1 Resolver en función de $\alpha, \beta > 0$

$$x_{n+2} - \alpha(1+\beta)x_{n+1} + \alpha\beta x_n = 1$$

- Ptos. equilibrio:

$$x - \alpha(1+\beta)x + \alpha\beta x = 1 \iff x - \alpha x = 1 \iff x = \frac{1}{1-\alpha} \quad \alpha \neq 1$$

- Pol. característico y parte homogénea:

$$\lambda^2 - \alpha(1+\beta)\lambda + \alpha\beta = 0 \iff \lambda = \frac{\alpha(1+\beta) \pm \sqrt{\alpha^2(1+\beta)^2 - 4\alpha\beta}}{2}$$

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1) $\alpha[\alpha(1+\beta)^2 - 4\beta] = 0 \iff \begin{cases} \alpha = 0 & \text{No, pues } \alpha, \beta > 0 \\ \alpha(1+\beta)^2 - 4\beta = 0 \iff \alpha = \frac{4\beta}{(1+\beta)^2} \Rightarrow \lambda = \frac{2\beta}{1+\beta} & \text{doble} \end{cases}$

$$x_n = (c_1 + c_2 n)\left(\frac{2\beta}{1+\beta}\right)^n + \frac{1}{1-\alpha}$$

Sin embargo, $\{x_n\} \to \frac{1}{1-\alpha} \Rightarrow \left|\frac{2\beta}{1+\beta}\right| = \frac{2\beta}{1+\beta} < 1 \iff 2\beta < 1+\beta \iff \beta < 1$

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2) $\alpha[\alpha(1+\beta)^2 - 4\beta] > 0 \iff \alpha > \frac{4\beta}{(1+\beta)^2} \Rightarrow$ 2 raíces reales

$\lambda_1, \lambda_2 > 0 \Rightarrow x_n = c_1\lambda_1^n + c_2\lambda_2^n + \frac{1}{1-\alpha}$

·) $\lambda_1 < 1 \Rightarrow \alpha(1+\beta) + \sqrt{\alpha^2(1+\beta)^2 - 4\alpha\beta} < 2 \iff$

$$\alpha^2(1+\beta)^2 - 4\alpha\beta < (2 - \alpha(1+\beta))^2 \iff \alpha > 1$$

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3) $\alpha[\alpha(1+\beta)^2 - 4\beta] < 0 \iff \alpha < \frac{4\beta}{(1+\beta)^2} \Rightarrow$

$$\lambda = \frac{\alpha(1+\beta) \pm \sqrt{4\alpha\beta - \alpha^2(1+\beta)^2}\, i}{2} \Rightarrow r^2 = \frac{\alpha^2(1+\beta)^2}{4} + \frac{4\alpha\beta - \alpha^2(1+\beta)^2}{4} =$$

$\alpha\beta \Rightarrow r = \sqrt{\alpha\beta}$

$$x_n = (\sqrt{\alpha\beta})^n [c_1\cos(n\omega) + c_2\,\mathrm{sen}(n\omega)] + \frac{1}{1-\alpha} \Rightarrow \alpha\beta < 1 \iff \alpha < \frac{1}{\beta}$$

$$\alpha < \min\left\{1/\beta, \frac{4\beta}{(1+\beta)^2}\right\}$$

③ Se considera el siguiente modelo de Samuelson:

(1) $y_n = C_n + I_n$

(2) $C_n = bI_{n-1}$ $0 < b < 1$

(3) $I_n = C_n - kC_{n-1} + G$ $k > 0$, $G =$ gasto público (dato fijo)

A) Sacar ec. recurrencia:

(3) $I_n \underset{(2)}{=} bI_{n-1} - kbI_{n-2} + G \Rightarrow G = I_{n+2} - bI_{n+1} + kbI_n$

B) Sacar lo que tienen que valer los parámetros.

- Ptos. equilibrio:

$$G = I - bI + kbI \Longleftrightarrow I = \frac{G}{1+(k-1)b}$$

- Parte homogénea:

$$p(\lambda) = \lambda^2 - b\lambda + kb = 0 \Longleftrightarrow \lambda = \frac{b \pm \sqrt{b^2 - 4kb}}{2}$$

1) $b(b-4k) = 0 \Longleftrightarrow \begin{cases} \cancel{b=0} \\ b = 4k \Longrightarrow \lambda = \frac{b}{2} = \frac{4k}{2} = 2k \end{cases}$

DOBLE

$$I_n = (c_1 + c_2 n)(2k)^n + \frac{G}{1+2k(k-1)}$$

$$\lim_{n\to+\infty} I_n = \frac{G}{1+2k(k-1)} \Longleftrightarrow 2k < 1 \Longleftrightarrow k < \frac{1}{2}$$

2) $b(b-4k)>0 \iff b>4k \implies \mathbb{R} \ni \lambda_1, \lambda_2 > 0$

$$x_n = c_1\lambda_1^n + c_2\lambda_2^n + \frac{G}{1+b(k-1)} \; *$$

Si $\lambda_1 \geq 1 \iff b \pm \sqrt{b^2-4kb} > 2 \iff 1+b(k-1) \leq 0$

Vemos que es el denominador de $*$ $\implies$ contradicción pues $1+b(k-1) \neq 0$ $\implies 0<\lambda_1, \lambda_2<1$

$$\lim_{n\to+\infty} x_n = \frac{G}{1+b(k-1)}$$

3) $b(b-4k)<0 \implies \frac{b}{4}<k<\frac{1}{b} \implies$ convergencia a $\frac{G}{1+b(k-1)}$