HOMEWORK 11

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Proposition. $\sum_{i=0}^{\infty} \frac{1}{i}$ *diverges.*

Proof. Suppose that the harmonic series converges to S. By definition, this means for all $\epsilon > 0$, there exists an $N \in \mathbb{N}$ such that for all $n \geq N$, $|S - \sum_{i=0}^{n} \frac{1}{i}| < \epsilon$

$$\begin{aligned}
S &= 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} + \dots \\
&= \left(1 + \frac{1}{2}\right) + \left(\frac{1}{3} + \frac{1}{4}\right) + \left(\frac{1}{5} + \frac{1}{6}\right) + \left(\frac{1}{7} + \frac{1}{8}\right) + \dots \\
&> \left(\frac{1}{2} + \frac{1}{2}\right) + \left(\frac{1}{4} + \frac{1}{4}\right) + \left(\frac{1}{6} + \frac{1}{6}\right) + \left(\frac{1}{8} + \frac{1}{8}\right) + \dots \\
&= 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} + \dots \\
&= S
\end{aligned}$$

Thus, $S > S$, which is absurd. ∎

Sources.
http://scipp.ucsc.edu/ haber/archives/physics116A10/harmapa.pdf

Date: May 9, 2017.