PAIRS TRADING: AN EMPIRICAL STUDY

Aitor Fiz

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Tutor: Dr. Alfonso Novales

Universidad Complutense de Madrid

Universidad del País Vasco

Universidad de Valencia

Universidad de Castilla-La Mancha

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Pairs Trading: An Empirical Study

Author:

Aitor Fiz

Supervisor:

Dr. Alfonso Novales

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[1]University of the Basque Country, Complutense University of Madrid, University of Castilla La Mancha and University of Valencia.

Abstract

The main objective of this study is to answer the following question: Is Pairs Trading a profitable strategy in an increasingly more sophisticated financial industry? To do so, we will carry out an empirical study by implementing a cointegration approach to Pairs Trading strategy. In previous literature, some authors have given an answer to this question taking into account another approach to Pairs Trading (the "distance approach"). Specifically, they concluded that there has been a declining trend in the profitability due to different reasons such as the rise in hedge fund activity or the lower confidence in the underlying convergence properties. However, we will try to answer the question by using the cointegration approach. Two more questions related to the market neutrality of Pairs Trading and the effects of volatility on the strategy will also be taken into account.

The empirical study is based on the analysis of Pairs Trading profitability on the firms belonging to Euro Stoxx 50 and DJIA during 2001-2014. The strategy is made up of two steps. In the first one, we identify the cointegrated pairs of stocks; i.e., the pairs that share a long-run equilibrium. In the second one, we detect, in a standard deviation metric, short-run deviations of the cointegrated pairs from their long-run equilibrium. By investing appropriately, a profit can be earned by using cointegration properties. Furthermore, we will carry out the study for different sub-periods as a robustness measure. Additionally, we will perform a sensitivity analysis and a research of Pairs Trading by sectors.

The main conclusion is that Pairs Trading is still a gross profitable strategy but the impact of trading fees has a very noticeable negative effect on the net result. However, Pairs Trading is still able to get net profits. Furthermore, we found evidence that Pairs Trading is not a market neutral strategy and that the volatility is a crucial variable on the profitability.

Keywords: Pairs trading; cointegration; mean reversion; hedge ratio; trading strategy.

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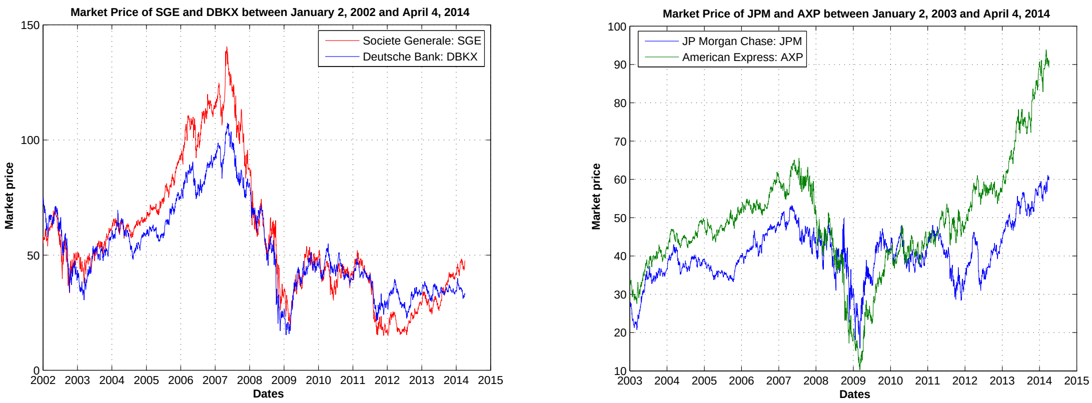
Chapter 1

Introduction

Pairs Trading is a well-known statistical arbitrage investing strategy which started in the early 1980s and it has been applied by many hedge funds during the last three decades. It was developed by the first *quants* in Wall Street. They actually were looking for statistical rules to find and take advantage of short-run deviations from a consistent long-run equilibrium between two determined assets. The very first methods to detect the potential trading pairs were mainly based on correlation and other non-parametric decision rules. Nonetheless, theses initial approaches to Pairs Trading were not taking into account a critical concept that is essential for the profitability of the strategy; the *mean reversion* property that takes place in a cointegration context. Therefore, by identifying trading pairs using a cointegration approach, places us in a significant better position with respect to the initial approaches.

Cointegration is a statistical relationship where two time series that are both integrated of same order d, $I(d)$, can be linearly combined to produce a single time series which is integrated of order $d - b$, where $b > 0$. When applying the definition to Pairs Trading, we refer to the case where two $I(1)$ stock price series are linearly combined to produce a stationary, or $I(0)$, portfolio time series; i.e., the portfolio satisfies the mean reversion property. The mean reversion value is actually the long-run equilibrium displayed by the pair/portfolio. Thereby, if the portfolio moves away from its long-run equilibrium, then we should invest appropriately in order to earn a profit from this purely short-run deviation.

Examples of cointegration can be found between the evolution of a future contract and its underlying asset as well as between interest rates with similar maturities[1]. Stocks also exhibit cointegrating relationships. As we know, cointegration takes place when two financial assets (*I(1)* time series) share a long-run equilibrium. Figure 1.1 is an example of possible cointegrating relationships. It shows, on one hand, the market price of Societe Generale and Deutsche Bank, two of the most significant banks in the Euro zone; and on the other, the market price of JP Morgan Chase and American Express. Although there seems that both pairs of stocks share a long-run equilibrium, only the pairs consisting on SGE and DBKX presents a cointegrating relationship according to the definition of cointegration given above.



(A) SGE and DBKX. (B) JPM and APX.

FIGURE 1.1: Examples of possible cointegrating relationship between two pairs of stocks.

In this study we will implement the Engle-Granger [13] and Johansen [23] methodologies to detect cointegrated pairs. Thus, the Engle and Granger's methodology is made up of two steps. In the first one, once we have checked that the time series two stocks are both I(1), we implement the following OLS regression

$$StockA_t = \alpha + \beta StockB_t + u_t \qquad u_t \sim \mathcal{N}(0,1) \tag{1.1}$$

[1]An important hedge fund run and co-founded by Nobel Prize in economics' Robert Merton, Long-Term Capital Management (LTCM), applied Pairs Trading using complex mathematical models to take advantage of fixed income arbitrage deals (they traded the pair consisting on the 29 and three quarter year old bond and the 30 year bond just issued by the Treasury). The fund earned high returns for several years (annualized return of over 21% (after fees) in its first year, 41% in the second year and 43% in the third year) but later lost US$4.6 billion in 1998 and was closed in early 2000.

where $StockA_t$ and $StockB_t$ are the market price in t of Stock A and B, respectively. In the second step, the estimated residual series $\{\hat{u}_t\}$ is then tested for stationarity using a unit root test (augmented Dickey-Fuller test). If it turns out that $\{\hat{u}_t\}$ is an *I(0)* time series, then stock A and B are cointegrated since we have found a linear combination of two non stationary variables that is stationary.

The *spread* is defined as

$$\hat{Spread}_t = StockA_t - \hat{\beta} StockB_t = \hat{\alpha} + \hat{u}_t \tag{1.2}$$

By applying OLS regression properties, we know that $E[\hat{u}] = 0$. We take expected value on 1.2

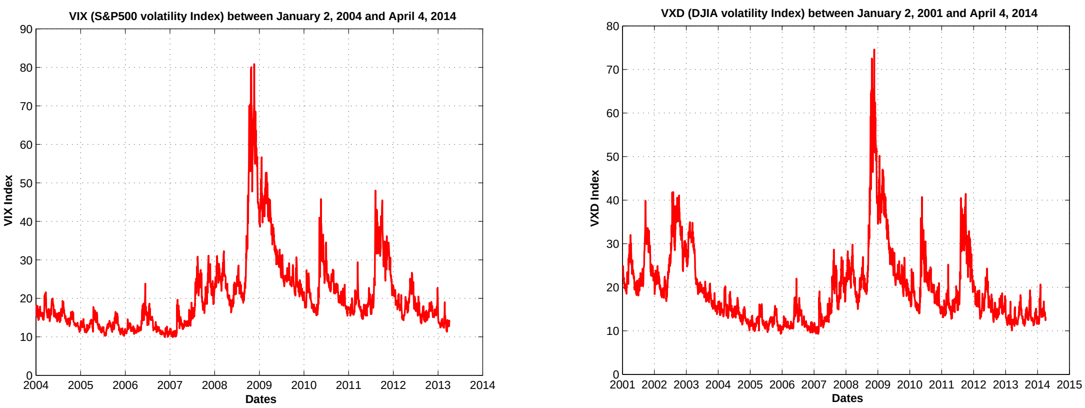
$$E[\hat{Spread}_t] = E[StockA_t - \hat{\beta} StockB_t)] = \hat{\alpha} \tag{1.3}$$

Equation 1.3 shows that a portfolio consisting of going *long* 1 monetary unit of stock A and *short* $\hat{\beta}$ monetary units of stock B has a long-run equilibrium value of $\hat{\alpha}$ and any deviations from this value are merely temporary fluctuations $\{\hat{u}_t\}$. Consequently, the portfolio will always revert back to its long-run equilibrium value since $\{\hat{u}_t\}$ is a mean-reverting time series.

This is how we identify the trading Pairs. Then, the trading strategy consists on open a trade in the portfolio as long as the spread hit two consecutive times a determined trigger level. The reason why we require the spread to hit two consecutive times the trigger is due to the fact that if we know that the spread is a mean-reverting time series, we expect it to revert back to its historical expectation. Therefore, the strategy begins when the spread is in its way back to its historical mean. Risk control strategies will also be taken into account.

In some studies such as Gatev et al. (2007) [18] and Do et al. (2009) [9], there are evidence that there has been a declining trend in the profitability from Pairs Trading. In those studies, the “distance approach” to Pairs Trading was taken into account. However, we will try to do the same study by implementing, in this case, the cointegration approach. Thus, we wonder if Pairs Trading strategy is still working nowadays.

There are also two significant questions that we are going to answer. The first one is related to the volatility. As we know, the spread must hit the trigger level in order to open a trade. Therefore, intuition tells us that Pairs Trading strategy might have a better performance in high volatile periods, such as financial crises. However, if there is extreme volatility, the risk control strategies will be activated to limit losses. Therefore, volatility will be a critical variable to study the profitability of this strategy. Just to get an overview of the volatility present in the market, figure 1.2 shows two volatility indices[2]. We will see how Pairs Trading performs in periods of extreme volatility.



(A) VIX Index. (B) VXD Index.

Figure 1.2: VIX and VXD indices.

The second question is about the market neutrality of Pairs Trading. Some authors such as Nath (2003) [25] and Lin *et al.* (2006) [24] , implicitly classify the Pairs Trading as a *market-neutral* strategy just because it belongs to the *"long/short"* equity investing strategies, even if the resulting portfolio (based on the cointegrating vector) may exhibit some market risk. A portfolio or a strategy is said to be *market-neutral* if its performance exhibits zero correlation with the significant market performance (a zero-beta portfolio). We will analyse whether the Pairs Trading is a market neutral strategy or not.

[2]VIX measures the volatility of the S&P500 index and is considered as a general measure of the level volatility around equity markets. VXD is the index that measures the volatility of the DJIA stock market index. The reason of including VXD index is that we will implement Pairs Trading on stocks belonging to DJIA index.

Finally, the structure of this study will be as follows. Firstly, in chapter two, we will see the four main methods to implement Pairs Trading. Furthermore, we will take a look to the last significant literature about this topic. Secondly, in the third chapter, we will define what cointegration is and how to test it. Thirdly, in chapter four, we will perform an empirical study of Pairs Trading strategy based on a cointegration approach. The structure followed in this empirical analysis will made up of a brief introduction, a description of data as well as the methodology implemented. Besides, we will implement Pairs Trading by sectors and a sensitivity analysis is going to be done. Lastly, the obtained results will be shown. Finally, in chapter five, we will end the study with the final remarks and conclusions.

Chapter 2

Literature Review

There are four main methods to implement Pairs Trading strategy. Namely, the conintegration method, the distance method, the stochastic spread method and the stochastic residual spread method.

The cointegration method, which is the one we are going to develop in this empirical study, is outlined in Vidyamurthy (2004) [32]. The non-parametric distance method is shown by Gatev, Goetzmann and Rouwenhorst (1999, 2007) [17] and [18] and Nath (2003) [25] for purposes of empirical testing. Finally, the stochastic spread and the stochastic residual spread methods are proposed more recently by Elliot, van Der Hoek and Malcolm (2005) [15] and Do, Faff and Hamza (2006) [10], respectively. These two latter methods seek to parametrize Pairs Trading by explicitly modelling the mean-reverting behaviour of the spread.

2.1 Cointegration method

We can find out the application of the cointegration method in Alexander and Dimitriu (2002) [1] and [2], Herlemont (2003) [20], Vidyamurthy (2004) [32], Lin *et al.* (2006) [24], Schmidt (2008) [31], and Puspaningrum (2012) [29].

The cointegration approach outlined in Vidyamurthy (2004) [32] is an attempt to parametrise a Pairs Trading strategy based on a cointegration framework as defined in Engle and Granger (1987) [13]. Vidyamurthy shows how to implement Pairs Trading strategy without empirical results.

Cointegration is a statistical relationship where two time series that are both integrated of same order d, $I(d)$, can be linearly combined to produce a single time series which is integrated of order $d-b$, where $b>0$. In its application to pairs trading, we refer to the case where two $I(1)$ stock price series are linearly combined to produce a stationary, or $I(0)$, portfolio time series.

Cointegration incorporates the property of mean reversion into a Pairs Trading framework which is the single most important statistical relationship required for success in the strategy. If the value of the cointegrating portfolio (consisting of a pair of stocks) is known to fluctuate around its equilibrium value, then any deviations from this value can be traded in order to get a profit by investing accordingly.

In order to test for cointegration, Vidyamurthy (2004) makes use of the the Engle and Granger's methodology. This cointegration test consists, first, of carrying out the following OLS regression

$$log(P_t^A) = \mu + \gamma log(P_t^B) + \epsilon_t \qquad \epsilon_t \sim \mathcal{N}(0, \sigma_\epsilon^2) \tag{2.1}$$

where $log(P_t^A)$ and $log(P_t^B)$ are the log prices of stocks A and B, respectively. Once the OLS regression is done and rearranging terms in equation 2.1, we get the *cointegrating equation*

$$log(P_t^A) - \hat{\gamma} log(P_t^B) = \hat{\mu} + \hat{\epsilon}_t \tag{2.2}$$

where $\hat{\gamma}$ is known as the *cointegrating coefficient* and the constant term $\hat{\mu}$ captures some sense of "premium" in stock A versus stock B[1]. The estimated residual series $\{\hat{\epsilon}_t\}$ is then tested for stationarity using the augmented Dickey-Fuller test (ADF). If it turns out that $\{\hat{\epsilon}_t\}$ is an *I(0)* time series, then it means that we have found out a unique[2] linear combination of two *I(1)* time series that is *I(0)* and, therefore, stocks A and B are cointegrated. Under this procedure, results are sensitive to the ordering of the variables. For instance, if instead $log(P_t^B)$ is regressed against $log(P_t^A)$ then a different residual series $\{\hat{\epsilon}_t'\}$ will be estimated

[1] As stated in [10], [13] and [31].

[2] Engle and Granger (1987) [13] proved that if a cointegrating relationship between two stocks does exist, then the cointegrating coefficient ($\hat{\gamma}$) is unique.

from the same sample. This issue can be resolved using the *t-statistics* from Engle and Yoo (1987) [14]. We will see it with more detail in chapter three.

The *spread* is defined as

$$Spr\hat{e}ad_t = log(P_t^A) - \hat{\gamma} log(P_t^B) = \hat{\mu} + \hat{\epsilon}_t \tag{2.3}$$

We take expected value on 4.2

$$E[Spr\hat{e}ad_t] = E[log(P_t^A) - \hat{\gamma} log(P_t^B)] = E[\hat{\epsilon}_t + \hat{\mu}] \tag{2.4}$$

By applying OLS regression properties, we know that $E[\hat{\epsilon}] = 0$. Therefore,

$$E[Spr\hat{e}ad_t] = E[log(P_t^A) - \hat{\gamma} log(P_t^B)] = \hat{\mu} \tag{2.5}$$

Equation 2.5 shows that a portfolio[3] consisting of going *long* 1 monetary unit of stock A and *short* $\hat{\gamma}$ monetary units of stock B has a long-run equilibrium value of $\hat{\mu}$ and any deviations from this value are merely temporary fluctuations $\{\hat{\epsilon}_t\}$. Consequently, the portfolio will always revert back to its long-run equilibrium value since $\{\hat{\epsilon}_t\}$ is an I(0) time series, i.e., a mean-reverting time series. Vidyamurthy (2004) [32] develops a trading strategy based on the mean reverting behaviour of the portfolio. The trading starts by opening a *long* position in the portfolio (buy 1 monetary unit of stock A and sell or short $\hat{\gamma}$ monetary units of stock B) when the spread is sufficiently below (threshold equals Δ) its long-run equilibrium $(\hat{\mu} - \Delta)$ and similarly, open a *short* position in the portfolio (sell or short 1 monetary unit of stock A and buy $\hat{\gamma}$ monetary units of stock B) when it is sufficiently above its long-run value $(\hat{\mu} + \Delta)$. Once the portfolio historical average reverts back to its long-run equilibrium, the portfolio positions are closed (by taking just the opposite positions) and we get a gross profit of Δ monetary units per trade.

At this point, the main question when developing a trading strategy is what value of Δ is going to maximise profits. Vidyamurthy (2004) [32] presents both a parametric approach and a non-parametric empirical approach for conducting this analysis.

[3] A portfolio built by using the "Cointegration Coefficients Weighted" (CCW) rule applied by Lin *et al.* (2006) [24] which consists of taking the cointegrating vector; i.e, $[1; -\hat{\gamma}]$, as the weights invested in stock A and stock B, respectively.

The first approach models the estimated residuals ($\hat{\epsilon}_t$) as an ARMA (Auto-Regressive Moving Average) process. Taking this assumption, Rice's formula, Rice (1945) [30], is used to calculate the rate of zero crossings and level crossings for different values of Δ in order to plot the profit function. The profit function is the profit per trade multiplied by the number of trades. The value of Δ which maximises the profit function is chosen as the trading trigger, so that it is the responsible for giving signals for opening positions in the cointegrated portfolio.

On the other hand, the non-parametric approach constructs an empirical distribution of zero and level crossings based on the estimation sample. The optimal value of Δ is chosen so as to maximise the profit function from the estimation sample. This value is then applied to real time portfolio construction. A fundamental assumption of this non-parametric approach to determining Δ is that the observed dynamics of the estimated residuals ($\hat{\epsilon}_t$) will continue into the future. This latter approach avoids a possible mis-specification since it is model-free. Furthermore, using Rice's formula to estimate the number of trades is not correct because it calculates the number of crossings without giving restrictions when the trade has opened. Instead of using Rice's formula, the first passage time of stationary time series should be used [29].

The method described by Vidyamurthy (2004) [32] may be exposed to errors arising from the econometric techniques applied. Firstly, the Engle and Granger's 2-step approach renders results sensitive to the ordering of variables, therefore the residuals may have different sets of statistical properties. Secondly, if the bivariate series are not cointegrated, the "cointegrating equation" results in *spurious estimators* which would have the effect of making any mean reversion analysis of the residuals unreliable. Do and Faff (2009) [9] also criticized the difficulty of cointegration approach in associating it with theories on asset pricing. Therefore, we will also make use of the Johansen test for cointegration in order to overcome these possible problems.

Lin *et al.* (2006) [24] proposed a Pairs Trading strategy based on a cointegration technique called the Cointegration Coefficients Weighted (CCW) rule. They also derived the minimum profit per trade using this technique. We will see their derivation in chapter three because, in fact, this is the procedure we are going to implement in this empirical study of Pairs Trading.

2.2 Distance method

Application of this method can be found out in Gatev *et al.* (1999 [17], 2007 [18]), Nath(2003) [25], Engelberg *et al.* (2008) [12], Do and Faff (2009) [9], Perlin (2009) [26], Huafeng *et al.* (2012) [21] and Pizzutilo (2013) [28]. However, Gatev *et al.* (1999, 2007) are the most cited papers in Pairs Trading. Under the distance approach, the co-movement in a pair is measured by what is referred to as the "distance", or the sum of squared differences between the two normalized price series.

