$$= \frac{x}{1+e^x} - \frac{x}{1+e^x} + \frac{dx}{1+e^x} \qquad \left[\begin{matrix} \frac{e^x\,dx}{(1+e^x)^2} = dv & -\frac{1}{1+e^x} = v \end{matrix} \right] =$$

Типовой рассчёт №3 вар. 5

а) $(1+e^x)\,y\,dy - e^y\,dx = 0$ — с раздел. перем.

$$\frac{y\cdot dy}{e^y} = \frac{dx}{1+e^x}$$

$$\int \frac{dx}{1+e^x} = \left[\begin{matrix} e^x = t \\ x = \ln t \\ dx = \frac{1}{t}\cdot dt \end{matrix} \right] = \int \frac{\frac{1}{t}\cdot dt}{t+1} = \int \frac{dt}{t(t+1)} =$$

$$= \int \frac{dt}{t^2+t} = \int \frac{dt}{(t+\frac{1}{2})^2 - (\frac{1}{2})^2} = -\int \frac{d(t+\frac{1}{2})}{(\frac{1}{2})^2 - (t+\frac{1}{2})^2} =$$

$$= -\frac{1}{1\cdot\frac{1}{2}} \cdot \ln\left| \frac{\frac{1}{2}+t+\frac{1}{2}}{\frac{1}{2}-t-\frac{1}{2}} \right| + C_1 = -\ln\left|\frac{t+1}{-t}\right| + C_1 =$$

$$= \ln\left(\frac{e^x}{e^x+1}\right) + C$$

$$\int \frac{y\cdot dy}{e^y} = \left[\begin{matrix} u = y & dy = du \\ \frac{dy}{e^y} = dv & -\frac{1}{e^y} = v \end{matrix} \right] = -\frac{y}{e^y} + \int \frac{dy}{e^y} =$$

$$= -\frac{y-1}{e^y} + C_2$$

$$\frac{-y-1}{e^y} = \ln\left(\frac{e^x}{e^x+1}\right) + C$$

общий интеграл

я случайно сделал из 4 варианта, простите)

b) $(x-y)dx + (x+y)dy = 0$ — однородное

$t = \frac{y}{x}$ $\quad y = tx \quad y' = t'x + t$

$dy = x\,dt + t\,dx$

$$(x-tx)dx + (x+tx)(x\,dt + t\,dx) = 0$$

$$x(1-t)dx + x^2(1+t)dt + x(t+t^2)dx = 0$$

$$x(t^2+1)dx + x^2(1+t)dt = 0$$

$$\frac{dx}{x} + \frac{1+t}{t^2+1}dt = 0$$

$$\ln|x| + \int\frac{t}{t^2+1}dt + \int\frac{dt}{t^2+1} = 0$$

$$\ln|x| + \ln\sqrt{t^2+1} + \operatorname{arctg} t + C^* = 0$$

$$\ln(x^2) + \ln\left(\frac{y^2}{x^2}+1\right) + 2\operatorname{arctg}\frac{y}{x} + C = 0$$

$$\ln(y^2+x^2) + 2\operatorname{arctg}\frac{y}{x} + C = 0$$

общий интеграл

c) ~~$xy' - 2y = 2x^4$ — линейное 1-го пор-ка~~

$y(0) = 0$

$y' = 2x(x^2+y)$ — линейное 1-го пор-ка

$y' - 2x\cdot y = 2x^3$

$y = u\cdot v \quad y' = u'v + u\cdot v'$

$u'v + u(v' - 2xv) = 2x^3$

1) $\frac{dv}{dx} = 2x \cdot v$

$\int \frac{dv}{v} = \int 2x \cdot dx$

$\ln|v| = x^2$

$v = e^{x^2}$

2) $\frac{du}{dx} e^{x^2} = 2x^3$

$du = \frac{2x^3}{e^{x^2}} dx$

$u = \left[\begin{array}{l} x^2 = t \\ dt = 2x \end{array} \right] = \frac{t\,dt}{e^t} = \ldots = -\frac{t+1}{e^t}$

