Non-dice games: incomplete nodes for laundering money and network crimes*

Leandro Marques†

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Abstract

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1 Introduction

Organized crime has a significant impact on the cost structure of enterprises as it necessitates the payment of bribes and protection fees to criminals. This not only raises the cost of doing business for entrepreneurs but also stifles economic growth (Blackburn et al., 2017). The dynamics between businesses, governments, and civil society have changed considerably over the last two decades, yet large organized crime chains have taken over commercial and banking structures, generating vast profits through the production, trafficking, and distribution of illegal rents. Recent scandals, such as the Panama, Paradise, and Pandora Papers, have illustrated the tension between creating a favorable economic climate for international business and preventing crimes such as corruption, money laundering, and tax offenses.

The EU Serious and Organised Crime Threat Assessment (2021) highlights the critical roles played in the criminal laundering chain, where criminals are skilled at exploiting the economy for their own purposes. Legal business structures are used to facilitate virtually all types of criminal activity, and all types of legal businesses are potentially vulnerable to exploitation by serious and organized crime (KLEEMANS, 2007).

In this context, this paper aims to explore the interconnections between innovation and regulation in the field of money laundering from an evolutionary game theory perspective. Specifically, we investigate how the interplay between innovation and regulation shapes the stability and development of the money laundering industry. Drawing on the Evolutionary Game Theory literature, we demonstrate how innovation alters the order and amplifies the network and materialism of money laundering, leading to the emergence and abundance of cooperation in a population of selfish individuals.

We also examine how risks related to financial innovation encourage the development of financial means and regulatory systems, and how this process is related to evolutionary games, where players decide and promote their strategies that ultimately define the stability and progress of the industry. By analyzing the regulation/innovation interconnections, we aim to achieve an enhanced reflection on the dynamics of long-term stability and financial development with the economics of crime.

The Literature Review section (2) provides a comprehensive overview of the economics of crime, including the roles played in the criminal laundering chain and the dynamics between businesses, governments, and civil society. It also discusses the Evolutionary Game Theory literature and the interconnections between innovation and regulation in the field of money laundering.

The Model Formulation section (3.1) presents our theoretical model based on evolutionary game theory, where we assume that in a given country or region, a sector is involved in illegal activities, and obtains a monetary profit if not caught in the money laundering scheme. The Results section (4) presents the equilibria for the replicator dynamics and the phase portrait of the model. Finally, the Conclusion section (6) summarizes the main findings and discusses their implications.

2 Literature Review

Several studies have been conducted on the economics of crime. It is observed that the dynamics between businesses, governments and civil society have changed considerably during the last two decades. Nevertheless, large organized crime chains have taken over commercial and banking structures, and the amount of profits generated as part of the production, trafficking and distribution of illegal rents is vast.

Recent scandals, such as Panama, Paradise and Pandora Papers, illustrate the tension between creating a favourable economic climate for international business and simultaneously preventing crimes such as corruption, money laundering and tax offences. A major report conducted by the EU Serious and Organised Crime Threat Assessment (2021) portrays in detail the roles played in the criminal laundering chain:

Criminals are highly skilled at exploiting the economy for their own purposes. Legal business structures, such as corporations or other entities, are used to facilitate virtually all types of criminal activity, impacting the nation. Criminals directly control or infiltrate legal business structures to facilitate their illegal activities. All types of legal businesses are potentially vulnerable to exploitation by serious and organised crime (KLEEMANS, 2007).

