HOMEWORK 1

LEANDRO RIBEIRO
(WORKED WITH KYLE FRANKE AND JOYCE GOMEZ)

Proposition 1.7. *If m is an integer, then 0 + m = m and 1 · m = m.*

Proof. By commutativity of addition (axiom 1.1.1), 0 + m = m + 0. Because of the identity element for addition (axiom 1.2), we have that m + 0 = m. We conclude that 0 + m = m + 0.

By commutativity of multiplication (axiom 1.1.4), 1 · m = m · 1. Because of the identity element for multiplication (axiom 1.3), we have that m · 1 = m. We conclude that 1 · m = m · 1

□

Proposition 1.8. *If m is an integer, then (-m) + m = 0*

Proof. By commutativity of addition (axiom 1.1.1), m + (-m) = (-m) + m. Since m + (-m) = 0 by axiom 1.4 (additive inverse), we can conclude (-m) + m = 0.

□

Proposition 1.10. *Let m, x_1 , x_2 ∈ **Z**. If m, x_1 , x_2 satisfy the equations m + x_1 = 0 and m + x_2 = 0, then x_1 = x_2 .*

Proof. Because both equations are equal to 0, we know that m + x_1 = 0 = m + x_2. By proposition 1.9, we can say that $x_1 = x_2$.

□

Proposition 1.12. *Let x ∈ **Z**. If x has the property that for each integer m, m + x = m, then x = 0.*

Proof. Let's add (-m) to both sides from the left: (-m) + (m + x) = (-m) + m. By associativity ((-m) + m) + x = (-m) + m. By proposition 1.8, (-m) + m = 0, so 0 + x = 0. Proposition 1.7 tells us that 0 + x = x, so we vonclude x = 0.

□

Proposition 1.13. *Let x ∈ **Z**. If x has the property that there exists an integer m such that m + x = m, then x = 0.*

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Proof. The proof for proposition 1.12 also proves this proposition. □

Proposition 1.16. *If m and n are even integers, then so are $m + n$ and mn.*

Proof. Since m and n are even there is j_1 and j_2 such that $j_1 \cdot 2 =$ m and $j_2 \cdot 2 =$ n. We now see that m + n = $j_1 \cdot 2 + j_2 \cdot 2 = (j_1 + j_2) \cdot 2$ thanks to proposition 1.6. Also, m · n = $(j_1 \cdot 2) \cdot (j_2 \cdot 2) = (j_1 \cdot 2 \cdot j_2) \cdot 2$ thanks to associativity of multiplication (axiom 1.1.5). Setting integers d := $j_1 \cdot 2 \cdot j_2$ and c := $j_1 + j_2$, we have that d · 2 = m · n and c · 2 = m + n. We conclude that 2 | m + n and 2 | m · n. □

Proposition 1.17. *(i) 0 is divisible by every integer.*
(ii) If m is an integer not equal to 0, then m is not divisible by 0.

Proof. (i) By proposition 1.14, we have that 0 · m = 0 for any integer m. Since 0 is an integer, we conclude that m | 0.

(ii) We assume m is divisible by 0. Then There is an integer j $\in$ **Z** such that j · 0 = m, but j · 0 = 0 by proposition 1.14, so m = 0. □

Proposition 1.18. *Let $x \in$ **Z**. If x has the property that for all $m \in$ **Z**, $mx = m$, then $x = 1$.*

Proof. By axiom 1.3, we have that m · x = m = m · 1. By cancellation (axiom 1.5), we can conclude x = 1. □