My formalization project

Howard Straubing, Soleil Repple, Ayden Lamparski, Nathan Hartdog

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0.1 Green's preorders (from `MyProject/GreensRelations.lean`)

Definition 1 (Green's R-preorder)**.** Let M be a monoid and $x, y \in M$. We define $x \leq_R y$ iff there exists $z \in M$ with $x \cdot z = y$. This relation is reflexive and transitive.

Lemma 2 (Reflexivity of $\leq_R$)**.** *For every $x \in M$, $x \leq_R x$.*

Lemma 3 (Transitivity of $\leq_R$)**.** *If $x \leq_R y$ and $y \leq_R z$, then $x \leq_R z$.*

0.2 Duality via `MulOpposite` (from `MyProject/MulOpposite.lean`)

Lemma 4 (Opposite monoid correspondence for R)**.** *For $x, y \in M$,*

$$RRel(\mathrm{op}\, x, \mathrm{op}\, y) \iff RRel(x, y).$$

Equivalently, the R-preorder on M corresponds to the R-preorder on MulOpposite M *via* op.