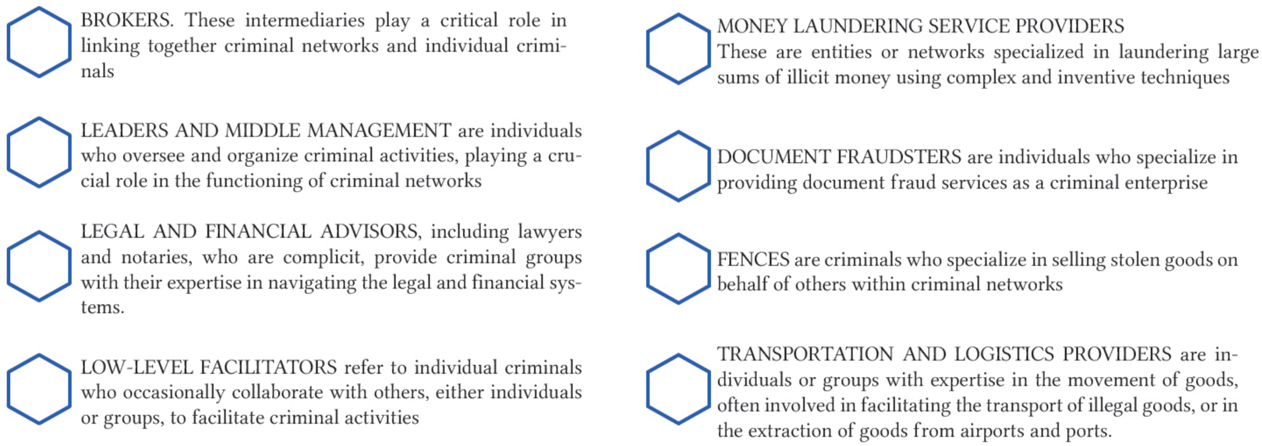

Figure 1: Critical Roles in Criminal Networks: High-Value Targets

According to Barone and Masciandaro (2011), criminals can use laundering not only to consume and spend money but also to invest money in both legitimate and illegal economic circuits. Moreover, the more their investments are successful and profitable, the more criminal groups gain strength, increasing the amount of corruption throughout the economy.

Naheem (2015), explains that today's criminals are digital natives. Almost all criminal activities practised today have some online component, and some phenomena have migrated altogether. Online marketplaces, both on the web and the deep web offer access to illicit goods and services. Digital platforms offer access to encrypted communications and payment solutions that are somewhat untraceable.

Furthermore, it is interesting to note what the Evolutionary Game Theory literature concludes about how innovation alters the order and amplifies the network and materialism of money laundering. The chronology demonstrates that innovation and regulation in this digital environment are intertwined in an evolutionary game process in which game members determine tactics that ultimately define the stability and progress of the industry. As a result, an evolutionary game theory examination of the interconnections between innovation and regulation is suitable for determining the genuine dynamics of financial development and stability.

Going in this direction, the emergence and abundance of cooperation in a population of selfish individuals is a challenging problem in evolutionary dynamics, according to (PAN et al., 2018).

Liu et al. (2019), on the other hand, analyzed that the amount of information an individual uses to update itself plays a key role in the formation of cooperation. That is, individuals who use less information are more likely to cooperate than those who use more information.

In this sense, risks related to financial innovation encourage the development of financial means and regulatory systems. Going further, there are strong indications that this process is related to evolutionary games, in which players decide and promote their strategies that finally shape the stability and development of this sector. It is in this way that an analysis of the "regulation/innovation" interconnections is appropriate to achieve an enhanced reflection on the dynamics of long-term stability and financial development with the economics of crime.

2.1 Model Formulation

Following Masciandaro's (1998) theoretical model, we assume an evolutionary game to answer the proposed problem. In this game, we assume that in a given country or region, a sector is involved in illegal activities, and obtains a monetary profit of $y(m-c)$ if not caught in the money laundering scheme. The law enforcement authority here has a cost $-c$ if it decides to fight crime.

In evolutionary game theory, we want to know how different strategies develop through time in a population of people. The system of differential equations we examined depicts the dynamics of two strategies (labeled 1 and 2) in a population, where the proportion of people using strategy 1 is represented by p and the proportion of individuals using strategy 2 is marked by q. The system's parameters indicate characteristics that influence the payoffs and costs associated with each approach.

To examine the system's long-term behavior, we sought for locations where $\dot{p} = \dot{q} = 0$, which reflect equilibria in which the proportions of individuals playing each strategy remain constant across time.