Implementing the simplest form of the Pairs Trading strategy involves two steps as can be seen in [17] or [21]. First, they match pairs based on normalized price differences ("distance") over a certain period. This is the *pairs formation period.* Once they have recognized the pairs, the *trading period* starts.

Specifically, on each day t, they compute each individual stock's normalized price (P_t^i) as

$$P_t^i = \prod_{\tau=1}^{t} 1 \times (1 + r_\tau^i)$$

where P_t^i is stock i's normalized price by the end of day t, τ is the index for all the trading days between the first trading day of the pairs formation period until day t, and r_τ^i is the stock's total return, dividends included, on day τ. After obtaining the normalized price series for each stock, at the end of the pairs formation period, we compute the following squared normalized price difference measure, called "distance", between stock i and stock j,

$$PD_{i,j} = \sum_{t=1}^{N_t} (P_t^i - P_t^j)^2$$

where $PD_{i,j}$ is the squared normalized price difference measure between stock i and stock j, N_t is the total number of trading days in the pairs formation period, P_t^i and P_t^j are the normalized prices for stock i and stock j, respectively on trading day t. If there are N stocks under consideration, we need to compute $\frac{N\times(N-1)}{2}$ normalized price differences. At this point, we can also compute the standard deviation of the normalized price difference as

$$\sigma(PD_{i,j}) = \frac{1}{N_t - 1} \sum_{t=1}^{N_t} \left\{ (P_t^i - P_t^j)^2 - \overline{(P_t^i - P_t^j)^2} \right\}$$

The next step during the pairs formation period is to identify pairs with the minimal normalized price differences, or "distance". They then pool all the pairs together and rank these pairs based on the pairwise normalized price difference. Once the pairs formation period has passed by, the "trading period" begins. The trading rules for opening and closing positions are based on a standard deviation metric. Each month, they consider a certain number of pairs with the smallest normalized price difference taken from the pairs formation period. If the stocks in the pair diverge by more than two standard deviations of the normalized price difference established during the estimation period, they buy the undervalued stock in the pair and sell the the overvalued one. Gatev *et al.* (1999 [17], 2007 [18]) wait one day after divergence before investing in order to mitigate the effects of *bid-ask* bounce and other market micro-structure induced irregularities. If turned out that the pair later converges, they unwind our position and wait for the pair to diverge again. Gatev *et al.* (1999, 2006) also provides results by industrial sector, where they restrict stocks to the same broad industry defined by S&P. This actually acts as a test for robustness of any net profits identified using the unrestricted sample of pair trades.

Gatev *et al.* (1998, 2007) proved that their pairs trading strategy after costs can be profitable. However, Do and Faff (2008) [9] by replicating the work done by Gatev *et al.* (1999, 2007) reported that the profit results of the strategy were declining. Moreover, Engelberg *et al.* (2008) [12] showed that the profitability from this strategy decreases exponentially over time.

Nath (2003) [25] also uses this Pairs Trading method to identify potential pair trades, although his approach does not identify mutually exclusive pairs. Nath (2003) keeps a record of distances for each pair, in an empirical distribution format so that each time t an observed distance crosses over the 15 percentile, a trade is opened for that pair. Contrary to Gatev *et al* (1999, 2007) it is possible under Nath's approach that one particular stock be traded against multiple stocks simultaneously. Another difference between Gatev *et al* (1999, 2007) and Nath (2003) is that in Gatev *et al* (1998, 2007) there are no risk management measures to prevent potential high losses. The risk management measure proposed in Nath (2003) is a stop-loss trigger to close the position whenever the distance hits the 5 percentile.

Nath (2003) also included two rules trying to improve the strategy. The first one is a maximum trading period in which all open positions are closed if distances have not reverted back to their equilibrium state inside a given time-frame. The second one is a rule which states that if any trades are closed early prior to the equilibrium reversion, then new trades on that particular pair are prohibited until such time as the distance of price series has reverted back. Moreover, Pizzutilo (2013) [28] analyses what happens with the profitability from Pairs Trading strategy when trading costs and restrictions to short selling are taken into account. He found that these constraints significantly affect the profitability of the strategy but that Pairs Trading still works by giving net profits. Nonetheless, he found evidence that restrictions to the number of shares that are allowed to be shorted have a relevant impact on the risk profile of the pairs portfolios.

As a final conclusion, the distance method exploits a statistical relationship between a pair of stocks, at a price level. As Do and Faff (2009) [9] noted, it is a model-free strategy and consequently, it has the advantage of not being exposed to model misspecification and mis-estimation. However, this non-parametric method lacks forecasting ability regarding the convergence time or expected holding period.

2.3 Stochastic Spread method

The application of this method, theory behind it and discussions about it can be found in Elliot *et al.* (2005) [15], Do *et al.* (2006) [10] and Herlemont (2008) [20].

Elliot *et al.* (2005) [15] outlined a method to Pairs Trading strategy which explicitly attempts to model the mean reverting behaviour of the spread in a continuous time setting. The observed spread, y_t, is defined as the difference between the two stocks prices instead of log prices. It is assumed that the observed spread is driven by a latent state variable x_k plus some measurement error captured by a Gaussian noise, ω_k. Therefore, the observed spread is defined as

$$y_k = x_k + H\omega_k$$

where x_k is the value of the state variable at time t_k for $k = 0, 1, 2, ...$ and $\omega_k \sim$ IID $\mathcal{N}(0, 1)$ with $H > 0$ being a constant measure of errors. The state variable x_k

is assumed to follow a mean reverting process since we are attempting to model the mean reverting behaviour of the spread, thus

$$x_{k+1} - x_k = (a - bx_k)\tau + \sigma\sqrt{\tau}\epsilon_{k+1} \tag{2.6}$$

where $\sigma > 0, a > 0, b > 0$ and $\{\epsilon_k\}$ is IID $\mathcal{N}(0,1)$ and independent of $\{\omega_k\}$.

Taking limits as k tends to infinite for the mean and the variance of the process, we get

$$\lim_{k\to\infty} \mu_k = \frac{a}{b} \tag{2.7}$$

$$\lim_{k\to\infty} \sigma_k^2 = \frac{\sigma^2\tau}{1-(1-b\tau)^2} \tag{2.8}$$

Therefore, the process mean reverts to $\mu = \frac{a}{b}$ with a mean reversion speed b and $x_k \sim \mathcal{N}(\mu_k, \sigma_k)$.

Equation 2.6 can also be written as

$$x_k = A + Bx_{k-1} + C\epsilon_k \tag{2.9}$$

with $A = a\tau \geq 0$, $0 < B = 1 - b\tau < 1$ and $C = \sigma\sqrt{\tau}$. The discrete process defined in 2.9 can be approximated by a continuous process, i.e. $x_k \approx X_t$ where $\{X_t|t \geq 0\}$ satisfies the following stochastic differential equation

$$dX_t = \rho(\mu - X_t)dt + \sigma dB_t \tag{2.10}$$

where $\rho = b$, $\mu = \frac{a}{b}$ and $\{B_t|t \geq 0\}$ is a standard Brownian motion.

Using the Ornstein-Uhlenbeck process as an approximation to 2.10, the first passage time result for X_t is proven to be

$$T = inf\{t \geq 0, X_t = \mu | X_0 = \mu + \frac{c\sigma}{\sqrt{2\rho}}\} = \hat{t}\rho \tag{2.11}$$

being $\hat{t}$

$$\hat{t} = 0.5ln\{1 + 0.5[\sqrt{(c^2 - 3)^2 + 4C^2} + C^2 - 3]\} \tag{2.12}$$

T refers to the time needed for the process X_t to reach the mean, μ, for the first time given that at time $t = 0$, the value for the process is $X_0 = \mu + \frac{c\sigma}{\sqrt{2\rho}}$. Variable c is a constant and coefficients A, B, C and D are estimated using the state space model and the Kalman filter[4] .

Elliot *et al.* (2005) [15] proposed a Pairs Trading strategy by firstly choose a positive value for c. They establish two alternative scenarios to enter a pair trade based on a constant bound defined as $\frac{c\sigma}{\sqrt{2\rho}}$. The first scenario occurs when the spread moves away form its mean and hits the upper bound, i.e., when $y_k \geq \mu + \frac{c\sigma}{\sqrt{2\rho}}$ and the second one happens when it hits the lower bound, i.e., $y_k \leq \mu - \frac{c\sigma}{\sqrt{2\rho}}$. Therefore, we should enter in a pair trade as long as the spread moves away from its mean value and hits the bound (upper or lower), knowing that the spread will revert back to its mean since it follows a mean reverting process. Once the pair trade has been opened, we should unwind the trade at time T, shown in equation 2.11.

Elliot *et al.* (2005) [15] did not give an explicit way of obtaining the optimal value of the constant c which is a crucial part of the definition of the threshold that triggers the strategy. Do *et al.* (2006) [10] pointed out that this model offers three relevant advantages from the empirical point of view, namely, it captures mean reversion which is key in pairs trading, it is a continuous time model and, as such, it is useful for forecasting purposes and the third advantage is that the model is fully tractable, being its parameters easily estimated by the Kalman filter in a state space setting. The estimator is a maximum likelihood estimator and optimal in the sense of minimum mean square error.

As we have mentioned before, the spread is defined in Elliot *et al.* (2005) [15] as the difference in prices. However, according to Do *et al.* (2006) [10], the observed spread should be defined as the the difference in log prices because, generally, the long-run mean of the price level difference in two stocks should not be constant, but widens as they increase and narrows as they decrease. The exception is when the stocks trade at similar price points. By defining the spread as the difference in log prices, this is no longer a problem. Despite the several advantages shown in Do *et al.* (2006) [10] related to this model, this method have a fundamental

[4] See Puspaningrum (2012) [29] for detailed information.

limitation in that it restricts the long-run relationship between the two stocks to one of return parity. In the long-run, the stock pairs chosen must provide the same return such that any departure from it will be expected to be corrected in the future[5]. This fact dramatically limits this model's generality as in practice it is rare to find two stocks with identical return series.

2.4 Stochastic Residual Spread method

This method can be found in Do *et al.* (2006) [10]. They propose a Pairs Trading strategy which differentiates itself from existing approaches by modelling mispricing at the *return level*, as opposed to the more traditional *price level*. The model also incorporates a theoretical foundation for a stock pairs pricing relationship in an attempt to remove *ad hoc* trading rules which are prevalent in previous studies. This approach assumes that there exists some long-run equilibrium in the relative valuation of two stocks measured by some spread. This mispricing is defined as the state of disequilibrium which is quantified by, what they call, a *residual spread function*: $G(R_t^A, R_t^B, U_t)$, where U denotes some exogenous vector potentially present in formulating the equilibrium. The term *residual spread* emphasizes that the function captures any excess over and above some long-run spread and may take non-zero values depending on the formulation of the spread. As in previous literature, trading positions are opened once the disequilibrium is sufficiently large and the expected correction time is sufficiently short.

This model is more general than the model described by Elliot *et al.* (2005) [15] but makes use of the same modelling and estimation framework. It uses a one-factor stochastic model to describe the state of mis-pricing or disequilibrium and incorporates a white Gaussian noise ($\omega_t \sim$ IID $\mathcal{N}(0,1)$) that contaminates its actual observation being measured by function $G(R_t^A, R_t^B, U_t)$. x is the state of mis-pricing (residual spread) with respect to a given long-run equilibrium relationship whose dynamic follows a Vasicek process

$$dx_t = \kappa(\theta - x_t)dt + \sigma dB_t \tag{2.13}$$

The residual spread function (observed mis-pricing) is then defined as follows

[5] See [10] p.8 for the proof.

$$y_t = G_t = x_t + \omega_t \tag{2.14}$$

At this point, Do *et al.* (2006) [10] argument the specification of G using asset pricing theory, specifically, the Arbitrage Pricing Theory model (APT). For instance, taking into account n-factors, we specify

$$R^i = E(R^i) + \beta r^m + \eta^i$$

where R^i is the observed return for the ith stock, $E(R^i)$ is its expected return, $\beta = \{\beta_1^i \beta_2^i \cdots \beta_n^i\}$ is the vector containing the sensitivities of the return of the ith stock to each risk factor, r_f is the risk free asset return,
$r^m = \{(R^1 - r_f)(R^2 - r_f) \cdots (R^n - r_f)\}$ is the vector of risk factor returns in excess over the risk free asset return and η^i is the idiosyncratic term which satisfies $E(\eta^i) = E(\eta^i \eta^j) = 0, \forall i$ and $\forall i \neq j$. We can also define a "relative" APT model on two stocks A and B. We can write it as follows

$$R^A = R^B + \Gamma r^m + e \tag{2.15}$$

where $\Gamma = \{(\beta_1^A - \beta_1^B)(\beta_2^A - \beta_2^B) \cdots (\beta_n^A - \beta_n^B)\}$ is a vector of exposure differentials and e is a residual noise term. Moreover, it is assumed that the equation in 2.15 keeps holding in all time periods, such that

$$R_t^A = R_t^B + \Gamma r_t^m + e_t \tag{2.16}$$

By solving 2.16 for e_t, we get the *residual spread function*, G_t

$$G_t = G(R_t^A, R_t^B, U_t) = e_t = R_t^A - R_t^B - \Gamma r_t^m \tag{2.17}$$

If Γ is known and r_t^m is specified, G_t is fully observable and a completely tractable model of mean-reverting relative pricing for two stocks A and B exists, which is then ready to be used for pairs trading. Besides, when Γ is a zero vector, we obtain the same model as in Elliot *et al.* (2005) [15].

As a final conclusion, Do *et al.* (2006) [10] formulate a continuous time model of mean reversion in the relative pricing between two assets where the relative pricing model has been adopted from the APT model of single asset pricing. However, this model does not make any assumptions regarding the validity of the APT model. Instead, it adapts the factor structure of the APT to derive a relative pricing framework without requiring the validity of the APT to the fullest sense[6]. Therefore, whereas a strict application of the APT may mean the long-run level of mispricing, θ, should be close to zero, a non-zero estimate does not serve to invalidate the APT or the pairs trading model as a whole. Rather it may imply that there is a firm specific premium commanded by one company relative to another, which could reflect such things such as managerial superiority. This could easily be incorporated into the model by simply adding or subtracting a constant term in the equilibrium function, G_t.

[6] See Schmidt (2008) [31] p.23.

Chapter 3

Pairs Trading Strategy

3.1 Pairs Trading strategy: A Cointegration Approach

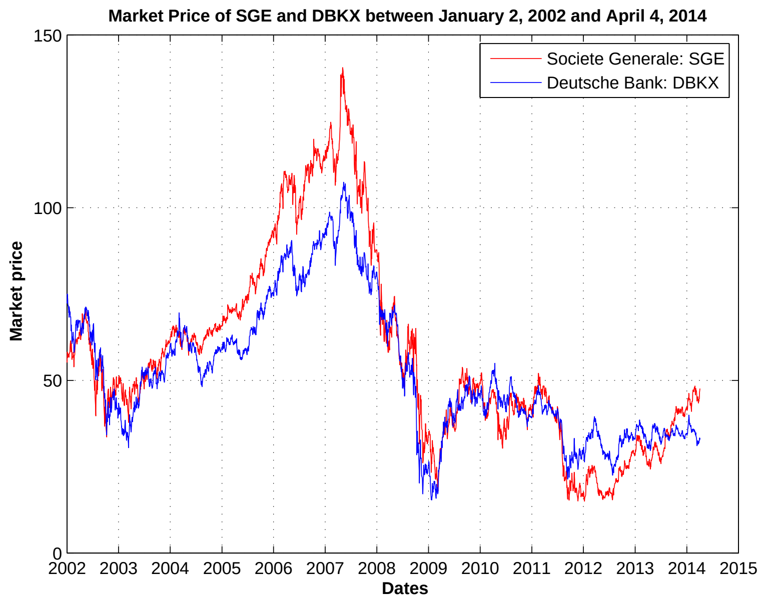
Definition of Cointegration linked to Classical Pairs Trading strategy.

3.2 A *“long/short”* equity investing strategy

3.3 Cointegration Tests

3.3.1 Engel-Granger

3.3.2 Johansen



(A) SGE and DBKX.

(B) Static picture of Pairs Trading between SGE and DBKX.

FIGURE 3.1: Example of a static picture of Pairs Trading between SGE and DBKX.

Chapter 4

An Empirical Study: Is Pairs Trading strategy still working?

4.1 Introduction

The above question is the same one as Binh Do and Robert Faff, two of the most significant authors about Pairs Trading, wonder in their article "Does Simple Pairs Trading Still Work?" [10] published in 2009. They implemented the "Distance Method[1]" and followed the same assumptions adopted by other important authors such as Gatev, Goetzmann and Rouwenhorst. Gatev *et al.* (1999 [17], 2007 [18]) also wondered the same question and concluded that there has been a declining trend in the profitability from Pairs Trading in the US markets due to the the rise in hedge fund activity. Moreover, Do and Faff (2009) [10] extended the original analysis of Gatev *et al.* (2007) [18] to June 2008, and they confirmed a continuation of the declining trend in profitability. However, contrary to popular belief, Do and Faff (2009) [10] found that the rise in hedge fund activity is not a plausible explanation for the decline. Instead, they observed that the underlying convergence properties are less reliable; there is an increased probability that a pair of close substitutes over the past 12 months (the *pairs formation period*) are no longer close substitutes in the subsequent half year (the *trading period*).

As we have stated before, both Gatev *et al.* (1999 [17], 2007 [18]) and Do and Faff (2009) [10] implemented a "distance approach" to Pairs Trading. Instead, in

[1]Showed in chapter 2, section 2.2.

this empirical study we are going to answer the same question but implementing the "cointegrating approach". Furthermore,, we will extend the analysed period to April 2014.

During financial meltdowns periods, correlations among stocks increase because almost all stock prices follow a bearish direction. This implies an increase in the market risk (stock prices moving against our positions) of well diversified portfolios.

For a Pairs Trading strategy to be successful, these two following points must be fulfilled:

1. Prices of two particular stocks A and B are *I(1)* and there exist a linear combination of them such that the spread defined as

$$\hat{Spread}_t = StockA_t - \hat{\beta} StockB_t = \hat{u}_t + \hat{\alpha} \tag{4.1}$$

   is *I(0)* and, consequently, shows a long-run equilibrium ($\hat{\alpha}$). $StockA_t$ and $StockB_t$ refers to the market price of Stock A and B, respectively.

2. The market price of at least one of these two stocks walks away from the long-run equilibrium (historical expectation of the spread, $\hat{\alpha}$) triggering an speculative strategy which consists of betting that the spread will revert back to its historical average value. And due to the fact that the spread is *I(0)*, we expect it to revert back to its historical mean value, by offering, then, a profit.

Intuition tells us that Pairs Trading strategy might have a better output in high volatile periods, such as financial crises, since, on one hand, they are periods where the correlation among assets increase[2] and, on the other, because of the high volatility itself[3]. In the results section, we will see that our expectations are fulfilled.

In the following empirical study we will analyse the Pairs Trading in detail. We will implement a top-down approach going from the general to the particular. Firstly, we will see how the Pairs Trading strategy behaves in terms of Profit and Loss (P&L) for different time intervals and for stocks belonging to the Dow Jones

[2]Resulting in strong co-movements among assets that are necessary for defining cointegrating relationships.

[3]Inciting the spread between two stocks to reach the trigger that begins the strategy.

Industrial Average (DJIA or DJ30) index and Euro Stoxx 50 (EU50) index. We will consider periods of extremely different nature and determinants such as the bull (market expansion: 2002-2007), the bear (financial meltdown: 2007-2009) and the sideways or horizontal markets. In all cases, time intervals will be large enough in order to define the cointegrating structures among stocks.

Secondly, we will carry out the same study but this time from a sectoral point of view in order to see what sector or kind of industry is more profitable. This is quite interesting given that that the co-movements of stocks belonging to the same industrial sector is expected to be higher that the ones for stocks belonging to different sectors. As we have stated in chapter three, strong co-movements between a certain pair of stocks is a main requirement for this strategy to succeed.

Thirdly, a sensitivity analysis will be introduced, consisting of changing some parameters that define the Pairs Trading strategy. These parameters will be the standard deviation of the spread (actually, the trigger of the strategy), the introduction of trading fees and the capital with which we enter in this strategy.

Finally, we will see what implications the legal ban of short selling, as well as its costs, has on the implementation of this strategy.

