

## HOMEWORK 11

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**Project 4.23 (Leibniz's formula).** Consider an operation denoted by  $'$  that is applied to symbols such as  $u, v, w$ . Assume that the operation  $'$  satisfies the following axioms:

$$\begin{aligned}(u + v)' &= u' + v', \\ (uv)' &= uv' + u'v, \\ (cu)' &= cu', \text{ where } c \text{ is a constant}\end{aligned}$$

Define  $w^{(k)}$  recursively by

- (i)  $w^{(0)} := w$
- (ii) Assuming  $w^{(n)}$  defined (where  $n \in \mathbb{Z}_{\geq 0}$ ), define  $w^{(n+1)} := (w^{(n)})'$ .

Prove:

$$\sum_{m=0}^k \binom{k}{m} u^{(m)} v^{(k-m)}.$$

Proof.

□

Sources.