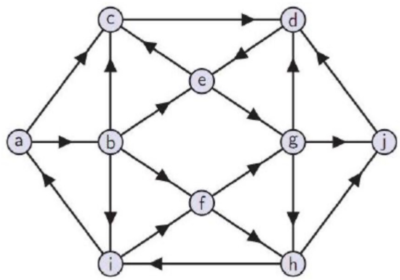
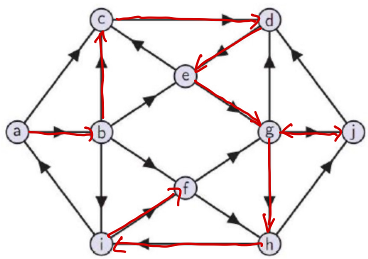
GNT EXERCISES

Task 03



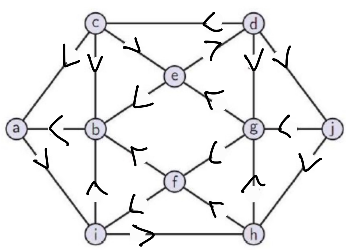
Is the graph in the figure strongly connected? Check using the algorithm from the presentation.

1) DFS (G) and store in order of visiting.



$L = \{A, B, C, D, E, G, J, H, I, F\}$

2) Compute G^T

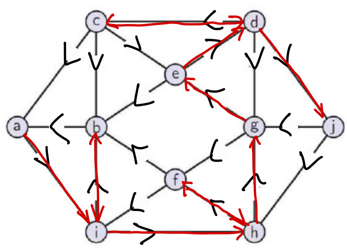


3) DFS(G^T)

3.1) If we reach a cycle, we have a SCC

3.2) Pop from the stack L the elements from SCC: $L = L \setminus SCC$

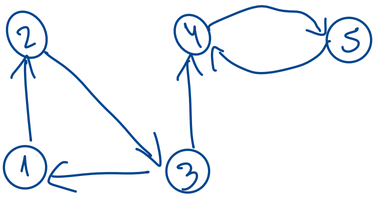
3.3) Repeat until $L = \emptyset$, over $G^T = G^T \setminus SCC$



$L = \{A, B, C, D, E, G, J, H, I, F\}$

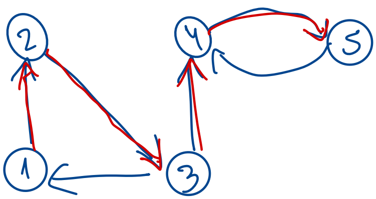
$SCC_1 = \{A, I, B, H, F, G, E, D, C, J\}$

We can see G is strongly connected, as we found 1 SCC.



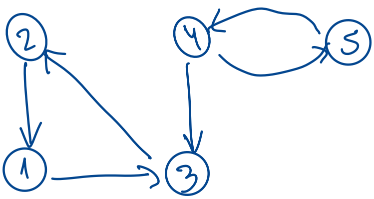
Check if the graph is strongly connected.

1)

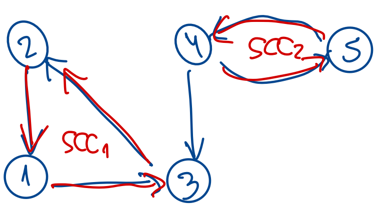


$L = \{1, 2, 3, 4, 5\}$

2)



3)



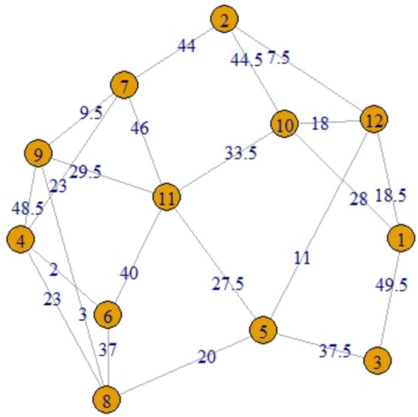
$L = \{\cancel{1}, \cancel{2}, \cancel{3}, \cancel{4}, \cancel{5}\}$

$SCC_1 = \{1, 2, 3\}$

$SCC_2 = \{4, 5\}$

Task 04

Find the MST in the graph. Use two methods.



Prim: When done in paper, the algorithm can be formulated the following way.

Prim(G, w, r) {

$S = V$, $A = \emptyset$

$A \cup \{e\}$, where e is the edge of less weight from the root r

$S = S \setminus \{r\}$

while($S \neq \emptyset$) {

$\forall v \in V \setminus S$ take edge of less weight e and mark it if it's not marked, and if it doesn't form a cycle.