To summarize, the structure of this empirical study will be as follows:

- Profitability of Pairs Trading strategy during different timeslots on Euro Stoxx 50 and Dow Jones 30 Index's listed firms.
- Sectoral analysis of Pairs Trading strategy.
- Sensitivity analysis of profitability from Pairs Trading strategy in terms of:
    1. Grid of values for the trigger: $\pm 1\sigma_{spread}$; $\pm 1.5\sigma_{spread}$; $\pm 2\sigma_{spread}$; $\pm 2.5\sigma_{spread}$.
    2. Introduction of transaction cost: Trading fees for buying and selling stocks.
    3. Volume (capital) invested in the cointegrated pair.
- Implications of banning short sales on Pairs Trading strategy and the costs (initial margins, interest costs and cash guarantees) of this practice.

4.2 Methodology

In this section we will explain the algorithm used in this empirical study. This algorithm carries out the first three points of the study; i.e., the analysis of the profitability of the Pairs Trading strategy as well as the the sectoral study and the sensitivity analysis.

The algorithm proceeds as follows:

1. Firstly, it checks if the two stocks are at least, separately, time series integrated of order 1, *I(1)*, during the analysed period. It takes into account the market price of stocks (not returns). This unit root test is implemented using the Augmented Dickey Fuller (ADF) test.

2. If both stocks overcome separately the ADF test, i.e., they are *I(1)*, we then perform the two cointegration tests explained in chapter three (Engel-Granger and Johansen).

3. In the Engle-Granger cointegration test we carry out an OLS regression to get the Hedge Ratio (the $\hat{\beta}$ of the regression) as well as the residuals that, actually, are defined as the time series of the spread. Hence, we run the following regression

$$StockA_t = \alpha + \beta StockB_t + u_t \qquad u_t \sim \mathcal{N}(0, \sigma_u^2)$$

4. Thereby, the spread in each period of time t is defined as

$$\hat{Spread}_t = \hat{u}_t + \hat{\alpha} = StockA_t - \hat{\beta} StockB_t. \tag{4.2}$$

   At this point, if the spread is a time series integrated of order 0 (we check it by means of the ADF test), *I(0)*; i.e., a time series with a mean reversion behaviour, we say that the pair consisting of stock A and B is cointegrated. As we have seen in the previous chapter, the Engle-Granger test gives us only a unique cointegrating relationship while the Johansen test is able to give us more than one (the algorithm exploits all possible cointegrating relationships). The cointegrating vector is $[1; -\hat{\beta}]$; which, in fact, it is also the vector of weights (monetary units) in which we invest in the cointegrated pair; i.e., for each monetary unit we invest in Stock A, we also invest $-\hat{\beta}$

monetary units in Stock B. The constant term $\hat{\alpha}$ captures some sense of premium in stock A versus stock B (see [10, p. 6]) and represents the long-run equilibrium of the cointegrated pair.

5. There is an initial period in which the cointegrating relationships must be defined. This period lasts 252 days (approx. 1 year) and there are no trades here. This is the *pairs formation period*. Once this period has passed by, the *trading period* starts and lasts until the last day in our sample size. However, the cointegrating relationships among pairs of stocks might not last forever. Therefore, each 6 months (approx. 126 days), the algorithm updates the cointegrating relationships among all stocks, taking into account all past information until this time.

6. So far we have not started the Pairs Trading strategy but we have managed to get the daily time series of the spread. We, then, calculate the historical expectation of the spread, i.e., the value at which the spread will revert back. As we saw in chapter three, this long-run equilibrium is $\hat{\alpha}$. The next step is to calculate, each day, the historical standard deviation of the spread up to the day before[4], σ_{spread}.

7. Once the *trading period* has begun, the algorithm continues by checking each day how far the today's spread is from its historical expectation ($\hat{\alpha}$). To do so, a standard deviation metric is applied. If today's spread hits two consecutive times a predetermined threshold, called "the trigger", (for instance, $trigger = \pm 1.5$ standard deviations from the spread's long-run equilibrium) the strategy starts. The reason why we require the spread to hit two consecutive times the trigger is due to the fact that if we know that the spread is a mean-reverting time series, we expect it to revert back to its historical expectation. Therefore, the strategy begins when the spread is in its way back to its historical mean.

8. If today's spread is greater than the predetermined threshold that we have considered, e.g., if $Spread_{today}$ is $\geq 1.5\sigma_{spread}$ from its long-run equilibrium, we then go *short* on the spread because we expect it to drop to its historical expectation since the spread is a time series integrated of order zero; i.e., it

[4]As we have mentioned in chapter three, cointegrating relationships are defined in extended periods of time. For instance, if our available data of stocks quotes begin on January 2, 2001, we can not implement the strategy just the day after (or in too close dates) because we would barely have data for the standard deviation of the spread. However, the underlying reason is not the lack of data, but that the cointegrating relationships are defined for long periods.

has a mean-reverting behaviour. Alternatively, if today's spread is smaller than the predetermined threshold, e.g., if $Spread_{today}$ is $\leq 1.5\sigma_{spread}$ from the long-run equilibrium, we go *long* on the spread because we expect it to increase in terms of the mean-reverting behaviour of an *I(0)* time series. To go *short* on the spread[5] means selling 1 monetary unit of $StockA$ and buying $\hat{\beta}$ monetary units of $StockB$. On the other hand, to go *long* on the spread means buying 1 monetary unit of $StockA$ and selling $\hat{\beta}$ monetary units of $StockB$.

9. Once the strategy has begun, the next step is to figure when to exit the trade. There are three possible scenarios to get out the trade; (a) and (c) are related to the implementation of risk management strategies and (b) is due to the fact that the cointegrating relationship has worked properly. Thus, the three alternative scenarios are:

   (a) The strategy starts when the spread is greater or smaller than the threshold (say, $\pm 1.5\sigma_{spread}$ from the long-run equilibrium). However, it can occur that the spread does not revert back to its historical expectation. If that happens, we might fall into huge potential losses. To prevent this situation, we get out of the trade if the spread keeps increasing or decreasing depending whether we are long or short on the spread. Therefore, we fix a stop loss order at a predetermined level (say, $\pm 2.5\sigma_{spread}$ from the long-run equilibrium depending on we are long or short on the spread[6]). Thus, once the strategy has been initialized, it must be ended (by unwinding the portfolio) whenever the spread keeps deviating ± 2.5 standard deviation from its long-run equilibrium.

   (b) If the cointegrating relationship works properly, the spread will go back to its average historical value and we will have earned a profit.

   (c) It may also happen that the spread fluctuates between the stop loss level and its historical mean without hitting none of them. If this period lasts a reasonable time, there is no problem. However, if it is not the case, this period meanwhile the spread is between these two bounds,

[5] Being the definition of the spread as in equation 4.2.

[6] The $\pm$ sign has to do with the initial strategy. If we are, for instance, *long* on the spread it is because we expect it to increase. If it does not increase, but decrease to a risky level, we then fix a stop loss order at a level of $-2.5\sigma_{spread}$ from the long-run equilibrium. Alternatively, if our position would have been to be *short* on the spread, then we must fix the stop loss order at $+2.5\sigma_{spread}$, because we are now protecting against increases in the spread

we are loosing the opportunity cost of taking the money and investing in other asset[7]. In addition, since the spread is a mean-reverting time series, we expect it to cross its average historical value in quite a lot times during the trading period. If it is not the case, we should get out of the trade because the spread is not showing a clear mean-reverting behaviour. Therefore, if in two months time (approx. 42 days), which is a reasonable period for the spread to hit either the stop loss or its long-run equilibrium, the spread has not hit any of these two bounds, we get out of the trade at closing prices of the following day (day 43) assuming a potential profit or loss.

10. The final step is to compute the results of each trade. It is useful to get some statistics from the *trading period* such as the P&L of the strategy, the average return per trade, the standard deviation of return per trade, the number of total trades, the number of days with at least one open position or with at least one trade and the average number of trades per day (including the ongoing and newly open trades). An example of these statistics can be found in Appendix B.

4.3 Data

In this study we considered data for the firms belonging to two of the most relevant indices in the world; Dow Jones Industrial Average (DJIA or DJ30 index) and Euro Stoxx 50 (EU50) index.

The EU50 Index provides a blue-chip representation of super-sector leaders in the Euro zone. The index covers 50 stocks from 12 Euro zone countries: Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, the Netherlands, Portugal and Spain. It is made up of fifty of the largest and most liquid stocks which currently represent more than 50% of the entire free float market capitalisation of all Euro zone equities. Therefore, Euro Stoxx 50 is considered to be a proxy of the overall Euro zone stock market.

[7]Actually, as in the following section will be noted, we will not consider the time value of money nor the possibility of using it to invest in another particular asset or strategy. Hence, the money is simply held in cash in order to invest it in another potential pair trade.

The components of this index that we considered are the ones listed in *Thomson Reuters* as of April 4, 2014[8]. We have taken from *Datastream* the daily closing prices for these firms from January 2, 2002 until April 4, 2014. Due to the fact that the *pairs formation period* lasts 1 year (approx. 252 days), the *trading period* starts not before January 2, 2003. GDF SUEZ was removed from this study because of its available data in *Datastream* start on July 6, 2005. Hence, we considered the remaining 49 firms of EU50 index in order to implement the Pairs Trading strategy.

On the other hand, the DJIA index is a price-weighted average of 30 blue-chip stocks that are generally the leaders in their industry (US). Therefore, it is a widely followed indicator of the stock market. The components of this index considered in this study are the ones listed in Reuters as of April 4, 2014[9]. We have taken from *Datastream* the daily closing prices for these firms from January 2, 2000 until April 4, 2014. Therefore, the *trading period* starts not before January 2, 2001. VISA was eliminated from this study due to the fact that its available data in *Datastream* begin on March 13, 2008. Thus, we considered the remaining 29 firms of DJIA index for carrying out the Pairs Trading strategy.

The chosen period is large (12 years and 3 months for the firms belonging to EU50 index and 14 years and 3 months for the firms belonging to DJ30 index) because of two reasons. The first one is that the cointegration structures must be defined during extended periods of time. The second one is related to the volatility. As we noted in the introduction of this chapter, volatility is needed for this strategy to succeed. Therefore, in such a large period of time, there are time intervals with extremely high volatility and others with low volatility. Thus, we can compare the performance of the Pairs Trading strategy in both scenarios.

We are applying the Pairs Trading strategy using the *daily closing prices*, instead of *bid-ask prices*. This will be likely to give us a higher profit than it should be because we are buying stocks at a price lower than it should be and selling at a price higher than it should be. However, this is actually not a relevant problem since we are in an extremely high liquidity context taking into account that the stocks considered are the most liquid firms in the Euro zone stock exchanges and in US stock exchange. Therefore, the bid-ask spread should be narrow. Thus, the effect of this issue should be insignificant.

[8]The list of stocks belonging to EU50 index can be seen in the cointegration matrices available in Appendix A: A.1, A.2 and A.3. The tickers are the ones provided by Datastream.

[9]The list of stocks belonging to DJ30 index can be seen in the cointegration matrices available in Appendix A: A.4, A.5 and A.6. The tickers are the ones provided by *Datastream*.

4.4 Results and conclusions

In this section we are going to present the final results as well as the conclusions of this empirical study. For the next three sections (4.4.1, 4.4.2 and 4.4.3) we will take into account the following *assumptions*:

1. The value for the variable that triggers the strategy is: $trigger = \pm 1.5\sigma_{spread}$ from the long-run equilibrium ($\hat{\alpha}$).

2. There are no trading fees for buying or selling stocks.

3. Each trading position is opened with a value of *10000* monetary units; i.e., we invest 10000 times the cointegrating vector $[1; -\hat{\beta}]$. Therefore, the quantity invested in a pair that fulfils the condition that triggers the strategy will be $[10000; -10000\hat{\beta}]$ monetary units[10] (€ or US$ depending whether the pair of stocks belongs to firms listed on the EU50 index or in the DJIA index, respectively). Thus, we will not mix both markets (US and Euro zone) in such a way that the pairs will be built separately by currencies (US$ and €). We will get then pairs from Euro Stoxx 50 index's listed firms and pairs from DJIA index's listed firms.

4. Short selling is allowed.

5. There are no initial margins, neither costs of borrowing nor cash guarantees for the short selling.

6. $\hat{\beta} > 0$; required to implement pairs trading strategy. Given that the prices of two cointegrated stocks share the same pattern, $\hat{\beta} > 0$ should be positive.

7. Time value of money is not considered in the sense that the amount of money that we are not investing in this strategy will be kept it as cash.

8. Risk management tools are implemented. Stop loss orders are fixed at $\pm 2.5\sigma_{spread}$ depending if we go *long* or *short* on the spread.

We will implement a sensitivity analysis to assumptions 1-3 in section 4.4.4; assumption 4 and 5 will be discussed in section 4.4.5.

[10] Due to the fact that we cannot buy/sell fractions of shares, we, instead, buy/sell the number of shares that multiplied by the market price of the pair of stocks is closer to the amounts of 10000 and $-10000\hat{\beta}$ monetary units, respectively.

4.4.1 Profitability of Pairs Trading strategy for EU50 Index's stocks

In this first section we analyse the Pairs Trading strategy for the firms belonging to the Euro Stoxx 50 stock market index. Different periods are going to be considered as a measure of the strategy's robustness.

4.4.1.1 January 2003 - April 2014

The performance of the strategy is stated in figure 4.1b and the statistics of the trading period can be seen in table B.1. For this period, and in order to get a general overview of the presence of cointegration, we find out 91 cointegrated pairs of stocks (see appendix A.1).

As we can see in figure 4.1a, this is a very volatile period (from January 2, 2003 until April 4, 2014) where strong rises and great falls have occurred in the Euro Stoxx 50 index. In addition, the analysed period is extremely long (12 years and 3 months, including the first year required for the *pairs formation period*) to perform any kind of equity investing strategy. Therefore, we are going to be able to check how well the Pairs Trading strategy performs in very different scenarios such as in stock exchange expansions and contractions.

If we take a look at figure 4.1b, where the P&L of this strategy is shown, we clearly see two different unambiguous behaviours. In bullish periods (2003-2008 and 2012-2014, generally speaking), long positions are clearly winning positions (specially, between 2003-2008). It is clear that in strong financial expansions (2003-2008), the market price of almost all firms increase. Note that we are implementing by definition a "*long/short*" investing strategy (Pairs Trading), i.e., we buy or sell the spread but in both cases we buy a certain stock and sell another. The problem at this point deals with the stock we are selling. If it takes a lot for the spread to revert back to its historical mean, it is quite likely that the short position enters into losses due to we are in a strong bull market.

In order to mitigate risks, we implemented two kind of strategies; on one hand, we used a risk management tool based on fixing a stop loss order at $\pm 2.5\sigma_{spread}$ level, and on the other, if the spread has not reverted back to its long-run equilibrium in two months time (approx. 42 days), we get out of the pair trade at the closing prices of the following day. Therefore, in so many trades where the losses coming from the short positions are big enough, the risk-control strategies are activated in such a way the losses are limited.

Taking everything into account, we see a very clear behaviour in the period between 2003 and 2008; long positions are giving huge profits while we are able to cut the losses resulting from the short sales (the red line is around a constant level). Instead, this fact does not happen in the other bullish trend of the Euro Stoxx 50 (mid-2012-2014). In this case, we see the red line decreasing instead of remaining constant at a certain level. Therefore, in some cases the risk-control strategies perform well in some periods and not that well in others.

On the other hand, an extremely bearish market took place between 2008 and 2009 where the opposite behaviour can be seen. In this period, the Euro Stoxx 50 index suffered huge losses, falling its market price by more than 50%. Thereby, during this period, the positions which give us profits are the short ones (red line increases a lot). Moreover, even the the risk-control strategies are incapable of containing the losses coming from the long positions (green line decreases dramatically) as opposed as in the case between 2003 and 2008 where the losses arising from short positions were controlled in the bull market.

It is also important to point out that in periods of relative calm, where there is not a well-defined trend (as in between mid-2009 and mid-2011), there are not many trades (green and blue line remain more or less constant at a certain level meaning that there are few trades). As we have mentioned in this chapter's introduction, we do need volatility to implement this strategy. Otherwise, the spread does not reach the trigger level required to begin the strategy.

Figure 4.1b sums up almost perfectly the behaviours that we will find out from now on in the following graphics. Namely,

1. In bull markets: Profits come practically from long positions and we try to limit the potential losses arising from short positions with risk-control strategies (stop loss and limiting the maximum period for the spread to revert back to its historical expected value).

    - Sometimes (as in period 2003-2008) we will achieve it (see the red line remaining constant around a constant level), but sometimes we will not (see the red line decreasing from 2012 to 2014).

2. In bear markets: Profits come practically form short positions and we try to cut the potential losses resulting from long positions with risk-control strategies.
   - Sometimes it is very difficult to limit losses (see the green line decreasing from 2008 until 2009) due to market conditions (extreme volatility).
3. In sideways or horizontal markets: P&L remain practically constant due to the fact that there is not exist the required volatility in order to reach the trigger necessary to begin the Pairs Trading strategy. Moreover, when the market trend is not well-defined, its more difficult to establish cointegrating relationships.

Thereby, we have found out a pattern that links the situation of the stock market index (bullish, bearish or sideways) to the behaviour of the profitability of Pairs Trading strategy implemented on firms that belong to this index. We will call this discovery "pattern 1" in order to differentiate it from another pattern, "pattern 2" (defined in the following paragraph), that there seems to be present in some time intervals throughout this empirical study.

Brooks and Kat (2002) [7] found evidences of significant correlation of classic *"long/short"* equity hedge funds indexes with equity market indexes such as S&P500, Dow Jones Industrial Average (DJIA), Russell 200 and NASDAQ. This fact suggests that Pairs Trading is not a *market-neutral* strategy. However, some authors such as Lin *et al.* [24] and Nath [25], implicitly classify the Pairs Trading as a *market-neutral* strategy just because it belongs to the *"long/short"* equity investing strategies, even if the resulting portfolio (based on the cointegrating vector) may exhibit some market risk. Nonetheless, it is important to note that Pairs Trading strategy is not initially defined to be a *market-neutral* strategy.

A portfolio or a strategy is said to be *market-neutral* if its performance exhibits zero correlation with the market performance (a zero-beta portfolio). The market is represented by a relevant stock market index (EU50 for the Eurozone and DJIA for the US in this empirical study).

In our particular case, the correlation between the Euro Stoxx 50 market price (representing the relevant market) and the profitability of the Paris Trading strategy (blue line) in figure 4.1b is noticeable during the first bull market (2003 to mid-2007). This is what we call "pattern 2"; i.e., the correlation between a relevant stock market index and the profitability of a "*long/short*" investing strategy (Pairs Trading) implemented on the stocks that are listed in this stock market index.

Nonetheless, if we really believe and prove that there is not a significant correlation between the stock market index and the profitability of the Pairs Trading implemented on stocks that belongs to this index, consequently we are arguing that the Pairs Trading strategy is a *market-neutral* strategy. Literature about whether the Pairs Trading might be a *market neutral* strategy or not can be found in Alexander and Dimitriu (2002) [2], Brooks and Kat (2002) [7], Nath (2003) [25], Lin *et al.* (2006) [24], Schmidt (2008) [31] and Do *et al.* (2009) [9].

