

IT Project Management – M.Sc. in Computer Science (2023/24 Winter Semester)

Cost Management – Project crashing

1. Find the critical path.

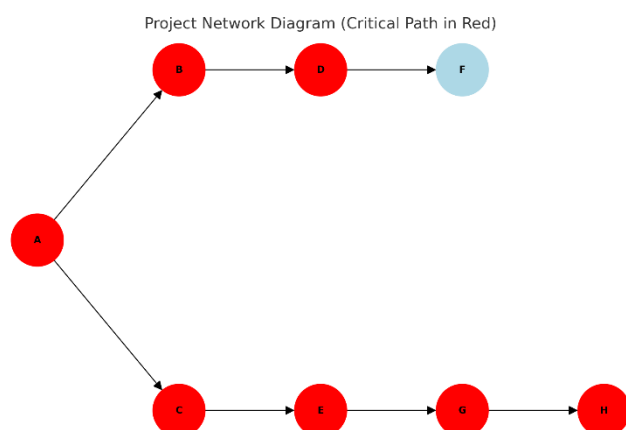
To determine which activity to crash in your project, first, you should calculate the slack or float for each activity. The formula for calculating slack or float is:

Slack (Float) = Latest Finish (LF) – Earliest Finish (EF) or equivalently, Slack (Float) = Latest Start (LS) – Earliest Start (ES)

Activity	Predecessor	Normal Time (weeks)	Earliest Start (ES)	Earliest Finish (EF)	Latest Start (LS)	Latest Finish (LF)	Slack (weeks)
A	-	4	0	4	0	4	0
B	A	5	4	9	4	9	0
C	A	4	4	8	4	8	0
D	B	6	9	15	9	15	0
E	C	6	8	14	8	14	0
F	D	5	15	20	20	25	5
G	E	7	14	21	14	21	0
H	G	4	21	25	21	25	0

This calculation helps identify which activities have some wiggle room in their schedule (i.e., non-critical activities with slack greater than zero) and which activities are on the critical path (i.e., activities with zero slack).

For our project, the activities on the critical path (those with zero slack) are the ones to consider for crashing, as delaying any of these activities will directly impact the project's completion time. The "Slack (weeks)" column indicates the amount of time each activity can be delayed without affecting the overall project duration. Activities with zero slack are on the critical path and cannot be delayed without impacting the project timeline.



The critical path for our project, based on the slack values calculated, is as follows:

Critical Path: A → B → C → D → E → G → H

This sequence of activities represents the longest path through the project and determines the shortest possible duration for completing the project. Each activity on this path has zero slack,

meaning any delay in these activities will directly delay the overall project completion.

In our project, Activity "F" is not on the critical path as it has a slack of 5 weeks, indicating that it can be delayed up to 5 weeks **without impacting the project's completion time**. All other activities must be completed on schedule to avoid delays.

Here are the durations for each activity on the critical path:

Activity A: Normal Time = 4 weeks

Activity B: Normal Time = 5 weeks

Activity C: Normal Time = 4 weeks

Activity D: Normal Time = 6 weeks

Activity E: Normal Time = 6 weeks

Activity G: Normal Time = 7 weeks

Activity H: Normal Time = 4 weeks

Now, let's sum these durations:

Total Critical Path Duration=4+5+4+6+6+7+4 is 36 weeks.

activity	Predecessor	Normal Time (weeks)	Crash Time (weeks)	Normal Cost (USD)	Crash Cost (USD)	Cost Slope (USD/week)
A	-	4	3	8,000	9,000	1,000
B	A	5	3	16,000	20,000	2,000
C	A	4	3	12,000	13,000	1,000
D	B	6	5	34,000	35,000	1,000
E	C	6	4	42,000	44,000	1,000
F	D	5	4	16,000	16,500	500
G	E	7	4	66,000	72,000	2,000
H	G	4	3	2,000	5,000	3,000

To determine the most cost-effective way to crash your project, you first need to understand the cost slope formula. The cost slope is calculated using the following formula:

$$\text{Cost Slope} = (\text{Crash Cost} - \text{Normal Cost}) / (\text{Normal Time} - \text{Crash Time})$$

This formula gives you the additional cost per unit of time saved by crashing an activity. It's essential for identifying which activity to crash when you want to reduce the project duration.

Total Direct Normal Cost: The sum of the normal costs of all the activities is \$196,000.

Indirect Cost: With an indirect cost of \$1,000 per week for a project duration of 36 weeks, the total indirect cost is \$36,000.

Total Cost for Normal Duration of the Project: Adding both direct and indirect costs, the total is \$196,000 + \$36,000 = \$232,000.

So, for the normal duration of 36 weeks, the total cost of the project is \$232,000.

2. Crashing

In the context of our project:

Calculate the Cost Slope: You start by calculating the cost slope for each activity using the above formula. This will tell you how much more it will cost to complete an activity faster than planned.

