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RELACIÓN 3: FUNCIONES SPLINE

① Determinar a, b, c para que

$$S(x) = \begin{cases} x^3 & 0 \le x \le 1 \\ \frac{1}{2}(x-1)^3 + a(x-1)^2 + b(x-1) + c & 1 \le x \le 3 \end{cases}$$

sea spline cúbico.

$S(x) \in S_3(0,1,3) \iff$ se verifica:

1) $S(x)$ es continua:

$S_0(1) = S_1(1) \iff 1^3 = \frac{1}{2}(1-1)^3 + a(1-1)^2 + b(1-1) + c$

$\boxed{c = 1}$

2) 1ª derivada continua (unión "suave"):

$S_0'(1) = S_1'(1) \iff 3 \cdot 1^2 = \frac{3}{2}(1-1)^2 + 2a(1-1) + b$ $\boxed{b = 3}$

$S_0'(x) = 3x^2 \qquad S_1'(x) = \frac{3}{2}(x-1)^2 + 2a(x-1) + b$

3) 2ª derivada continua (ser de clase 2):

$S_0''(1) = S_1''(1) \iff 6 \cdot 1 = 3(1-1) + 2a$ $\boxed{a = \frac{6}{2} = 3}$

$S_0''(x) = 6x \qquad S_1''(x) = 3(x-1) + 2a$

$$S(x) = \begin{cases} x^3 & 0 \le x \le 1 \\ \frac{1}{2}(x-1)^3 + 3(x-1)^2 + 3(x-1) + 1 & 1 \le x \le 3 \end{cases}$$