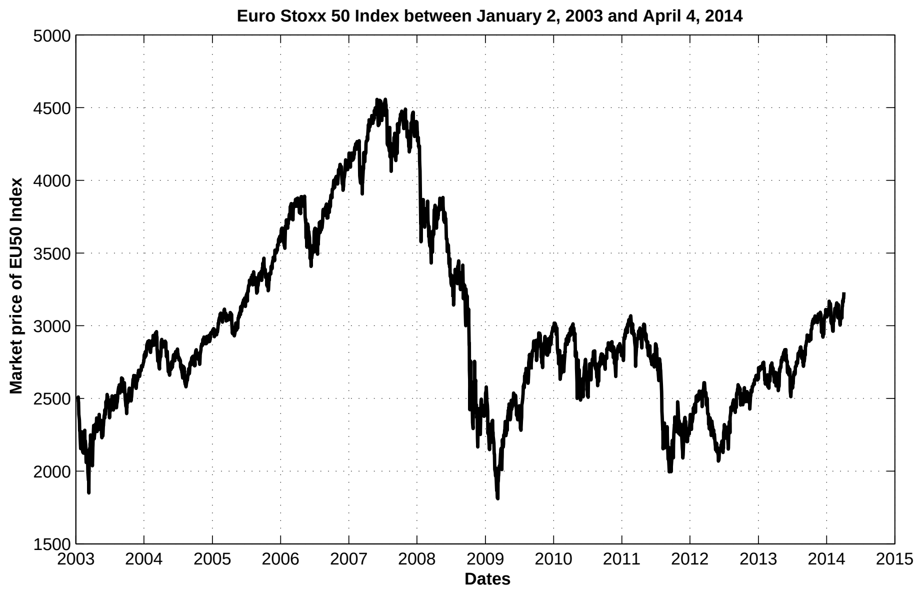
Just to illustrate it in a very easy and simple way, the linear correlation coefficient (ρ) between each of the three time series in figure 4.1b and the EU50 index is respectively

- $\rho_{long,EU50index} = 0.7938$
- $\rho_{short,EU50index} = -0.5577$
- $\rho_{long+short,EU50index} = -0.2588$

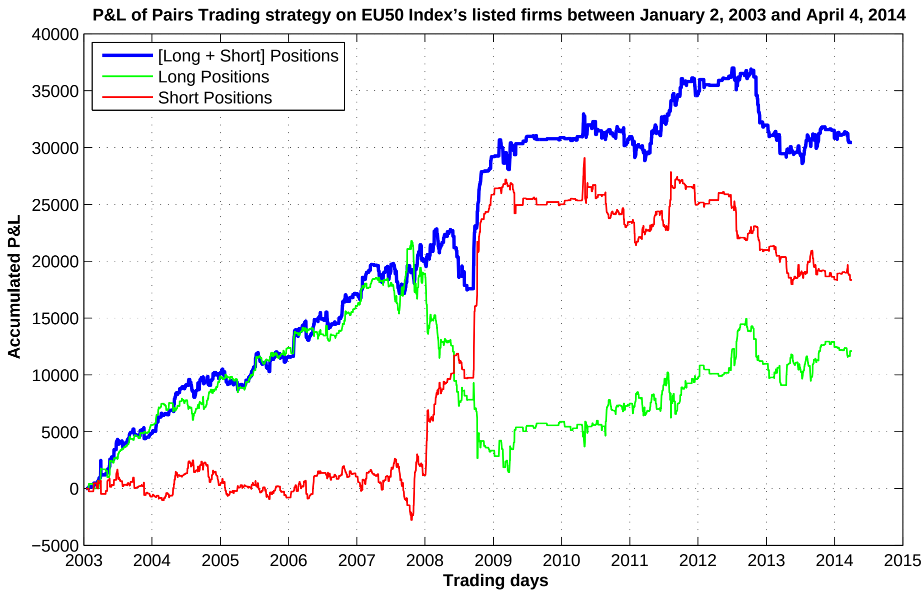
The performance of the portfolio (blue line) is slightly negatively correlated with the market ($\rho_{long+short,EU50index} = -0.2588$) suggesting that the Pairs Trading is not a *market-neutral* strategy. However, this is a very strong statement and a deeper analysis would be strongly needed to confirm it.

At this point, it is important to note that to find out reliable patterns (such as the two patterns defined above) in any kind of trading strategy is really interesting to make money because it implies that we are risking money taking advantage of probabilities. If any fact is more likely to occur, we should then invest accordingly.

Finally, the strategy gives us a gross profit of €30445.24 and an average return per trade of 0.26% ($\sigma_{return} = 3.12\%$) throughout 2374 trades (appendix B.1). Actually, the total profit is not a relevant number because it depends on the volume invested in the cointegrated pairs. Instead, the average return per trade and the standard deviation of return give us more useful information.



(A) Market price of Euro Stoxx 50 Index.



(B) Gross P&L of Pairs Trading strategy.

Figure 4.1: Euro Stoxx 50 Index and P&L of Pairs Trading strategy between January 2, 2003 and April 4, 2014.

4.4.1.2 January 2003 - January 2007

The output of this strategy is stated in figure 4.2b and the statistics of the trading can be seen in table B.2. In this period we are able to find out 220 cointegrated pairs of stocks (appendix A.2).

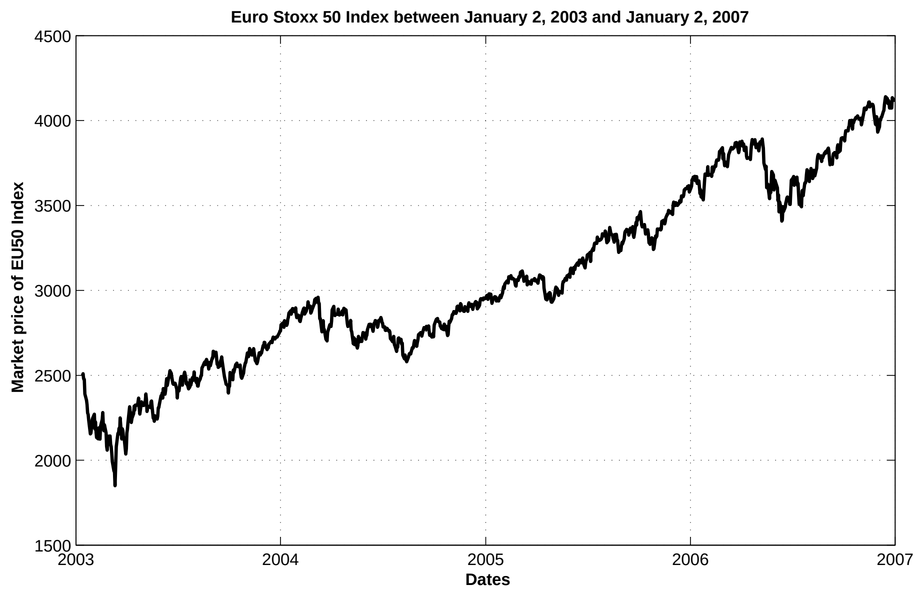
In this case, we have chosen a period with a very well defined bullish trend (see figure 4.2a) in order to check the performance of the Pairs Trading strategy. Taking into account the patterns that we have identified in the previous sub-section, it is easy to understand figure 4.2b.

Since we are in a bull market, our concerns will be related to the short positions. As we see in figure 4.2b, our strategy tried and succeeded in limiting the losses coming from the short sales at the same time that it was able to let running the profits arising from the long positions (note the red line remaining constant around zero level and see the increasing green line). This was "pattern 1".

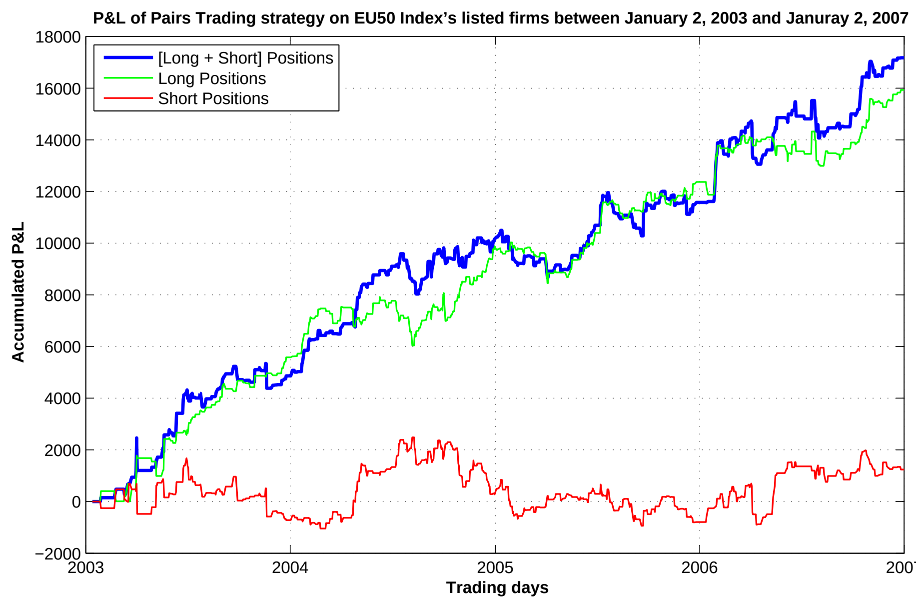
During this period, "pattern 2" seems to be present again since there seems to be correlation between the stock market index (EU50) and the profitability of the Pairs Trading strategy (blue line) carried out on the stocks listed in EU50 index for this period. If this pattern is proven to be really true, Pairs Trading, even though it is a "long/short" equity strategy, has not been a *market-neutral* strategy during this period. A further analysis is needed to confirm this fact but just to get an overview, the linear correlation coefficient (ρ) between each of the three time series in figure 4.2b and the EU50 index is respectively

- $\rho_{long,EU50index} = 0.9690$
- $\rho_{short,EU50index} = 0.0709$
- $\rho_{long+short,EU50index} = 0.9431$

The strategy worked well and it earned gross profits of €17175.53 and an average return per trade of 0.41% ($\sigma_{return} = 2.44\%$) during 844 trades (table B.2).



(A) Market price of Euro Stoxx 50 Index.



(B) Gross P&L of Pairs Trading strategy.

FIGURE 4.2: Euro Stoxx 50 Index and P&L of Pairs Trading strategy between January 2, 2003 and January 2, 2007.

4.4.1.3 January 2007 - April 2014

The output of this strategy is stated in figure 4.3b and the statistics of the trading can be seen in table B.3. There are 110 cointegrated pairs of stocks during this period (appendix A.3). We included GDF SUEZ since the availability of its market price starts in *Datastream* on July 6, 2005.

This period contains the great fall related to the beginning of the financial crisis as well as the theoretical[11] W-shaped recuperation period. Hence, we are considering two very different market scenarios.

Once again, we observe "pattern 1" during the entire analysed period. There is an initial period (2007-2008) with a non-defined trend in which the strategy offered gross profits. Then, between 2008 and early 2009, just in the middle of the financial crash, the long positions start to loose money, as expected, while the opposite occurs with the short positions. In spite of we are implementing a risk management strategy based on fixing a stop loss order, it does not work properly judging from the strong losses coming from long positions. Yet, it is strongly recommended to implement this kind of risk-control strategies. Otherwise, we might incur in huge potential losses.

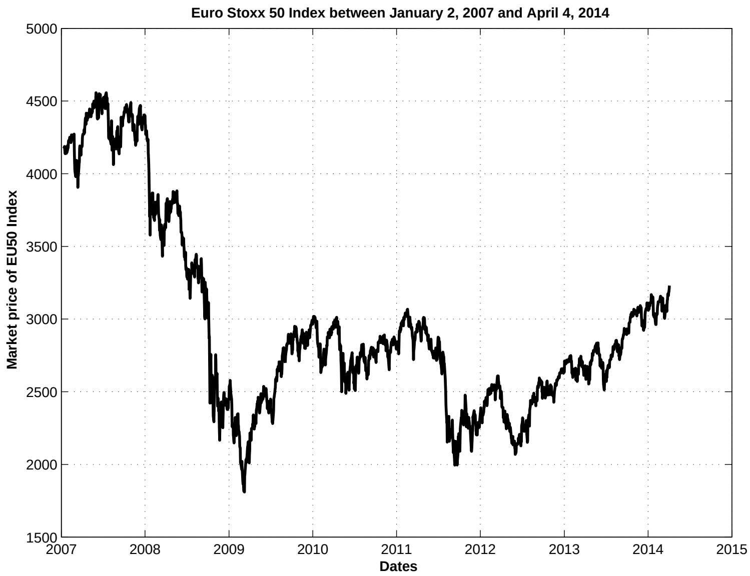
From mid-2009 until 2012, there is again a period with a non-defined trend in which the accumulated P&L remain more or less constant, excepting a peak in mid-2011 in which there was a strong fall in the Euro Stoxx 50 index. Finally, from 2012 onwards, there is a bullish trend in the Euro Stoxx 50 index which allows us to see again the "pattern 1" (profits coming from long positions increase while losses coming from short positions also increase).

Regarding "pattern 2", correlation between the stock market index (EU50) and the profitability of the strategy (blue line) is not as clear as before. However, it seems that during 2007-2014, the profitability of the strategy (blue line) followed an inverse path with respect to the EU50 index. If this had been true, the EU50 index might have been used as a hedge instrument for the Pairs Trading strategy (e.g., using futures or options contracts on the EU50 index). However, this is just a simple supposition and we should analyse with detail this supposed inverse correlation and

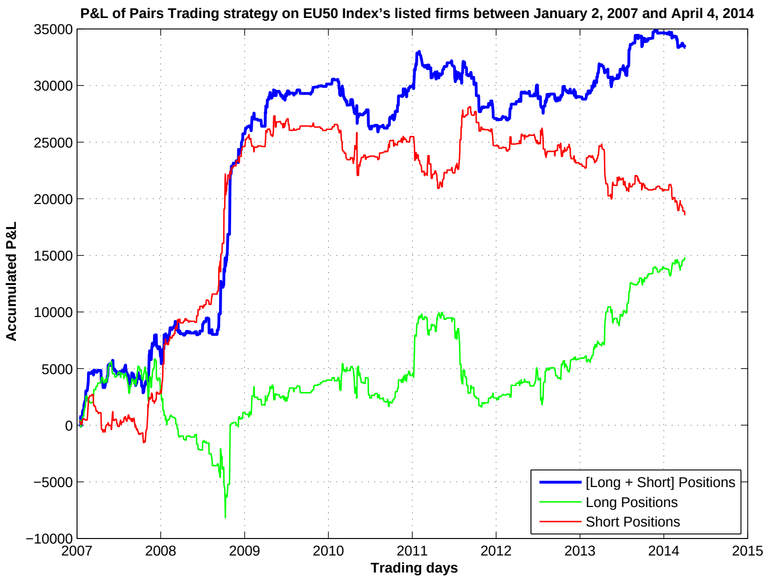
[11]We say theoretical due to the fact that the W-shaped recuperation needs to be confirmed and reach levels of 4500 points in the Euro Stoxx 50 index before becoming a real W-shaped recuperation.

its implications. The linear correlation coefficients are $\rho_{long,EU50index} = -0.0578$, $\rho_{short,EU50index} = -0.9452$ and $\rho_{long+short,EU50index} = -0.8264$.

During this period the Pairs Trading strategy made a profit of €33383.51 and a mean return per trade of 0.37% ($\sigma_{return} = 3.33\%$) throughout 1818 trades (table B.3).



(A) Market price of Euro Stoxx 50 Index



(B) P&L of Pairs Trading strategy

FIGURE 4.3: Euro Stoxx 50 Index and P&L of Pairs Trading strategy between January 2, 2007 and April 4, 2014

4.4.2 Profitability of Pairs Trading strategy for DJ30 Index's stocks

In this section we implement the same analysis as before but now for the stocks belonging to Dow Jones Industrial Average stock market index. We are going to do so for different periods. We have taken into account the same assumptions as before (section 4.4.1).

4.4.2.1 January 2001 - April 2014

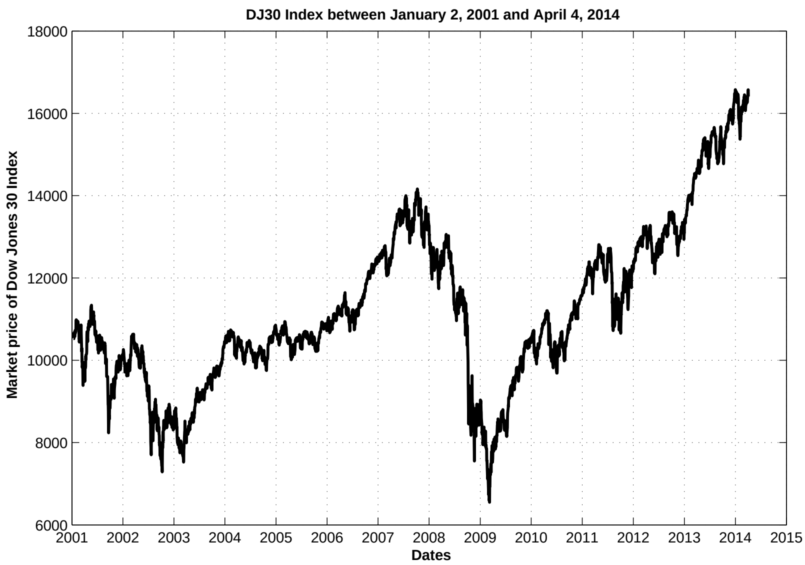
The performance of this strategy is stated in figure 4.4b and the statistics of the trading can be seen in table B.4. We are able to find out 26 cointegrated pairs of stocks (appendix A.4).

In the most actively-traded US firms, we also find out the "pattern 1" between the kind of market (bull, bear or sideways) and the Pairs Trading strategy. Firstly, there was a downward trend period during 2001-2003 in which P$L came practically from the short positions and the strategy was able to mitigate the negative effect over profits from the long positions (specifically, in 2001-2002). Therefore, since the very beginning, the strategy gave gross profits. In 2003 began a very strong bull market which lasted 5 years until early 2008. During this period, the profits came from the long positions but the strategy was not able to limit the losses coming from the short positions (note the decreasing red line in this period). As expected, the green line, which collects the profits from long positions, starts to increase; and it does it until mid2008. However, the risk management strategy is not able to limit properly the losses and the red line also begins to decrease from 2002 until mid-2007. However, the profits from long positions are smaller than the losses from short positions; that is why the blue line (net profits) decreases during 2004-2006 time interval.

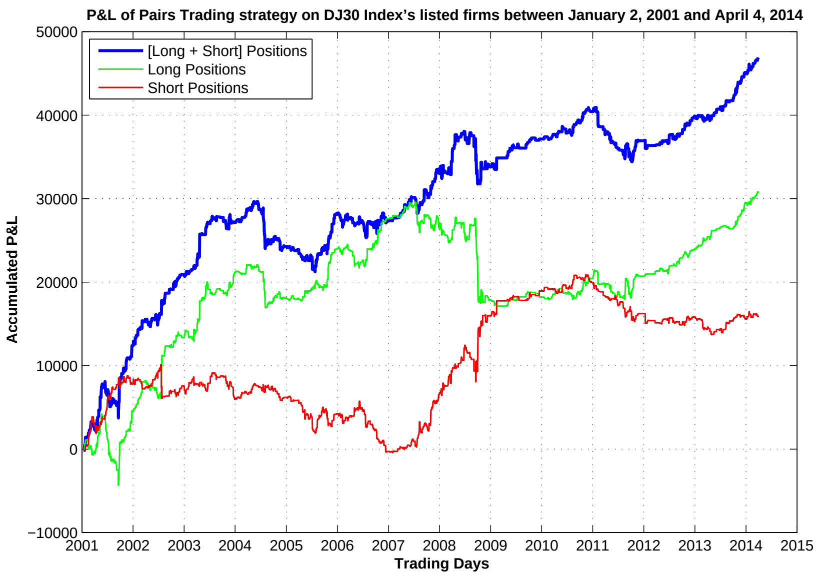
In mid-2007, the financial crisis began. And obviously, the situation of the strategy is inverted. Thereby, the long position started to give losses and the profits came from the short positions. This happened until early 2009 where began another bull market, and as expected, the situation of the strategy changed again. Finally, since early 2011 we observe the same pattern that occurred between 2002 and 2007 in a quite similar bull market, but in this case it appears that the risk management strategy worked fine by limiting losses.

This time, "pattern 2" is not that clear as in previous cases. The linear correlation coefficients are $\rho_{long,DJ30index} = 0.6145$, $\rho_{short,DJ30index} = 0.1044$ and $\rho_{long+short,DJ30index} = 0.5151$.

During this very volatile period, the pairs trading strategy was able to gain US\$46611.35 and an average mean return of 0.37% ($\sigma_{return} = 3.27\%$) along 2514 trades (table B.4).



(A) Market price of Dow Jones 30 Index.



(B) Gross P&L of Pairs Trading strategy.

FIGURE 4.4: Dow Jones 30 Index and P&L of Pairs Trading strategy between January 2, 2001 and April 4, 2014.

