rule:

- $EDT^{(t)}(p) = EDT^{(t-1)}(q) + (|x_q - x_p|, |y_q - y_p|)$, where q characterizes the minimum value of all the distances from each neighborhood pixel to the center pixel given by: $q \Rightarrow \min \|EDT^{(t-1)}(q') + (|x_{q'} - x_p|, |y_{q'} - y_p|)\|_2$
- the $min\|.\|$ operator for the neighborhood should also consider the value of p itself. We should make sure that we are propagating the minimum distance. This becomes important when updating pixels of the distance map containing values other thatn ∞.
- The updating (propagation) procedurre is carreid out in two passes.

Pass 1 - Upward Sequential Pass

The distance map is scanned line by line from the bottom line to the top line using three different **sub-masks**. For each line, all three sub-masks whose union is the lower half of the complete Euclideean mask are used successively.

- The sub-mask (A) is first applied successively from left to right at each pixel of the current line.
- The sub-mask (B) is applied successively from left to right at each pixel of the current line.
- The sub-mask © is applied successively from right to left at each pixel of the current line.



Scan each line bottom to top.

```
for i = m:-1:1
```

First update (A)

```
    for j = 1:n
        vals = [];
        if(img(i,j))
            if i == m

            elseif j == 1
                for k = 1:2
                    vals = [vals,norm(dist{i+1,j-1+k}+edm_8{3,k+1})];
                end
                [minVal,idx] = min(vals);
                dist(i,j) = {dist{i+1,j-1+idx} + [abs(i+1-i),abs(j-1+idx-j)]};
            elseif j == n
                for k = 1:2
                    vals = [vals,norm(dist{i+1,j-2+k}+edm_8{3,k})];
                end
                [minVal,idx] = min(vals);
                dist(i,j) = {dist{i+1,j-2+idx} + [abs(i+1-i),abs(j-2+idx-j)]};
            else
                for k = 1:3
                    vals = [vals,norm(dist{i+1,j-2+k}+edm_8{3,k})];
                end
                [minVal,idx] = min(vals);
                dist(i,j) = {dist{i+1,j-2+idx} + [abs(i+1-i),abs(j-2+idx-j)]};
            end
        end
    end
```

Second Update (B)

```
    for j = 1:n
        if(img(i,j))
            if j ~= 1
                val1 = norm(dist{i,j});
                val2 = norm(dist{i,j-1}+[1,0]);
                min_ = min(val1,val2);
                if min_ == val2
```

```
                    dist(i,j) = {dist{i,j-1}+[0,1]};
                end
            end
        end
    end
```

Third Update (C)

```
    for j = n:-1:1
        if(img(i,j))
            if(j~=n)
                val1 = norm(dist{i,j});
                val2 = norm(dist{i,j+1}+[1,0]);
                min_ = min(val1,val2);
                if min_ == val2
                    dist(i,j) = {dist{i,j+1}+[0,1]};
                end
            end
        end
    end
end
```

The complexity of this procedure applied on a binary image of size $W \times H$ is $\mathcal{O}(W \times H)$ since the updating rule can be applied in **constant time**. The upward pass propagates correct Euclidean distance values from the lowest line that contains a point in Γ.

Note: Final values $EDT^{(t1)}(p)$ in the intermediate distance map are correct and are threrefore not to be updated in a further iteration.

```
show_Edist(img,dist)
title("Distance Map after Upward Pass")
```

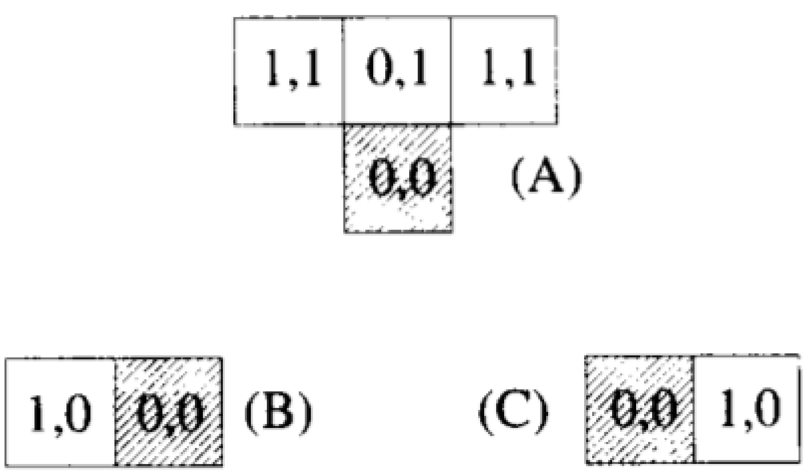
Distance Map after Upward Pass

3,3	3,2	3,1	3,0	3,1	3,2	3,3
2,3	2,2	2,1	2,0	2,1	2,2	2,3
1,3	1,2	1,1	1,0	1,1	1,2	1,3
0,3	0,2	0,1	0,0	0,1	0,2	0,3
0,3	0,2	0,1	0,0	0,1	0,2	0,3
Inf,Inf	Inf,Inf	Inf,Inf	Inf,Inf	Inf,Inf	Inf,Inf	Inf,Inf
Inf,Inf	Inf,Inf	Inf,Inf	Inf,Inf	Inf,Inf	Inf,Inf	Inf,Inf
Inf,Inf	Inf,Inf	Inf,Inf	Inf,Inf	Inf,Inf	Inf,Inf	Inf,Inf

Pass 2 - Downward Sequential Pass

The Euclidean distance map is again scanned line by line from top to bottom. For each line, the upper half of the mask is used via a decomposition in thress sub-masks.