$u = -\frac{x^2+1}{e^{x^2}} + C$

$y = u \cdot v = -x^2 - 1 + C \cdot e^{x^2}$ - общее реш-е

$y(0) = 0$

$C - 1 = 0$

$C = 1$

$y = -x^2 - 1 + e^{x^2}$ - частное решение

d) $xy\,dy = (y^2 + x)dx$ - с интегрирующим множителем

$(y^2 + x)dx - xy\,dy = 0$

$\frac{\partial P}{\partial y} = 2y$

$\frac{\partial Q}{\partial x} = -y$

$F(x) = \int -\frac{1}{xy}(2y + y) = \int -\frac{3}{x} = -3 \cdot \ln|x|$

$\mu(x) = e^{\int}$

$$\cancel{\frac{\partial Q}{\partial x}} = -y \cdot e^{-\frac{3}{x}} + y \cdot x \cdot e^{-\frac{3}{x}} \cdot \frac{3}{x^2}$$

$$\mu(x) = \frac{1}{x^3}$$

$$\left(\frac{y^2}{x^3} + \frac{1}{x^2}\right)dx - \frac{y}{x^2}dy = 0$$

$$\frac{\partial P}{\partial y} = \frac{2y}{x^3}$$

$$\frac{\partial Q}{\partial x} = \frac{2y}{x^3}$$

$$\frac{\partial u}{\partial x} = \frac{y^2}{x^3} + \frac{1}{x^2}$$

$$du = \left(\frac{y^2}{x^3} + \frac{1}{x^2}\right)dx$$

$$u = -\frac{1}{2}\frac{y^2}{x^2} - \frac{1}{x} + C(y)$$

$$\frac{\partial u}{\partial y}\left(-\frac{y^2}{2x^2} - \frac{1}{x} + C(y)\right) = -\frac{y}{x^2}$$

$$-\frac{y}{x^2} + C'(y) = -\frac{y}{x^2}$$

$$C(y) = C_1$$

b)

$$u = -\frac{y^2}{2x^2} - \frac{1}{x} + C_1$$

a) $xy'' - y' = x^2 e^x$ №1

$y' = p(x)$ $\qquad y''(x) = p'(x)$

$xp' - p = x^2 e^x \quad |:x$

$p' - \frac{p}{x} = x \cdot e^x$

$p = u \cdot v \quad | \quad p' = u'v + uv'$

$u'v + u(v' - \frac{v}{x}) = x \cdot e^x$

$\frac{dv}{v} = \frac{dx}{x}$

$v = x$

$\frac{du}{dx} \cdot x = x \cdot e^x$

$u = e^x + C_1$

$p = x \cdot e^x + C_1 x$

$y' = x \cdot e^x + C_1 x$

$y = \int x \cdot e^x \, dx + \frac{C_1}{2} \cdot x^2 + C_2$

$x = u \quad dx = du$

$e^x dx = dv \quad e^x = v$

$y = x \cdot e^x - e^x + C_1^* x^2 + C_2$

b) $y'' - 14y' + 53y = 53x^3 - 42x^2 + 59x - 14$

ОО: $y'' - 14y' + 53y = 0$

$k^2 - 14k + 53 = 0$

$D = 196 - 212 = -16$

$k_1 = \frac{14 + 4i}{2} = 7 + 2i \qquad k_2 = 7 - 2i$

$y_{oo} = e^{7x}(C_1 \cdot \cos 2x + C_2 \sin 2x)$

$y_{чн} = C_1(x) y_1 + C_2(x) y_2$

$y_{чн} = C_1(x) \cdot e^{7x} \cdot \cos 2x + C_2(x) \cdot e^{7x} \cdot \sin 2x$

$C_1'(x) e^{7x} \cos 2x + C_1'(x) \cdot e^{7x} \cdot \sin 2x = 0$

(2) $2C_2'(x)\left(-e^{7x}\cdot\frac{\sin^2 2x}{\cos 2x}+e^{7x}\cdot\cos 2x\right)$

