Proposition 10.27. *Given any $r \in \mathbb{R}_{>0}$, the number $\sqrt{r}$ is unique in the sense that, if x is a positive real number such that $x^2 = r$, then $x = \sqrt{r}$.*

Proof. Suppose u and v are such that $u^2 = r$ and $v^2 = r$. Let $w = sup\{x \in \mathbb{R} | x^2 < r\}$. We will show u = w = v. For any $x \in A := \{x \in \mathbb{R} | x^2 < r\}$ we see that $x^2 < u^2 = r$. If $x < 0$, then clearly $x < u$. If $x \geq 0$, then proposition 10.5 ($x < u$ if and only if $x^2 < u^2$) ensures that $x < u$. Since w is the least upper bound of A, we conclude that $w \leq u$. But $w^2 = r = u^2$. By proposition 10.5 again it must be the case that w = u. Similarly, v = w. $\square$