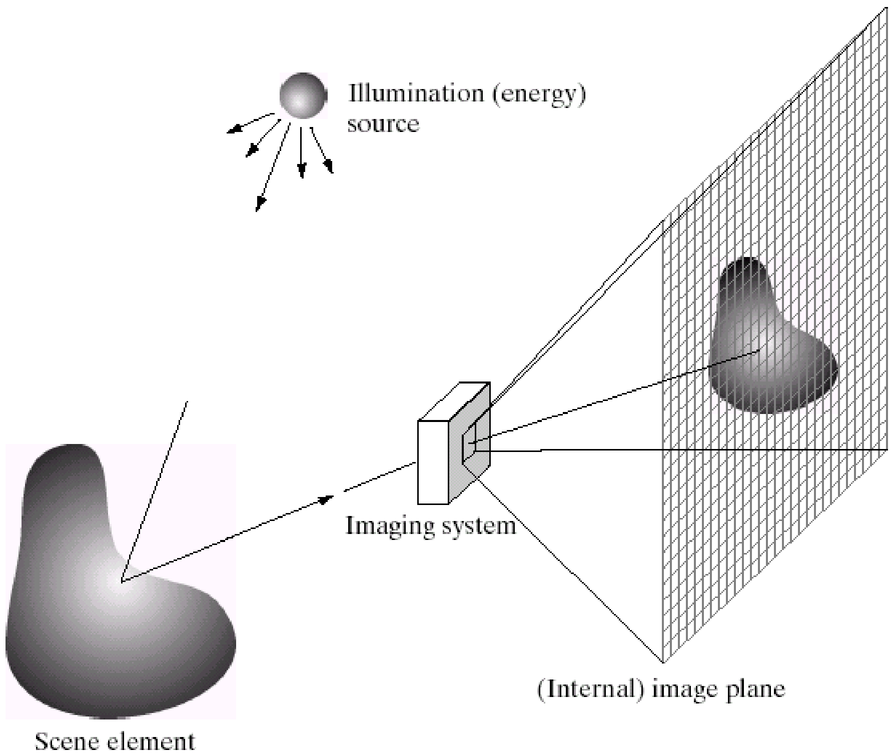
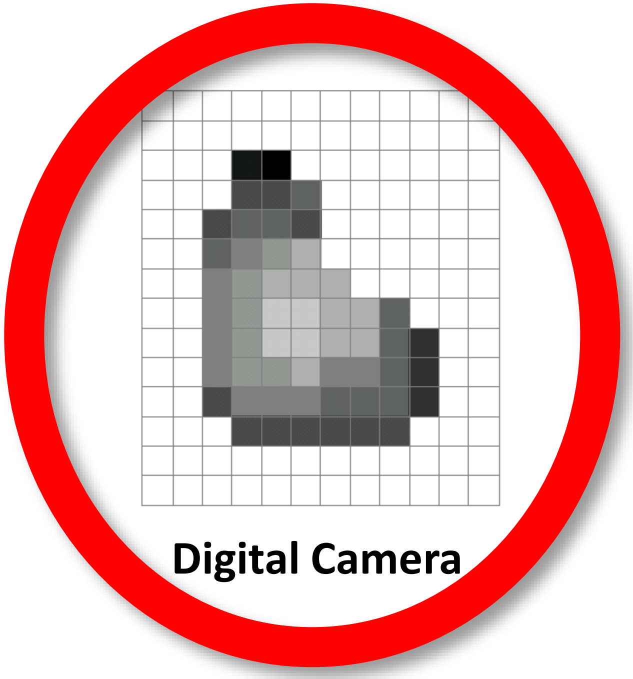
Image Processing Fundamentals

Prof. H. Sahli
VUB-ETRO
2021-2022

What is an image?

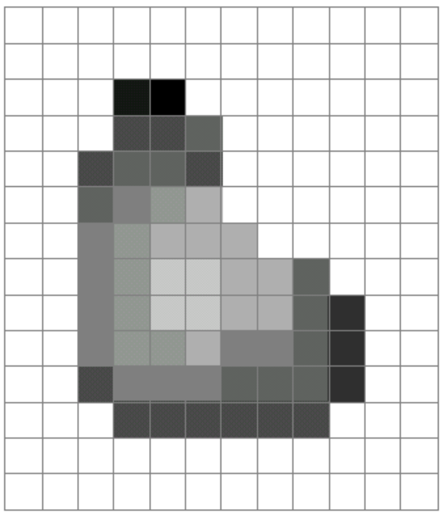




Source: A. Efros

What is an image?

