вар.5 КР N1

а) $6xdx - 6ydy = 2x^2ydx - 3xy^2dx$

$3x(2+y^2)dx = 2y(3+x^2)dy$

$\frac{3x}{x^2+3}dx = \frac{2y}{y^2+2}dy$

$\frac{3}{2}\frac{d(x^2+3)}{x^2+3} = \frac{d(y^2+2)}{y^2+2}$

$\frac{3}{2}\ln(x^2+3) = \ln|C\cdot(y^2+2)|$

$(x^2+3)^{\frac{3}{2}} = C\cdot(y^2+2)$

общий интеграл

б) $2y' = \frac{y^2}{x^2} + 6\frac{y}{x} + 3$

$2dy = (\frac{y^2}{x^2} + 6\frac{y}{x} + 3)dx$

$t = \frac{y}{x}$ $y = tx$ $y' = t'x + t$

$dy = xdt + tdx$

$2xdt + 2tdx = (t^2 + 6t + 3)dx$

$\frac{dt}{t^2+4t+3} = \frac{1}{2}\frac{dx}{x}$

$\frac{dt}{(t+3)(t+1)} = \frac{1}{2}\frac{dx}{x}$

$\frac{A}{t+3} + \frac{B}{t+1} = \frac{At + A + Bt + 3B}{(t+3)(t+1)}$

$A + B = 0$

$A + 3B = 1$

$A = -B$

$B = \frac{1}{2}$ $A = -\frac{1}{2}$

$$\frac{dt}{t+3} + \frac{dt}{t+1} = \frac{dx}{x}$$

$$\ln\left|\frac{t+1}{t+3}\right| = \ln|Cx|$$

$$\frac{\frac{y}{x}+1}{\frac{y}{x}+3} = Cx$$

$$Cy + 3Cx = \frac{y}{x} + 1$$

общий интеграл

б) $y' - \frac{y}{x+2} = x^2 + 2x \qquad y(-1) = \frac{3}{2}$

$y = u \cdot v \qquad y' = u'v + uv'$

$u'v + u\left(v' - \frac{v}{x+2}\right) = x^2 + 2x$

1) $\frac{dv}{v} = \frac{dx}{x+2}$

$v = x+2$

2) $\frac{du}{dx}(x+2) = x(x+2)$

$du = x\,dx$

$u = \frac{x^2}{2} + C$

$y = \frac{x^3}{2} + x^2 + Cx + 2C$

$\frac{3}{2} = -\frac{1}{2} + 1 - C + 2C$

$1 = C$

$y = \frac{x^3}{2} + x^2 + x + 2$

ЧАСТНОЕ РЕШ

2) $xy' - y = -y^2(\ln x + 2)\ln x \mid :y^2 \quad y(1) = 1$

$x\frac{y'}{y^2} - \frac{1}{y} = -(\ln x + 2)\ln x$

$z = \frac{1}{y} \quad z' = -\frac{1}{y^2}\cdot y'$

$-x\cdot z' + z = -(\ln x + 2)\cdot \ln x \mid :(-x)$

$z' + \frac{z}{x} = \frac{(\ln x + 2)\ln x}{x}$

$z = u\cdot v \quad z' = u'v + u\cdot v'$

$u'v + u\left(v' + \frac{v}{x}\right) = \frac{(\ln x + 2)\cdot \ln x}{x}$

1) $\frac{dv}{v} = -\frac{dx}{x}$

$v = \frac{1}{x}$

2) $\frac{du}{dx}\cdot\frac{1}{x} = \frac{(\ln^2 x + 2)\cdot \ln x}{x}$

$du = (\ln^2 x + 2\ln x)\,dx$

$\ln^2 x = p \qquad \frac{2\ln x}{x} = dp$

$dx = dq \qquad x = q$

$u = x\cdot\ln^2 x - 2\int \ln x\,dx + 2\int \ln x\,dx + C$

$u = x\cdot\ln^2 x + C$

$y = \ln^2 x + \frac{C}{x}$

$1 = C$

$y = \ln^2 x + \frac{1}{e}$

ЧАСТНОЕ РЕШ.

g) (C

Sin

=

=

g) $(\cos 2y \cdot \cos^2 y - x)y' = \sin y \cdot \cos y$

$y|_{x=\frac{1}{4}} = \frac{\pi}{3}$

$\sin y \cdot \cos y \, dx + (x - \cos 2y \cdot \cos^2 y)\, dy = 0$

P $\qquad$ Q

$\frac{\partial P}{\partial y} = \cos^2 y - \sin^2 y = \cos 2y$

$\frac{\partial Q}{\partial x} = 1$

$\frac{1}{P}\left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) = \frac{2\sin^2 x}{\cos y \cdot \sin y} = 2\frac{\sin y}{\cos y}$

$= F(y)$

$\mu(y) = e^{\int 2\operatorname{tg} y \, dy} = e^{-2\ln|\cos y|} = \frac{1}{\cos^2 y}$

$\frac{\partial P}{\partial y} = \frac{\cos^2 y + \sin^2 y}{\cos^2 y} = \frac{1}{\cos^2 y}$

