1 Methods of Proof

- A **direct proof** proceeds by establishing a chain of implications $P \Rightarrow P_1 \Rightarrow \cdots \Rightarrow P_n \Rightarrow Q$ leading directly from P to Q.
- A **proof by contradiction** assumes that the hypothesis P is true, just as in a direct proof, but then supposes that the conclusion Q is false.
- Closely related to the method of proof by contradiction is **proof by contrapositive**: The negative of a statement P is the statement "it is not the case that P", abbreviated symbolically as $\neg P$ and pronounced "not P".

1.1 Mathematical Induction

First Principle of Mathematical Induction

Let $S(n)$ be a statement about the positive integer n. If

1. the **basis step** holds: $S(1)$ is true, and
2. the **induction step** holds: for all $k \geq 1$, the truth of **induction hypothesis** $S(k)$ implies the truth of $S(k+1)$,

then $S(n)$ is true for all $n \geq 1$.

Second Principle of Math. Ind. (Strong Induction)

Let $S(n)$ be a statement about the positive integer n. If

1. the **basis step** holds: $S(1)$ is true, and
2. the **induction step** holds: the truth of all **induction hypotheses** $S(1), S(2), \ldots, S(k)$ implies the truth of $S(k+1)$,

then $S(n)$ is true for all $n \geq 1$.

2 Combinatorics

2.1 Subsection 1

Definition — Theorem

Description: Text

$$\mathbf{v} \cdot \mathbf{w} = 0 \iff \alpha = \frac{\pi}{2} \iff \mathbf{v} \perp \mathbf{w}$$

3 Asymptotics and Big-O Notation

3.1 Subsection 1

Definition — Theorem

Description: Text

$$\mathbf{v} \cdot \mathbf{w} = 0 \iff \alpha = \frac{\pi}{2} \iff \mathbf{v} \perp \mathbf{w}$$

4 Graphs

4.1 Subsection 1

Definition

Isomorphic:

Definition

Planar graph (PG): all crossings avoidable, i.e. representable in 2D

Definition

Maximum planar graph (MPG): and triangular

Euler's Formula

E

Definition

Complete graph: all vertices are connected

Definition

Complete bipartite graph: all vertices in each of two parts are connected

Definition

Circuit: a path that begins and ends at the same vertex, i.e. a closed loop

Definition

Cycle: a circuit that doesn't repeat any vertices

Definition

Hamiltonian Cycle: a cycle that visits every vertex once

Many (but not all) MPGs have a HC.

Definition

Eulerian Cycle: a cycle that uses every edge once

Definition — Theorem

Description: Text

$$\mathbf{v} \cdot \mathbf{w} = 0 \iff \alpha = \frac{\pi}{2} \iff \mathbf{v} \perp \mathbf{w}$$

5 Connectivity, Trees, Cycles

5.1 Subsection 1

Definition — Theorem

Description: Text

$$\mathbf{v} \cdot \mathbf{w} = 0 \iff \alpha = \frac{\pi}{2} \iff \mathbf{v} \perp \mathbf{w}$$

6 Eulerian and Hamiltonian Cycles

6.1 Subsection 1

Definition — Theorem

Description: Text

$$\mathbf{v} \cdot \mathbf{w} = 0 \iff \alpha = \frac{\pi}{2} \iff \mathbf{v} \perp \mathbf{w}$$

7 Spanning Trees

7.1 Subsection 1

Definition — Theorem

Description: Text

$$\mathbf{v} \cdot \mathbf{w} = 0 \iff \alpha = \frac{\pi}{2} \iff \mathbf{v} \perp \mathbf{w}$$

8 Maximum Flow and Minimum Cut

8.1 Subsection 1

Definition — Theorem

Description: Text

$$\mathbf{v} \cdot \mathbf{w} = 0 \iff \alpha = \frac{\pi}{2} \iff \mathbf{v} \perp \mathbf{w}$$

9 Matchings in Bipartite Graphs

9.1 Subsection 1

Definition — Theorem

Description: Text

$$\mathbf{v} \cdot \mathbf{w} = 0 \iff \alpha = \frac{\pi}{2} \iff \mathbf{v} \perp \mathbf{w}$$

References

[1] Poole, David. *Linear Algebra: A Modern Introduction*. 2012.

[2] Shanghai Jiao Tong University. *Discrete Mathematics*. 2024. URL: https://www.coursera.org/learn/discrete-mathematics.