



UNIVERSIDADE FEDERAL DO TRIÂNGULO MINEIRO
DEPARTAMENTO DE MATEMÁTICA APLICADA - ICTE
Lista 08 - Fundamentos de Matemática Elementar

01. Se $f(x) = \arctg x$, prove que $f(x) + f(y) = f\left(\frac{x+y}{1-xy}\right)$

02. Seja $f(x) = tg x$. Verifique a igualdade $f(2x) = \frac{2f(x)}{1-[f(x)]^2}$.

03. Determine o domínio das seguintes funções:

a) $y = \arccos \frac{2x}{1+x}$

b) $y = \arcsen\left(\log_{10} \frac{x}{10}\right)$

c) $y = \sqrt{\sen 2x}$

d) $y = 2 + \sen x$

e) $y = tg(x - 30^\circ)$

04. Dada a função $f(x) = 2\senh x - 3tgh x$, calcule $f(2)$, $f(-1)$ e $f(0)$.

05. Prove as identidades:

a) $1 - tgh^2 u = \sec h^2 u$

b) $1 - \cotgh^2 u = -\cossech^2 u$

06. Resolva a equação $\sen(3x - \pi) = -\frac{1}{2}$, sendo $U = \mathbb{R}$.

07. Resolva a equação $2\sen^2 x - 5\sen x + 3 = 0$.

08. Determine o conjunto solução da equação $tg^2 x = \sqrt{3} \cdot tg x$.

09. Simplifique as expressões:

a) $E = \frac{tg x + \cotg x}{\cossec x}$

$$\text{b)} \quad F = \frac{2 - \operatorname{sen}^2 x}{\cos^2 x} - \operatorname{tg}^2 x$$

$$10. \text{ Sabendo-se que } \operatorname{cotg} x = \frac{3}{4} \text{ e } \pi < x < \frac{3\pi}{2}, \text{ calcule } M = \frac{4 - 2\operatorname{sen} x}{\cos^2 x}.$$

$$11. \text{ Resolva a equação } \operatorname{sen}^2 x + 4 \cos x = -4$$

$$12. \text{ Sendo } a, b \in]0^\circ, 90^\circ[, \operatorname{sen} a = \frac{1}{3} \text{ e } \cos b = \frac{1}{2}, \text{ determine:}$$

$$\text{a)} \quad \operatorname{sen}(a+b)$$

$$\text{b)} \quad \cos(a+b)$$

13. Demonstre:

$$\text{a)} \quad \frac{\cos(a-b) - \cos(a+b)}{\operatorname{sen}(a+b) + \operatorname{sen}(a-b)} = \operatorname{tg} b$$

$$\text{b)} \quad \operatorname{sen}(a+b) \cdot \operatorname{sen}(a-b) = \operatorname{sen}^2 a - \operatorname{sen}^2 b$$

RESPOSTAS

3.

$$\text{a)} \quad D_y = \left[-\frac{1}{3}, 1 \right]$$

$$\text{b)} \quad D_y = \{x \in \mathbb{R} \mid 1 \leq x \leq 100\}$$

$$\text{c)} \quad D_y = \bigcup_{n \in \mathbb{Z}} \left[n\pi, n\pi + \frac{\pi}{2} \right]$$

$$\text{d)} \quad D_y = \mathbb{R}$$

$$\text{e)} \quad D_y = \{x \in \mathbb{R} \mid x \neq 120^\circ + k \cdot 180^\circ\}$$

4.

$$f(2) = \frac{e^4 - e^{-4} - 3e^2 + 3e^{-2}}{e^2 + e^{-2}} \quad f(-1) = \frac{e^{-2} - e^2 - 3e^{-1} + 3e}{e^{-1} + e} \quad f(0) = 0$$

$$6. \quad S = \left\{ x \in \mathbb{R} \mid x = \frac{13\pi}{18} + \frac{2k\pi}{3} \text{ ou } x = \frac{17\pi}{18} + \frac{2k\pi}{3}, k \in \mathbb{Z} \right\}$$

$$7. S = \left\{ x \in \mathbb{R} \mid x = \frac{\pi}{2} + 2k\pi, k \in \mathbb{Z} \right\}$$

$$8. S = \left\{ x \in \mathbb{R} \mid x = k\pi \text{ ou } x = \frac{\pi}{3} + k\pi, k \in \mathbb{Z} \right\}$$

9.

$$\text{a) } E = \sec x$$

$$\text{b) } F = 2$$

$$10. M = \frac{140}{9}$$

$$11. S = \{ x \in \mathbb{R} \mid x = \pi + 2k\pi, k \in \mathbb{Z} \}$$

12.

$$\text{a) } \frac{1+2\sqrt{6}}{6}$$

$$\text{b) } \frac{2\sqrt{2}-\sqrt{3}}{6}$$