HOMEWORK 11

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Proposition 9.12. *Let A and B be sets. There exists an injection from A to B if and only if there exists a surjection from B to A.*

Proof. ($\Rightarrow$) Suppose there's an injection $f : A \to B$. Fix $a_o \in A$. Observe that for $b \in f(a)$ there's a unique $a_b \in A$ such that $f(a_b) = b$. Define $g : B \to A$ by $g(b) = \begin{cases} a_o, \, b \notin f(a) \\ a_b, \, b \in f(a) \end{cases}$ is surjective by definition.

($\Leftarrow$) Suppose $g : B \to A$ is surjective. For each $a \in A$, there is at least one $b \in B$ such that $g(b) = a$. For each $a \in A$, fix some such $b_a \in B$. Define $f : A \to B$ by $f(a) = b_a$. Let's check if f is injective. Suppose $a_1 \neq a_2$. Then, $g(a_1) \neq g(a_2)$. $\square$

Proposition 10.9. *Let $x \in \mathbb{R}$ be such that $0 \leq x \leq 1$, and let $m, n \in \mathbb{N}$ be such that $m \geq n$. Then $x^m \leq x^n$.*

Proof. Here we have three cases. $x = 0$, $x = 1$, and $0 < x < 1$.
Case 1. x = 0. $0^m = 0 = 0^n$.
Case 2. x = 1. $1^m = 1 = 1^n$
Case 3. $0 < x < 1$. We have two different subcases here, since either $m = n$ or $m > n$.

Subcase 1. m = n. Here $x^m = x^n$

Subcase 2. $m > n$. Take $y > 1$ such that $\frac{1}{y} = x$. Because $m > n$, we have $y^m > y^n$. By proposition 8.40 (ii), we know $\frac{1}{y^m} < \frac{1}{y^n}$. Thus, $\frac{1}{y^m} = \frac{1^m}{y^m} = (\frac{1}{y})^m = x^m < \frac{1}{y^n} = \frac{1^n}{y^n} = (\frac{1}{y})^n = x^n$ $\square$

Proposition 10.16. *If the sequence (x_k) converges to L, then*

$$\lim_{k \to \infty} x_{k+1} = L.$$

Proof. $\square$

Proposition 10.14. *If (x_k) converges to L and to L' then $L = L'$.*

Proof. $\square$

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