- A grid (matrix) of intensity values



=

255	255	255	255	255	255	255	255	255	255	255	255
255	255	255	255	255	255	255	255	255	255	255	255
255	255	255	20	0	255	255	255	255	255	255	255
255	255	255	75	75	75	255	255	255	255	255	255
255	255	75	95	95	75	255	255	255	255	255	255
255	255	96	127	145	175	255	255	255	255	255	255
255	255	127	145	175	175	175	255	255	255	255	255
255	255	127	145	200	200	175	175	95	255	255	255
255	255	127	145	200	200	175	175	95	47	255	255
255	255	127	145	145	175	127	127	95	47	255	255
255	255	74	127	127	127	95	95	95	47	255	255
255	255	255	74	74	74	74	74	74	255	255	255
255	255	255	255	255	255	255	255	255	255	255	255
255	255	255	255	255	255	255	255	255	255	255	255

(common to use one byte per value: 0 = black, 255 = white)

Source: A. Efros

Image filtering

- Modify the pixels in an image based on some function of a local neighborhood of each pixel

10	5	3
4	5	1
1	1	7

Local image data

Some function →

	7	

Modified image data

Source: L. Zhang

Linear filtering

- One simple version: linear filtering (cross-correlation, convolution)
  - Replace each pixel by a linear combination of its neighbors
- The prescription for the linear combination is called the “kernel” (or “mask”, “filter”)

10	5	3
4	6	1
1	1	8

Local image data

0	0	0
0	0.5	0
0	1	0.5

kernel

	8	

Modified image data

Source: L. Zhang

Cross-correlation

Let F be the image, H be the kernel (of size 2k+1 x 2k+1), and G be the output image

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v]F[i+u, j+v]$$

This is called a **cross-correlation** operation:

$$G = H \otimes F$$

Convolution

- Same as cross-correlation, except that the kernel is “flipped” (horizontally and vertically)

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v]F[i-u,j-v]$$

This is called a **convolution** operation:

- Convolution is **commutative** and **associative**

$$G = H * F$$

Linear filters: examples

Original

$*$

0	0	0
0	1	0
0	0	0

$=$

Identical image

Source: D. Lowe

Linear filters: examples

Original

$$* \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} =$$

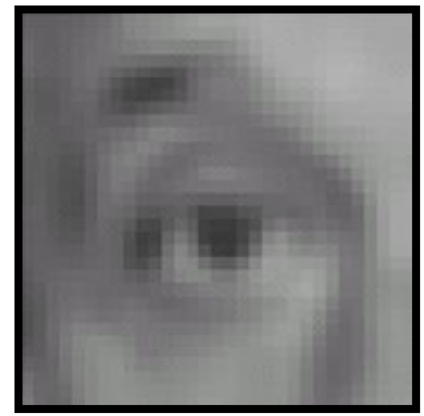
Shifted left
By 1 pixel

Source: D. Lowe

Linear filters: examples

Original

$$* \ \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} =$$



Blur (with a mean filter)

Linear filters: examples

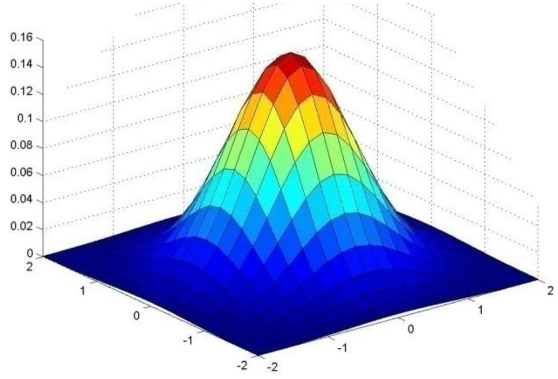
Original

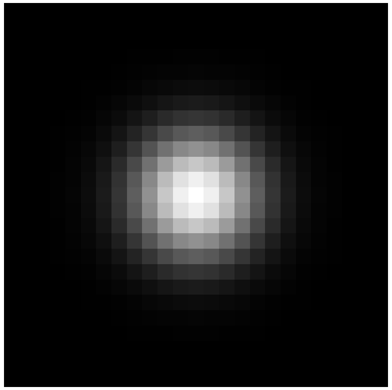
$$* \left(\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \right) =$$

Sharpening filter
(accentuates edges)

