UNIVERSIDADE FEDERAL DO TRIÂNGULO MINEIRO
DEPARTAMENTO DE MATEMÁTICA APLICADA - ICTE
Lista 8

1. Mostre que:

a) $\tan(-\theta) = -\tan(\theta), \quad \forall \theta \neq \dfrac{\pi}{2} + k\pi, \quad k \in \mathbb{Z}.$

b) $\tan(a - b) = \dfrac{\tan a - \tan b}{1 + \tan a \cdot \tan b}.$

2. Simplifique a expressão

$$\frac{\sin(\pi - x) - \cos\left(\frac{\pi}{2} - x\right) - \tan(2\pi - x)}{\tan(\pi - x) - \cos(2\pi - x) + \sin\left(\frac{\pi}{2} - x\right)}.$$

3. Mostre que são verdadeiras as seguintes identidades trigonométricas:

a) $\sin 2x = 2 \sin x \cos x, \quad \forall x \in \mathbb{R};$

b) $\cos 2x = \cos^2 x - \sin^2 x, \quad \forall x \in \mathbb{R};$

4. Mostre as seguintes identidades:

a) $\cos^2 x = \dfrac{1 + \cos 2x}{2};$

b) $\sin^2 x = \dfrac{1 - \cos 2x}{2};$

c) $\cos(A)\cos(B) = \dfrac{\cos(A + B) + \cos(A - B)}{2};$

d) $\sin(A)\sin(B) = \dfrac{\cos(A - B) - \cos(A + B)}{2};$

e) $\sin(A)\cos(B) = \dfrac{\sin(A + B) + \sin(A - B)}{2}.$

5. Assinale a alternativa verdadeira.

a) $\tan^2(x) + \sec^2(x) = 1, \quad \forall x \in \left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$

b) $\sin\left(\dfrac{\pi}{12}\right) = \dfrac{\sqrt{2}}{6}$

c) $\tan(x) = \dfrac{1}{\sin(2x)} - \dfrac{\cos(2x)}{\sin(2x)}, \quad \forall x \in \left(0, \dfrac{\pi}{2}\right)$

d) $\tan(2x) = \dfrac{2\tan x}{1+\tan^2 x}, \quad \forall x \in \left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$

e) $\cos\left(\dfrac{11\pi}{6}\right) = -\dfrac{\sqrt{3}}{2}$

6. Sabendo que $\sin x = \frac{1}{3}$ e $0 < x < \frac{\pi}{2}$, calcule

$$\frac{1}{\text{cossec } x + \text{cotg } x} + \frac{1}{\text{cossec } x - \text{cotg } x}.$$

7. Mostre que $(1 + \text{cotan}^2 x)(1 - \cos^2 x) = 1$, para todo $x \neq k\pi$.

8. Verifique se as afirmações abaixo são verdadeiras ou falsas. Se a alternativa for verdadeira, justifique a resposta. Se a alternativa for falsa, dê um contra exemplo.

a) Todo felino é um gato;

b) $\sin(\pi + x) = \sin\pi + \sin x, \quad \forall x \in \mathbb{R};$

c) $\sin(\pi^2) = [\sin\pi]^2;$

d) $\sec^2 x - \tan^2 x = 1, \quad \forall x \neq \frac{\pi}{2} + k\pi, \quad k \in \mathbb{Z};$

e) $\dfrac{\cos 2x}{2} = \cos x, \quad \forall x \in \mathbb{R};$

f) $1 + \text{cotan}^2 x = \text{cossec}^2 x, \quad \forall x \neq k\pi \in \mathbb{Z};$

g) $2\sin 3x = \sin 6x, \quad \forall x \in \mathbb{R};$

h) $\dfrac{\sin x}{n} = \text{six} = 6;$

i) $\sin\left(\dfrac{y}{2}\right)\cos\left(\dfrac{2x+y}{2}\right) = \dfrac{\sin(x+y) - \sin(x)}{2}, \quad \forall x, y \in \mathbb{R};$

j) $\tan(x+y) = \tan(x) + \tan(y), \quad \forall x, y \neq \frac{\pi}{2} + k\pi, \quad k \in \mathbb{Z};$

k) $\sin\left(\dfrac{y}{2}\right)\sin\left(\dfrac{2x+y}{2}\right) = \dfrac{\cos(x) - \cos(x+y)}{2}, \quad \forall x, y \in \mathbb{R};$

l) $\sin(ab) = \sin a \cdot \sin b, \quad \forall a, b \in \mathbb{R}$;

m) $0 < \cos(2) < 1$;

n) Se $n \in \mathbb{Z}$, então $-1 < \cos(n) < 1$;

o) Se $n \in \mathbb{Z}$, então $\cos(n\pi) = (-1)^n$;