$\frac{\partial Q}{\partial x} = \frac{\cos^2 x + (x - \cos 2y \cdot \cos^2 y) \cdot 2\cos x \cdot \sin x}{\cos^4 x}$

$= \cos^3 x + x \sin 2x -$

$$\frac{\sin y}{\cos y}\,dx + \left(\frac{x}{\cos^2 y} - \cos 2y\right) dy = 0$$

$$\frac{\partial P}{\partial y} = \frac{\cos^2 y + \sin^2 y}{\cos^2 x} = \frac{1}{\cos^2 y}$$

$$\frac{\partial Q}{\partial x} = \frac{1}{\cos^2 y}$$

$$\frac{\partial u}{\partial x} = \frac{\sin y}{\cos y}$$

$$\int du = \frac{\sin y}{\cos y} \int dx$$

$$u = \frac{\sin y}{\cos y} x + C(y)$$

$$\frac{\partial}{\partial y}\left(\frac{\sin y}{\cos y} x + C(y)\right) = \frac{x}{\cos^2 y} - \cos 2y$$

$$\frac{x}{\cos^2 y} + C'(y) = \frac{x}{\cos^2 y} - \cos 2y$$

$$C'(y) = -\cos 2y$$

$$C(y) = -\int \cos 2y\, dy$$

$$C(y) = -\frac{1}{2}\sin 2y + C_1$$

$$u = \operatorname{tg} y \cdot x - \frac{1}{2}\sin 2y + C_1 = C_2 - C_1 = C$$

$$\operatorname{tg}\frac{\pi}{3} \cdot \frac{1}{4} - \frac{1}{2}\sin\frac{2\pi}{3} = C$$

$$0 = \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = C$$

$tg^2 y \cdot x - \frac{1}{2}\sin 2y = 0$

ЧАСТНЫЙ ИНТЕГРАЛ

е) $\left(y^2 + \frac{y}{\cos^2 x}\right)dx + (2xy + tg\,x)dy = 0$

$\frac{\partial P}{\partial y} = 2y + \frac{1}{\cos^2 x}$

$\frac{\partial Q}{\partial x} = 2y + \frac{1}{\cos^2 x}$

$\frac{\partial u}{\partial x} = y^2 + \frac{y}{\cos^2 x}$

$du = \int y^2 dx + y\int \frac{1}{\cos^2 x}dx$

$u = y^2 x + y \cdot tg\,x + C(y)$

$\frac{\partial}{\partial y}(y^2 x + y\,tg\,x + C(y)) = 2xy + tg\,x$

$2yx + tg\,x + C'(y) = 2xy + tg\,x$

$C(y) = C_1$

$u = y^2 x + y \cdot tg\,x = C_2 - C_1 = C$

№ 2

а) $tg\,x \cdot y'' - y' + \frac{1}{\sin x} = 0$

$p = y' \qquad p' = y''$

$tg\,x \cdot p' - p = -\frac{1}{\sin x} \quad | : tg\,x$

$p' - p \cdot ctg\,x = -\frac{\cos x}{\sin^2 x}$

$p = u \cdot v \quad p' = u'v + u \cdot v'$

$u'v + u \cdot v' - v \cdot ctg x) = -\frac{\cos x}{\sin^2 x}$

1) $\frac{dv}{v} = \frac{\cos x}{\sin x} dx$

$v = \sin x$

2) $\frac{du}{dx} \sin x = -\frac{\cos x}{\sin^2 x}$

$du = -\frac{\cos x}{\sin^3 x} dx$

$u = \frac{1}{2\sin^2 x} + C_1$

$p = \frac{1}{2\sin x} + C_1 \sin x$

$y' = \frac{1}{2} \frac{1}{\sin x} + C_1 \cdot \sin x$

$\int dy = \frac{1}{2} \int \frac{1}{\sin x} \cdot dx + \int C_1 \cdot \sin x \, dx$

$p = \frac{1}{\sin x} \quad dp = -\frac{1}{\sin^2 x}$

$dq = dx \quad q = x$

$y = \frac{1}{2} \ln \left| tg \frac{x}{2} \right| - C_1 \cos x + C_2$

общее рсш

б) $y'' = 32 \sin^2 y \cdot \cos y + 1$ $\quad y(1) = \frac{\pi}{2}$

$y'(1) = 4$

$p(y) = y'$ $\quad y'' = p_y p'_y$

$p \cdot p' = 32 \sin^3 y \cdot \cos y + 1$

$p \cdot dp = (32 \sin^3 y \cdot \cos y + 1)\, dy$

$\frac{p^2}{2} = 32 \int \sin^3 y \cdot \cos y\, dy + \int dy$

$\frac{p^2}{2} = 32 \int \sin^3 y\, d(\sin y) + y + C_1$

$\frac{p^2}{2} = 32 \cdot \frac{\sin^4 y}{4} + y + C_1$

$p^2 = 16 \cdot \sin^4 y + 2y + C_1$

$(y')^2 = 16 \cdot \sin^4 y + 2y + C_1$

$y' = \pm \sqrt{16 \cdot \sin^4 y + 2y + C_1}$

$\pm \frac{dy}{\sqrt{16 \sin^4 y + 2y + C_1}} = dx$?