Source: D. Lowe

Gaussian Kernel

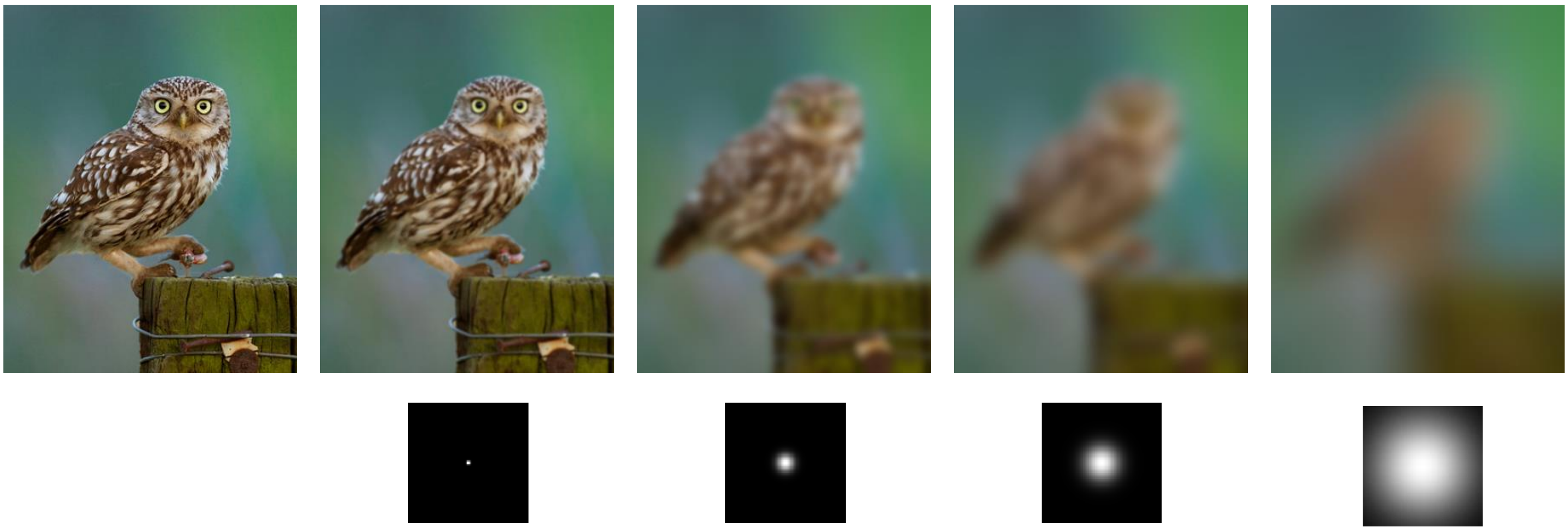




$$G_\sigma = \frac{1}{2\pi\sigma^2} e^{-\frac{(x^2+y^2)}{2\sigma^2}}$$

Source: C. Rasmussen

Gaussian filters

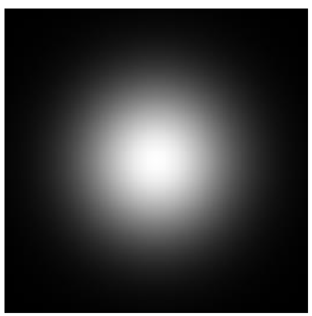


σ = 1 pixel σ = 5 pixels σ = 10 pixels σ = 30 pixels

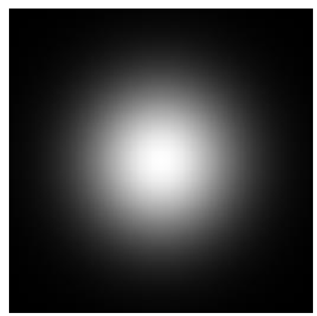
Source: K. Grauman

Gaussian filter

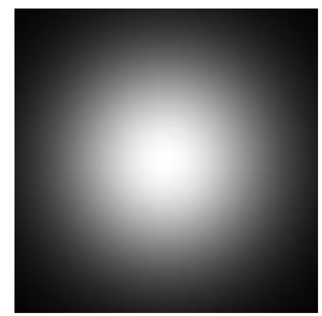
- Removes "high-frequency" components from the image (low-pass filter)
- Convolution with self is another Gaussian



