$(x^2-\sin^2 y)dx + x\sin 2y\, dy = 0$

д.з. 4

20.09.23

№4155

$y'' = x + \sin x$

$y' = \int (x+\sin x)dx$

$y' = \frac{x^2}{2} - \cos x + C_1$

$y = \frac{x^3}{6} - \sin x + C_1 x + C_2$

№4156

$y'' = \operatorname{arctg} x$

$y' = \int \operatorname{arctg} x\, dx$

$u = \operatorname{arctg} x$ $\quad du = \frac{1}{1+x^2}\, dx$

$dv = dx$ $\quad v = x$

$y' = x\cdot \operatorname{arctg} x - \frac{1}{2}\ln(x^2+1) + C_1$

$y = \int (x \operatorname{arctg} x\, dx - \frac{1}{2}\int \ln(x^2+1)dx + \int C_1)dx$

$u = \operatorname{arctg} x$ $\quad du = \frac{dx}{1+x^2}$

$dv = x\,dx$ $\quad v = \frac{x^2}{2}$

$y = \frac{x^2}{2}\cdot \operatorname{arctg} x - \frac{1}{2}\left(\int 1\,dx - \int \frac{1}{1+x^2}dx\right) - \frac{1}{2}\int \ln(x^2+1)dx + \int C_1 dx$

$y = \frac{x^2}{2}\operatorname{arctg} x - \frac{1}{2}x + \frac{1}{2}\operatorname{arctg} x - \frac{1}{2}\int \ln(x^2+1)dx + C_1 x + C_2$

$u = \ln(x^2+1)$ $\quad du = \frac{1}{x^2+1}$

$dv = dx$ $\quad v = x$

$y = \frac{1}{2}\operatorname{arctg} x\cdot(x^2+1) - \frac{1}{2}x - \frac{1}{2}x\cdot \ln(x^2+1) + \frac{1}{4}\cdot\ln(x^2+1) + C_1 x + C_2$

№ 4157

$y'' = \ln x$

$y' = \int \ln x \, dx$

$u = \ln x \quad du = \frac{dx}{x}$

$dv = dx \quad v = x$

$y' = x \cdot \ln x - x + C_1^*$

$y = \int x \cdot \ln x \, dx - \int x \, dx + \int C_1^* dx$

$u = \ln x \quad du = \frac{dx}{x}$

$dv = x\,dx \quad v = \frac{x^2}{2}$

$y = \frac{x^2}{2} \cdot \ln x - \frac{1}{4} x^2 - \frac{x^2}{2} + C_1^* x + C_2^* \quad | \cdot 4$

$y = x^2 (2\ln x - 3) + C_1 x + C_2$

№ 4158

$x y'' = y'$

$y' = p \quad y'' = p'$

$x p' = p$

$x \frac{dp}{dx} = p$

$\frac{dp}{p} = \frac{dx}{x}$

$p = C_1 x$

$y' = C_1 x$

$y'' = C_1 \frac{x^2}{2} + C_2$

$y'' =$

p'

p

1)