Focus on the Critical Path: When considering crashing, you should focus on activities on the critical path. The critical path is the longest path through the project in terms of time, and any delay in these activities will delay the entire project.

Excluding Non-Critical Activities: Since 'F' is not on the critical path, it's not a priority for crashing. Crashing activities not on the critical path won't help in reducing the overall project duration. Therefore, it's more efficient to focus your resources on crashing activities on the critical path where a reduction in time directly impacts the project's completion time.

By following this approach, you can identify the most cost-effective activities to crash in order to expedite your project completion while incurring the minimum additional cost.

When multiple activities have the same lowest cost slope, the decision on which activity to crash should also consider other factors, such as the feasibility of crashing (based on the difference between normal and crash times) and the impact on subsequent activities.

Since multiple activities on the critical path have the same cost slope of \$1,000, let's re-evaluate and consider all such activities to determine the best option for crashing by one week. We should crash the one that can be completed within its crash time limit and whose crashing would effectively reduce the overall project duration. Let's identify all such activities.

Based on the re-evaluation, there are four activities on the critical path with a cost slope of \$1,000 and can potentially be crashed by at least one week. These activities are:

Activity A: Can be crashed by 1 week.

Activity C: Can be crashed by 1 week.

Activity D: Can be crashed by 1 week.

Activity E: Can be crashed by 2 weeks.

In determining which activity to crash in a project, several factors come into play. Ideally, the choice revolves around which activity can be accelerated most efficiently and practically within the specific context of your project. For instance, if Activity E could be expedited easily and without significant risks, and if it's more practical to allocate additional resources to it, it might be a preferable choice. This is particularly relevant considering Activity E can be crashed by up to 2 weeks, providing greater flexibility.

However, in the context of your project, our focus shifts to the activity on the critical path that offers the least additional cost for crashing and can feasibly be shortened by at least one week. This is where Activity "A" comes into the picture. It has a cost slope of \$1,000 per week, which means crashing Activity "A" will incur an additional cost of \$1,000 for every week it's expedited.

Given this scenario, to reduce the overall duration of the project by one week, crashing Activity "A" emerges as the most cost-effective strategy (because Activity 'A' is the starting activity which is

affecting to all other activities). This recommendation is based on the assumption that it's feasible to crash Activity "A" by one week considering its normal and crash times. By doing so, you can achieve the goal of shortening the project timeline with a minimal increase in cost, ensuring that the critical path is effectively managed.

1st Iteration

To evaluate the cost-effectiveness of crashing Activity A in our project, considering an indirect cost of 1,000 \$ per week, we will perform the following analysis:

- Crashing Activity, A: Based on the provided data, crashing Activity A by one week will reduce the project duration by one week. This crashing involves additional costs, which we can calculate using the cost slope.
- Cost Slope for Activity A: The cost slope for Activity A is 1,000 \$ per week. This means it costs an additional 1,000 \$ to expedite Activity A by one week.
- Indirect Costs: The indirect cost of the project is 1,000 \$ per week. By crashing the project by one week, you save one week of indirect costs.
- Total Cost Implication: The total cost implication of crashing Activity A includes the additional direct cost (due to crashing) and the savings in indirect costs (due to reduced project duration).

Let's calculate the net cost impact of crashing Activity A by one week.

Additional Direct Cost for Crashing Activity A:

Cost Slope for Activity A = \$1,000 per week.

We are crashing it by 1 week.

Therefore, Additional Direct Cost = Cost Slope × Crashing Duration

Additional Direct Cost = \$1,000/week × 1 week = \$1,000.

Savings in Indirect Costs by Reducing Project Duration:

Indirect Cost = \$1,000 per week.

By crashing Activity, A, we reduce the project duration from 36 weeks to 35 weeks.

Therefore, Savings in Indirect Costs = Indirect Cost per week × Reduction in Duration

Savings in Indirect Costs = \$1,000/week × 1 week = \$1,000.

Total Cost Impact of Crashing:

Total Cost Impact = Additional Direct Cost - Savings in Indirect Costs

Total Cost Impact = \$1,000 - \$1,000 = \$0.

Total Cost for Crashed Project Duration:

Original Total Direct Normal Cost = \$196,000.

New Indirect Cost for reduced duration (35 weeks) = $\$1,000/\text{week} \times 35 \text{ weeks} = \$35,000$.

Total Cost for Crashed Project Duration = Original Total Direct Normal Cost + New Indirect Cost + Total Cost Impact of Crashing

Total Cost for Crashed Project Duration = $\$196,000 + \$35,000 + \$0 = \$231,000$.

By crashing Activity A by one week, the new total cost for the project would be \$231,000. This reflects the savings of \$1,000 in indirect costs, offsetting the additional \$1,000 spent on crashing Activity A, resulting in a net zero increase in the total project cost.