$*$



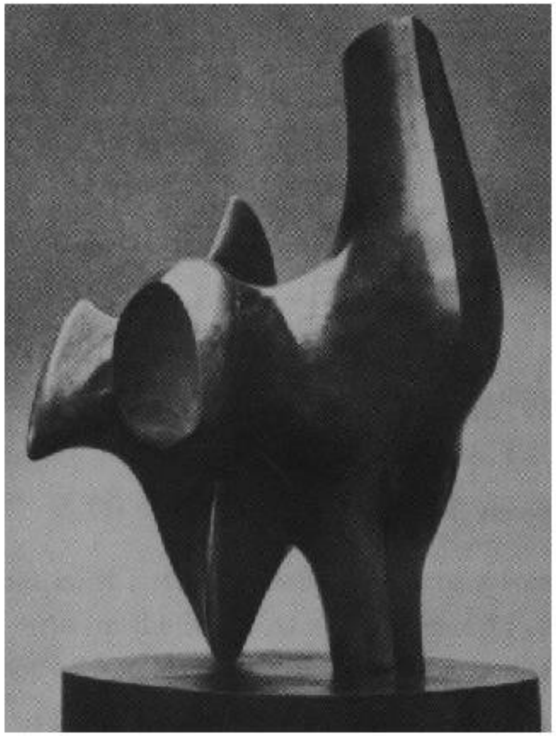
$=$

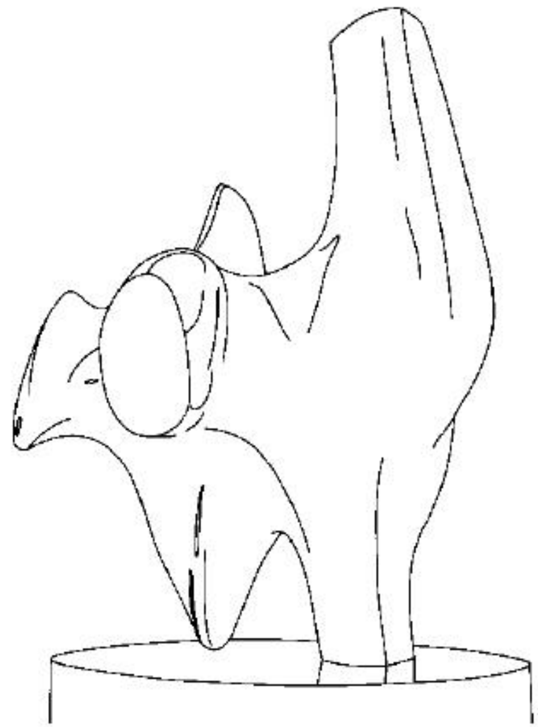


– Convolving two times with Gaussian kernel of width σ = convolving once with kernel of width $\sigma\sqrt{2}$

Source: K. Grauman

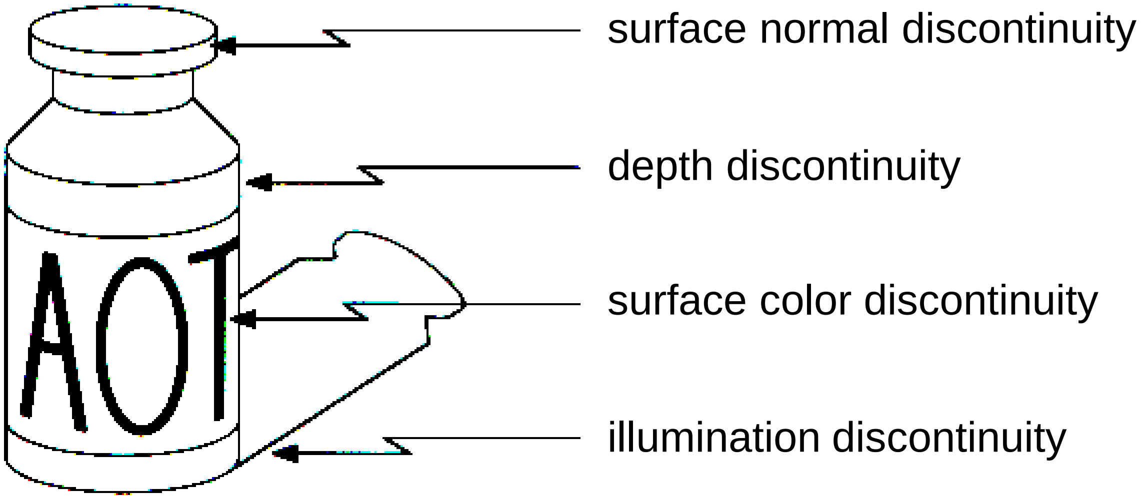
Edge detection





- Convert a 2D image into a set of curves
  - Extracts salient features of the scene
  - More compact than pixels