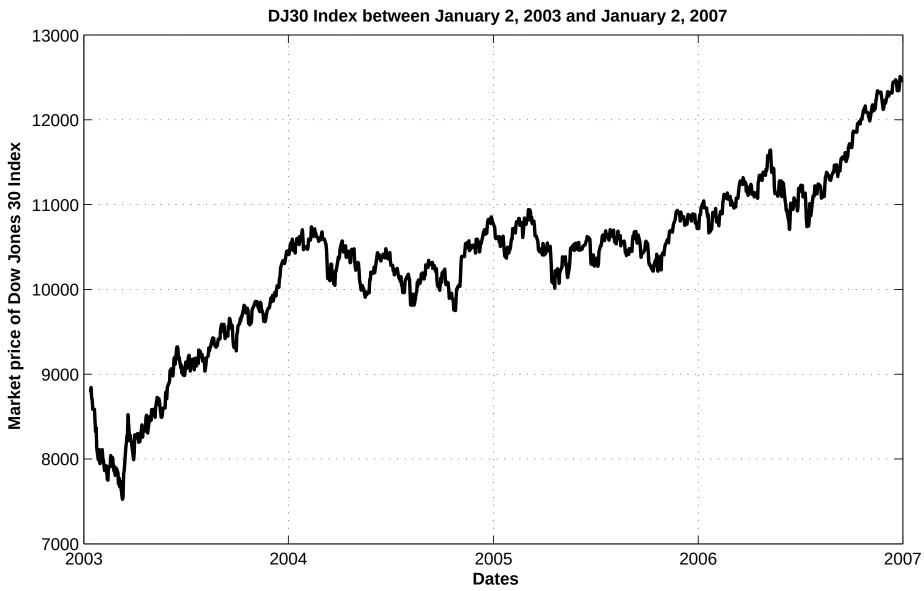
4.4.2.2 January 2003 - January 2007

The output of this strategy is stated in figure 4.5b and the statistics of the trading can be seen in table B.5. During this period we have found out that there are 31 cointegrated pairs of stocks (appendix A.5).

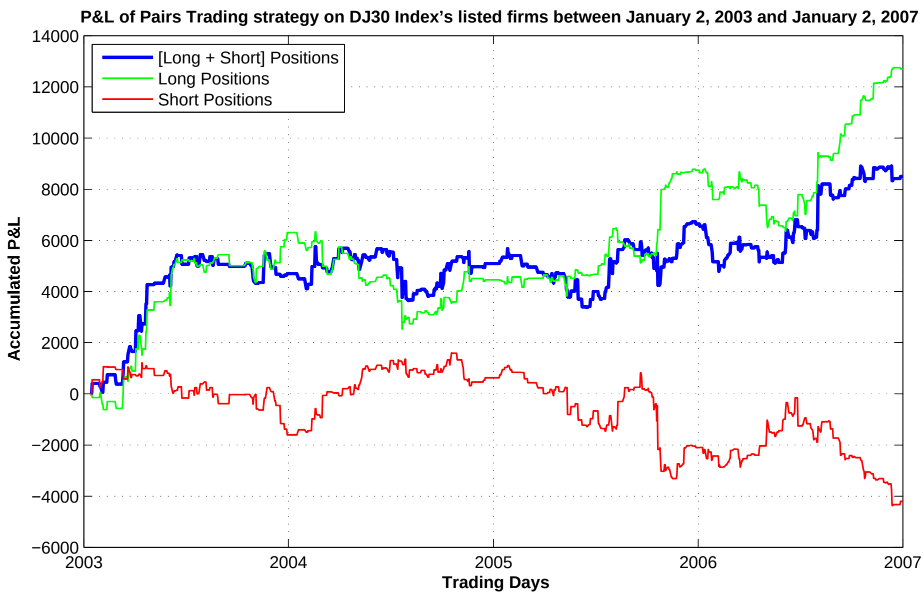
We have chosen a bullish period (January 2, 2003- January 2, 2007). If we take a look to DJIA index (figure 4.5a) we can identify three periods. The first one (March, 2003 - early 2004) with a strong upward trend where long positions made big profits and the risk management strategy consisting of a stop loss order fixed at $\pm 2.5\sigma$ worked properly by cutting the losses of the short positions. As figure 4.5a suggest, we can define a second period (2004-2006) as a sideways trend period. As expected, in this period there are not so many trades because, as we have mentioned in this chapter's introduction, we do need volatility because the spread between two stocks must reach the trigger level ($\pm 1.5\sigma_{spread}$ from the spread's historical average value). Therefore, the almost flat lines for the P&L from the long and short positions was an expected fact. And, finally, the third period (mid 2006 - early 2007) was quite similar to the first one. Nonetheless, in this case the risk management strategy did not work as well as in the first period judging from the decreasing red line, meaning that the losses arising from short positions were not limited in a right way.

In regard to "pattern 2", there seems to be correlation between DJIA index market price and the profitability of Pairs Trading during the entire period (blue line). Again, a further analysis of this issue is needed to extract trustful conclusions. The linear correlation coefficients are $\rho_{long,DJ30index} = 0.8508$, $\rho_{short,DJ30index} = -0.6869$ and $\rho_{long+short,DJ30index} = 0.8118$.

Taking everything into account, the strategy made a profit of US\$8502.23 and a mean return of 0.21% ($\sigma_{return} = 2.65\%$) during 816 trades (table B.5).



(A) Market price of Dow Jones 30 Index.



(B) Gross P&L of Pairs Trading strategy.

FIGURE 4.5: Dow Jones 30 Index and P&L of Pairs Trading strategy between January 2, 2003 and April 4, 2014.

4.4.2.3 January 2007 - April 2014

The output of this strategy is stated in figure 4.6b and the statistics of the trading can be seen in table B.6. We find out 35 cointegrated pairs of stocks (appendix A.6).

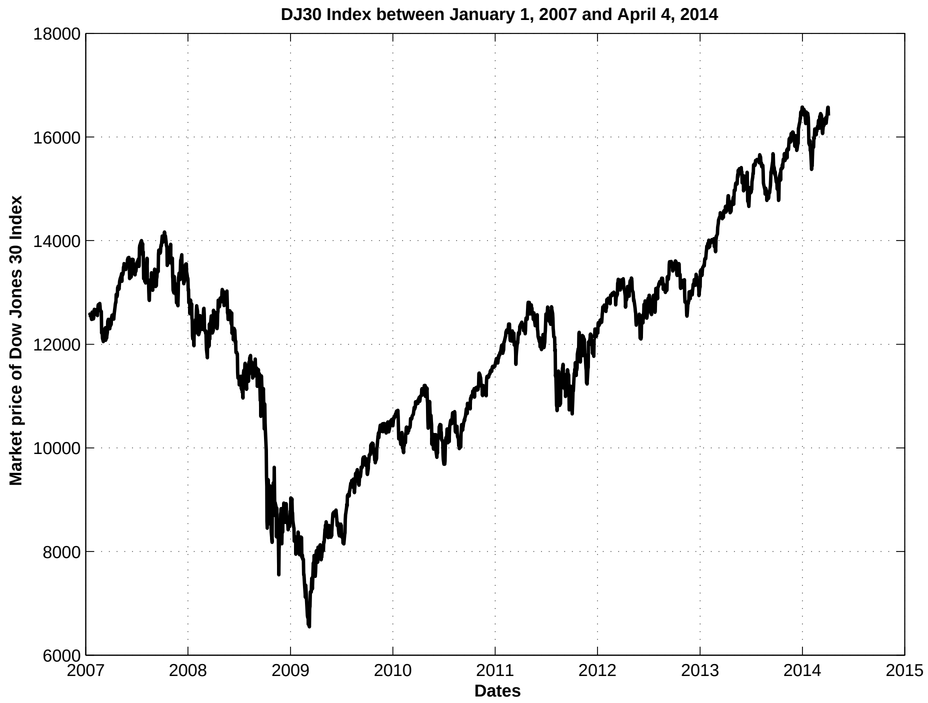
We clearly have two different markets. The first one is a bear market (2007 - early 2009) and the second one is a bull market (early 2009 - April 2014).

Once again, what we in fact observe in the bear market is what we had expected to ("pattern 1"); i.e., the short positions making huge profits and the long positions incurring in losses (stop loss strategy did not work fine in this sub-period).

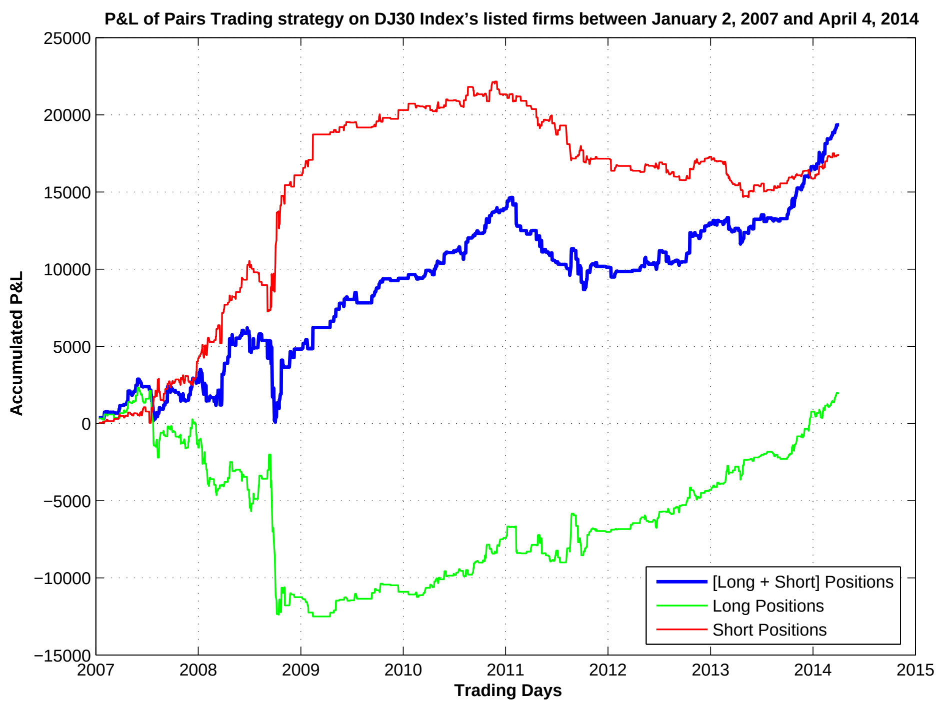
In the bull market we also see what we had expected to, which in fact is the long positions making big profits and the short positions offering losses. However, in this case, the stop loss strategy did not work too bad judging from what we observe in the figure 4.6b.

From February 2009 onwards, we also notice that there seems to be a slightly positive correlation between the DJIA stock market index and the profitability of the strategy (blue line), suggesting that the Pairs Trading is not a *market-neutral* strategy. However, it is not as clear as in some previous cases and a deeper analysis of correlation should be done about this topic. The linear correlation coefficients are $\rho_{long,DJ30index} = 0.8434$, $\rho_{short,DJ30index} = -0.3058$ and $\rho_{long+short,DJ30index} = 0.3503$.

Finally, the Pairs Trading strategy showed a gross profit of US$19356.34 and an average mean return of 0.33% ($\sigma_{return} = 3.46\%$) throughout 1156 trades (table B.6).



(A) Market price of Dow Jones 30 Index.



(B) Gross P&L of Pairs Trading strategy.

Figure 4.6: Dow Jones 30 Index and P&L of Pairs Trading strategy between January 2, 2007 and April 4, 2014.

4.4.3 Sectoral analysis

The reason behind doing a sectoral study has to do with the fact that cointegration occurs when a pair of stocks follows a very similar co-movement or pattern during a determined time interval. We might expect that firms belonging to the same sector follow a similar co-movement. Therefore, it may be interesting to know in which sector, the Pairs Trading might offer greater rewards.

Therefore, in this section we are going to analyse in which industrial sector the Pairs Trading strategy behaves better in terms of P&L and management of risks. We carry out this study for the stocks belonging to Euro Stoxx 50 index and to Dow Jones 30 index (excluding GDF SUEZ and VISA[12]). The *assumptions* taken into account are the same ones as those stated at the very beginning of the results section (4.4). The analysed period extends from January 2, 2003 to April 4, 2014.

Firstly, we define 4 macro sectors. Namely, financial sector, "type 1" industrial sector (the heavy industry), "type 2" industrial sector (the rest of the industry) and consumer goods sector. The following list[13] contains the kind of firms belonging to each macro sector:

- Financial sector: banks, financial services firms and insurance firms.
- "Type 1" industrial sector (the heavy industry): chemicals, oil & gas and utilities.
- "Type 2" industrial sector (the rest of the industry): industrial goods & services, auto mobiles & parts, conglomerates, construction & materials, real estate, telecommunication, media, software and technological firms.
- Consumer goods sector: retail, food & beverages, personal & household goods, healthcare and apparel firms.

The sectoral analysis has also been done in previous literature. Gatev *et al.* (1999 [17], 2007 [17]) performed Pairs Trading ("distance method") within four Standard & Poor's major industry groups: Utilities, Financials, Transportation and

[12]See section 4.3.

[13]The sub-sectors of these 4 macro sectors are based on the definition applied by STOXX and S&P (the firms responsible for making the EU50 index and the DJ30 index, respectively) for categorising each firm in each sector.

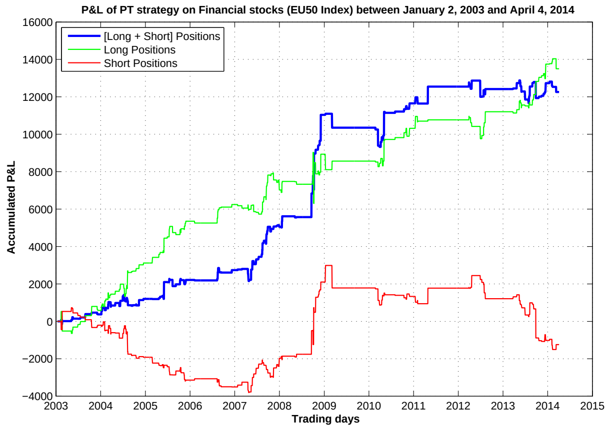
Industrials. They documented statistically significant profit across all the four groups, however greater profits are found in Utilities and Financials. For Do and Faff (2009) [10] this is entirely expected because utility firms face rather stable demands, their products have low differentiation and electric utility suppliers have been subject to some form of rate regulation, therefore there is a great deal of homogeneity amongst the utilities. Financials, on the other hand, are sensitive to common macroeconomic factors such as interest rates and unemployment shocks, hence, their share prices are likely to move together. In the following sectoral analysis, we test if this industry based pattern also holds for the "cointegration approach" instead of the "distance approach" to Pairs Trading, even though the sectors are not exactly defined in the same way.

4.4.3.1 Euro Stoxx 50 Index's listed firms

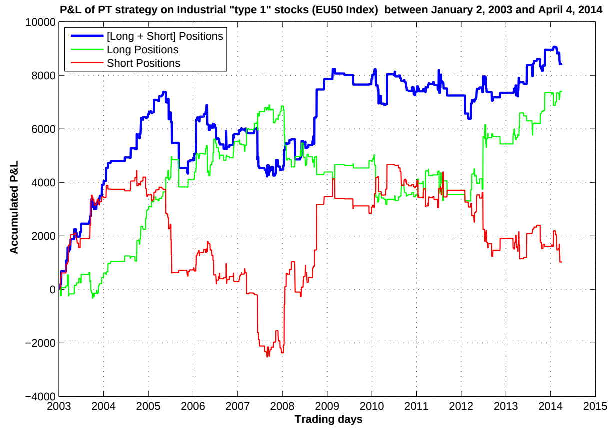
The results from implementing Pairs Trading by sectors can be viewed in figure 4.7 and table 4.1. There are 12 firms in the financial sector, 10 in the "type 1" industrial sector, 18 in the "type 2" industrial sector and 9 in the consumer goods sector. All sectors provided gross profits. We really observe the same behaviour as in Gatev *et al.* (2007) [18] and Do *et al.* (2009) [10] but we should point out some issues.

The financial sector offered the greatest average return per trade (0.70%) with, by far, the lowest total number of trades (352). Then, the "type 1" industrial sector (the heavy industry), which contains the utility firms, has the second best results in terms of average return per trade (0.25%) closely followed by the consumer goods sector (0.24%) and by the "type 1" industrial sector (0.20%). However, the lowest standard deviation of return per trade is found in the heavy industry. Another key point to pay attention to is the pattern displayed by the strategy in figure 4.7. This figure can give us a roughly idea of the level of risk assumed during the trading period. Even though we have implemented risk control strategies, all sectors present isolated significant losses that we should not overlook; specially, in the case of the consumer goods sector. The risk management strategy did not work properly since it was not able to control losses in some time intervals.

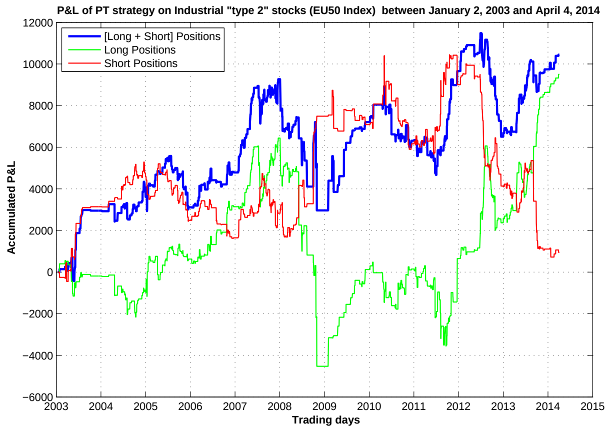
Note that we are assuming no transaction costs. However, if we incorporate them, gross profit might become a net loss in some sectors.



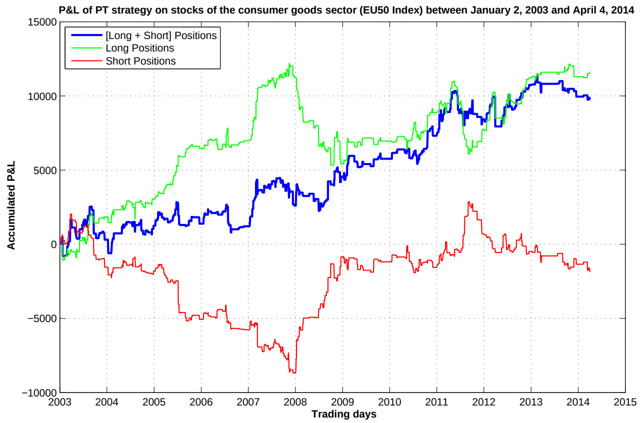
(A) Financial sector.



(B) “Type 1” industrial sector.



(C) “Type 2” industrial sector.



(D) Consumer goods sector.

FIGURE 4.7: Sectoral analysis of Pairs Trading strategy. Stocks belonging to Euro Stoxx 50 index.

STATISTICS	Financial sector	Industrial “type 1” sector	Industrial “type 2” sector	Consumer goods sector
Gross P$L ([Long + Short] Positions	12259.04	8426.2	10447.89	9763.73
# of Trades	352	686	1022	806
Average return per trade	**0.70%**	**0.25%**	**0.20%**	**0.24%**
SD (σ_{return}) of Return per trade.	3.19%	2.52%	3.71%	3.47%
Number of days with at least one open position	725	1108	1667	1628
(% of total trading days)	(24.75%)	(37.83%)	(56.88%)	(55.58%)
Number of days with at least one trade	165	306	456	375
(% of total trading days)	(5.63%)	(10.45%)	(15.57%)	(12.80%)
Average number of trades each day	2.13	2.24	2.24	2.15

TABLE 4.1: Sectoral analysis results. Stocks belonging to Euro Stoxx 50 Index.

4.4.3.2 Dow Jones 30 Index's listed firms

The performance from implementing Pairs Trading by sectors is shown in figure 4.8 and table 4.2. There are 4 firms in the financial sector, 7 in the "type 1" industrial sector, 7 in the "type 2" industrial sector and 11 in the consumer goods sector. All sectors provided gross profits. Once again, patterns identified by Gatev *et al.* (2007) [18] and Do *et al.* (2009) [10] are shown when implementing Paris Trading in a cointegration approach.

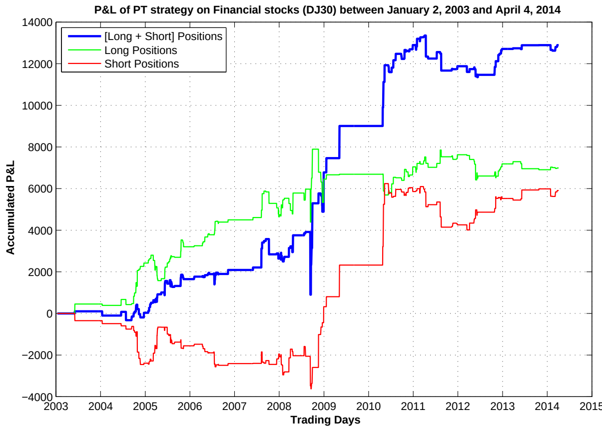
As before, we found out that the financial sector is the one that provides by far the largest average return per trade. However, its standard deviation is also the greatest one among all sector taken into account. Gatev *et al.* (2007) [18] implemented their sectoral study in the US markets and concluded that utility firms and financial sector were the most profitable sectors when carrying out a Pairs Trading strategy. We also reach the same conclusion from a cointegration point of view since the "type 1" industrial sector, which is the one containing the utility firms, is the second best sector in order to implement this strategy, behind the financial sector.

