Control Theory and Practice
Advanced Course

Computer Exercise:
DECOUPLING &
GLOVER-MCFARLANE ROBUST
LOOP-SHAPING

Anders Hansson, Alf Isaksson och Magnus Jansson, January 1999

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1 Introduction

The purpose of this computer exercise is to investigate different procedures for multivariable control design. The process is the same as in the computer exercise on multivariable systems, *i.e.* the four-tank process. First, we will investigate static and dynamic decoupling. The control performance will be evaluated. Then, the design will be robustified using a method proposed by Glover and McFarlane.

Preparations: Chapters 8.3 and 10.5 in Glad, T. and Ljung, L.: "Control theory—multivariable and nonlinear methods".

Presentation: All problems in this exercise should be solved, but only the tasks on the report form should be handed in. The report form and the date when it should be handed in can be found on the course website. The exercise should be performed in pairs of students.

2 Theoretical overview

This section provides the theory that you will need to solve the problems. It is based on the course book.

2.1 Decentralized control and decoupling

A fundamental problem in multivariable control is that the input and output signals are coupled. This means that if one input is changed then, in general, all outputs are affected. A measure of the strength of the coupling in a multivariable system $(G(s))$ is given by the *Relative Gain Array*, RGA of the transfer matrix G, defined as:

$$\text{RGA}(G(i\omega)) := G(i\omega). * \left[G^{-1}(i\omega) \right]^{T},$$

where ".∗" denotes element wise multiplication. In decentralized control the RGA can help us to determine which inputs and outputs that are suitable to pair. Two rules of thumb:

1. Find a pairing such that the diagonal elements in $\text{RGA}(G(i\omega_c))$ are as close to 1 as possible, where ω_c is the intended cross-over frequency.

2. Avoid pairings that correspond to negative elements in $\text{RGA}(G(0))$.

If it is not possible to find a suitable pairing of inputs and outputs, one can try to make a better system using *decoupling*. Consider the following example: one output is a function of the difference of two inputs, while another output depends on the sum of these two inputs. In this case, it is suitable to introduce two new inputs which denote the sum and the difference respectively of the two original inputs. This is the main idea in decoupling. Generally, decoupling is performed in the following way. Introduce

the new variables $\tilde{y} = W_2 y$ and $\tilde{u} = W_1^{-1} u$, so that the transfer function from $\tilde{u}$ to $\tilde{y}$ becomes

$$\tilde{G}(s) := W_2(s)G(s)W_1(s),$$

where we try to design $\tilde{G}$ as diagonal as possible. Typically, we let $W_2 = I$. The idea is to find a $\tilde{G}$ with more suitable input and output pairings than the original system G. In general, a completely diagonal $\tilde{G}$ is not realizable. However, we can try to design $\tilde{G}$ to be as decoupled as possible in a certain frequency range with the dynamical transfer matrices $W_1(s)$ and $W_2(s)$. Alternatively, we can decouple the system for one frequency, *e.g.* $\omega = 0$ or $\omega = \omega_c$, with constant matrices W_1 and W_2.

2.2 Glover-McFarlane robust loop-shaping

The control can be robustified using the method proposed by Glover and McFarlane. It is described in Chapter 10.5 in the course book. A summary of the design procedure, step by step, is given below.

1. Start by pairing the input and output signals in such a way that the system becomes as diagonal as possible. A useful mathematical tool is RGA (relative gain array).

2. Design an initial controller using pre-compensation W_1 and post-compensation W_2. To start with, we can typically choose $W_2 = I$ and $W_1 = W_{dc}F_{diag}$ where W_{dc} decouples the system at a suitable frequency (*i.e.* $\omega = 0$ or the intended ω_c) and $F_{diag}(s)$ is a diagonal lead-lag controller designed using classical loop-shaping (*c.f.* computer exercises 1 and 2).

3. Robust stabilization. Design the Glover-McFarlane controller F_r for the system $G_s = W_2 G W_1$. If $\gamma > 4$, redesign W_1.

