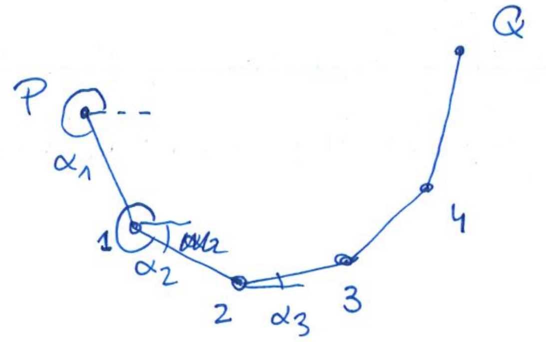


$N = 5$

$P = [p_x, p_y]$

$Q = [q_x, q_y]$

$$\left.\begin{aligned} x_1 &= p_x + L\cos\alpha_1 \\ y_1 &= p_y + L\sin\alpha_1 \end{aligned}\right\} \qquad \left.\begin{aligned} x_2 &= x_1 + L\cos\alpha_2 \\ y_2 &= y_1 + L\sin\alpha_2 \end{aligned}\right\}$$

$$\begin{cases} x_i = p_x + \sum_{j=1}^{i} L\cos\alpha_j \\ y_i = p_y + \sum_{j=1}^{i} L\sin\alpha_j \end{cases}$$

In the last link, we close the kinematic chain:

$$q_x = x_5 = p_x + \sum_{j=1}^{N} L\cos\alpha_j$$

$$q_y = y_5 = p_y + \sum_{j=1}^{N} L\sin\alpha_j$$

$$\boxed{\begin{aligned} q_x - p_x - \sum_{j=1}^{N} L\cos\alpha_j &= 0 \\ q_y - p_y - \sum_{j=1}^{N} L\sin\alpha_j &= 0 \end{aligned}}$$

The kinematic chain must comply these equations

Besides complying with the compatibility equations, the goal is to minimize 2 potentials: gravitatory and torsional

For every link:

$$V_i = g y_i = g \left[ P_y + \sum_{j=1}^{i} L \sin\alpha_j \right]$$

For the ~~pot~~ gravitatory potential, constant term can be ignored

$$V_1 = Lg \sin\alpha_1$$
$$V_2 = Lg \sin\alpha_1 + Lg \sin\alpha_2$$
$$V_3 = Lg \sin\alpha_1 + Lg \sin\alpha_2 + Lg \sin\alpha_3$$
$$V_4 = Lg \sin\alpha_1 + Lg \sin\alpha_2 + Lg \sin\alpha_3 + Lg \sin\alpha_4$$

$$\boxed{V = \sum_{j=1}^{N-1} (N-j) Lg \sin\alpha_j}$$

For torsional potential, the standard form is $K = \frac{1}{2} k (\theta_i - \theta_j)^2$

$$K_1 = \frac{1}{2} k (\alpha_2 - \alpha_1)^2$$

$$\boxed{K = \sum_{j=1}^{N-1} \frac{1}{2} k (\alpha_{j+1} - \alpha_j)^2}$$

$$\boxed{f(\alpha_i) = V + K}$$

This last expression is not applicable to 3D space, and can return false values if, for example $\alpha_1 = 0$ and $\alpha_2 = 2\pi$, so it should be replace ~~d~~ with the absolute value of the ~~vector~~ cross product or a similar function