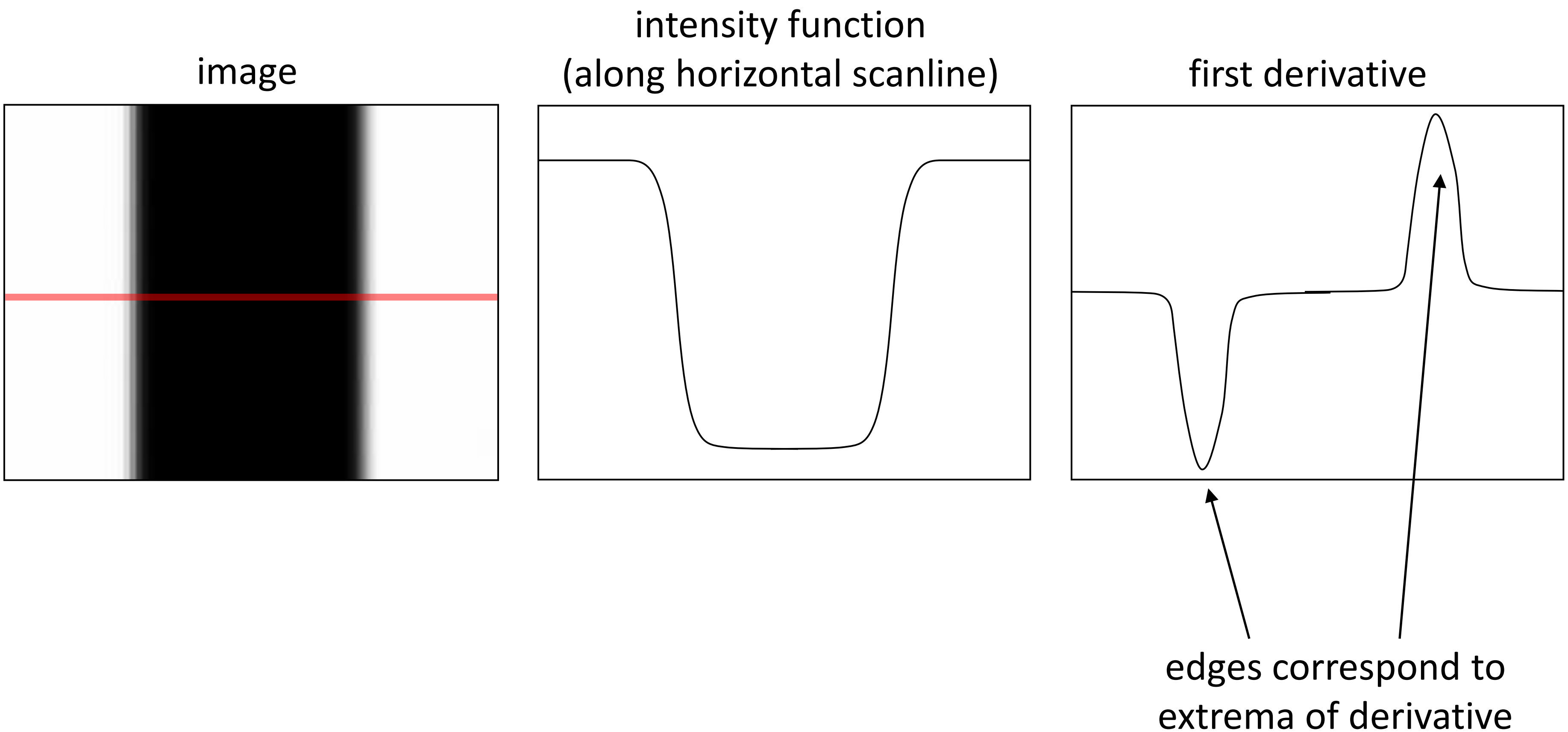
Origin of Edges



- Edges are caused by a variety of factors

Characterizing edges

- An edge is a place of rapid change in the image intensity function



Source: L. Lazebnik

Derivatives of 1D functions

First Derivative

(not centered at x)

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \approx f(x+1) - f(x) \quad (h{=}1) \Rightarrow \text{mask: } [-1 \quad 1]$$

$$\text{mask } \mathbf{M} = [-1, 0, 1]$$ (centered at x)

Second Derivative

$$f''(x) = \lim_{h \to 0} \frac{f'(x+h) - f'(x)}{h} \approx f'(x+1) - f'(x) =$$

$$f(x+2) - 2f(x+1) + f(x) \quad (h{=}1)$$

(centered at x+1)

Replace x+1 with x (i.e., centered at x):

$$f''(x) \approx f(x+1) - 2f(x) + f(x-1)$$

$$\Downarrow$$

$$\text{mask: } [1 \quad -2 \quad 1]$$

Image derivatives

- How can we differentiate a *digital* image F[x,y]?
  - Option 1: reconstruct a continuous image, *f,* then compute the derivative
  - Option 2: take discrete derivative (finite difference)

$$\frac{\partial f}{\partial x}[x,y] \approx F[x+1,y] - F[x,y]$$

Implement this as a linear filter

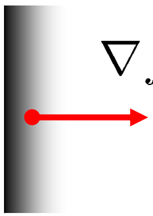
$$\frac{\partial f}{\partial x}: \underset{H_x}{\begin{array}{|c|c|c|}\hline \quad & \quad & \quad \\ \hline \quad & \quad & \quad \\ \hline \quad & \quad & \quad \\ \hline \end{array}} \qquad \frac{\partial f}{\partial y}: \underset{H_y}{\begin{array}{|c|c|c|}\hline \quad & \quad & \quad \\ \hline \quad & \quad & \quad \\ \hline \quad & \quad & \quad \\ \hline \end{array}}$$

Source: S. Seitz

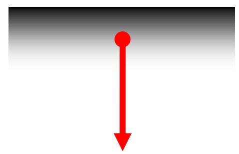
Image gradient

- The *gradient* of an image: $\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$

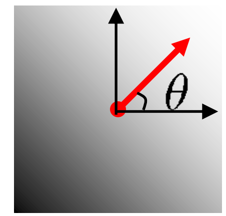
The gradient points in the direction of most rapid increase in intensity



$\nabla f = \left[\frac{\partial f}{\partial x}, 0\right]$



$\nabla f = \left[0, \frac{\partial f}{\partial y}\right]$



$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$

The *edge strength* is given by the gradient magnitude:

$$\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

The gradient direction is given by:

