HOMEWORK 11

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Proposition 9.12. *Let A and B be sets. There exists an injection from A to B if and only if there exists a surjection from B to A.*

Proof. ($\Rightarrow$) Suppose there's an injection $f : A \to B$. Fix $a_o \in A$. Observe that for $b \in f(a)$ there's a unique $a_b \in A$ such that $f(a_b) = b$. Define $g : B \to A$ by $g(b) = \begin{cases} a_o, \, b \notin f(a) \\ a_b, \, b \in f(a) \end{cases}$ is surjective by definition.

($\Leftarrow$) Suppose $g : B \to A$ is surjective. For each $a \in A$, there is at least one $b \in B$ such that $g(b) = a$. For each $a \in A$, fix some such $b_a \in B$. Define $f : A \to B$ by $f(a) = b_a$. Let's check if f is injective. Suppose $a_1 \neq a_2$. Then, $g(a_1) \neq g(a_2)$. $\square$

Proposition 10.9. *Let $x \in \mathbb{R}$ be such that $0 \leq x \leq 1$, and let $m, n \in \mathbb{N}$ be such that $m \geq n$. Then $x^m \leq x^n$.*

Proof. $\square$

Proposition 10.16. *If the sequence (x_k) converges to L, then*

$$\lim_{k \to \infty} x_{k+1} = L.$$

Proof. $\square$

Proposition 10.14. *If (x_k) converges to L and to L' then $L = L'$.*

Proof. $\square$

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