We assume that the criminal sector must launder at least a fraction $0 < \lambda < 1$ of its illegal profits, while the rest will be reinvested in illegal markets. We also assume that each laundering operation has a cost to the criminal sector, represented by the price of money laundering services.

The net value of organized crime activities can be considered as a whole, although it is well recognized that organized crime groups are becoming increasingly heterogeneous and dynamically organized in structural terms, moving towards integrated networks away from the well-known pyramidal monopolies.

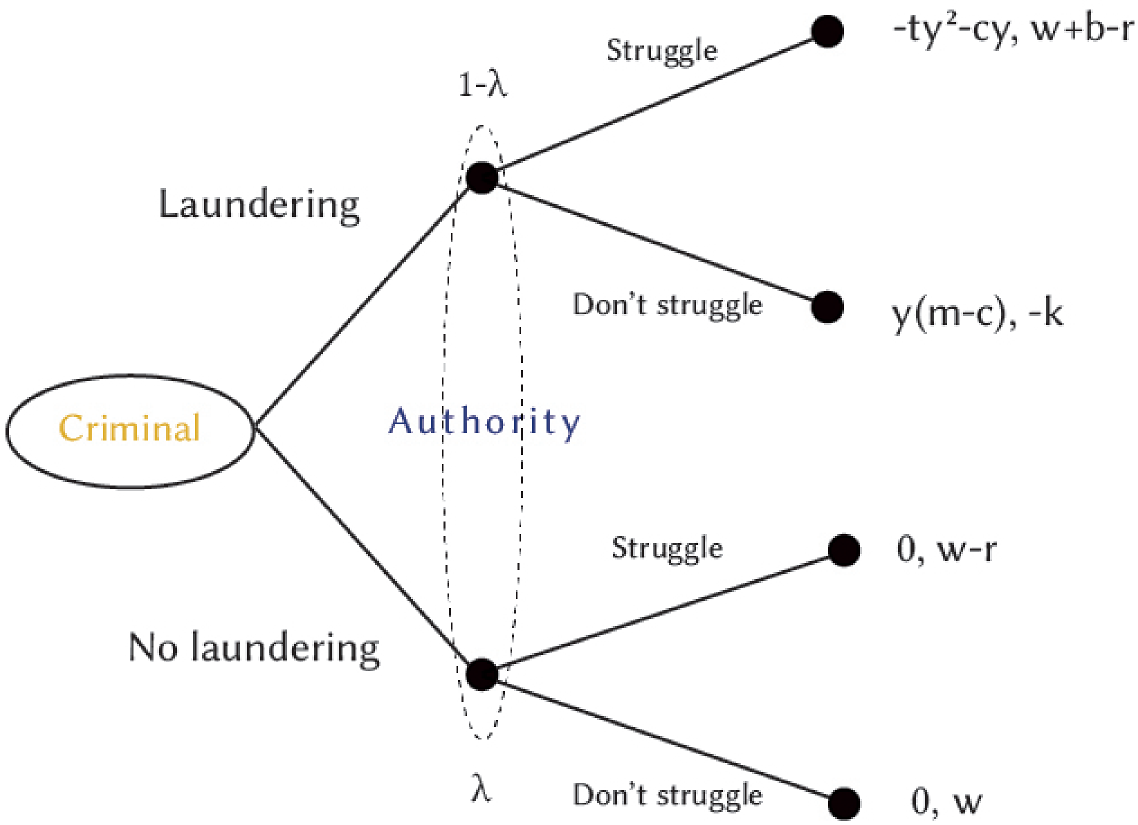

Figure 2: Evolutionary Game Model

It is already well known that each individual or criminal group that produces income from illegal activities faces a special type of transaction cost, linked to the fact that the use of these revenues increases the probability of discovery of the crime and, therefore, of incrimination (13). If the criminal decides to launder money and the police authority does not fight the crime, he will have his cost $k < 0$. If the criminal launders and the authority combats it, he will have a bonus known as $b > 0$.

