Clasificar y estudiar los elementos euclídeos de las siguientes cónicas

$-2\sqrt{2} + 12x + 3\sqrt{2}x^2 + 4y + 2\sqrt{2}xy + 3\sqrt{2}y^2 == 0$

In[ ]:=

$$mm := \begin{pmatrix} -2\sqrt{2} & 6 & 2 \\ 6 & 3\sqrt{2} & \sqrt{2} \\ 2 & \sqrt{2} & 3\sqrt{2} \end{pmatrix}$$

$$pp := \begin{pmatrix} 3\sqrt{2} & \sqrt{2} \\ \sqrt{2} & 3\sqrt{2} \end{pmatrix}$$

```
CharacteristicPolynomial[mm, x]
```
polinomio característico

```
CharacteristicPolynomial[pp, x]
```
polinomio característico

```
Eigenvalues[pp]
```
autovalores

```
Eigenvectors[pp]
```
autovectores

Out[ ]= $-128\sqrt{2} + 48x + 4\sqrt{2}x^2 - x^3$

Out[ ]= $16 - 6\sqrt{2}x + x^2$

Out[ ]= $\{4\sqrt{2}, 2\sqrt{2}\}$

Out[ ]= $\{\{1, 1\}, \{-1, 1\}\}$

Se trata de una elipse

In[ ]:=

```
{0, 1, 1}.mm.{1, x, y}
{0, -1, 1}.mm.{1, x, y}
Solve[{{0, 1, 1}.mm.{1, x, y} == 0, {0, -1, 1}.mm.{1, x, y} == 0}, {x, y}]
```
resuelve

Out[ ]= $8 + 4\sqrt{2}x + 4\sqrt{2}y$

Out[ ]= $-4 - 2\sqrt{2}x + 2\sqrt{2}y$

Out[ ]= $\{\{x \to -\sqrt{2}, y \to 0\}\}$

ejes : $8 + 4\sqrt{2}x + 4\sqrt{2}y == 0, \; -4 - 2\sqrt{2}x + 2\sqrt{2}y = 0$

centro := $(-\sqrt{2}, 0)$

Sistema de Referencia

$\{(-\sqrt{2}, 0), \text{Normalizar}\{\{1, 1\}, \{-1, 1\}\}\}$

$n := \begin{pmatrix} 1 & 0 & 0 \\ -\sqrt{2} & 1/\sqrt{2} & -1/\sqrt{2} \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$ es la matriz de cambio de sistema de referencia

$a = 4\sqrt{2}\, x^2 + 2\sqrt{2}\, y^2,$

con $\text{Det}[mm] = -128\sqrt{2} = -16\, a$, así

$1 == x^2/2 + y^2/4$

In[∘]:= Transpose[n].mm.n
transposición

Out[∘]= $\{\{-8\sqrt{2}, 0, 0\}, \{0, 4\sqrt{2}, 0\}, \{0, 0, 2\sqrt{2}\}\}$

Focos

F1 : $\{-\sqrt{2}, 0\} + \sqrt{2}$ Normalize[{-1, 1}]

F2 : $\{-\sqrt{2}, 0\} - \sqrt{2}$ Normalize[{-1, 1}]

F1 : $\{-1-\sqrt{2}, 1\}$

F2 : $\{1-\sqrt{2}, -1\}$

Otro ejercicio

$4x^2 + y^2 + + 4xy + 2x + 1 == 0$

In[∘]:= mm := $\begin{pmatrix} 1 & 1 & 0 \\ 1 & 4 & 2 \\ 0 & 2 & 1 \end{pmatrix}$

m := $\begin{pmatrix} 4 & 2 \\ 2 & 1 \end{pmatrix}$

```
CharacteristicPolynomial[mm, x]
```
polinomio característico

```
CharacteristicPolynomial[m, x]
```
polinomio característico

```
Eigenvalues[m]
```
autovalores

```
Eigenvectors[m]
```
autovectores

Out[∘]= $-1 - 4x + 6x^2 - x^3$

Out[∘]= $-5x + x^2$

Out[∘]= {5, 0}

Out[∘]= {{2, 1}, {-1, 2}}

Es una parábola

In[∘]:=

```
{0, 2, 1}.mm.{1, x, y}
```

Out[∘]= $2 + 10x + 5y$

eje := 2 + 10 x + 5 y == 0

In[∘]:=

```
Solve[{4 x^2 + y^2 + 4 x y + 2 x + 1 == 0, 2 + 10 x + 5 y == 0}, {x, y}]
```
resuelve

Out[∘]= $\{\{x \to -\frac{29}{50}, y \to \frac{19}{25}\}\}$

Matriz cambio sistema de referencia : $\begin{pmatrix} 1 & 0 & 0 \\ -\frac{29}{50} & 2/\sqrt{5} & -1/\sqrt{5} \\ \frac{19}{25} & 1/\sqrt{5} & 2/\sqrt{5} \end{pmatrix}$

In[∘]:=

p := $\begin{pmatrix} 1 & 0 & 0 \\ -\frac{29}{50} & 2/\sqrt{5} & -1/\sqrt{5} \\ \frac{19}{25} & 1/\sqrt{5} & 2/\sqrt{5} \end{pmatrix}$