$y_{чн}=Ax^3+Bx^2+Cx+D$

$y'_{чн}=3Ax^2+2Bx+C$

$y''_{чн}=6Ax+2B$

$53Ax^3+(53B-42A)x^2+(53C-28B+6A)x+53D-14C+2B=$

$=53x^3-42x^2+59x-14$

x^3	$53A=53$	$A=1$
x^2	$53B-42A=-42$	$B=0$
x	$53C-28B+6A=59$	$C=1$
1	$53D-14C+2B=-14$	$D=0$

$y_{ч}$

$y_{он}=$

$y_{чн} = \cancel{1}\, x^3 + x$

$y_{он} = e^{7x}(C_1 \cdot \cos 2x + C_2 \sin 2x) + x^3 + x$

c) $y'' + 2y' + 2y = \dfrac{e^{-x}}{\cos x}$

$\cancel{y_{чн}} =$ ОО: $y'' + 2y' + 2y = 0$

$k^2 + 2k + 2 = 0$

$D = 4 - 8 = -4$

$k_1 = \dfrac{-2+2i}{2} = -1+i$

$k_2 = -1-i$

$y_{оо} = e^{-x} \cdot (C_1 \cos x + C_2 \sin x)$

ОН: $y_{чн} = C_1(x) e^{-x} \cdot \cos x + C_2(x) e^{-x} \sin x$

$$\begin{cases} C_1'(x) e^{-x} \cdot \cos x + C_2'(x) e^{-x} \sin x = 0 \\ -C_1'(x) e^{-x} \cos x - C_1'(x) e^{-x} \sin x - C_2'(x) e^{-x} \sin x + C_2'(x) e^{-x} \cdot \cos x = \dfrac{e^{-x}}{\cos x} \end{cases}$$

$C_1'(x) = -C_2'(x) \operatorname{tg} x$

$C_2'(x) \left(\dfrac{\sin^2 x}{\cos x} + \cos x \right) = \dfrac{1}{\cos x}$

$C_2'(x) = 1$

$C_2(x) = x$

$C_1'(x) = -\operatorname{tg} x$

$C_1(x) = -\ln|\cos x|$

$y_{чн} = -\ln|\cos x| \cdot e^{-x} \cdot \cos x + x \cdot e^{-x} \sin x$

$y_{он} = e^{-x}(C_1 \cos x + C_2 \sin x) - \ln|\cos x| \cdot e^{-x} \cdot \cos x + x \cdot e^{-x} \sin x$

№3

a)
$$\begin{cases} x' = x - y \\ y' = -4x + 4y \end{cases} \qquad \begin{matrix} x(0) = 2 \\ y(0) = 7 \end{matrix}$$

$y' = x' - x''$

$x' - x'' = -4x + 4(x - x')$

$x' - x'' = -4x'$

$x'' - 5x' = 0$

$k^2 - 5k = 0$

$k_1 = 0 \quad k_2 = 5$

$x(t) = C_1 + C_2 e^{5t}$

$x'(t) = 5C_2 e^{5t}$

$y = x - x' = C_1 + C_2 e^{5t} - 5C_2 e^{5t} = C_1 - 4C_2 e^{5t}$

b) $\det(A - \lambda E) = 0$

$$\begin{vmatrix} 1-\lambda & -1 \\ -4 & 4-\lambda \end{vmatrix} = (\lambda - 1)(\lambda - 4) - 4 = \lambda^2 - 5\lambda = 0$$

$\lambda_1 = 0 \quad \lambda_2 = 5$

1) $\lambda = 0$
$$\begin{pmatrix} 1 & -1 \\ -4 & 4 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$\alpha = \beta$

$\Gamma_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

2) $\lambda = 5$
$$\begin{pmatrix} -4 & -1 \\ -4 & -1 \end{pmatrix} \sim \begin{pmatrix} -4 & -1 \\ 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} -4 & -1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\alpha = -\frac{1}{4}\beta$$

$$\Gamma_2 = \begin{pmatrix} -1 \\ 4 \end{pmatrix}$$

$$\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = C_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} + C_2 \begin{pmatrix} -1 \\ 4 \end{pmatrix} e^{5t}$$

$$\begin{cases} x(t) = C_1 - C_2 e^{5t} \\ y(t) = C_1 + 4C_2 e^{5t} \end{cases}$$