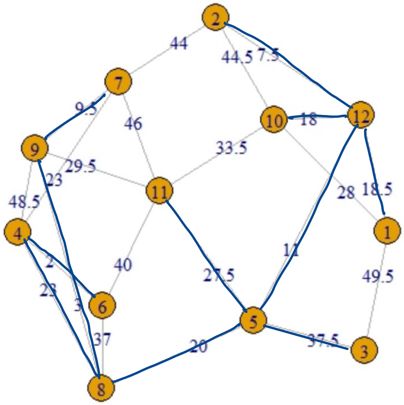
u = vertex incident with e in the path

$A \cup \{e\}$

$S = S \setminus \{u\}$

}

}



Let $r=1$, $A=\emptyset$, $S=\{1..12\}$ $V\setminus S=\{1\}$

1) $S=\{2..12\}$ $A=\{\{1,12\}\}$ $V\setminus S=\{1,12\}$

2) $S=\{3,..11\}$ $A=\{\{1,12\},\{12,2\}\}$ $V\setminus S=\{1,12,2\}$

3) $S=\{3,4,6...11\}$ $A=\{\{1,12\},\{12,2\},\{12,5\}\}$ $V\setminus S=\{1,12,2,5\}$

4) $S=\{3,4,6..9,11\}$ $A=\{\{1,12\},\{12,2\},\{12,5\},\{12,10\}\}$ $V\setminus S=\{1,12,2,5,10\}$

5) $S=\{3,4,6,7,9,11\}$ $A=\{\{1,12\},\{12,2\},\{12,5\},\{12,10\},\{5,8\}\}$ $V\setminus S=\{1,12,2,5,10,8\}$

6) $S=\{3,4,6,7,11\}$ $A=\{\{1,12\},\{12,2\},\{12,5\},\{12,10\},\{5,8\},\{8,9\}\}$
$V\setminus S=\{1,12,2,5,10,8,9\}$

7) $S=\{3,4,6,11\}$ $A=\{\{1,12\},\{12,2\},\{12,5\},\{12,10\},\{5,8\},\{8,9\},\{9,7\}\}$
$V\setminus S=\{1,12,2,5,10,8,9,7\}$

8) $S=\{3,6,11\}$ $A=\{\{1,12\},\{12,2\},\{12,5\},\{12,10\},\{5,8\},\{8,9\},\{9,7\},\{8,4\}\}$
$V\setminus S=\{1,12,2,5,10,8,9,4\}$

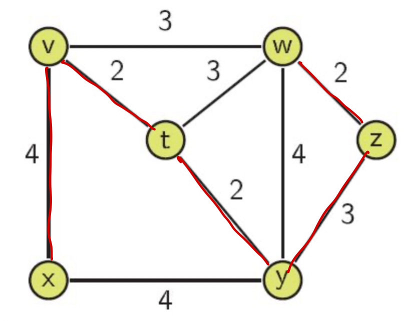
9) $S=\{3,11\}$ $A=\{\{1,12\},\{12,2\},\{12,5\},\{12,10\},\{5,8\},\{8,9\},\{9,7\},\{8,4\},\{4,6\}\}$
$V\setminus S=\{1,12,2,5,10,8,9,4,6\}$

10) $S=\{3\}$ $A=\{\{1,12\},\{12,2\},\{12,5\},\{12,10\},\{5,8\},\{8,9\},\{9,7\},\{8,4\},\{4,6\},\{5,11\}\}$
$V\setminus S=\{1,12,2,5,10,8,9,4,6,11\}$

11) $S=\emptyset$ $A=\{\{1,12\},\{12,2\},\{12,5\},\{12,10\},\{5,8\},\{8,9\},\{9,7\},\{8,4\},\{4,6\},\{5,11\},\{5,3\}\}$

Kruskal:

```
Kruskal(G,w){
    A=∅
    ∀v∈V, Makeset(v)
    Sort edges by weight in non-decreasing order in a queue Q.
    while(Q≠∅){
        {u,v}=Q.pop
        if(u∉set(v)){
            A=A∪{u,v}
            Union(set(u), set(v))
        }
    }
}
```



$Q=\{\cancel{\{v,t\}}, \cancel{\{t,y\}}, \cancel{\{w,z\}}, \cancel{\{y,z\}}, \cancel{\{t,w\}}, \cancel{\{v,w\}}, \cancel{\{x,v\}}, \cancel{\{x,y\}}, \cancel{\{y,w\}}\}$