9. Se $f(x) = \arctan x$, prove que

$$f(x) + f(y) = f\left(\frac{x+y}{1-xy}\right).$$

10. Seja $f(x) = \tan x$. Verifique a igualdade

$$f(2x) = \frac{2f(x)}{1-[f(x)]^2}$$

11. Determine o domínio das seguintes funções:

a) $f(x) = \arccos\left(\dfrac{2x}{1+x}\right)$

b) $f(x) = \arcsin\left(\log_{10}\left(\dfrac{x}{10}\right)\right)$

c) $f(x) = \sqrt{\sin 2x}$

d) $f(x) = 2 + \sin x$

e) $f(x) = \tan(x - 30^o)$

f) $f(x) = \dfrac{1}{1+\tan x}$

g) $f(x) = \sec x + \tan x$

h) $f(x) = \sec^2 x$

i) $f(x) = \tan^2 x$

j) $f(x) = \dfrac{1}{1+\tan^2 x}$

k) $f(x) = \dfrac{\sin x}{x}$

l) $f(x) = \sec(\pi x)$

m) $f(x) = \tan\left(\dfrac{x}{2}\right)$

n) $f(x) = \sec(2x)$

o) $f(x) = \text{cosec }(3x)$

12. Dada a função $f(x) = 2\sinh x - 3\tanh x$, calcule $f(2)$, $f(-1)$ e $f(0)$.

13. Prove as identidades:

a) $1-\tanh^2 u = \text{sech}^2 u$

b) $1-\text{cotgh}^2 u = -\text{cossech}^2 u$

14. Resolva a equação

$$\sin(3x-\pi) = -\frac{1}{2}.$$

15. Resolva a equação

$$2\sin^2 x - 5\sin x + 3 = 0.$$

16. Determine o conjunto solução da equação

$$\tan^2 x = \sqrt{3}\tan x.$$

17. Simplifique as expressões:

a) $E = \dfrac{\tan x + \text{cotan} x}{\text{cossec} x}$.

b) $F = \dfrac{2-\sin^2 x}{\cos^2 x} - \tan^x$.

18. Sabendo-se que $\text{cotan}\ x = \dfrac{3}{4}$ e $\pi < x < \frac{3\pi}{2}$, calcule $M = \dfrac{4-2\sin x}{\cos^2 x}$.