Contrary to what we observe in the Euro zone, in the US market, there has systematically been less total trades in all the 4 sectors. This might be due to the fact that there are less firms in the DJIA index, and besides, there are less cointegrated firms for this period[14]. This might lead to less trading opportunities.

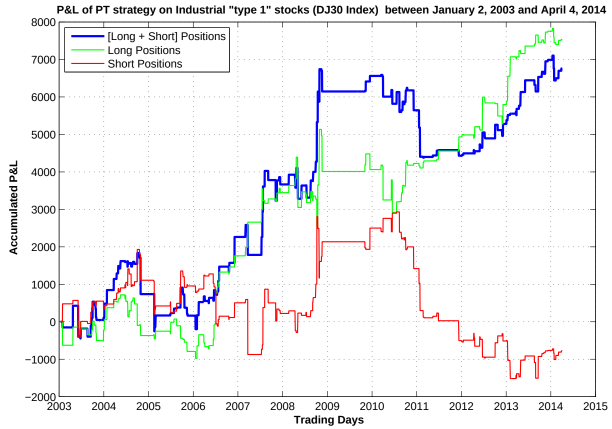
Another significant fact is that the short sales ended in losses in three sector excepting from the financial one where the short sales began to give huge profits from end-2008 onwards. Among others, this particular issue is the responsible that these sectors are less profitable than the financial one. Therefore, it seems that the financial sector is the one that better behaves in terms of, first, arising trading opportunities by hitting the trigger, and, second, reverting back to its historical mean value.

Taking everything into account and regarding the risk profile of the strategy (figure 4.8), generally speaking, we can conclude that the risk management tool used is not sufficient to limit losses.

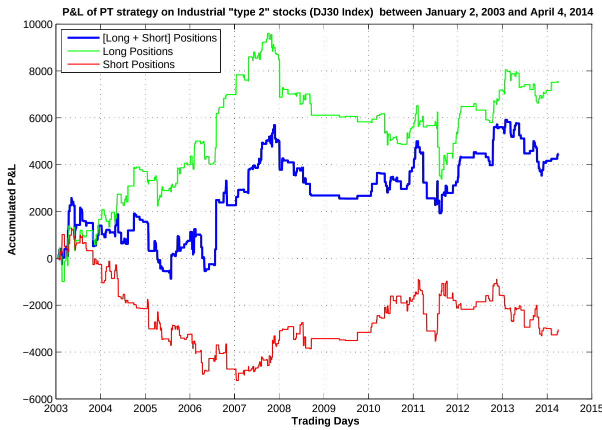
[14] 91 cointegrated pairs for the EU50 index's listed firms vs. 26 cointegrated pairs for the DJ30 index's listed firms. Note that the cointegrated pairs are updated every 6 months.



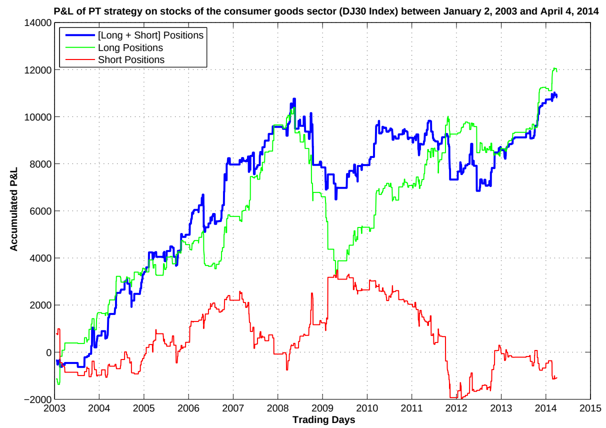
(A) Financial sector.



(B) "Type 1" industrial sector.



(C) "Type 2" industrial sector.



(D) Consumer goods sector.

FIGURE 4.8: Sectoral analysis of Pairs Trading strategy. Stocks belonging to Dow Jones 30 index.

STATISTICS	Financial sector	"Type 1" industrial sector	"Type 2" industrial sector	Consumer goods sector
Gross P$L ([Long + Short] Positions	12892.00	6760.00	4462.02	10821.55
# of trades	322	330	672	752
Average return per trade	**0.80%**	**0.41%**	**0.13%**	**0.29%**
SD (σ_{return}) of Return per trade.	4.57%	3.20%	3.21%	2.75%
Number of days with at least one open position	696	1133	1287	1499
(% of total trading days)	(23.79%)	(38.68%)	(43.94%)	(51.18%)
Number of days with at least one trade	152	161	311	342
(% of total trading days)	(5.19%)	(5.50%)	(10.62%)	(11.68%)
Average number of trades each day	2.12	2.05	2.16	2.2

TABLE 4.2: Sectoral analysis results. Stocks belonging to Dow Jones 30 Index.

4.4.4 Sensitivity analysis

In this section we are going to see the effects over the profitability of the Pairs Trading strategy when we change some of the parameters that define this strategy. Namely,

1. Grid of values for the trigger: $\pm 1\sigma_{spread}$; $\pm 1.5\sigma_{spread}$; $\pm 2\sigma_{spread}$; $\pm 2.5\sigma_{spread}$.
2. Introduction of transaction cost: Trading fees.
3. Volume (capital) invested in each cointegrated pair.

4.4.4.1 Trigger level

As we already know, the starting point of the strategy is defined by the historical standard deviation of the spread calculated up to the day before we want to implement the strategy[15]. So far, we have carried out the previous analysis taking into account a value for the trigger of $\pm 1.5\sigma_{spread}$. Therefore, a strategy begins when the spread hits two consecutive times the trigger level: ± 1.5 standard deviations from its historical expectation (long-run equilibrium). In this section we are going to check what happens with the profitability of this strategy when we consider different values for the trigger, i.e., ($\pm 1\sigma_{spread}$; $\pm 1.5\sigma_{spread}$; $\pm 2\sigma_{spread}$; $\pm 2.5\sigma_{spread}$).

We are going to pay attention to the effects of changing the trigger level on the following variables:

- Profitability of the strategy ([Long + Short] positions).
- Number of trades.
- Average Return per Trade.
- Number of days with at least one open position including the % of total trading days.
- Number of days with at least one Trade including the % of total trading days.

[15] Each day we compute whether the spread has hit two consecutive times the trigger or not.

- Average number of Trades each day.

We show, in figure 4.9 and table 4.3, the final result of the strategy implemented for the assets belonging to Euro Stoxx 50 stock market index for the period between January 2, 2003 and April 4, 2014.

Between 2003 and end-2007, there were a bull market on the Euro Stoxx 50 index. This was a period with a relatively low volatility (asymmetry behaviour in financial markets[16]). As we have pointed out in previous sections, we do need volatility to implement this strategy. If we set the trigger at a high level (say, $\pm 2\sigma_{spread}$ or even greater as in figures 4.9c and 4.9d), we need periods of great volatility if we want the strategy to work[17]. That is why in figure 4.9c and 4.9d we observe very few movements until mid-2007 (few trades). This might be one reason to explain why the gross profits are reduced when the values for the trigger level increase. Furthermore, in 2008, when a period of extremely high volatility started (figure 4.1a), there were many more trades (red and green lines are widened). This observed fact might confirm our expectations in the sense that relatively high volatility is needed in order to carry out this strategy.

There is no doubt that as we reduce the trigger level, it is easier for the spread to reach this level. As we can expect, the lower the trigger level, the higher the risk we are assuming because, even though the spread is a mean-reverting time series, it is not that difficult for the spread to reach small trigger values and to walk far away from it. We see this fact in figure 4.9a where the behaviour of the strategy, in terms of P&L, becomes more erratic for small values of the trigger[18]. Moreover, in this context of small values for the trigger, it is very difficult for the risk management strategy to perform properly (and it is more complicated as long as the distance between the trigger and the stop loss level is greater[19]).

It is clear that if we reduce the value for the variable that initiates the strategy, there will be more trades. But this trades will also be riskier that the trades implemented with a greater trigger value. This is because it is relatively easier for the spread to reach values of $1\sigma_{spread}$, but it is not that easy to hit levels of

[16]The return volatility is lower in strong bull markets than in strong bear markets (positive and high correlation in the left tail of the returns distribution).

[17]Otherwise, there will not be any trade because the spread will not reach the trigger level.

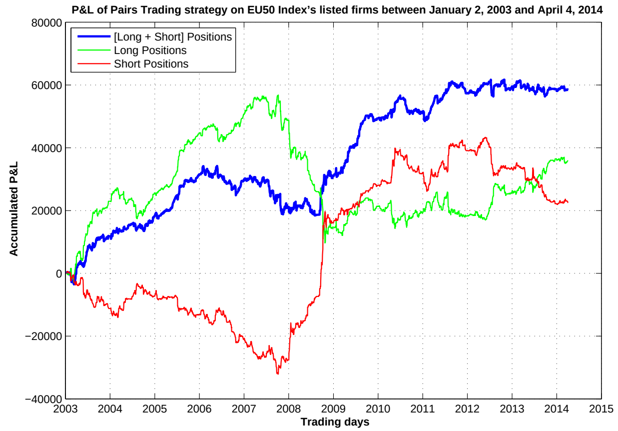
[18]Note the strongly decreasing red line in the case $trigger = \pm 1\sigma_{spread}$.

[19]In all cases, the stop loss has been fixed at $\pm 2.5\sigma_{spread}$, excluding the case of $trigger = \pm 2.5\sigma_{spread}$, where we have raised it to $\pm 3\sigma_{spread}$.

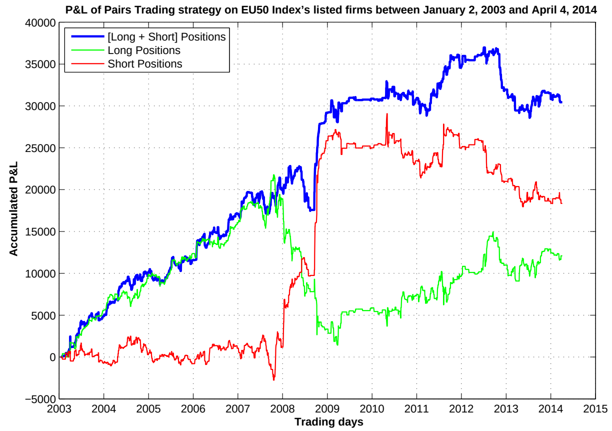
$\pm 2\sigma_{spread}$ or $\pm 2.5\sigma_{spread}$. This means that if the spread hits a value of $\pm 2\sigma_{spread}$ or $\pm 2.5\sigma_{spread}$ it is extremely likely that it reverts back to its historical average because it is a mean-reverting time series. Hence, it makes sense to get greater mean returns for greater values for the trigger. Actually, as we see in table 4.3, fixing a higher level for the trigger leads us to a greater average return per trade. This happens due to the fact that the nature of an I(0) time series and its mean-reverting behaviour. Generally speaking, it is more likely that an I(0) time series that has hit a level of $\pm 2\sigma_{spread}$ reverts back to its historical average, than the same I(0) time series reverts back to its historical mean once it has hit a level of $\pm 1\sigma$. This is the reason why we expect the strategy to offer greater average returns per trade for greater trigger levels (the probabilities of success grow up when the level for the trigger increases but, at the same time, more volatility is needed to hit the trigger).

Our expectations are confirmed in table 4.3. As the trigger level increases...

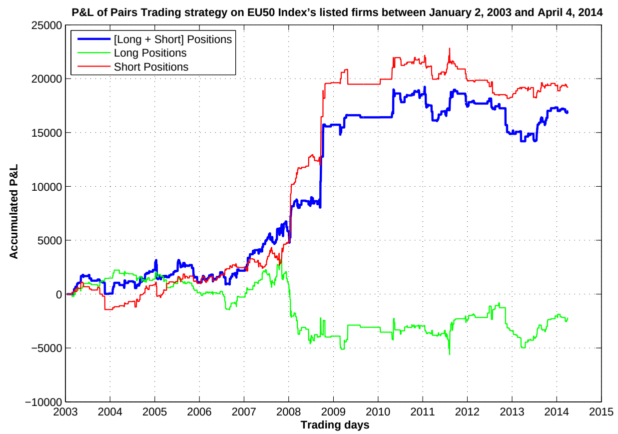
- ... the gross profits decrease.
- ... the total number of trades decreases.
- ... the average return per trade increases.
- ... the number of days with at least one open position decreases.
- ... the number of days with at least one trade decreases.
- ... the average number of trades per day decreases.



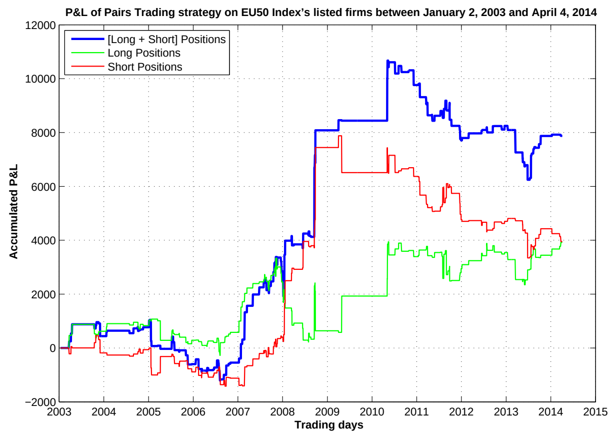
(A) Trigger $= \pm 1\sigma_{spread}$.



(B) Trigger $= \pm 1.5\sigma_{spread}$.



(C) Trigger $= \pm 2\sigma_{spread}$.



(D) Trigger $= \pm 2.5\sigma_{spread}$.

FIGURE 4.9: Sensitivity analysis of Pairs Trading strategy when modifying the value for the trigger. Stocks belonging to Euro Stoxx 50 index.

Grid of values for the Trigger	$\pm 1\sigma_{spread}$	$\pm 1.5\sigma_{spread}$	$\pm 2\sigma_{spread}$	$\pm 2.5\sigma_{spread}$
Gross P&L ([Long + Short] Positions	58415.67	30445.24	16972.66	7875.76
# of trades	5758	2374	930	344
Average return per trade	**0.20%**	**0.26%**	**0.37%**	**0.46%**
SD ($\sigma return$) of Return per trade.	3.56%	3.12%	3.35%	3.61%
Number of days with at least one open position (% of total trading days)	2850 (97.30%)	2257 (77.06%)	1243 (42.44%)	548 (18.71%)
Number of days with at least one trade (% of total trading days)	1742 (59.47%)	917 (31.31%)	413 (14.10%)	161 (5.50%)
Average number of trades each day (new and already held positions)	3.31	1.76	0.81	0.26

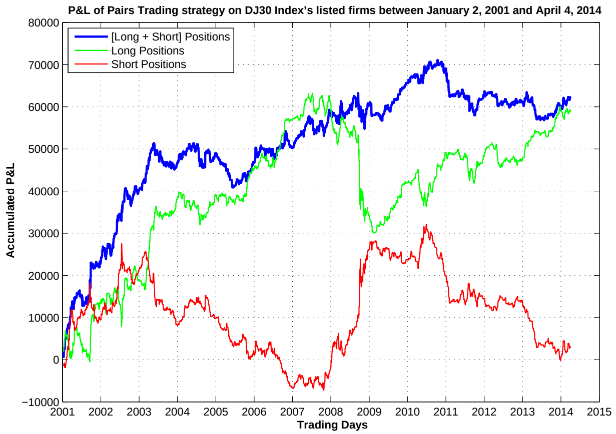
TABLE 4.3: Statistics for different trigger levels. Assets belonging to Euro Stoxx 50 index.

At this point, it is time to see what happens with the assets belonging to DJIA index. The analysed period is between January 2, 2001 and April 4, 2014. We summarize everything in figure 4.10 and table 4.4.

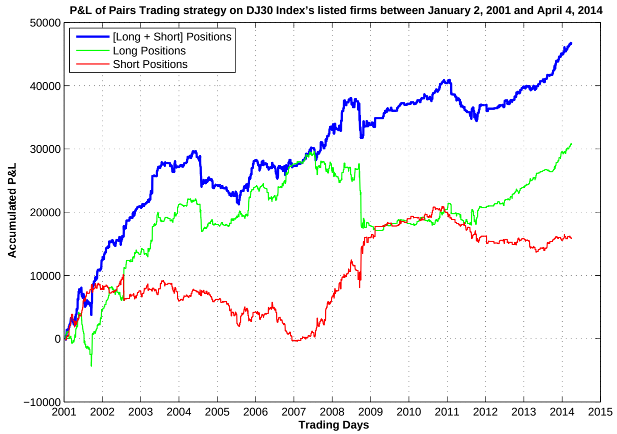
Although we are now doing the sensitivity analysis with the US stocks, we really observe the same patterns as for the Europeans ones. This was expected because the stocks taken into account in this analysis are the blue chips from US and the Euro zone. Hence, it was expected that the strategy behaves in an extremely similar way. We really establish the same relationships as before among the trigger and the rest of the variables. Namely, the greater the trigger level...

- ... the lower the gross profits.
- ... the lower the total number of trades.
- ... the greater the average return per trade.
- ... the lower the number of days with at least one open position.
- ... the lower the number of days with at least one trade.
- ... the lower the average number of trades each day.

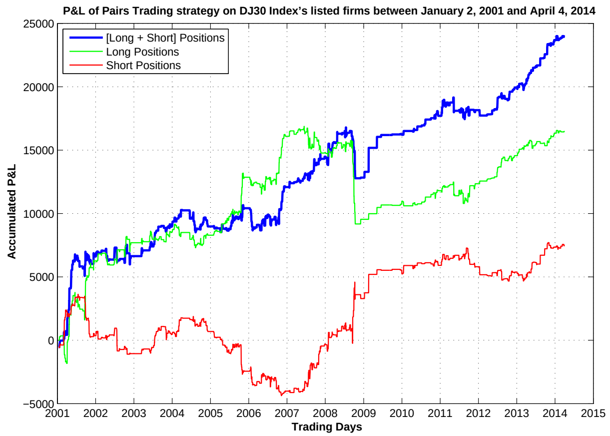
The factors that explain these relationships are the same ones as before for the Euro zone stocks.



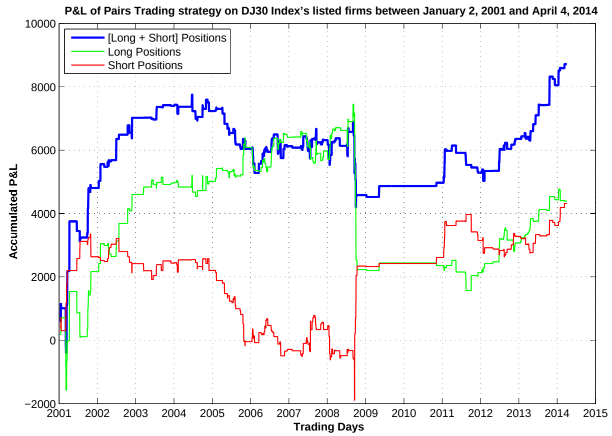
(A) Trigger $= \pm 1\sigma_{spread}$.



(B) Trigger $= \pm 1.5\sigma_{spread}$.



(C) Trigger $= \pm 2\sigma_{spread}$.



(D) Trigger $= \pm 2.5\sigma_{spread}$.

FIGURE 4.10: Sensitivity analysis of Pairs Trading strategy when modifying the value for the trigger. Stocks belonging to Dow Jones 30 index.

Grid of values for the Trigger	$\pm 1\sigma_{spread}$	$\pm 1.5\sigma_{spread}$	$\pm 2\sigma_{spread}$	$\pm 2.5\sigma_{spread}$
Gross P&L ([Long + Short] Positions	62023.79	46611.35	23936.52	8712.98
# of trades	6174	2514	1196	494
Average return per trade	**0.20%**	**0.37%**	**0.40%**	**0.43%**
SD (σ_{return}) of Return per trade.	3.72%	3.27%	2.95%	2.96%
Number of days with at least one open position (% of total trading days)	3422 (99.19%)	2730 (79.13%)	1636 (47.42%)	712 (20.64%)
Number of days with at least one trade (% of total trading days)	1913 (55.45%)	1009 (29.25%)	542 (15.71%)	225 (6.52%)
Average number of trades each day (new and already held positions)	3.23	1.95	1.01	0.70

Table 4.4: Statistics for different trigger levels. Assets belonging to DJIA index.