As has been pointed out recently, organized crime groups tend to invest a lot of energy in diversifying money laundering procedures from primitive to overly complex schemes (1). Money laundering is the necessary interface between licit and illicit markets. On the one hand, some of the illegal funds need to be spent: minimizing the risks of incrimination comes at a price; the criminal sector has to pay a price. On the other hand, we assume that some of the dirty money will be reinvested in the illegal market without concealment. For example, in all illicit services, cash is by definition the currency of choice, operating in a closed circuit separate from the legitimate market.

Reinvestment in criminal markets is a distinctive feature of genuine organized crime groups, given their tendency to specialize. Organized crime tends to acquire specialized functions to increase its illegal business. Economic analysis has pointed out that transaction costs can be minimized through effective money laundering actions, a means of concealment that separates financial flows from their illegal origin (2). Given that money laundering is not a free lunch, each agent will choose its ideal behaviour, deciding what type of concealment procedure should be used.

Reinvestment in criminal markets is a distinctive feature of genuine organised crime groups, given their tendency to specialise. Organised crime tends to acquire specialised functions to increase its illegal business.

			Authority	
			λ	$1 - \lambda$
			S	DS
	θ	L	$-ty^2 - cy,\ w + b - r$	$(m - c)y, -k$
Criminal				
	$1 - \theta$	NL	$0,\quad w - r$	$0,\ w$

Figure 3: Matrix Game

It is important to note here that for the outcome of this evolutionary game, we want to find equilibrium in the steady state if only if

$$0 < \lambda = \frac{m - c}{ty^2 + cy + m - c} < 1$$

By setting the replicator functions and the Jacobian matrix, we could find the most likely inner equilibrium to be

$$\left(\frac{m-c}{ty^2+cy+m-c}, \frac{r}{w+b+k}\right)$$

3 Nada ainda

3.1 Finding Equilibria

To find the points at which $\dot{p} = \dot{q} = 0$, we need to simultaneously solve the two differential equations:

$$\begin{aligned} \dot{q} &= q(1-q)[(1-p)(m-c)y - p(ty^2+cy)] = 0 \\ \dot{p} &= p(1-p)[q(w+b+k-r) - (1-q)r] = 0 \end{aligned} \tag{1}$$

There are three possible cases:

- $q = 0$ or $q = 1$: In this case, the first equation reduces to $\dot{q} = 0$, which means that q is constant over time. To find the values of p at which q is constant, we need to solve the second equation for p when $q = 0$ and when $q = 1$. This gives us two sets of equilibria:
  - When $q = 0$, we have $p = 0$ or $p = 1$.
  - When $q = 1$, we have $p = (w+b+k-r)/(2w+b+k-r)$ or $p = 1$.
- $p = 0$ or $p = 1$: In this case, the second equation reduces to $\dot{p} = 0$, which means that p is constant over time. To find the values of q at which p is constant, we need to solve the first equation for q when $p = 0$ and when $p = 1$. This gives us two sets of equilibria:
  - When $p = 0$, we have $q = 0$ or $q = 1$.
  - When $p = 1$, we have $q = \left(1 - \frac{c}{my}\right)$ or $q = 1$.
- q and p are both non-zero and non-one: In this case, we need to solve both equations simultaneously for p and q. This is a bit more complicated and typically requires numerical

methods or graphical analysis.

To summarize, the equilibria for $\dot{p} = \dot{q} = 0$ are:

- When $q = 0$, we have $p = 0$ or $p = 1$.
- When $q = 1$, we have $p = (w + b + k - r)/(2w + b + k - r)$ or $p = 1$.
- When $p = 0$, we have $q = 0$ or $q = 1$.
- When $p = 1$, we have $q = \left(1 - \frac{c}{my}\right)$ or $q = 1$.
- In the case of non-zero and non-one values of p and q, the equilibria must be found using numerical methods or graphical analysis.

