Ejercicio 28-03-23

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Problema Sea el sistema dinámico matricial:

$$\vec{x}_{n+1} = \begin{pmatrix} 2 & -1 & -1 \\ 2 & 3 & 2 \\ 1 & 1 & 2 \end{pmatrix} \vec{x}_n.$$

Si

$$\vec{x}_n = \begin{pmatrix} a_n \\ b_n \\ c_n \end{pmatrix},$$

escribe la ecuación lineal de tercer orden que verifica c_n.

$$\left.\begin{aligned} a_{n+1} &= 2a_n - b_n - c_n \\ b_{n+1} &= 2a_n + 3b_n + 2c_n \\ c_{n+1} &= a_n + b_n + 2c_n \end{aligned}\right\} \quad (1)$$

Sustituimos $b_n = c_{n+1} - a_n - 2c_n$ en el resto de ecuaciones de (1):

- $a_{n+1} = 2a_n - c_{n+1} + a_n + 2c_n - c_n = 3a_n + c_n - c_{n+1}$

- $b_{n+1} = c_{n+2} - a_{n+1} - 2c_{n+1} = 2a_n + 2c_n + 3c_{n+1} - 3a_n - 6c_n$

$$\Longleftrightarrow$$

$$a_{n+1} = -2a_n - 2c_n - 3c_{n+1} + 3a_n + 6c_n + c_{n+2} - 2c_{n+1} =$$
$$a_n + 4c_n - 5c_{n+1} + c_{n+2}$$

$$\left.\begin{aligned} a_{n+1} &= 3a_n + c_n - c_{n+1} \\ a_{n+1} &= a_n + 4c_n - 5c_{n+1} + c_{n+2} \end{aligned}\right\} (2)$$

$$3a_n - a_n + c_n - 4c_n - c_{n+1} + 5c_{n+1} - c_{n+2} = 0 \iff$$

$$2a_n - 3c_n + 4c_{n+1} - c_{n+2} = 0 \iff$$

$$a_n = \frac{3c_n - 4c_{n+1} + c_{n+2}}{2}$$

Sustituimos a_n en la 2ª ecuación de (2)

$$a_{n+1} = \frac{3c_{n+1} - 4c_{n+2} + c_{n+3}}{2} =$$

$$\frac{3c_n - 4c_{n+1} + c_{n+2}}{2} + 4c_n - 5c_{n+1} + c_{n+2} \iff$$

$$3c_{n+1} - 4c_{n+2} + c_{n+3} = 3c_n - 4c_{n+1} + c_{n+2} + 8c_n - 10c_{n+1} + 2c_{n+2} \iff$$

$$\boxed{c_{n+3} - 7c_{n+2} + 17c_{n+1} - 11c_n = 0}$$