In[ ]:=
```
Transpose[p].mm.p
```
transposición

Out[ ]= $\left\{\left\{0, 0, -\frac{1}{\sqrt{5}}\right\}, \{0, 5, 0\}, \left\{-\frac{1}{\sqrt{5}}, 0, 0\right\}\right\}$

$$-\frac{2}{\sqrt{5}} y + 5 x^{\wedge}2 == 0, \quad 2 c y == x^{\wedge}2, \quad c = \frac{1}{5\sqrt{5}}$$

$$\text{FOCO} := \left\{-\frac{29}{50}, \frac{19}{25}\right\} + \frac{1}{10\sqrt{5}} \frac{1}{\sqrt{5}} \{-1, 2\} == \left\{-\frac{3}{5}, \frac{4}{5}\right\}$$

In[18]:=

$$\left\{-\frac{29}{50}, \frac{19}{25}\right\} + \frac{1}{10\sqrt{5}} \frac{1}{\sqrt{5}} \{-1, 2\}$$

In[12]:=

$$\left\{-\frac{29}{50}, \frac{19}{25}\right\} + \frac{1}{10\sqrt{5}} \frac{1}{\sqrt{5}} \{1, -2\}$$

Out[12]= $\left\{-\frac{14}{25}, \frac{18}{25}\right\}$

In[13]:=

$$2\left\{-\frac{29}{50}, \frac{19}{25}\right\} - \left\{-\frac{14}{25}, \frac{18}{25}\right\}$$

Out[13]= $\left\{-\frac{3}{5}, \frac{4}{5}\right\}$

$$\text{Directriz} := \left\{-\frac{14}{25}, \frac{18}{25}\right\} + L\,(\{2, 1\})$$

$$2 + x == 2 y$$

In[21]:=
```
Solve[{2 + x == 2 y, 4 x^2 + y^2 + 4 x y + 2 x + 1 == 0}, {x, y}]
```
resuelve

Out[21]= $\left\{\left\{x \to -\frac{14}{25} - \frac{2\,i}{25}, y \to \frac{18}{25} - \frac{i}{25}\right\}, \left\{x \to -\frac{14}{25} + \frac{2\,i}{25}, y \to \frac{18}{25} + \frac{i}{25}\right\}\right\}$

Otro ejemplo

$$-3 + 2 x + x^2 - 6 y - 6 x y + y^2 == 0$$

In[•]:=

$$mm := \begin{pmatrix} -3 & 1 & -3 \\ 1 & 1 & -3 \\ -3 & -3 & 1 \end{pmatrix}$$

$$m := \begin{pmatrix} 1 & -3 \\ -3 & 1 \end{pmatrix}$$

```
CharacteristicPolynomial[mm, x]
```
polinomio característico

```
CharacteristicPolynomial[m, x]
```
polinomio característico

```
Eigenvalues[m]
```
autovalores

```
Eigenvectors[m]
```
autovectores

Out[•]= $32 + 24\,x - x^2 - x^3$

Out[•]= $-8 - 2\,x + x^2$

Out[•]= {4, -2}

Out[•]= {{-1, 1}, {1, 1}}

In[•]:=

```
{0, -1, 1}.mm.{1, x, y}
{0, 1, 1}.mm.{1, x, y}
Solve[{{0, -1, 1}.mm.{1, x, y} == 0, {0, 1, 1}.mm.{1, x, y} == 0}, {x, y}]
```
resuelve

Out[•]= $-4 - 4\,x + 4\,y$

Out[•]= $-2 - 2\,x - 2\,y$

Out[•]= {{x → -1, y → 0}}

ejes : -4 - 4 x + 4 y == 0, -2 - 2 x - 2 y = 0
centro := (-1, 0)

$$n := \begin{pmatrix} 1 & 0 & 0 \\ -1 & -1/\sqrt{2} & 1/\sqrt{2} \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$$ es la matriz de cambio de sistema de referencia

In[•]:=

```
Transpose[n].mm.n
```
transposición

Out[•]= {{-4, 0, 0}, {0, 4, 0}, {0, 0, -2}}

Ecuación reducida : -4 + 4 x^2 - 2 y^2 == 0

1 = x^2 - y^2/2

Focos

```
F1 := (-1, 0) + √3 (-1/√2, 1/√2)
F2 := (-1, 0) - √3 (-1/√2, 1/√2)
```

In[•]:=

```
Solve[x^2 - 6 x y + y^2 == 0, {x, y}]
```

resuelve

Solve: Equations may not give solutions for all "solve" variables.

In[•]:=

```
{0, x, y}.mm.{0, x, y}
```

In[•]:=

```
Solve[x^2 - 6 x y + y^2 == 0, {x, y}]
```

resuelve

Solve: Equations may not give solutions for all "solve" variables.

Out[•]= $\{\{y \to 3x - 2\sqrt{2}\,x\}, \{y \to 3x + 2\sqrt{2}\,x\}\}$

In[•]:=

```
Simplify[{0, 1, 3 - 2 √2}.mm.{1, x, y} == 0]
```

simplifica

```
Simplify[{0, 1, 3 + 2 √2}.mm.{1, x, y} == 0]
```

simplifica

Out[•]= $3\sqrt{2}\,(1+x) == 4 + 4x + \sqrt{2}\,y$

Out[•]= $2\left(4+3\sqrt{2}\right)(1+x) == 2\sqrt{2}\,y$

Asíntotas

$3\sqrt{2}\,(1+x) == 4 + 4x + \sqrt{2}\,y$

$\left(8+6\sqrt{2}\right)(1+x) == 2\sqrt{2}\,y$

Otra forma de dar las asíntotas es como rectas que pasan por el centro y llevan las direcciones asintóticas :

```
a1 := (-1, 0) + L (1, 3 - 2 √2)
a2 := (-1, 0) + L (1, 3 + 2 √2)
```