4 Results

In this study, we investigate the equilibrium correspondence and dynamics of a system that models the behaviour of individuals or criminal groups involved in money laundering activities. The dynamical system is defined by two variables, p and q, where p represents the proportion of dirty money laundered, and q signifies the proportion of money reinvested in the illegal market. By generating a phase portrait illustrating the vector field, nullclines, and equilibrium points, we aim to understand the system's behaviour and its implications for organized crime groups and law enforcement authorities.

4.1 Vector Field

The phase portrait exhibits a vector field that displays the direction and magnitude of change in p (laundered money) and q (reinvested money) at each point in the phase plane. The trajectory of the system can be inferred from the direction of the arrows, while the color and size represent the rate of change (magnitude). The vector field provides valuable insights into the regions of the phase plane where the equilibrium between money laundering and reinvestment is stable or unstable, with implications for both criminal groups and law enforcement strategies.

4.2 Nullclines

The p-nullcline (green dashed line) and q-nullcline (blue dashed line) represent the curves where the p or q derivatives are zero, respectively. The equilibrium points of the system occur at the intersections of these nullclines. Analyzing the nullclines facilitates the identification of the system's equilibrium points and provides insights into the system's behavior in the vicinity of these points, which may have implications for the effectiveness of money laundering and reinvestment strategies pursued by criminal groups.

4.3 Equilibrium Points

We identified five equilibrium points for the dynamical system: $(0, 0)$, $(1, 1)$, $(1, q)$, $(p, 1)$, and (p^*, q^*). These points represent steady-state conditions where both the p and q derivatives are zero, suggesting a balance between money laundering and reinvestment in the illegal market. We examined the stability of these equilibrium points by analyzing the behaviour of the system around them.

$(0, 0)$ and $(1, 1)$: The phase portrait demonstrates that the system tends to move away from both these points, indicating that they are unstable equilibrium points. This suggests that criminal groups are unlikely to find a stable balance between money laundering and reinvestment when either of these activities is completely dominant.

$(1, q)$ and $(p, 1)$: The stability of these points is contingent on the parameter values, which may represent factors such as the costs and benefits associated with money laundering and reinvestment, as well as law enforcement efforts. Further analysis using linearization techniques or numerical simulations is necessary to determine the stability of these points under various conditions.

(p^*, q^*): As an interior equilibrium point, the stability of (p^*, q^*) depends on the parameter values. To investigate its stability, we recommend employing linearization techniques, such as the Jacobian matrix, or conducting numerical simulations. The stability of this point may have implications for the optimal balance between money laundering and reinvestment for criminal groups, as well as the effectiveness of law enforcement efforts to combat these activities.