$+ C_1^x \mid \cdot 4$

N4159

$y'' = y' + x$ $\qquad y' = p \quad y'' = p'$

$p' = p + x$

$p' - p = x$

$p = u \cdot v \qquad p' = u'v + uv'$

$u'v + u(v' - v) = x$

1) $\frac{dv}{v} = dx$

$v = e^x$

2) $\frac{du}{dx} \cdot e^x = x$

$du = x \cdot e^{-x} \cdot dx$

$u = x \qquad du = dx$

$dv = e^{-x} dx \qquad v = -e^{-x}$

$u = -xe^{-x} + \int e^{-x} dx$

$u = e^{-x}(-x-1) + C$

$p = C_1 e^x - x - 1$

$y' = C_1 \cdot e^x - x - 1$

$y = C_1 \cdot e^x - \frac{x^2}{2} - x + C_2 x$

N4160

$y'' = \frac{y'}{x} + x$ $\qquad y' = p \quad y'' = p'$

$p' - \frac{p}{x} = x$ $\qquad p = uv \quad p' = u'v + uv'$

$u'v + u(v' - \frac{v}{x}) = x$

1) $$\frac{dv}{v} = \frac{dx}{x}$$

$$v = x$$

2) $$\frac{du}{dx} x = x$$

$$du = dx$$

$$u = x + C_1$$

$$p = x^2 + C_1 x$$

$$y' = x^2 + C_1 x$$

$$y'' = \frac{x^3}{3} + C_1 \frac{x^2}{2}$$

№ 4161

$$(1+x^2)y'' + (y')^2 + 1 = 0$$

$$y' = p \qquad y'' = p'$$

$$(1+x^2)p' + p^2 + 1 = 0 \quad \cancel{|:(1+x^2)}$$

$$\cancel{p'} \quad (1+x^2)dp + (p^2+1)dx = 0$$

$$\frac{dp}{p^2+1} + \frac{dx}{x^2+1} = 0$$

$$\operatorname{arctg} p + \operatorname{arctg} x + C_1 = 0$$

$$\operatorname{arctg}(y') + \operatorname{arctg} x + C_1 = 0$$

$$y' = -tg(\operatorname{arctg} x + C_1)$$

$$dy = -tg(\operatorname{arctg} x + C_1)\,dx$$

№4161

$x y'' = y' \ln \frac{y'}{x}$ $\qquad p = y' \qquad p'' = y'$

$x p' = p \cdot \ln \frac{p}{x}$

$p \cdot \ln \frac{p}{x} dx - x \cdot dp = 0$

$t = \frac{p}{x} \qquad p = tx \qquad p' = t'x + t$

$p \cdot tx \cdot \ln t \, dx - x^2 dt - xt\,dx = 0 \qquad dp = x\,dt + t\,dx$

$tx(\ln t - 1)\, dx - x^2 dt = 0$

$$\frac{dx}{x} - \frac{dt}{t(\ln t - 1)} = 0$$

$$\frac{dx}{x} - \frac{d(\ln t - 1)}{\ln t - 1} = 0$$

$$\ln|x| - \ln|\ln t - 1| = \ln C^*$$

$$\frac{x}{\ln \frac{p}{x} - 1} = C^*$$

$$\frac{\ln \frac{p}{x} - 1}{\cancel{\ }} = Cx$$

$$\frac{y'}{x} \cdot \frac{1}{e} = e^{Cx}$$

$$\frac{y'}{\cancel{x}} = x \cdot e^{Cx+1}$$

$$dy = x \cdot e^{Cx+1} dx$$

$u = x \qquad du = dx$

$dv = e^{Cx+1} dx \qquad v = \frac{1}{C} \cdot e^{Cx+1}$

$$\int dy = \frac{1}{C} x e^{Cx+1} - \frac{1}{C} \int e^{Cx+1} dx$$

$$y = \frac{1}{C} x e^{Cx+1} - \frac{1}{C^2} \cdot e^{Cx+1}$$

№ 4163

$(y'')^2 = y'$

$y' = p = \cancel{\frac{dy}{dx}}$

$y'' = p \cdot \cancel{p'} \frac{dp}{dy}$

$p^2 (p')^2 = p$

$p (p')^2 = 1$

№

№ 4164

$2x y' y'' = (y')^2 + 1$

$y' = p \qquad y'' = p'$

$2x p p' = p^2 + 1$

$2x p \, dp = (p^2 + 1) dx$

$\frac{2p \, dp}{p^2 + 1} = \frac{dx}{x}$

$p^2 + 1 = C_1 x$

$(y')^2 + 1 = C_1 x$

$y' = \pm \sqrt{C_1 x - 1}$

$dy = +\sqrt{C_1 x - 1} \, dx$ или $dy = -\sqrt{C_1 x - 1} \, dx$

$y = \pm \frac{1}{C_1} \cdot \cancel{\frac{3}{2}} \frac{2}{3} \cdot \sqrt{(C_1 x - 1)^3} + C_2$

№4165

$$y'' - 2\operatorname{ctg}x \, y' = \sin^3 x$$

$$p = y' \qquad p' = y''$$

$$p' - 2\operatorname{ctg}x \cdot p = \sin^3 x$$

$$p = u \cdot v \qquad p' = u'v + uv'$$

$$u'v + u(v' - 2\operatorname{ctg}x \cdot v) = \sin^3 x$$

① $$\frac{dv}{v} = 2\frac{\cos x}{\sin x}\,dx$$

$$v = \sin^2 x$$

$1 - 2\sin^2 = \cos 2x$

$\frac{1-\cos 2x}{2}$

② $$du \cdot \sin^2 x = \sin^3 x \cdot dx$$

$$du = \sin x \cdot dx$$

$$u = -\cos x + C_1$$

$$p = -\sin^2 x \cdot \cos x + C_1 \sin^2 x$$

$$y' = -\sin^2 x \cdot \cos x + C_1 \cdot \sin^2 x$$

$$dy = -\sin^2 x \cdot d(\sin x) + C_1 \cdot \sin^2 x \cdot dx$$

$$y = -\frac{\sin^3 x}{3} + C_1 \int \sin^2 x \, dx$$

$$y = -\frac{\sin^3 x}{3} + \frac{C_1}{2}\int (1 - \cos 2x)\,dx$$

$$y = -\frac{\sin^3 x}{3} + \frac{C_1}{2}x - \frac{C_1}{4}\sin 2x + C_2$$

№4166

$$1 + (y')^2 = 2yy''$$

$$y' = p \qquad y'' = p \cdot p'$$

$1+p^2 = \cancel{2p\,p'}\ 2y p \cdot p'$

$\frac{dy}{y} = \frac{2p}{1+p^2}\, dp$

$C_1 y = 1+p^2$

$C_1 y = 1+(y')^2$

$y' = \pm\sqrt{C_1 y - 1}$

$\pm \frac{dy}{\sqrt{C_1 y - 1}} = dx$

$\pm \frac{2}{C_1}\sqrt{C_1 y - 1} = x + C_2$

№4167

$(y')^2 + 2yy'' = 0$

$y' = p \quad y'' = p \cdot p'$

$p^2 + 2y \cdot p p' = 0$

$\frac{dy}{y} + \frac{2\,dp}{p} = 0$

$p^2 = \frac{C_1}{y}$

$p = \pm\sqrt{\frac{C_1}{y}} = \pm\frac{C_1^*}{\sqrt{y}}$

$y' = \pm\frac{C_1^*}{\sqrt{y}}$

$\pm\frac{1}{C_1^*}\sqrt{y}\, dy = dx$

$$\pm \frac{1}{C_1} \cdot \frac{2}{3} \cdot y\sqrt{y} = x + C_2$$

№ 4168

$$a^2 y'' - y = 0$$

$$p = y' \qquad y'' = p \cdot p'$$

$$a^2 \cdot p \cdot p' - y = 0$$

$$a^2 p\,dp = y\,dy$$

$$a^2 \frac{p^2}{2} = \frac{y^2}{2} + C_1^*$$

$$\frac{a^2}{\cancel{2}} (y')^2 = \frac{y^2}{\cancel{2}} + C_1$$

$$y' = \pm \sqrt{\frac{y^2}{a^2} + \frac{C_1}{a^2}}$$