Note: If Activity A cannot be crashed any further (because it has reached its crash duration limit), or if crashing it again would not be cost-effective, you would then move to the next activity with the lowest cost slope on the critical path. You would repeat the analysis, taking into account the new activity's cost slope and its potential for crashing.

Keep in mind that as you crash activities and shorten the project duration, the critical path may change if the total duration of a parallel path becomes longer than the current critical path. Therefore, you must re-evaluate the critical path after each crashing decision to ensure that you are still focusing on the correct sequence of activities.

Since, Activity A has a normal duration of 4 weeks and a crash duration of 3 weeks, then we can only crash Activity A by 1 week without exceeding its crash limit. Once Activity A has been crashed by 1 week, it has reached its crash duration and cannot be crashed any further.

For Activity A, we can see it has already been crashed from its normal duration of 4 weeks to its crash duration of 3 weeks by adding extra resources or by other means. Once Activity A is at 3 weeks duration, attempting to crash it further (3 weeks) is not possible without violating the constraints that define its crash time.

To proceed with further crashing and find the optimal solution, we will iterate through the activities with the lowest crash slopes that can still be crashed. We'll continue the process until we reach the optimal project duration where further crashing is either not possible or no longer cost-effective.

To iterate the process of crashing to find the optimal duration or cost, we follow these steps:

- Review the remaining crash potential for each activity on the critical path.
- Select the activity with the lowest cost slope that can still be crashed.
- Calculate the additional direct cost for crashing the selected activity.
- Calculate the savings in indirect costs by reducing the project duration by one more week.
- Determine the net cost impact of crashing.
- Update the total cost and project duration.
- Repeat the process until no more crashing is possible or it no longer results in a net benefit.

We have already crashed Activity A by one week, reducing the project duration to 35 weeks and the total cost remained at \$232,000 due to the indirect cost saving balancing the additional direct cost.

Let's proceed with the next iterations:

Activity Crashed	Additional Direct Cost	Indirect Cost Savings	Net Cost Impact	Total Cost After Crashing	Project Duration After Crashing
E (1st Crash)	\$1,000	\$1,000	\$0	\$231,000	34 weeks
E (2nd Crash)	\$1,000	\$1,000	\$0	\$231,000	33 weeks
C	\$1,000	\$1,000	\$0	\$231,000	32 weeks
D	\$1,000	\$1,000	\$0	\$231,000	31 weeks

Each time an activity is crashed (E twice, then C and D), we incur an additional direct cost of \$1,000. However, each crash also saves \$1,000 in indirect costs by reducing the project duration by one week. Therefore, the net cost impact of each crash is \$0, and the total cost of the project remains constant at \$231,000. As a result of this process, the project duration has been successfully reduced from 35 weeks to 31 weeks without increasing the total project cost beyond \$231,000. This iterative crashing process can continue until either all crashable activities reach their crash limit, or the cost of crashing exceeds the benefits. In this scenario, each crash has been cost-neutral, effectively reducing the project duration without additional costs.

In conclusion, a project duration of 31 weeks at a total cost of \$231,000 is the most efficient outcome based on the crashing process applied to our project, considering both the cost implications and the crash limits of the activities involved.

3. Other

The selection of the critical path in our project:

Parallel Paths: In our project, there are different sequences of activities that can be pursued in parallel. For instance, one path could be A-B-D-F, and another could be A-C-E-G-H. Each of these paths has a total duration, calculated by adding the durations of each activity in the sequence.

Identifying the Longest Path: The critical path is the longest path from the start to the finish of the project. It's not just any path with zero slack, but specifically the one that takes the longest time to complete. The total project duration is dictated by this path because you cannot complete the project until all tasks on this path are finished.

Zero Slack and Its Implications: On the critical path, activities will have zero slack, meaning there is no leeway to delay these activities without affecting the overall project duration. Activities not on the critical path may have slack, which means they can be delayed without impacting the project's completion date.

Why Zero Slack Alone Is Not the Only Criterion: There might be activities in the project with zero Slack that are not part of the longest path. These activities, even though critical in their own sequence, do not determine the shortest possible completion time of the entire project.

The Unique Nature of the Critical Path: The critical path is uniquely characterized by being the longest sequence of activities (in terms of duration) with zero slack. This is why it is crucial in project management; any delay in an activity on the critical path directly impacts the project completion date.

Focus on the Critical Path for Crashing: When considering crashing (speeding up tasks), focus on the critical path. This is because shortening any activity on this path will directly reduce the overall project duration.

In our project, the critical path was identified as $A \rightarrow B \rightarrow C \rightarrow D \rightarrow E \rightarrow G \rightarrow H$. This sequence represents the longest path through the project network with zero slack, hence its importance in determining the total duration of the project. Activities on this path are prime candidates for crashing if our aim is to reduce the project's completion time.

Note prepared by;

H.L.N.Himanshi