Ejercicio 18-04-23

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Problema Dadas las siguientes matrices

$$a) \begin{pmatrix} 6 & 7 \\ -3 & -2 \end{pmatrix},$$

$$b) \begin{pmatrix} 2 & 1 & -1 \\ -1 & 1 & 2 \\ -1 & -2 & -1 \end{pmatrix},$$

$$c) \begin{pmatrix} 3 & 0 & 0 & 0 \\ 2 & 4 & -2 & 2 \\ 1 & -2 & -1 & 3 \\ 2 & 0 & 0 & -4 \end{pmatrix}.$$

Determina, el radio espectral, la norma matricial subordinada a la norma de la suma $|\cdot|_1$, y la norma matricial subordinada a la norma del máximo $|\cdot|_\infty$.

$$A = \begin{pmatrix} 6 & 7 \\ -3 & -2 \end{pmatrix}$$

$$P_A(\lambda) = \begin{vmatrix} 6-\lambda & 7 \\ -3 & -2-\lambda \end{vmatrix} = (\lambda-6)(\lambda+2)+21 =$$

$$\lambda^2+2\lambda-6\lambda-12+21 = \lambda^2-4\lambda+9=0 \iff \lambda = \begin{cases} 2+\sqrt{5}i \\ 2-\sqrt{5}i \end{cases}$$

$$\rho(A) = \text{máx}\{|\lambda| \,/\, \lambda\in\sigma(A)\} \,,\; \sigma(A) = \{2\pm\sqrt{5}\}$$

$$|\lambda_1| = |\lambda_2| = \sqrt{2^2+(\sqrt{5})^2} = 3 \implies \boxed{\rho(A)=3}$$

$$\|A\|_1 = \text{máx}_{i=1..K}\{|C_i|_1 \,/\, C_i \text{ es columna de } A\}$$

$$\left.\begin{array}{l} |C_1|_1 = 6+|-3| = 9 \\ |C_2|_1 = 7+|-2| = 9 \end{array}\right\} \implies \boxed{\|A\|_1 = 9}$$

$$\|A\|_\infty = \text{máx}_{i=1..K}\{|F_i| \,/\, F_i \text{ es fila de } A\}$$

$$\left.\begin{array}{l} |F_1|_1 = 6+7 = 13 \\ |F_2|_1 = |-3|+|-2| = 5 \end{array}\right\} \implies \boxed{\|A\|_\infty = 13}$$

$$B = \begin{pmatrix} 2 & 1 & -1 \\ -1 & 1 & 2 \\ -1 & -2 & -1 \end{pmatrix}$$

$$P_B(\lambda) = \begin{vmatrix} 2-\lambda & 1 & -1 \\ -1 & 1-\lambda & 2 \\ -1 & -2 & -1-\lambda \end{vmatrix} =$$

$$(\lambda-2)(1-\lambda)(1+\lambda) - 2 - 2 - (1-\lambda) + 4(2-\lambda) + (-1-\lambda)$$

$$= (\lambda - 2)(1-\lambda^2) - 4 - 1 + \lambda + 8 - 4\lambda - 1 - \lambda =$$

$$\cancel{\lambda} - \lambda^3 - 2 + 2\lambda^2 - 4 - 1 + \lambda + 8 - 4\lambda - 1 - \cancel{\lambda} =$$

$$-\lambda^3 + 2\lambda^2 - 3\lambda = 0 \Longleftrightarrow \lambda = \begin{cases} 1+\sqrt{2}\,i \\ 1-\sqrt{2}\,i \\ 0 \end{cases}$$

$$|\lambda_1| = |\lambda_2| = \sqrt{1^2 + (\sqrt{2})^2} = \sqrt{3} \quad |\lambda_3| = 0 \Longrightarrow \boxed{\rho(B) = \sqrt{3}}$$

$$\boxed{\|B\|_1 = \max\{(2+1+1), (1+1+2), (1+2+1)\} = 4}$$

$$\boxed{\|B\|_\infty = \max\{(2+1+1), (1+1+2), (1+2+1)\} = 4}$$

$$C = \begin{pmatrix} 3 & 0 & 0 & 0 \\ 2 & 4 & -2 & 2 \\ 1 & -2 & -1 & 3 \\ 2 & 0 & 0 & -4 \end{pmatrix}$$

$$P_C(\lambda) = \begin{vmatrix} 3-\lambda & 0 & 0 & 0 \\ 2 & 4-\lambda & -2 & 2 \\ 1 & -2 & -1-\lambda & 3 \\ 2 & 0 & 0 & -4-\lambda \end{vmatrix} =$$

$$(3-\lambda)\begin{vmatrix} 4-\lambda & -2 & 2 \\ -2 & -1-\lambda & 3 \\ 0 & 0 & -4-\lambda \end{vmatrix} =$$

$$(\lambda-3)(4+\lambda)\begin{vmatrix} 4-\lambda & -2 \\ -2 & -1-\lambda \end{vmatrix} =$$

$$(\lambda-3)(4+\lambda)\left[(\lambda-4)(1+\lambda)-4\right] =$$

$$(4\lambda+\lambda^2-12-3\lambda)(\lambda+\lambda^2-4-4\lambda-4) =$$

$$(\lambda^2+\lambda-12)(\lambda^2-3\lambda-8) =$$

$$\lambda^4 - 3\lambda^3 - 8\lambda^2 + \lambda^3 - 3\lambda^2 - 8\lambda - 12\lambda^2 + 36\lambda + 96 =$$

$$\lambda^4 - 2\lambda^3 - 23\lambda^2 + 28\lambda + 96 = 0 \iff \lambda = \begin{cases} 3 \\ -4 \\ \dfrac{3+\sqrt{41}}{2} \\ \dfrac{3-\sqrt{41}}{2} \end{cases}$$

$$\begin{array}{r|rrrrr}
 & 1 & -2 & -23 & 28 & 96 \\
3 & & 3 & 3 & -60 & -96 \\
\hline
-4 & 1 & 1 & -20 & -32 & \boxed{0} \\
 & & -4 & 12 & 32 & \\
\hline
 & 1 & -3 & -8 & \boxed{0} &
\end{array}$$

$$\lambda^2 - 3\lambda - 8 = 0 \iff \lambda = \begin{cases} \dfrac{3+\sqrt{41}}{2} \\ \dfrac{3-\sqrt{41}}{2} \end{cases}$$

$$\rho(C) = \text{máx}\left\{3, 4, \frac{3+\sqrt{41}}{2}, \left|\frac{3-\sqrt{41}}{2}\right|\right\} = \frac{3+\sqrt{41}}{2}$$

$$\|C\|_1 = \text{máx}\{(3+2+1+2), (0+4+2+0), (0+2+1+0), (0+2+3+4)\} = 9$$

$$\|C\|_\infty = \text{máx}\{(3+0+0+0), (2+4+2+2), (1+2+1+3), (2+0+0+4)\} = 10$$