4.4.4.2 Trading fees

So far we have assumed that there are no fees for buying or selling stocks. However this is not a real fact. The effect of introducing trading fees is clear; they reduce the profits of the strategy. The key question is in what quantity profits are reduced. The answer is that it depends on our broker. There are plenty of brokers and almost each of them carries out a different policy regarding the trading fees. In some cases, the trading fees are simply a flat rate but in other cases they are a % of the total volume invested in each trade.

In order to quantify the effect of trading fees on Pairs Trading strategy for EU50 index's listed firms, we are going to take a real brokerage firm with real trading fees. Specifically, we considered the European firm "X-Trade Brokers, Inc."[20] which charges a rate of 0.10% of the total volume of each trade with a minimum of €10 per trade. We then considered, just for simplifying the calculations, a flat rate of €10 since we invest [€10000; $-$€10000$\hat{\beta}$], being $\hat{\beta} > 0$ and, particularly, a number or grid of numbers that the investor can choose (see section 4.4.4.3). Due to the fact that we are operating in Euro zone firms; i.e., in €, it make sense to implement the strategy using a European brokerage firm in order to eliminate the currency risk (in case that € is not the investor's local currency). Otherwise, we would be forced to apply the corresponding exchange rate whenever a trade is opened or closed.

Table 4.5 summarizes the effects of introducing trading fees on the Paris Trading applied on stocks belonging to EU50 Index. As we can see, the effects on the strategy's profitability are quite noticeable. The net average return per trade drops by 0.1% in every single period for the long and short positions and, consequently, it decreases 0.2% for the net positions ([Long + Short]). Actually, in some cases, gross profit and gross average return per trade become negative (short positions from Jan. 2, 2003 to Jan. 2, 2007). Although we keep on having net profits in the net positions, the introduction of trading fees has a dramatic effect on the profitability of the strategy. Trading fees do only affect the strategy's profitability (P&L and average return per trade); the remaining statistics such as the number of trades or the standard deviation of return per trade prevail constant. Actually, the statistics for these latter variables can be seen in Appendix B: B.1, B.2 and B.3.

[20] X-Trade Brokers, Inc. is a brokerage firm registered and supervised by the securities and exchange commissions of various European countries such as UK's FCA or Germany's BaFin.

RESULTS from January 2, 2003 to April 4, 2014	**{Long Positions}**	**{Short Positions}**	**{[Long + Short] Positions}**
Gross P&L	12088.58	18356.70	30445.24
Net P&L	218.58	6486.70	6705.24
Gross Average return per trade	0.10%	0.15%	0.26%
Net Average return per trade	0.00%	0.05%	0.06%
RESULTS from January 2, 2003 to January 2, 2007			
Gross P&L	15934.15	1241.38	17175.53
Net P&L	11714.15	-2978.62	8735.53
Gross Average return per trade	0.38%	0.03%	0.41%
Net Average return per trade	0.28%	-0.07%	0.21%
RESULTS from January 2, 2007 to April 4, 2014			
Gross P&L	14811.60	18571.91	33383.51
Net P&L	5721.60	9481.91	15203.51
Gross Average return per trade	0.16%	0.20%	0.37%
Net Average return per trade	0.06%	0.10%	0.17%

TABLE 4.5: Results of introducing trading fees (0.1% of total volume of each trade, min. €10) on Pairs Trading strategy. Stocks belonging to EU50 Index.

In order to carry out the study of the implications of trading fees on Pairs Trading performance in DJIA index's listed firms, we also consider a real stock brokerage firm with real trading fees. Particularly, we considered the US brokerage firm "Zecco Trading"[21] which is an on-line stock brokerage that offers US stock trades charging a flat rate of US$4.95 per stock traded, regardless of the size of the account, the trading frequency or the number of traded shares. Furthermore, the same rate applies to market, limit, stop and stop limit orders. In this case, we are operating in US firms; i.e., in US$. Hence, it is very interesting to implement the Paris Trading strategy using a US brokerage in order to remove the currency risk (in case that US$ is not the investor's local currency).

Table 4.6 collects the effects of introducing trading fees on the Paris Trading applied on stocks belonging to DJIA Index. Trading fees applied to US stocks trades are practically half than in the Eurozone. Hence, the effects on the strategy's profitability are not that significant. As before, trading fees only affect the strategy's profitability; the remaining statistics of the strategy can be seen in Appendix B: B.4, B.5 and B.6.

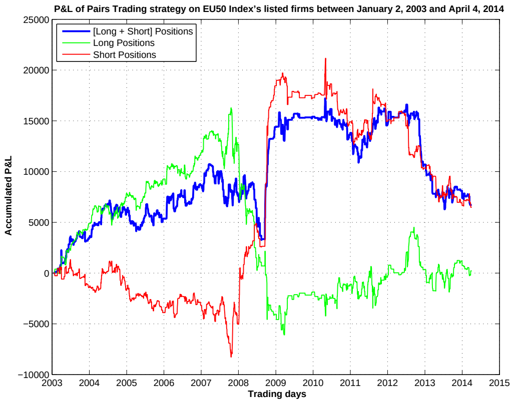
In section 4.4.4.1, we carried out a sensitivity analysis of the trigger level. As we proved, the lower the trigger level, the greater the number of trades. Obviously, as the number of trades increase, the trading fees also increase in a multiplying way. Hence, we should be careful if we set a small value for the trigger since it means that the number of trades, as we saw in the previous section, increase. At this time, it is key to remark that before implementing this strategy is crucial, first, to take into account the commission of the broker and, second, to set the value of σ_{spread} because it is possible to get a gross profit but a net loss due to the introduction of trading fees.

Figure 4.11 collects the evolution of the accumulated P&L of Pairs Trading strategy implemented in EU50 and DJIA index's listed firm during the periods analysed in sections 4.4.1 and 4.4.2, taking into account trading fees of €10 (for Euro zone stocks) and US$4.95 (for US stocks) for each trade. As we can see, the shape of each sub-figure is very close to the one that does not take into account trading fees. However, the risk profile (maximum loss reached by the strategy) increases.

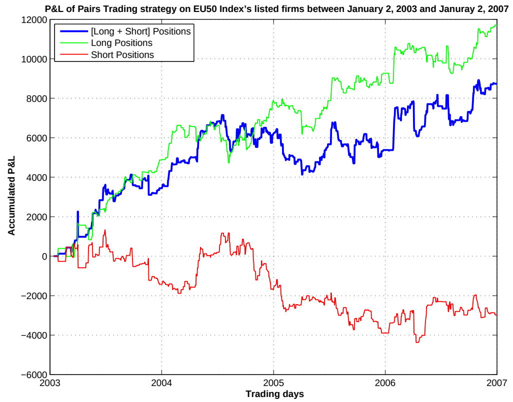
[21] Zecco Trading, Inc. is a member of US Financial Industry Regulatory Authority (FINRA) and Securities Investor Protection Corporation (ISIPC).

RESULTS from January 2, 2001 to April 4, 2014	**{Long Positions}**	**{Short Positions}**	**{[Long + Short] Positions}**
Gross P&L	30718.07	15893.27	46611.35
Net P&L	24495.92	9671.12	34167.05
Gross Average return per trade	0.24%	0.13%	0.37%
Net Average return per trade	0.19%	0.08%	0.27%
RESULTS from January 2, 2003 to January 2, 2007			
Gross P&L	12705.29	-4203.06	8502.23
Net P&L	10685.69	-6222.67	4463.03
Gross Average return per trade	0.31%	-0.10%	0.21%
Net Average return per trade	0.26%	-0.15%	0.11%
RESULTS from January 2, 2007 to April 4, 2014			
Gross P&L	1951.93	17404.40	19356.34
Net P&L	-909.17	14543.30	13634.14
Gross Average return per trade	0.03%	0.30%	0.33%
Net Average return per trade	-0.02%	0.25%	0.24%

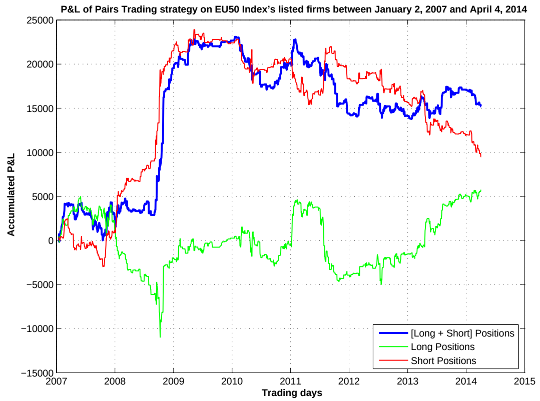
TABLE 4.6: Results of introducing trading fees (flat rate at US$4.95) on Pairs Trading strategy. Stocks belonging to DJIA Index.



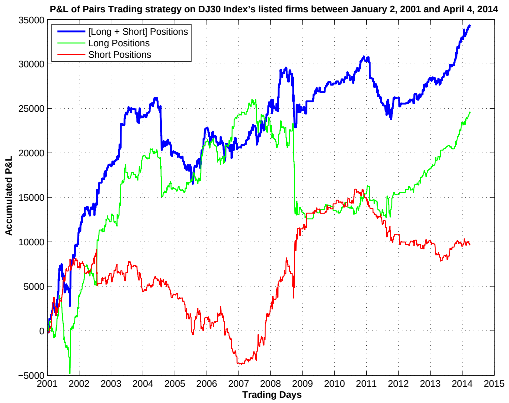
(A) PT on EU50 index's listed firms between Jan. 2, 2003 and Apr. 4, 2014.



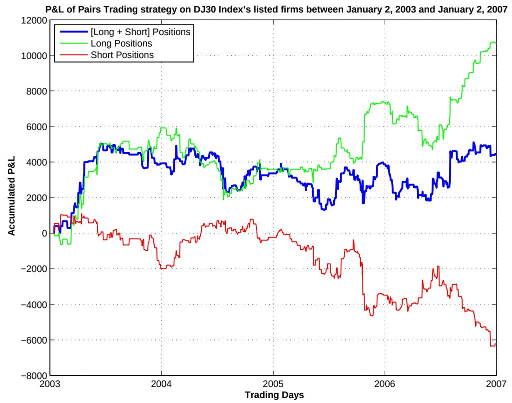
(B) PT on EU50 index's listed firms between Jan. 2, 2003 and Jan. 2, 2007.



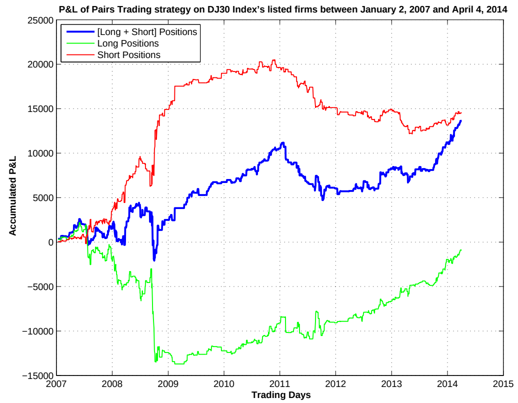
(C) PT on EU50 index's listed firms between Jan. 2, 2007 and Apr. 2, 2014.



(D) PT on DJ30 index's listed firms between Jan. 2, 2001 and Apr. 4, 2014.



(E) PT on DJ30 index's listed firms between Jan. 2, 2003 and Jan. 2, 2007.



(F) PT on DJ30 index's listed firms between Jan. 2, 2007 and Apr. 4, 2014.

FIGURE 4.11: Results of implementing PT strategy defined as in sections 4.4.1 and 4.4.2 with the incorporation of trading fees.

4.4.4.3 Capital

The volume with which we enter in each trade is a very important variable. So far, in all previous analysis, we considered a volume per trade of €10000 for the Euro zone stocks, and a volume per trade of US$10000 for the US stocks.

As we have seen, the cointegrating vector $[1; -\hat{\beta}]$ is what we use as the monetary units invested in a cointegrated pair[22]. The cointegrating vector is obtained by estimating the following OLS regression:

$$StockA_t = \alpha + \beta StockB_t + u_t \qquad u_t \sim \mathcal{N}(0,1) \tag{4.3}$$

The estimated β, $\hat{\beta}$, is

$$\hat{\beta} = \frac{cov(StockA, StockB)}{var(StockB)}$$

If $\hat{\beta}$ equals 1, then the strategy is self-funded[23] in the sense that we use the money from the short sale (we sell the borrowed shares) to fund the long position. Therefore, values of $\hat{\beta}$ closer to 1 are extremely interesting. Since two cointegrated stocks share the same patter, there is no doubt that $\hat{\beta}$ should be positive since $cov(StockA, StockB)$ should also be positive and $var(StockB)$ is always a positive number. Actually, all $\hat{\beta}$'s in this study were strictly positive for the cointegrated pairs. However, apart from the cointegration point of view, given the kind of firms we used in this study, the *covariance* between a certain pair of stocks is also expected to be positive. And this is so because we are taking into account, on one hand, the 50 largest and most liquid firms in the Euro zone, and on the other, the 30 most traded stocks in the US Stock Exchange.

The importance of the needed capital required to implement the Pairs Trading strategy depends on the value of $\hat{\beta}$. Note that the β in equation 4.3 tells us in how many monetary units the market price of stock A will increase, when the market price of stock B increases in one monetary unit, *ceteris paribus*. Therefore, the estimated β, $\hat{\beta}$, depends on the bands in which the prices of the cointegrated stocks are trading, i.e., its estimated value depends on the magnitude of the firms'

[22] Following Lin's *et al.* (2003) [24] Cointegration Coeffiecientes weighted (CCW) rule.

[23] As long as there are no transaction costs.

stock market price. Thus, if we carry out an OLS regression as in 4.3, and stock A has been trading within a €400-500 band during the sample size and stock B has been trading within a €4-8 band, we expect a value for $\hat{\beta}$ much greater than one because of the interpretation of $\hat{\beta}$. The reason why we expect a $\hat{\beta}$ much greater than one is based on the fact that it is relatively easy for the stock that is trading within the €400-500 band to increase in 1 monetary unit, but it is not that easy to increase 1 monetary unit for the firm that is trading within the €4-8 band. Thereby, remembering the interpretation of $\hat{\beta}$, an increment of 1 monetary unit in the market price of stock B should result in an increment by more than 1 monetary units in the price of Stock A. The alternative argument applies when the stock A is trading in the low-band and the stock B is trading in the high-band. In this new context, we expect positive but close to zero values for $\hat{\beta}$. Therefore, depending on the way we implement the regression, we might get high or low values for $\hat{\beta}$.

To summarize, the amount of money required for implement this strategy (assuming no transaction costs) depends on the value of $\hat{\beta}$. Namely,

1. If $0 < \hat{\beta} < 1$: The strategy is not self-funded and there is a % of the long position that is not funded by the short sale.

2. If $\hat{\beta} = 1$: The strategy is completely self-funded.

3. If $\hat{\beta} > 1$: The strategy is completely self-funded and we get extra cash from the short sale.

Therefore, an investor can restrict his/her trades to those cointegrated pairs that show a determined value or grid of values for $\hat{\beta}$ based on his/her preferences or availability of capital.

Just to illustrate the effect of the volume per trade on the strategy's profitability, we will take into account the strategy implemented with stocks belonging to DJIA index between January 2, 2001 and April 4, 2014. The gross profits obtained were US$31724.18. Nonetheless, these profits were obtained by taking into account a volume per trade of US$10000. If we now consider a volume of US$1000 per trade (ten times less), the profits are reduced by ten times. Alternatively, if the invested volume per trade is now US$100000 (ten times more), the profits increase by ten times. The conclusion is that risking more money leads to greater potential profits (if the strategy performs well) or greater potential losses (if the strategy performs wrong).

4.4.5 Short Selling. Legal ban and implications of its cost on Pairs Trading profitability

The ban of short selling is a significant point when implementing Pairs Trading strategy. During all this empirical study, we assumed that short selling is allowed and that there are no initial margins, neither costs of borrowing nor cash guarantees for the short selling.

First of all, we should distinguish between short selling and naked short selling. While short selling is the practice of selling a stock that the seller does not own (in fact, borrows it) and then purchases it later, naked short selling is the practice of selling a stock short, without first borrowing the shares (as occurs in a conventional short sale). Therefore, short sales or naked short sales are interesting strategies when we anticipate falls in stock prices.

Suppose that we identify a cointegrating relationship between two stocks. Then, the spread between them walks away from its historical expectation reaching a level greater than the trigger (say, $+1.5\sigma_{spread}$). Hence, the strategy begins by going short on the spread. This involves selling stock A and buying β times stock B. However, if short selling is not allowed, we cannot implement a Pairs Trading strategy[24] due to the fact we cannot short sell stock B.

Consequently, the legal ban of short selling is a very important risk[25] in Pairs Trading strategy because the strategy itself consists of buying one stock and selling another. And, unless we have in portfolio the stock we want to sell, we are forced to use the short selling.

So far we did not include in the empirical study the possibility that the legislators ban the practice of short selling. Moreover, we did not take into account the costs of this practice. Namely, initial margins, interest costs and cash guarantees.

Nonetheless, in Pizzutilo (2013) [28], all of these variables were taken into account and the conclusions were that they, in fact, significantly affect the payoffs from pairs trading even though net excess returns remain largely positive. Additionally, he found evidences that restrictions to the number of shares that are allowed to be shorted have a significant impact on the risk profile of the strategy. During the analysed period there were some dates in which the short selling was banned in US

[24]Unless we have Stock A in portfolio, so that we can sell it.
[25]Operational risk.

and in the Euro zone due to the extremely high volatility in the financial markets. Just to illustrate it with an example, on September 19, 2008 the U.S. Securities and Exchange Commission (SEC) banned short selling in 799 stocks. However, this prohibition lasted only until October 9, 2008. In Europe, the ban lasted much longer. For instance, in Spain, the very last regulation on short selling was the ban of this practice between July 23, 2012 and January 31, 2013. Therefore, the short selling was not allowed for the spanish stocks belonging to Euro Stoxx 50 index. The same happened in some other European countries such as Germany, Italy and France. More information about this topic can be found in Beber *et al.* (2011) [5]

As a final remark, this is an operational risk (legal ban of short selling) that affects Pairs Trading and that we should not ignore before implementing this strategy.

Chapter 5

Conclusions

The main objective of this study was to answer empirically three important questions.

The first one is related to the observed declining trend in the profitability of Pairs Trading documented in previous studies from a "distance approach" point of view. We proved that Pairs Trading is a profitable gross strategy when implementing a cointegration approach. However, trading fees have a significant effect on the profitability. Even so, Pairs Trading keeps on being a profitable net strategy. Thus, this fact is line with the observed pattern in previous literature, judging from the low (but positive) average net returns that we have obtained in the empirical study.

The second one has to do with the market neutrality of Pairs Trading. Some authors include Pairs Trading within the *market neutral* investing strategies. Actually, by doing a quick internet search, one is able to find that Pairs Trading is defined as a market neutral strategy. However, by carrying out a simple analysis of this topic (applying a correlation approach), we found that the profitability of Pairs Trading implemented on the stocks belonging to significant stock market indices (Euro Stoxx 50 and DJIA) shows correlation with their respective index. Therefore, Pairs Trading seems not to be a *market neutral* investing strategy.

The third one deals with the effects of volatility on the profitability of Pairs Trading. We proved that volatility is actually needed for this strategy to work. In order to see the effects, we implemented a sensitivity analysis regarding the volatility. As the trigger level increase, the spread finds more difficulties in order to hit it. In fact, when setting the trigger at an elevated level (say, $\pm 2\sigma_{spread}$ or $\pm 2.5\sigma_{spread}$), there are very few trades and high volatility is needed (2008 financial crisis) for the strategy to work. The conclusion is that in periods of low volatility, we might set the trigger at relatively low levels, and in periods of high volatility, it is preferred to fix it at greater levels (otherwise, the risk control strategies will be activated more times than they should be).

Furthermore, the implementation of Pairs Trading by industrial sectors tells us that the financial sector is the most profitable one. As Do *et al.* (2009) [9] stated, this was expected since financial stocks are sensitive to common macroeconomic factors such as interest rates and unemployment shocks; hence, their share prices are likely to move together but not in the same proportion, allowing the conditions required for a successful Pairs Trading strategy.