$\{v\}\ \{t\}\ \{y\}\ \{w\}\ \{z\}\ \{x\}$ $\qquad A=\emptyset$

1) $\{v,t\}\ \{y\}\ \{w\}\ \{z\}\ \{x\}$ $\qquad A=\{\{v,t\}\}$

2) $\{v,t,y\}\ \{w\}\ \{z\}\ \{x\}$ $\qquad A=\{\{v,t\},\{t,y\}\}$

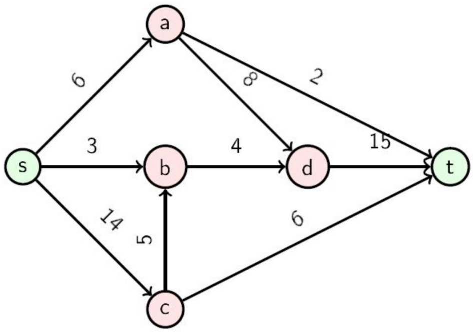
3) $\{v,t,y\}\ \{w,z\}\ \{x\}$ $\qquad A=\{\{v,t\},\{t,y\},\{w,z\}\}$

4) $\{v,t,y,w,z\}\ \{x\}$ $\qquad A=\{\{v,t\},\{t,y\},\{w,z\},\{y,z\}\}$

5) $\{v,t,y,w,z,x\}$ $\qquad A=\{\{v,t\},\{t,y\},\{w,z\},\{y,z\},\{x,v\}\}$

Task 09

For the network, find the maximum flow function and the value of the maximum flow, from vertex s to t.



Ford-Fulkerson (G, s, t) {

$\forall (u,v) \in E,\ f(u,v) = 0$

Build residual network G_f

while ($\exists$ augmenting path p from s to t in G_f) {

$c_f(p) = \min\{c_f(u,v) \mid (u,v) \in P\}$

for($(u,v) \in P$) {

if($(u,v) \in E$) $\quad f(u,v) = f(u,v) + c_f(p)$

else $\quad f(v,u) = f(v,u) - c_f(p)$

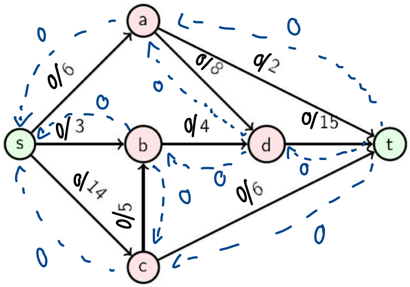
}

}

return $\sum_{p} c_f(p) = \sum_{v \in V} f(s,v)$

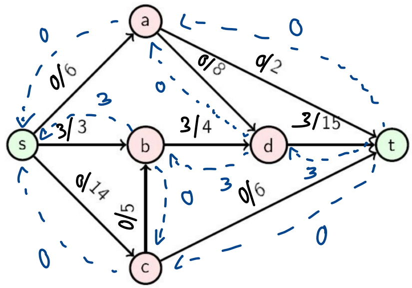
}

G_f



1) $p = (s, b, d, t)$

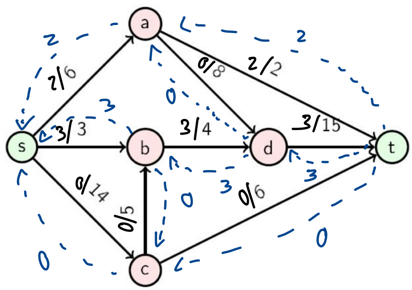
$c_f(p) = 3$



2)

$p = (s, a, t)$

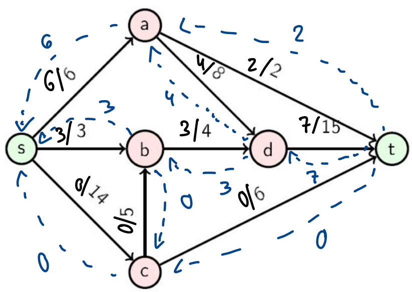
$c_f(p) = 2$



3)

$p = (s, a, d, t)$

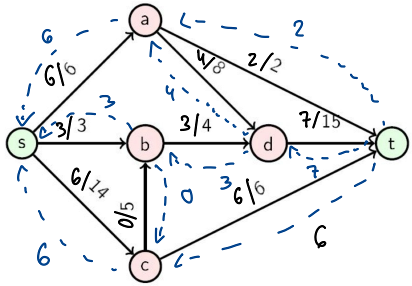
$c_f(p) = 4$



4)

$p = (s, c, t)$

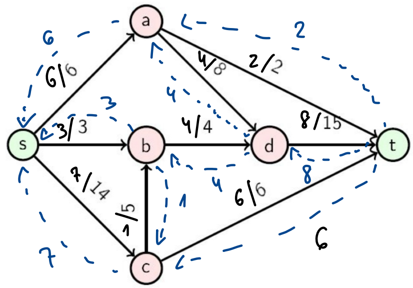
$c_f(p) = 6$



5)

$p = (s, c, b, d, t)$

$c_f(p) = 1$



We can see there are no more augmenting paths $\Rightarrow$

$$|f| = \sum_{p} c_f(p) = 3 + 2 + 4 + 6 + 1 = 16$$