19. Resolva a equação $\sin^2 x + 4\cos x = -4$.

20. Sendo $a, b \in]0,\pi[$ e $\sin a = \frac{1}{3}$ e $\cos b = \frac{1}{2}$, determine

a) $\sin(a+b)$

b) $\cos(a+b)$

21. Mostre que

a) $\dfrac{\cos(a-b)-\cos(a+b)}{\sin(a+b)+\sin(a-b)} = \tan b$

b) $\sin(a+b)\sin(a-b) = \sin^2 a - \sin^2 b$

22. Calcule:

a) $\arcsin 1$

b) $\arcsin\left(\frac{1}{2}\right)$

c) $\arcsin\left(\frac{\sqrt{3}}{2}\right)$

d) $\arctan 1$

e) $\arctan(-1)$

f) $\arctan(\sqrt{3})$

23. Se $0 \leq x \leq 2\pi$ e $\tan x \geq 0$, é possível afirmar que $0 \leq x < \pi$?

24. Esboce o gráfico das seguintes funções:

a) $f(x) = \sin 2x$

b) $f(x) = 2\cos 3x$

c) $f(x) = \cos x + \sin x$

d) $f(x) = \sin x \cos x$

e) $f(x) = |\cos x|$

f) $f(x) = \dfrac{\sin x}{x}$

g) $f(x) = \cos(x+1)$

h) $f(x) = 1 + 2\sin x$

i) $f(x) = x + \cos x$

j) $f(x) = x\cos x$

k) $f(x) = \cos(x+\pi)$

l) $f(x) = \sin(x+\pi/2)$

m) $f(x) = \sin^2(10x) + \cos^2(10x)$

n) $f(x) = \sin(x-1)$

25. Relacione as funções da primeira coluna que são iguais às funções da segunda coluna:

A) $f_1(x) = \cos x \sin x$	1) $g_1(x) = \dfrac{1 - \cos 2x}{2}$
B) $f_2(x) = 1 - \sin^2 x$	2) $g_2(x) = \dfrac{2\tan\left(\frac{x}{2}\right)}{1 + \tan^2\left(\frac{x}{2}\right)}$
C) $f_3(x) = \sin^2 x$	3) $g_3(x) = \dfrac{1 - \tan^2\left(\frac{x}{2}\right)}{1 + \tan^2\left(\frac{x}{2}\right)}$
D) $f_4(x) = \sin x$	4) $g_4(x) = \dfrac{\sin 2x}{2}$
E) $f_5(x) = \cos x$	5) $g_5(x) = \tan^2(x)$
F) $f_6(x) = \sec^2 x - 1$	6) $g_6(x) = \cos^2 x$

26. Qual o domínio da função $f(x) = \frac{\sin x}{x}$? Com o auxílio de uma calculadora, calcule $f(x)$ para os seguintes valores de x:

a) $x = 1$ b) $x = 10^{-1}$ c) $x = -10^{-2}$ d) $x = 10^{-8}$ e) $x = 10^{-20}$

27. Verifique se as afirmações abaixo são verdadeiras ou falsas. Se a alternativa for verdadeira, justifique a resposta. Se a alternativa for falsa, dê um contra exemplo.

a) $\cos(x + 2\pi) = \cos(x), \quad \forall x \in \mathbb{R};$

b) $\sin(x + 2\pi) = \sin(x), \quad \forall x \in \mathbb{R};$

c) $\sin(x + \pi) = -\sin(x), \quad \forall x \in \mathbb{R};$

d) $\sin(-x) = \sin(x), \quad \forall x \in \mathbb{R};$

e) $\tan(-x) = -\tan(x), \quad \forall x \neq \dfrac{\pi}{2} + k\pi,\ k \in \mathbb{Z};$

f) $\cos(-x) = \cos(x), \quad \forall x \in \mathbb{R};$

g) $\dfrac{\sin 2x}{x} = \sin 2$

h) $\sin(x + 1) - \sin(x) = 2\sin\left(\dfrac{1}{2}\right)\cos\left(\dfrac{2x + 1}{2}\right), \quad \forall x \in \mathbb{R}.$

Respostas:

1. a) Definição

b) basta desenvolver os termos

2) -1

5) c)

8)

a) F b) F c) F d) V e) F f) F

g) F h) F i) V j) F k) V l) F

m) V n) V o) V

11.a) $D_f = \left[-\frac{1}{3}, 1\right]$

b) $D_f = \{x \in \mathbb{R} | 1 \leq x \leq 100\}$

c) $D_f = \bigcup_{n \in \mathbb{Z}} \left[n\pi, n\pi + \frac{\pi}{2}\right]$

d) $D_f = \mathbb{R}$

e) $D_f = \left\{x \in \mathbb{R} | x \neq \frac{2\pi}{3} + k\pi,\ k \in \mathbb{Z}\right\}$

f) $D_f = \left\{x \in \mathbb{R} | x \neq -\frac{\pi}{2} + k\pi,\ k \in \mathbb{Z}\right\}$

g) $D_f = \left\{x \in \mathbb{R} | x \neq \frac{\pi}{2} + 2k\pi,\ k \in \mathbb{Z}\right\}$

h) $D_f = \left\{x \in \mathbb{R} | x \neq \frac{\pi}{2} + k\pi,\ k \in \mathbb{Z}\right\}$

i) $D_f = \left\{x \in \mathbb{R} | x \neq \frac{\pi}{2} + k\pi,\ k \in \mathbb{Z}\right\}$

j) $D_f = \left\{x \in \mathbb{R} | x \neq -\frac{\pi}{2} + k\pi,\ k \in \mathbb{Z}\right\}$

k) $D_f = \{x \in \mathbb{R} | x \neq 0\}$

l) $D_f = \left\{x \in \mathbb{R} | x \neq -\dfrac{1}{2} + k,\ k \in \mathbb{Z}\right\}$

m) $D_f = \{x \in \mathbb{R} | x \neq \pi + 2k\pi,\ k \in \mathbb{Z}\}$

n) $D_f = \left\{x \in \mathbb{R} | x \neq \dfrac{\pi}{4} + k\dfrac{\pi}{2},\ k \in \mathbb{Z}\right\}$

o) $D_f = \left\{x \in \mathbb{R} | x \neq \dfrac{k\pi}{3},\ k \in \mathbb{Z}\right\}$

12. $f(2) = \dfrac{e^4 - e^{-4} - 3e^2 + 3e^{-2}}{e^2 + e^{-2}}$, $f(-1) = \dfrac{e^{-2} - e^2 - 3e^{-1} + 3e}{e^{-1} + e}$ e $f(0) = 0$.