4. The final controller is $F(s) = W_1 F_r W_2$.

3 Exercises

In this computer exercise the four-tank process will be investigated. Please recall the MATLAB functions introduced in the exercise on multivariable systems.

NB: numerical problems in MATLAB can occur if you work with systems as transfer functions (`tf` objects in MATLAB). It is therefore important that you instead work with state space representations (`ss`). When performing multiplication and division of systems, it is highly recommended to use the function `minreal`, which creates an equivalent system where all canceling pole-zero pairs or non minimal state dynamics are eliminated. Numerical properties can depend on the MATLAB version that you use.

3.1 Static decoupling

Static decoupling is obtained by choosing $W_2(s) = I$ and $W_1(s) = G^{-1}(0)$. This implies that $\tilde{G}(s) = G(s)G^{-1}(0)$ is decoupled at $s = 0$.

The problems below should be solved both for the minimum phase and non-minimum phase case. It is suitable to start with the former.

Exercise 3.1.1. Calculate the static decoupling for the system and plot the Bode diagrams of $\tilde{G}(s)$ for verification.

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Exercise 3.1.2. Design a diagonal controller $\tilde{F}(s)$ for $\tilde{G}(s)$. Design the controllers $\tilde{f}_i(s)$ as PI controllers. The intended phase margin is $\varphi_m = \pi/3$. The intended crossover frequency ω_c is 0.1 rad/s for the minimum phase case and 0.02 rad/s for the non-minimum phase case.

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The controller is now given by

$$F(s) = G^{-1}(0)\tilde{F}(s)$$

Exercise 3.1.3. Calculate the singular values of the sensitivity function

$$S(s) = (I + G(s)F(s))^{-1}$$

and the complementary sensitivity function

$$T(s) = (I + G(s)F(s))^{-1}G(s)F(s)$$

Is the design good with respect to sensitivity and robustness?

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Exercise 3.1.4. Simulate the closed-loop system in Simulink by typing `closedloop`. A Simulink window will appear where the block diagram is shown. Make sure that the variables `F` and `G` in the MATLAB work-space contain the controller and the process respectively. Go to the Simulation meny and click Start. On the screen the unit step responses from the references to the outputs $y_1(t)$ (at $t = 100$) and $y_2(t)$ (at $t = 500$) are plotted together with the inputs. The time instant of the steps can be modified by clicking on the step blocks and changing the Step time. The total simulation time can be modified by changing the Stop time in the menu Simulation/Parameters. Simulation

data is saved in the variables `uout` and `yout` in the MATLAB workspace. Is the control good? Are the outputs coupled?

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Now perform the exercises above for the non-minimum phase case.

Exercise 3.1.5. Describe the most important differences between the two cases.

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3.2 Dynamical decoupling

Dynamical decoupling can be obtained for example by choosing $W_2(s) = I$ and $W_1(s)$ in such a way that $\tilde{G}(s) = G(s)W_1(s)$ is a diagonal matrix. The conditions for $\tilde{G}(s)$ to be diagonal are the following:

$$g_{11}(s)w_{12}(s) + g_{12}(s)w_{22}(s) = 0$$
$$g_{21}(s)w_{11}(s) + g_{22}(s)w_{21}(s) = 0$$

where $w_{ij}(s)$ denote the elements of the matrix $W_1(s)$. Since there are four unknowns and and two equations we have additional degrees of freedom. A suitable procedure is to let the diagonal elements of $W(s)$ be equal to one if the RGA of $G(s)$ indicates the pairings (u_1, y_1) and (u_2, y_2). Accordingly, for other pairings it is suitable to set the other two elements equal to one. For the case $w_{11}(s) = w_{22}(s) = 1$, we have

$$w_{12}(s) = -g_{12}(s)/g_{11}(s)$$
$$w_{21}(s) = -g_{21}(s)/g_{22}(s)$$

Divisions with `ss` object is not possible in MATLAB. After the divisions have been performed, we can return to state space descriptions using the function `ss`. (Notice that analytically, $\tilde{G}(s)$ is diagonal. However, numerically we can have off diagonal elements of size 10^{-16}, which can cause problems if we work with `tf` objects in MATLAB).