Appendix A

Cointegration Matrices

In this Appendix, cointegration matrices are going to be shown. A cointegration matrix is a symmetric matrix composed of zeros and ones. A zero means that there are no cointegrating relationships between a determined pair of stocks while a one means that, at least, the pair shows one cointegrating relationship.

For the two stock market indices taken into account (Euro Stoxx 50 and Dow Jones Industrial Average), these are the cointegration matrices for their listed firms for every analysed period. It is important to point out that the cointegrating relationships among all possible pairs of stocks are updated every 6 months by taking into account all past information, and once the *pairs formation period* has finished (see methodology section in 4.2).

A.1 Cointegration Matrices for Euro Stoxx 50 Index's listed firms

A.1.1 January 2003 - April 2014

GDF SUEZ was deleted since its available data in *Datastream* begins on July 6, 2005. Therefore, we have 49 firms and a cointegration matrix of size 49x49. We have found out 91 cointegrated pairs of stocks.

	B:ABI	F:TAL	F:SQ@F	D:SIEX	E:SCH	D:BAYX	F:OR@F	F:BNP	D:DAIX	D:BASX	F:LVMH	D:SAPX	E:IND	I:ENI	E:TEF	D:ALVX	D:BMWX	E:BBVA	D:DTEX	H:UNIL	F:MIDI	F:AIRS	I:UCG	H:ING	I:ISP	I:ENEL	F:QT@F	F:SGE	F:BSN	E:IBE	F:DG@F	D:DBKX	D:DPWX	F:AIR	F:ORA	H:ASML	D:MU2X	I:G	D:EONX	F:EX@F	E:REP	H:PHIL	F:GOB	F:CRFR	H:UBL	F:EI	D:RWEX	CRGI	D:VO3X
B:ABI	0	0	0	0	0	0	1	0	0	1	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1
F:TAL	0	0	0	0	1	0	0	1	0	0	0	0	0	1	0	0	0	1	0	0	1	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	1	0	1	0	0	0	0	1	0
F:SQ@F	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
D:SIEX	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
E:SCH	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
D:BAYX	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F:OR@F	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1
F:BNP	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0
D:DAIX	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
D:BASX	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
F:LVMH	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
D:SAPX	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
E:IND	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
I:ENI	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0
E:TEF	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
D:ALVX	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
D:BMWX	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
E:BBVA	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0
D:DTEX	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
H:UNIL	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	1	0	0	1
F:MIDI	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0
F:AIRS	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
I:UCG	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
H:ING	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
I:ISP	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	1	0
I:ENEL	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
F:QT@F	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0
F:SGE	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0
F:BSN	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
E:IBE	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F:DG@F	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
D:DBKX	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
D:DPWX	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F:AIR	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1
F:ORA	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
H:ASML	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
D:MU2X	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
I:G	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
D:EONX	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
F:EX@F	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
E:REP	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0
H:PHIL	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F:GOB	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
F:CRFR	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
H:UBL	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F:EI	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
D:RWEX	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
CRGI	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
D:VO3X	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

TABLE A.1: Cointegration matrix for EU50 Index's listed firms between January 2, 2003 and April 4, 2014.

A.1.2 January 2003 - January 2007

During this period, GDF SUEZ was also deleted due to missing data in *Datastream* until July 6, 2005. Consequently, we have 49 firms and a cointegration matrix of size 49x49. In this case we have found out 220 cointegrated pairs of stocks.

	B:ABI	F:TAL	F:SQ@F	D:SIEX	E:SCH	D:BAYX	F:OR@F	F:BNP	D:DAIX	D:BASX	F:LVMH	D:SAPX	E:IND	I:ENI	E:TEF	D:ALVX	D:BMWX	E:BBVA	D:DTEX	H:UNIL	F:MIDI	F:AIRS	I:UCG	H:ING	I:ISP	I:ENEL	F:QT@F	F:SGE	F:BSN	E:IBE	F:DG@F	D:DBKX	D:DPWX	F:AIR	F:ORA	H:ASML	D:MU2X	I:G	D:EONX	F:EX@F	E:REP	H:PHIL	F:GOB	F:CRFR	H:UBL	F:EI	D:RWEX	CRGI	D:VO3X
B:ABI	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0
F:TAL	0	0	1	0	0	0	0	0	0	1	0	0	0	1	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	1	0	1	0
F:SQ@F	0	0	0	0	1	1	0	1	0	1	1	1	1	1	0	0	1	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	1	1	0	1	0	1	0	1	1	1	1	0
D:SIEX	0	0	0	0	1	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0
E:SCH	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	1	0	1	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0
D:BAYX	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F:OR@F	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F:BNP	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	1	0	1	0	1	1	1	1	1	1	0	1	0	0	0	1	1	0	0	0	1	0	0	1	1	1	0
D:DAIX	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
D:BASX	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	1	0	0	0	1	0	0	0	0	0	1	0	0	1	0	0	0	1	1	0	0	0	0	0	0	1	1	1	0
F:LVMH	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	1	0	0	0	0	1	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0
D:SAPX	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
E:IND	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
I:ENI	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	1	1	0	0	0	0	0	0	1	1	1	0
E:TEF	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
D:ALVX	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0	1	0	0	1	0	0	0	0	0	1	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0
D:BMWX	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0
E:BBVA	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	1	0	1	1	1	1	1	0	0	1	0	0	0	1	1	0	0	0	1	0	1	1	1	1	0
D:DTEX	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
H:UNIL	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F:MIDI	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	1	1	1	0	1	0	0	1	0	0	0	1	1	0	0	0	1	0	0	1	1	1	0
F:AIRS	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
I:UCG	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0
H:ING	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
I:ISP	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	1	0	1	0	0	0	1	1	0	0	0	1	0	0	0	0	1	0
I:ENEL	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F:QT@F	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	1	0	1	0	0	0	1	1	0	0	0	0	0	0	1	1	1	0
F:SGE	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	1	0	0	0	1	1	0	0	0	0	0	0	1	1	1	0
F:BSN	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
E:IBE	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
F:DG@F	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	1	0	0	0	0	0	0	0	1	1	0
D:DBKX	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0	0	1	0	0	1	1	1	0
D:DPWX	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F:AIR	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	1	1	1	1	0
F:ORA	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
H:ASML	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
D:MU2X	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
I:G	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	1	1	1	1	0
D:EONX	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0
F:EX@F	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
E:REP	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
H:PHIL	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F:GOB	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
F:CRFR	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
H:UBL	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F:EI	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0
D:RWEX	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
CRGI	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
D:VO3X	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

TABLE A.2: Cointegration matrix for EU50 Index's listed firms between January 2, 2003 and January 2, 2007.

A.1.3 January 2007 - April 2014

For this last analysed period, all firms belonging to Euro Stoxx 50 Index are included. Thereby, we have a cointegration matrix of size 50x50. We have been capable of finding out 110 cointegrated pairs of stocks.

	B:ABI	F:TAL	F:SQ@F	D:SIEX	E:SCH	D:BAYX	F:OR@F	F:BNP	D:DAIX	D:BASX	F:LVMH	D:SAPX	E:IND	I:ENI	E:TEF	D:ALVX	D:BMWX	E:BBVA	D:DTEX	F:GSZ	H:UNIL	F:MIDI	F:AIRS	I:UCG	H:ING
B:ABI	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0
F:TAL	0	0	0	0	1	0	0	1	0	0	0	0	0	1	0	1	0	1	1	0	0	1	0	1	1
F:SQ@F	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
D:SIEX	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
E:SCH	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
D:BAYX	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F:OR@F	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
F:BNP	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	1	0	0	0
D:DAIX	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
D:BASX	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
F:LVMH	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
D:SAPX	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0
E:IND	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
I:ENI	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	1	1
E:TEF	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
D:ALVX	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
D:BMWX	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
E:BBVA	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
D:DTEX	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	1
F:GSZ	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
H:UNIL	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F:MIDI	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
F:AIRS	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
I:UCG	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
H:ING	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
I:ISP	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
I:ENEL	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F:QT@F	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F:SGE	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F:BSN	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
E:IBE	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F:DG@F	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
D:DBKX	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
D:DPWX	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F:AIR	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F:ORA	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
H:ASML	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
D:MU2X	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
I:G	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
D:EONX	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F:EX@F	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
E:REP	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
H:PHIL	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F:GOB	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F:CRFR	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
H:UBL	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F:EI	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
D:RWEX	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
CRGI	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
D:VO3X	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

	I:ISP	I:ENEL	F:QT@F	F:SGE	F:BSN	E:IBE	F:DG@F	D:DBKX	D:DPWX	F:AIR	F:ORA	H:ASML	D:MU2X	I:G	D:EONX	F:EX@F	E:REP	H:PHIL	F:GOB	F:CRFR	H:UBL	F:EI	D:RWEX	CRGI	D:VO3X
B:ABI	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0
F:TAL	1	1	0	1	0	1	0	1	0	0	0	0	0	1	0	1	0	0	1	1	0	0	0	1	0
F:SQ@F	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
D:SIEX	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
E:SCH	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
D:BAYX	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F:OR@F	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F:BNP	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0
D:DAIX	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
D:BASX	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
F:LVMH	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
D:SAPX	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
E:IND	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
I:ENI	1	0	0	1	0	0	0	1	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	1	0
E:TEF	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
D:ALVX	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
D:BMWX	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
E:BBVA	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	1	0
D:DTEX	1	1	0	1	0	1	1	0	0	0	0	0	0	1	0	1	0	0	1	1	0	0	0	1	0
F:GSZ	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	1	1	0	0
H:UNIL	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	1	0	0	1
F:MIDI	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0
F:AIRS	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
I:UCG	1	1	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
H:ING	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
I:ISP	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	1	0	0	0	1	0
I:ENEL	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0
F:QT@F	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F:SGE	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0
F:BSN	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
E:IBE	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	1	0	0	0	0	0
F:DG@F	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
D:DBKX	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
D:DPWX	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F:AIR	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1
F:ORA	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0
H:ASML	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
D:MU2X	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
I:G	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0
D:EONX	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F:EX@F	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0
E:REP	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
H:PHIL	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F:GOB	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F:CRFR	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
H:UBL	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F:EI	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
D:RWEX	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
CRGI	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
D:VO3X	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table A.3: Cointegration matrix for EU50 Index's listed firms between January 2, 2007 and April 4, 2014.

A.2 Cointegration Matrices for Dow Jones 30 Index's listed firms

A.2.1 January 2001 - April 2014

During this period, VISA was deleted since its available data in *Datastream* start on March 13, 2008. Therefore, we have 29 firms and a cointegration matrix of size 29x29. We have found out 26 cointegrated pairs of stocks.

	U:XOM	@MSFT	U:JNJ	U:GE	U:WMT	U:JPM	U:CVX	U:PG	U:IBM	U:PFE	U:VZ	U:T	U:KO	U:MRK	U:DIS	@INTC	@CSCO	U:HD	U:UTX	U:MCD	U:AXP	U:BA	U:MMM	U:UNH	U:GS	U:CAT	U:DD	U:NKE	U:TRV
U:XOM	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
@MSFT	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
U:JNJ	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0
U:GE	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
U:WMT	0	0	0	0	0	0	1	0	1	0	0	0	1	0	1	0	0	1	1	1	0	0	1	0	0	0	0	1	1
U:JPM	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
U:CVX	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
U:PG	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	1	0	0	0
U:IBM	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
U:PFE	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
U:VZ	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
U:T	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
U:KO	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	1
U:MRK	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
U:DIS	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
@INTC	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
@CSCO	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
U:HD	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
U:UTX	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
U:MCD	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
U:AXP	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0
U:BA	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
U:MMM	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
U:UNH	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
U:GS	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
U:CAT	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
U:DD	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
U:NKE	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
U:TRV	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

TABLE A.4: Cointegration matrix for DJIA Index's listed firms between January 2, 2001 and April 4, 2014.

A.2.2 January 2003 - January 2007

As in the previous case, VISA was deleted due to missing data until March 13, 2008. Consequently, we have 29 firms and a cointegration matrix of size 29x29. We find out 31 cointegrated pairs of stocks in this period.

	U:XOM	@MSFT	U:JNJ	U:GE	U:WMT	U:JPM	U:CVX	U:PG	U:IBM	U:PFE	U:VZ	U:T	U:KO	U:MRK	U:DIS	@INTC	@CSCO	U:HD	U:UTX	U:MCD	U:AXP	U:BA	U:MMM	U:UNH	U:GS	U:CAT	U:DD	U:NKE	U:TRV
U:XOM	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
@MSFT	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
U:JNJ	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
U:GE	0	0	0	0	0	1	1	1	0	0	0	0	0	1	1	0	0	1	1	1	1	1	0	1	1	1	0	1	0
U:WMT	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
U:JPM	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
U:CVX	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0
U:PG	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	1	1	0	0	0	0	0	0	1	0
U:IBM	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
U:PFE	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
U:VZ	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
U:T	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1
U:KO	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
U:MRK	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
U:DIS	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0
@INTC	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
@CSCO	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
U:HD	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
U:UTX	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
U:MCD	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
U:AXP	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
U:BA	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
U:MMM	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
U:UNH	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
U:GS	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
U:CAT	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
U:DD	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
U:NKE	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
U:TRV	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

TABLE A.5: Cointegration matrix for DJIA Index's listed firms between January 2, 2003 and January 2, 2007.

A.2.3 January 2007 - April 2014

VISA was not taken into account due to missing data until March 13, 2008. Therefore, we have 29 firms and a cointegration matrix of size 29x29. We are able to find out 35 cointegrated pairs of stocks in this period.

	U:XOM	@MSFT	U:JNJ	U:GE	U:WMT	U:JPM	U:CVX	U:PG	U:IBM	U:PFE	U:VZ	U:T	U:KO	U:MRK	U:DIS	@INTC	@CSCO	U:HD	U:UTX	U:MCD	U:AXP	U:BA	U:MMM	U:UNH	U:GS	U:CAT	U:DD	U:NKE	U:TRV
U:XOM	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
@MSFT	0	0	1	0	0	0	0	1	0	0	0	0	0	1	0	1	0	0	0	0	1	1	1	1	0	0	0	0	0
U:JNJ	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
U:GE	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
U:WMT	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
U:JPM	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
U:CVX	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
U:PG	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	1
U:IBM	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
U:PFE	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
U:VZ	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1
U:T	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
U:KO	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
U:MRK	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
U:DIS	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1
@INTC	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	1	0	1	1	1	0
@CSCO	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
U:HD	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
U:UTX	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
U:MCD	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
U:AXP	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0
U:BA	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
U:MMM	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
U:UNH	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
U:GS	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
U:CAT	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
U:DD	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
U:NKE	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
U:TRV	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

TABLE A.6: Cointegration matrix for DJIA Index's listed firms between January 2, 2007 and April 4, 2014.

Appendix B

Statistics of Pairs Trading strategy

In this appendix we are going to show the statistics of Pairs Trading strategies implemented in Chapter 4. We will consider statistics only for the long and short positions and also for the cointegrated portfolios ([long + short] positions).

The statistics include information about the gross P&L, total number of trades, gross average return per trade, standard deviation of return per trade, the number of days with at least one open position and with at least one trade and, finally, the average number of trades per day (including the ongoing and newly open trades).

B.1 Statistics of Pairs Trading strategy implemented on Euro Stoxx 50 Index's listed firms

B.1.1 January 2003 - April 2014

Statistics for Long Positions	
Gross P&L	12088.58
# of total trades	1187
Gross Average Return per trade	0.1%
SD (σ_{return}) of Return per trade	3.2%

Statistics for Short Positions	
Gross P&L	18356.66
# of total trades	1187
Gross Average Return per trade	0.15%
SD (σ_{return}) of Return per trade	3.1%

Statistics for [Long + Short] Positions	
Gross P&L	30445.24
# of total trades	2374
Gross Average Return per trade	0.26%
SD (σ_{return}) of Return per trade	3.12%
Number of days with at least one open position	2257 (77.06% of total trading days)
Number of days with at least one trade	917 (31.31% of total trading days)
Average number of trades each day	2.59

Table B.1: Statistics between January 2, 2003 and April 4, 2014. EU50 index's listed firms.

B.1.2 January 2003 - January 2007

Statistics for Long Positions	
Gross P&L	15934.15
# of total trades	422
Gross Average Return per trade	0.38%
SD (σ_{return}) of Return per trade	2.07%

Statistics for Short Positions	
Gross P&L	1241.38
# of total trades	422
Average Return per trade	0.03%
SD (σ_{return}) of Return per trade	2.17%

Statistics for [Long + Short] Positions	
Gross P&L	17175.53
# of total trades	844
Average Return per Trade	0.41%
SD (σ_{return}) of Return per trade	2.44%
Number of days with at least one open position	866 (83.67% of total trading days)
Number of days with at least one trade	343 (33.43% of total trading days)
Average number of trades each day	2.44

TABLE B.2: Statistics between January 2, 2003 and January 2, 2007. EU50 index's listed firms.

B.1.3 January 2007 - April 2014

Statistics for Long Positions	
Gross P&L	14811.6
# of total trades	909
Gross Average Return per trade	0.16%
SD (σ_{return}) of Return per trade	3.45%

Statistics for Short Positions	
Gross P&L	18571.91
# of total trades	909
Gross Average Return per trade	0.2%
SD (σ_{return}) of Return per trade	3.09%

Statistics for [Long + Short] Positions	
Gross P&L	33383.51
# of total trades	1818
Gross Average Return per trade	0.37%
SD (σ_{return}) of Return per trade	3.33%
Number of days with at least one open position	1449 (76.87% of total trading days)
Number of days with at least one trade	673 (35.7% of total trading days)
Average number of trades each day	2.7

Table B.3: Statistics between January 2, 2007 and April 4, 2014. EU50 index's listed firms.

B.2 Statistics of Pairs Trading strategy implemented on Dow Jones 30 Index's listed firms

B.2.1 January 2001 - April 2014

Statistics for Long Positions	
Gross P&L	30718.07
# of total trades	1257
Gross Average Return per trade	0.24%
SD (σ_{return}) of Return per trade	3.04%

Statistics for Short Positions	
Gross P&L	15893.27
# of total trades	1257
Gross Average Return per trade	0.13%
SD (σ_{return}) of Return per trade	2.88%

Statistics for [Long + Short] Positions	
Gross P&L	46611.35
# of total trades	2514
Gross Average Return per trade	0.37%
SD (σ_{return}) of Return per trade	3.27%
Number of days with at least one open position	2730 (79.13% of total trading days)
Number of days with at least one trade	1009 (29.25% of total trading days)
Average number of trades each day	2.49

Table B.4: Statistics between January 2, 2001 and April 4, 2014. DJIA index's listed firms.

B.2.2 January 2003 - January 2007

Statistics for Long Positions	
Gross PP&L	12705.29
# of total trades	408
Average Return per trade	0.31%
SD (σ_{return}) of Return per trade	2.45%

Statistics for Short Positions	
Gross P&L	− 4203.06
# of total trades	408
Average Return per trade	− 0.1%
SD (σ_{return}) of Return per trade	2.32%

Statistics for [Long + Short] Positions	
Gross P&L	8502.23
# of total trades	816
Gross Average Return per trade	0.21%
SD (σ_{return}) of Return per trade	2.65%
Number of days with at least one open position	887 (85.70% of total trading days)
Number of days with at least one Trade	330 (31.88% of total trading days)
Average number of trades each day	2.47

TABLE B.5: Statistics between January 2, 2003 and January 2, 2007. DJIA index's listed firms.

B.2.3 January 2007 - April 2014

Statistics for Long Positions	
Gross P&L	1951.93
# of total trades	578
Gross Average Return per trade	0.03%
SD (σ_{return}) of Return per trade	3.08%

Statistics for Short Positions	
Gross P&L	17404.4
# of total trades	578
Gross Average Return per trade	0.3%
SD (σ_{return}) of Return per trade	3.15%

Statistics for [Long + Short] Positions	
Gross P&L	19356.34
# of total trades	1156
Average Return per trade	0.33%
SD (σ_{return}) of Return per trade	3.46%
Number of days with at least one open position	1395 (74.01% of total trading days)
Number of days with at least one trade	470 (24.93% of total trading days)
Average number of trades each day	2.46

TABLE B.6: Statistics between January 2, 2007 and April 4, 2014. DJIA index's listed firms.

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