14. $S = \left\{x \in \mathbb{R} \mid x = \dfrac{13\pi}{18} + \dfrac{2k\pi}{3} \text{ ou } x = \dfrac{17\pi}{18} + \dfrac{2k\pi}{3},\quad k \in \mathbb{Z}\right\}$

15. $S = \left\{x \in \mathbb{R} \mid x = \dfrac{\pi}{2} + 2k\pi,\quad k \in \mathbb{Z}\right\}$

16. $S = \left\{x \in \mathbb{R} \mid x = k\pi \text{ ou } x = \dfrac{\pi}{3} + k\pi,\quad k \in \mathbb{Z}\right\}$

17. a) $E = \sec x$

b) $F = 2$

18. $M = \frac{140}{9}$

19. $S = \{x \in \mathbb{R} \mid x = \pi + 2k\pi,\ k \in \mathbb{Z}\}$

20. a) $\dfrac{1 + 2\sqrt{6}}{6}$

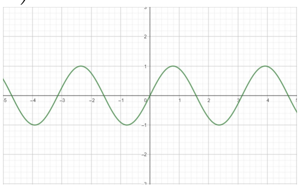
b) $\dfrac{2\sqrt{2} - \sqrt{3}}{6}$

22. a) $\dfrac{\pi}{2}$ b) $\dfrac{\pi}{6}$ c) $\dfrac{\pi}{3}$ d) $\dfrac{\pi}{4}$ e) $-\dfrac{\pi}{4}$ f) $\dfrac{\pi}{3}$

23. não

24.

a)



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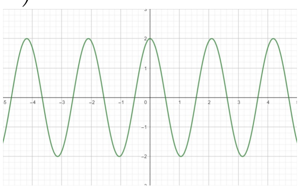

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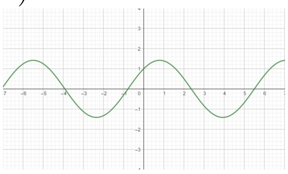

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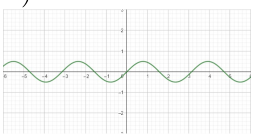

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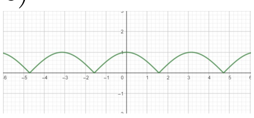

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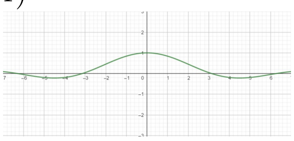

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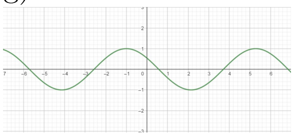

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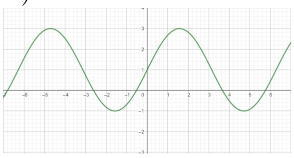

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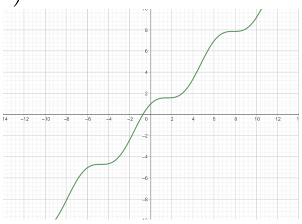

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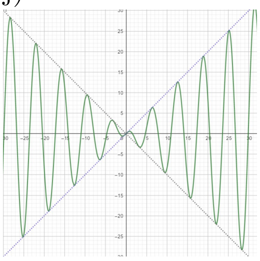

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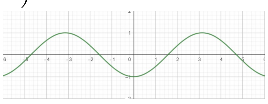

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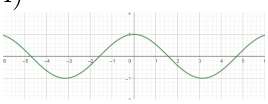

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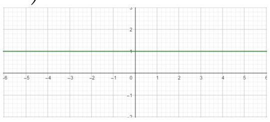

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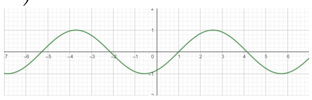

25. A-4, B-6, C-1, D-2, E-3, F-5

27. a) V, b) V c) V d) F e) V f) V g) F h) V