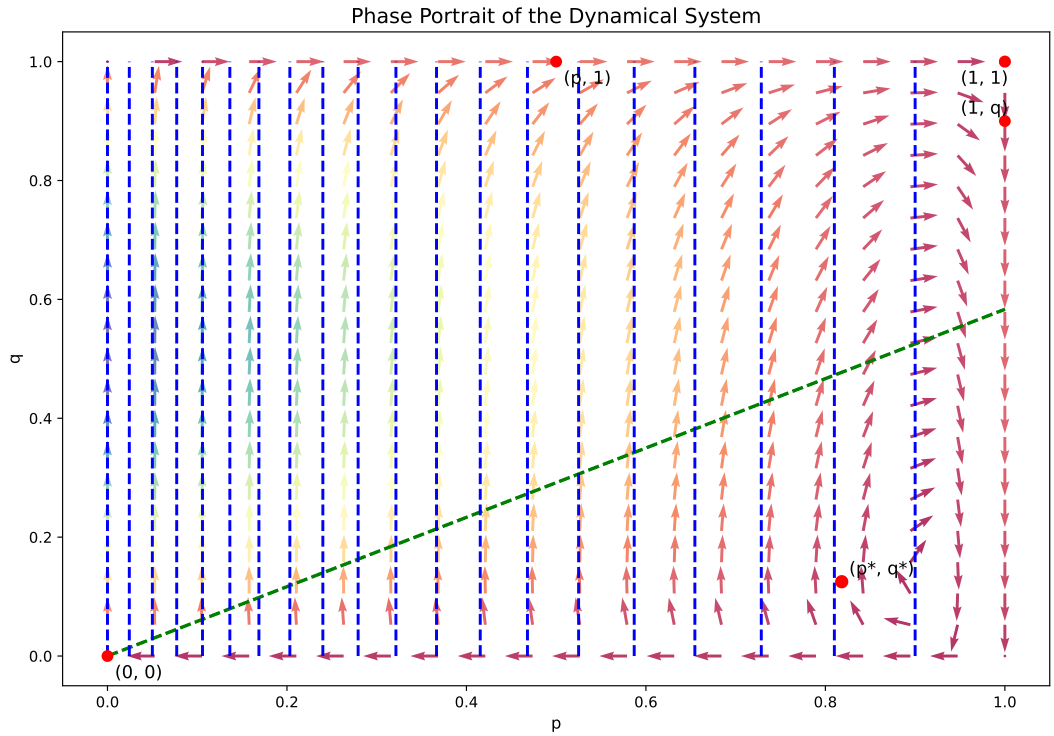

Figure 4: Directional Field Plot

Our findings reveal valuable insights into the equilibrium correspondence and the overall dynamics of the investigated dynamical system, which models the money laundering and reinvestment behavior of individuals or criminal groups. By understanding the system's behavior around the equilibrium points, researchers and practitioners can better predict the response of criminal groups and law enforcement authorities under different conditions and parameter values.

Furthermore, investigating the impact of various factors, such as the level of resources devoted to law enforcement and the sophistication of money laundering techniques, could enhance our understanding of the complex interplay between criminal activities and the efforts to combat them. Ultimately, this research can inform the development of more effective policies and interventions aimed at disrupting the financial flows of organized crime groups and reducing the negative social consequences associated with their illicit activities.

To summarize, our study highlights the importance of understanding the equilibrium correspondence and dynamics of systems that model complex criminal activities such as money laundering. By applying mathematical modeling and analysis techniques, we can gain insights into the behavior of such systems, identify equilibrium points, and assess their stability under different conditions. Such insights can inform the development of more effective strategies for combating organized crime and reducing its negative impact on society.

5 Discussions

5.1 Types and Bayesian Games

The paper shows that sequential sampling equilibrium can be extended to accommodate Bayesian games, where players have different types with different beliefs and payoffs. Specifically, sequential sampling equilibrium can be extended to games described by $\langle I, A, \Theta, \rho, u \rangle$, where I denotes the finite set of players, A is the (finite) set of action profiles, $\Theta = \times_{i \in I} \Theta_i$ is the (finite) set of type profiles, where Θ_i represents player i's possible types, $u : A \times \Theta \to \mathbb{R}$ is the payoff function, and $\rho \in \Delta(\Theta)$ is the distribution over types.

Each player i with type θ_i learns about the joint distribution of opponents' action profiles and type profiles, $q_i \in \Delta(A_{-i} \times \Theta)$, by sequentially sampling from q_i at cost c_i and stopping according to the earliest optimal stopping time τ_{i,θ_i}.

The paper shows that a sequential sampling equilibrium in this setting corresponds to a fixed point such that $\sigma_{i,\theta_i} = \mathbb{E}q_i[\sigma^{i,\theta_i}(\mu_i | y_{\tau_{i,\theta_i}})]$, where $\sigma^{i,\theta_i}(\mu_i)$ is a selection of best responses given belief μ_i, and $qi, \theta_i(a_{-i}, \theta) = \rho(\theta) \times_{j \neq i} \sigma_{j,\theta_j}(a_j)$ for every $a_{-i} = (a_j)j \neq i \in A{-}i$ and $\theta = (\theta_j)_{j \in I} \in \Theta$.