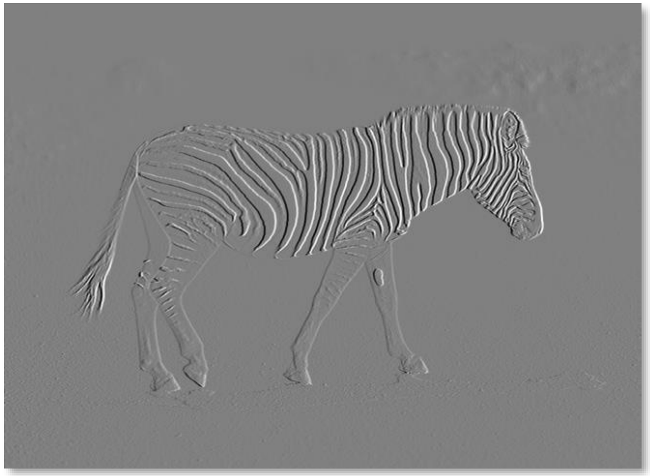
$$\theta = \tan^{-1}\left(\frac{\partial f}{\partial y} / \frac{\partial f}{\partial x}\right)$$

- how does this relate to the direction of the edge?

Source: Steve Seitz

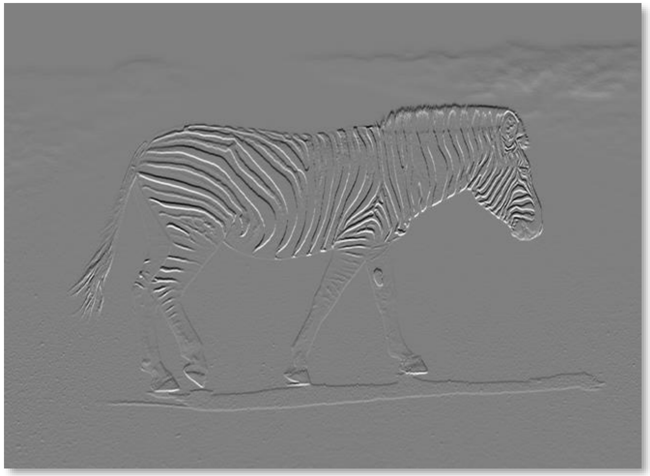
Image gradient

f



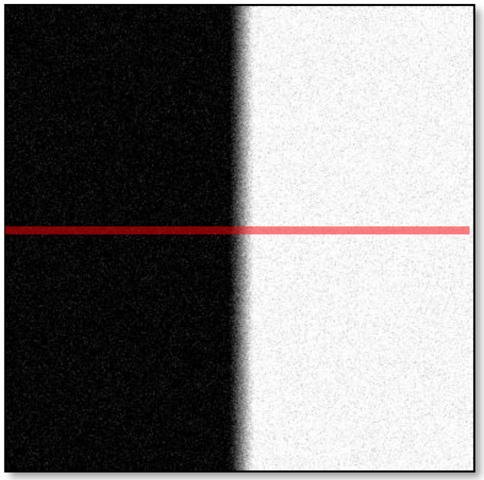
$\frac{\partial f}{\partial x}$

Magnitude $\|\nabla f\| = \sqrt{(\frac{\partial f}{\partial x})^2 + (\frac{\partial f}{\partial y})^2}$



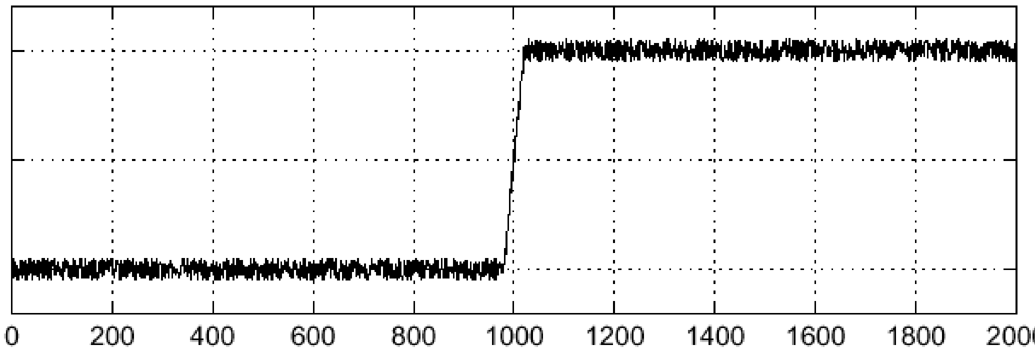
$\frac{\partial f}{\partial y}$

Effects of noise

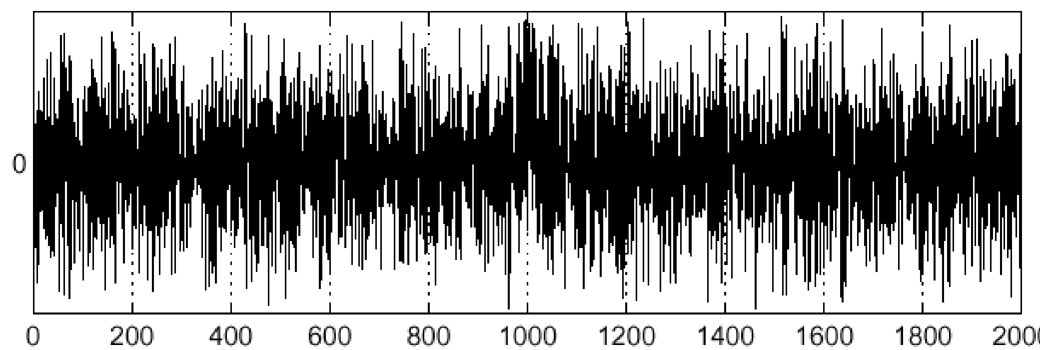


Noisy input image

$$f(x)$$



$$\frac{d}{dx}f(x)$$



Where is the edge?

