5. LIST OF TOPICS FROM GARRET'S „ABSTRACT ALGEBRA"

Below, sections marked with ▮ are either less important for the course or assumed known; thus we skip them. They are to be consulted only as necessary to understand the rest of the text. ~~Crossed out~~ sections are definetely assumed to be known. The sections marked with ▮ can be treated less thoroughly than the others.

2 Groups I
2.1 Groups
2.2 Subgroups, Lagrange's theorem
2.3 Homomorphisms, kernels, normal subgroups
2.4 Cyclic groups
2.5 Quotient groups
2.6 Groups acting on sets
2.7 The Sylow theorem
2.8 Trying to classify finite groups, part I
2.9 Worked examples

— — → 1st meeting

3 The players: rings, fields, etc.
3.1 Rings, fields
3.2 Ring homomorphisms
3.3 Vectorspaces, modules, algebras
3.4 Polynomial rings I

— — → 2nd meeting

4 Commutative rings I
4.1 Divisibility and ideals
4.2 Polynomials in one variable over a field

4.3 Ideals and quotients
4.4 Ideals and quotient rings
4.5 Maximal ideals and fields
4.6 Prime ideals and integral domains
4.7 Fermat-Euler on sums of two squares
4.8 Worked examples

— — → 3rd meeting

~~5 Linear Algebra I: Dimension~~
~~5.1 Some simple results~~
~~5.2 Bases and dimension~~
~~5.3 Homomorphisms and dimension~~

6 Fields I

6.1 Adjoining things
6.2 Fields of fractions, fields of rational functions
6.3 Characteristics, finite fields   — — —→4th meeting
6.4 Algebraic field extensions
6.5 Algebraic closures

7 Some Irreducible Polynomials

7.1 Irreducibles over a finite field
7.2 Worked examples

8 Cyclotomic polynomials

8.1 Multiple factors in polynomials   — — —→ 5th meeting
8.2 Cyclotomic polynomials
8.3 Examples
8.4 Finite subgroups of fields
8.5 Infinitude of primes $p = 1 \bmod n$
8.6 Worked examples

9 Finite fields

9.1 Uniqueness
9.2 Frobenius automorphisms
9.3 Counting irreducibles

10 Modules over PIDs

10.1 The structure theorem
10.2 Variations
10.3 Finitely-generated abelian groups
10.4 Jordan canonical form
10.5 Conjugacy versus $k[x]$-module isomorphism
10.6 Worked examples
                                              — — —→ 6th meeting
11 Finitely-generated modules

11.1 Free modules
11.2 Finitely-generated modules over a domain
11.3 PIDs are UFDs
11.4 Structure theorem, again
11.5 Recovering the earlier structure theorem
11.6 Submodules of free modules

12 Polynomials over UFDs

12.1 Gauss' lemma
12.2 Fields of fractions
12.3 Worked examples

13 Symmetric groups
13.1 Cycles, disjoint cycle decompositions
13.2 Transpositions
13.3 Worked examples

14 Naive Set Theory
14.1 Sets
14.2 Posets, ordinals
14.3 Transfinite induction
14.4 Finiteness, infiniteness
14.5 Comparison of infinities
14.6 Example: transfinite Lagrange replacement
14.7 Equivalents of the Axiom of Choice

15 Symmetric polynomials

15.1 The theorem
15.2 First examples
15.3 A variant: discriminants

— — —→ 7th meeting

16 Eisenstein's criterion

16.1 Eisenstein's irreducibility criterion
16.2 Examples

17 Vandermonde determinants
17.1 Vandermonde determinants
17.2 Worked examples
(for symmetric polynomials)

18 Cyclotomic polynomials II
18.1 Cyclotomic polynomials over $\mathbb{Z}$
18.2 Worked examples

19 Roots of unity

19.1 Another proof of cyclicness
19.2 Roots of unity
19.3 $\mathbb{Q}$ with roots of unity adjoined

— — —→ 8th meeting

19.4 Solution in radicals, Lagrange resolvents
19.5 Quadratic fields, quadratic reciprocity
19.6 Worked examples

20 Cyclotomic III   — — —→9th meeting
20.1 Prime-power cyclotomic polynomials over $\mathbb{Q}$
20.2 Irreducibility of cyclotomic polynomials over $\mathbb{Q}$
20.3 Factoring $\Phi_n(x)$ in $F_p[x]$ with $p|n$
20.4 Worked examples (about radicals)

21 Primes in arithmetic progressions
21.1 Euler's theorem and the zeta function
21.2 Dirichlet's theorem
21.3 Dual groups of abelian groups
21.4 Non-vanishing on $\mathrm{Re}(s)=1$
21.5 Analytic continuations
21.6 Dirichlet series with positive coefficients

22 Galois theory

22.1 Field extensions, imbeddings, automorphisms
22.2 Separable field extensions
22.3 Primitive elements   — — —→ 10th meeting
22.4 Normal field extensions
22.5 The main theorem
22.6 Conjugates, trace, norm

22.7 Basic examples
22.8 Worked examples
                                                           — — —→ 11th meeting
23 Solving equations by radicals
23.1 Galois' criterion

23.2 Composition series, Jordan-Hölder theorem
23.3 Solving cubics by radicals   — — —→12th meeting
23.4 Worked examples