The paper also notes that different assumptions on players' beliefs will give rise to different equilibria. By applying similar arguments, behavioral implications can be obtained by comparing across types. For instance, if the payoff to action a_i is higher for type θ_i than for θ_i', everything else equal, in every sequential sampling equilibrium type θ_i chooses action a_i more often and faster (in the sense of Theorem 4) than type θ_i'.

Overall, the extension of sequential sampling equilibrium to Bayesian games allows for a more realistic representation of decision-making problems where players have different types and beliefs. The approach provides a useful tool for analyzing and understanding decision-making behavior in more complex games.

6 Conclusion

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Tables

Appendix A. Interior Equilibria

The following Python code generates the phase portrait plot:

```
import numpy as np
import matplotlib.pyplot as plt

#Define the system of differential equations
def dP_dt(P, t=0):
    p, q = P
    dq_dt = q*(1-q)((1-p)(m-c)y-p(ty**2+cy))
    dp_dt = p*(1-p)(q(w+b+k-r)-(1-q)*r)
return [dp_dt, dq_dt]

#Define the parameter values
m = 10
c = 1
y = 1
t = 1
w = 1
b = 0.5
k = 0.1
r = 0.2

#Calculate the fifth equilibrium point
p_star = (m-c)/(ty**2 + cy + m - c)
q_star = r/(w+b+k)

#Define the range of p and q values
p_range = np.linspace(0, 1, 20)
q_range = np.linspace(0, 1, 20)

#Create a grid of p and q values
P, Q = np.meshgrid(p_range, q_range)

#Calculate the derivatives at each point in the grid
dp, dq = dP_dt([P, Q])

#Normalize the derivatives to create a unit vector field
magnitude = np.sqrt(dp2 + dq2)
magnitude[magnitude == 0] = 1e-6
dp_norm = dp / magnitude
dq_norm = dq / magnitude

#Create a plot of the vector field
fig, ax = plt.subplots(figsize=(12, 8))
ax.quiver(P, Q, dp_norm, dq_norm, magnitude, cmap='Spectral', alpha=0.8)

#Add the nullclines
p_nullcline = Q*(w+b+k-r)/(2w+b+k-r)
q_nullcline = (1-c/my)/(1+((1-P)(ty+c))/(P + 1e-6))
ax.plot(p_range, p_nullcline, 'g--', linewidth=2)
ax.plot(q_nullcline, q_range, 'b--', linewidth=2)

#Add the equilibria
```

```python
ax.plot([0, 1], [0, 1], 'ro', markersize=7)
ax.plot([1, 1], [(1-c/my), 1], 'ro', markersize=7)
ax.plot([(w+b+k-r)/(2(w+b+k-r)), 1], [1, 1], 'ro', markersize=7)
ax.plot(p_star, q_star, 'ro', markersize=8)

#Add labels and captions
ax.set_xlabel('p', fontsize=11)
ax.set_ylabel('q', fontsize=11)
ax.set_title('Phase Portrait of the Dynamical System', fontsize=14)

# Add equilibrium captions on the plot
ax.annotate('(0, 0)', xy=(0, 0), xytext=(5, -15), textcoords='offset points', fontsize=12, color='
    black')
ax.annotate('(1, 1)', xy=(1, 1), xytext=(-30, -15), textcoords='offset points', fontsize=12, color
    ='black')
ax.annotate('(1, q)', xy=(1, 1-c/(my)), xytext=(-30, 5), textcoords='offset points', fontsize=12,
    color='black')
ax.annotate('(p, 1)', xy=((w+b+k-r)/(2(w+b+k-r)), 1), xytext=(5, -15), textcoords='offset points',
     fontsize=12, color='black')
ax.annotate('(p*, q*)', xy=(p_star, q_star), xytext=(5, 5), textcoords='offset points', fontsize
    =12, color='black')

ax.legend(loc='upper left', fontsize=12)

#Save the figure as a PDF
plt.savefig('phase_portrait.pdf', format='pdf', bbox_inches='tight')
#Show the plot
plt.show()
```

Listing 1: Python code for phase portrait plot