If, by some reason, the static gain of $\tilde{G}(s)$ happens to be negative, this can be modified by changing signs of $W_1(s)$. If $W_1(s)$ becomes non-proper (for example if it contains derivations), we can still realize the dynamical decoupling for frequencies up to approximately 10 times the intended cross-over frequency ω_c using the following modification

$$W_1(s) \leftrightarrow W_1(s)\frac{10\omega_c}{s + 10\omega_c}$$

The problems below should be solved both for the minimum phase and non-minimum phase case. It is suitable to start with the former.

Exercise 3.2.1. Calculate a dynamical decoupling $W(s)$ for the system and plot the Bode diagrams of $\tilde{G}(s)$ for verification.

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Exercise 3.2.2. Design a controller $\tilde{F}(s)$ for $\tilde{G}(s)$. Design the controllers $\tilde{f}_i(s)$ as PI controllers so that we have phase margin $\varphi_m = \pi/3$. The intended cross-over frequency ω_c is 0.1 rad/s for the minimum phase case and 0.02 rad/s for the non-minimum phase case.

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The controller is now given by

$$F(s) = W_1(s)\tilde{F}(s)$$

Exercise 3.2.3. Calculate the singular values of the sensitivity function

$$S(s) = (I + G(s)F(s))^{-1}$$

and the complementary sensitivity function

$$T(s) = (I + G(s)F(s))^{-1}G(s)F(s)$$

Is the design good with respect to sensitivity and robustness?

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Exercise 3.2.4. Simulate the closed-loop system in Simulink. Is the control good? Are the outputs coupled?

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Now solve the above problems for the non-minimum phase case.

Exercise 3.2.5. Describe the most important differences between the two cases.

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Exercise 3.2.6. Which type of decoupling is the best for the minimum phase system and the non-minimum phase system respectively? What are the advantages and disadvantages of the static and dynamical decoupled designs?

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3.3 Glover-McFarlane robust loop-shaping

In the above exercises we combined static and dynamical decoupling with PI control. In this exercise we will continue with the design that showed to be best for each of the two cases. Alternatively, we could start all over as described above in 2.2, but we will not do that. The reason is that the Glover-McFarlane method works well for reasonably well-tuned nominal controllers.

Therefore assume that we have a nominal loop gain

$$L_0(s) = G(s)W_1(s)\tilde{F}(s)$$

obtained in the exercises above. The Glover-McFarlane method adds a link $F_r(s)$ to this loop gain so that the final controller becomes

$$F(s) = W_1(s)\tilde{F}(s)F_r(s)$$

In MATLAB this link is calculated with the function

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[Fr,gam] = rloop(L0,alpha)
```

A suitable choice for `alpha` is 1.1.

The problems below should be solved both for the minimum phase and non-minimum phase case. It is suitable to start with the former.

Exercise 3.3.1. Calculate L_0 corresponding to the best previous design procedure and plot the Bode diagrams to verify that L_0 has the intended cross-over frequency ω_c and that it is reasonably decoupled at ω_c. For the minimum phase case, ω_c is 0.1 rad/s and for the non-minimum phase case it is 0.02 rad/s.

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Exercise 3.3.2. Calculate the Glover-McFarlane controller for L_0. Are you satisfied with the γ that you have obtained?

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The controller is now given by

$$F(s) = W_1(s)\tilde{F}(s)F_r(s)$$

Exercise 3.3.3. Calculate the singular values of the sensitivity function

$$S(s) = (I + G(s)F(s))^{-1}$$

and the complementary sensitivity function

$$T(s) = (I + G(s)F(s))^{-1}G(s)F(s)$$

Describe the differences between the robust design and the nominal design in terms of the sensitivity functions. Is the robust design better with respect to sensitivity and robustness?

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Exercise 3.3.4. Simulate the closed-loop system in Simulink. Compare with the result that you obtained simulating the nominal design. What are the differences and similarities?

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Now solve the above problems for the non-minimum phase case.

Exercise 3.3.5. Describe the most important differences between the two cases.

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