Source: S. Seitz

Associative property of convolution

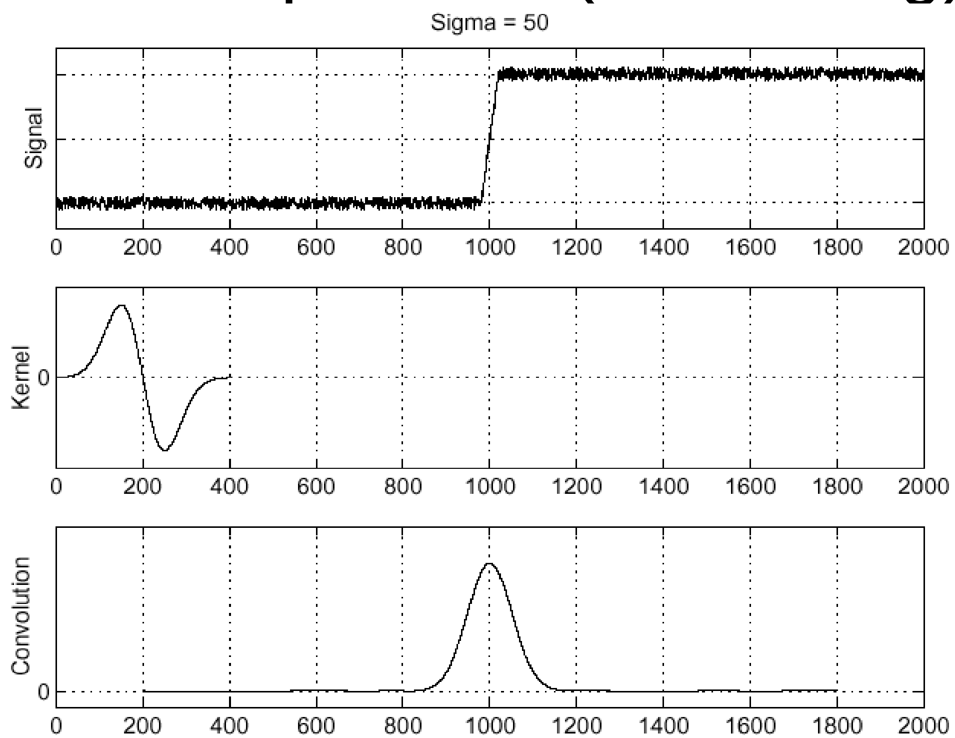
- Differentiation is convolution, and convolution is associative: $\frac{d}{dx}(f * h) = f * \frac{d}{dx}h$

- This saves us one operation (smoothing):