- The first sub-mask(A) is applied on each pixel of the current line from left to right.
- The sub-mask (B) is applied on each pixel of the current line from left to right.
- The sub-mask (C) is applied on each pixel of the current line from right to left.



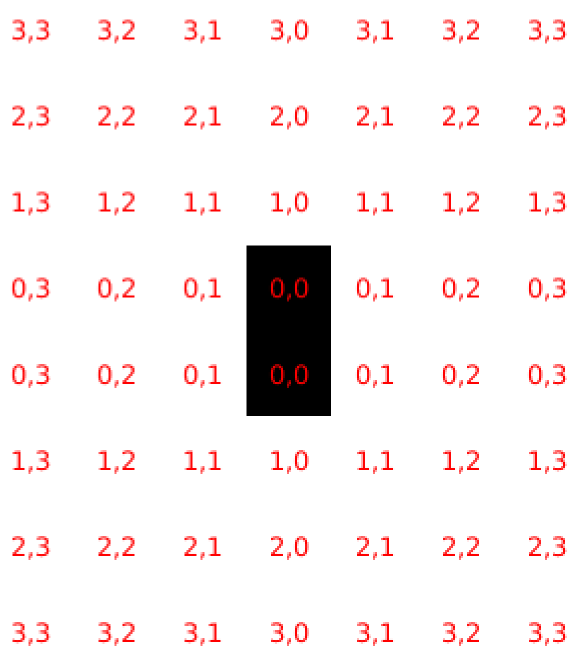
```
for i = 1:m
    for j = 1:n
        vals = [];
        if(img(i,j))
            if i == 1

            elseif j == 1
                for k = 1:2
                    vals = [vals,norm(dist{i-1,j-1+k}+edm_8{3,k+1})];
                end
                [minVal,idx] = min(vals);
                minVal1 = norm(dist{i-1,j-1+idx} + [abs(i-1-i),abs(j-1+idx-j)]);
                minVal2 = norm(dist{i,j});
                if min(minVal1,minVal2)== minVal1
                    dist(i,j) = {dist{i-1,j-1+idx} + [abs(i-1-i),abs(j-1+idx-j)]};
                end
            elseif j == n
                for k = 1:2
                    vals = [vals,norm(dist{i-1,j-2+k}+edm_8{3,k})];
                end
                [minVal,idx] = min(vals);
                minVal1 = norm(dist{i-1,j-2+idx} + [abs(i-1-i),abs(j-2+idx-j)]);
                minVal2 = norm(dist{i,j});
                if min(minVal1,minVal2) == minVal1
                    dist(i,j) = {dist{i-1,j-2+idx} + [abs(i-1-i),abs(j-2+idx-j)]};
                end

            else
                for k = 1:3
                    vals = [vals,norm(dist{i-1,j-2+k}+edm_8{3,k})];
                end
                [minVal,idx] = min(vals);
                minVal1 = norm(dist{i-1,j-2+idx} + [abs(i-1-i),abs(j-2+idx-j)]);
                minVal2 = norm(dist{i,j});
                if min(minVal1,minVal2) == minVal1
                    dist(i,j) = {dist{i-1,j-2+idx} + [abs(i-1-i),abs(j-2+idx-j)]};
                end
            end
        end
    end
    for j = 1:n
        if(img(i,j))
            if j ~= 1
                val1 = norm(dist{i,j});
                val2 = norm(dist{i,j-1}+[1,0]);
                min_ = min(val1,val2);
                if min_ == val2
                    dist(i,j) = {dist{i,j-1}+[0,1]};
                end
            end
        end
    end
    for j = n:-1:1
```

```
        if(img(i,j))
            if(j~=n)
                val1 = norm(dist{i,j});
                val2 = norm(dist{i,j+1}+[1,0]);
                min_ = min(val1,val2);
                if min_ == val2
                    dist(i,j) = {dist{i,j+1}+[0,1]};
                end
            end
        end
    end
end

show_Edist(img,dist);
```



Finally, convert the (x, y) data to actual euclidean distance, so that the distance transform could be used in other procedures such as Medial Axis Transform and shortest path computation.

```
dMap = zeros(8,7);
for i = 1:m
    for j = 1:n
        dMap(i,j) = norm(dist{i,j});
    end
end
figure
imshow(dMap,[],'InitialMagnification','fit')
```