f

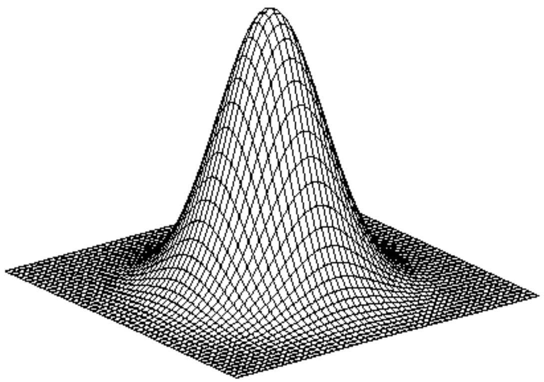
$\frac{d}{dx}h$

$f * \frac{d}{dx}h$



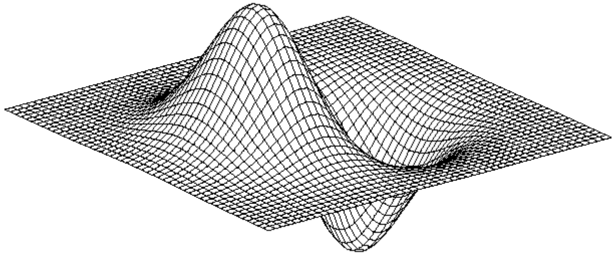
Source: S. Seitz

2D edge detection filters



Gaussian

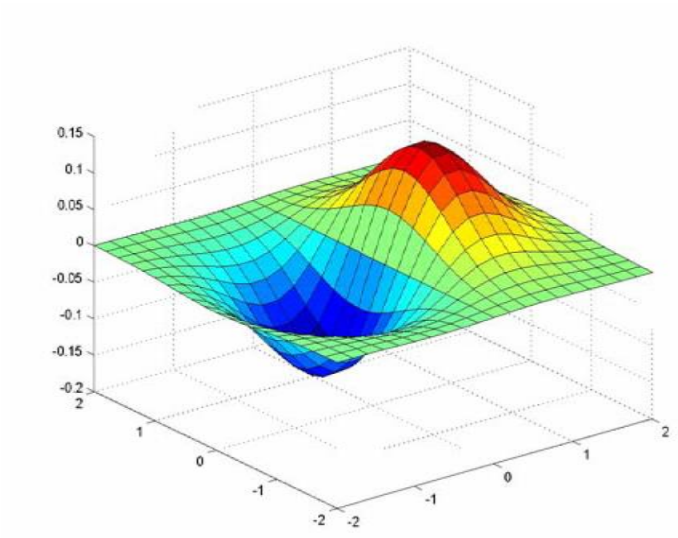
$$h_\sigma(u,v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{2\sigma^2}}$$



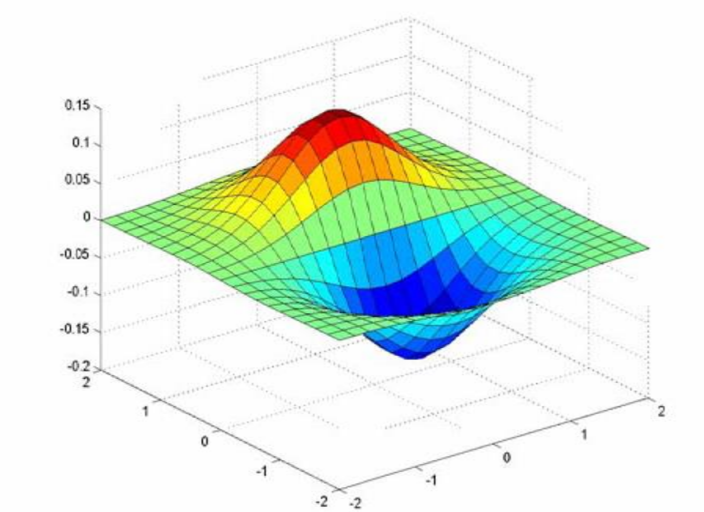
derivative of Gaussian (x)

$$\frac{\partial}{\partial x} h_\sigma(u,v)$$

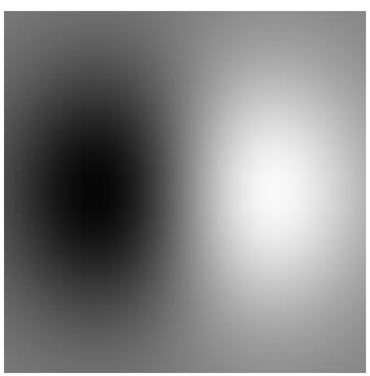
Derivative of Gaussian filter

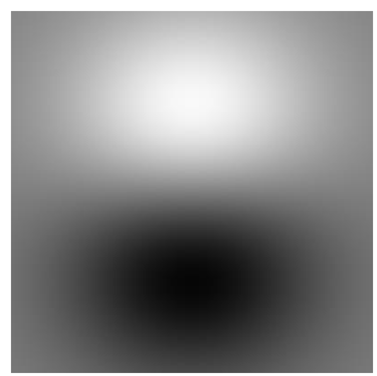


x-direction



y-direction





The Sobel operator

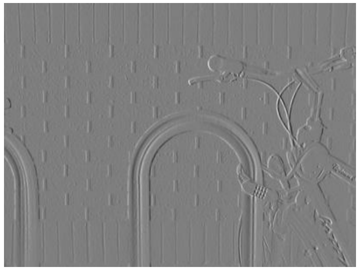
- Common approximation of derivative of Gaussian

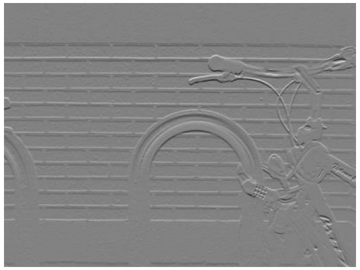
$$\frac{1}{8}\begin{array}{|c|c|c|}\hline -1 & 0 & 1 \\ \hline -2 & 0 & 2 \\ \hline -1 & 0 & 1 \\ \hline \end{array} \qquad \frac{1}{8}\begin{array}{|c|c|c|}\hline 1 & 2 & 1 \\ \hline 0 & 0 & 0 \\ \hline -1 & -2 & -1 \\ \hline \end{array}$$

$$s_x \qquad\qquad\qquad s_y$$

- The standard defn. of the Sobel operator omits the 1/8 term
  - doesn't make a difference for edge detection
  - the 1/8 term **is** needed to get the right gradient value

Sobel operator: example





Source: Wikipedia