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"An Asset Allocation Strategy through Cointegration"

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Firma dello studente

Matteo Fronelli

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"It is not the strongest of the species that survives, nor the most intelligent that survives. It is the one that is most adaptable to change." Charles Darwin, "Origin of Species"

Section 1. Introduction

The world of Asset Allocation has changed during the last years and this change is just at the beginning. Nowadays, the performances of many active funds are damaged by the increasing efficiency of the market and observing the U.S. financial markets, we can try to imagine the future of the investing system.

Obviously, the viewpoints are numerous, but looking at the data it is clear that something is happening.

In the recent years, we are seeing an historic shifting from Active to Passive Management and a strong increase in the number of financial products based on the idea of not *"to beat the market"[1]*, but just to follow it. The investors are choosing with their decisions the winning strategy in this special era of Investing.

The consequences of this trend are unpredictable and only the future events will tell us the impact of this phenomenon on the financial markets.

In this dissertation, we study the Theory of Cointegration, firstly introduced by Robert F. Engle and Clive W.J.Granger in the late eighties, and we try to adapt it to a passive investment strategy. The idea is to test a statistical technique in an investing environment in order to achieve a portfolio that replicates the trend of an index, understanding the method and evaluating strengths and weaknesses with real data.

Using simple words, the Cointegration is the relationship between two variables, which is stable over time, and it indicates that their trends are linked together into a defined range, "like a complicated love story between two lovers never too far, but never too close."[2]

The use of this technique in the econometric analysis is well known and different applications have been developed.

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[1] The expression *"to beat the market"* is used in the common language to identify the competition to outperform the average return of the market.
[2] This fascinating metaphor was firstly found in a paper of Riccardo Lucchetti, dated in 2000, and in our opinion is the proof that even in a boring world of numbers, it is possible to find romanticism.

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The choice to focus the work on a passive investment strategy was influenced by the growing importance that the financial newspapers and the research's world are dedicated to it during the recent years.

It is clear to us that this work is just a brave attempt in the complex world of the Econometrics and like every beginners, our purpose is only to receive the approval of our masters and of experts.

Citing the words of Bernardo De Chartes, a French philosopher of the XII centuries: "We are like dwarfs sitting on the shoulders of giants. We see more, and things that are more distant, than they did, not because our sight is superior or because we are taller than they, but because they raise us up, and by their great stature add to ours."[3]

1.1 Structure of the Work

The work is divided in five sections and it is organized as follows.

Section 1 shows the Theory Background.
In this first part, we give an extensive explanation of the theory environment, summing up the different views of the asset management, outlining the connection between the investing framework and the theories about the Market Efficiency.
The two theories developed by Eugene Fama and Charles Dow are explained and analyzed looking at the current evolution of the financial markets.

Section 2 is dedicated to the study of Engle-Granger procedure and the tests related.
The theory, formalized more than thirty years ago, spent several years to be accepted, becoming in the recent period a fundamental pillar for the study of non-stationary time series. The researchers theorized several different formulation of the theory, connecting it with others, like the ARCH-GARCH's family model.
For their studies, the two Authors won the Nobel Prize in the 2003.

Section 3 defines the possible applications of Cointegration in finance, explaining also the indexing replication problem and gives a survey of different methods.

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[3] The first quote of this phrase can be found in "Metalogicon", written by John of Salisbury in the 1159.

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The use of Cointegration has spread from academic analysis, where is widely in the macroeconomic framework, to financial system, giving the birth to new strategies, as the pair trading between two financials assets.

Section 4 is an empirical case study of the asset allocation approach tracking the Dow Jones Industrial Average. Using 6 years of daily observations of the Index and the 25 Constituents, we start the study with the explanation of the Engle-Granger procedure.

The empirical analysis follows the work and the OLS methodology firstly applied by Alexander and Dumitru (2002)[4], in order to create a portfolio that can achieve the same return of the Index. As suggested by Stock (1987)[5], in presence of non-stationary time series the OLS estimation may be biased and for this reason, we developed another regression based on the Dynamic OLS, in order to achieve a correct specification of the coefficients.

The focus of the last part is on the comparison between 4 portfolios, with different time of rebalance.

Section 5 concludes the research, summing up the results.

The work has to be viewed as a building block for more complex and diversified strategies, taking into account the possible modifications/implementations to the procedure itself and the connections with other types of investing strategy.

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[4] 'The Cointegration Alpha: Enhanced Index Tracking and Long – Short Equity Market Neutral Strategies'
[5] "Asymptotic properties of least squares estimators of cointegrating vectors."

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"Past performance is not indicative of future results, which may vary" Almost every financial investments' disclaimer.

Section 2. Theory Back Ground

2.1 The World we live in

Talking about Economy, a common phrase that is always true is that "the consumer makes the market". Buying one product instead of another, the choices of the consumers influence the decisions and survival of the companies.

In the last years, the world of asset allocation was characterized by constant outflows from the active investment funds to passive ones. The financial press has dedicated a great attention to this trend and many researches have been written analyzing the different features and reasons of the phenomenon.

According to a report of Pwc, dated in end of 2015, passively management funds have received $2.2 trillion dollars during the last ten years, passing from 9% to 23% .The amount is expected to increase even more, reaching the $5 trillion dollars invested by the 2020.[6]
Moreover, another analysis, published by Bank of America Merrill Lynch's research group on 29 August 2016, outlines as the upward trend is destined to last. In this part of the year, only the 14% of the active managers have reached their goal outperforming the market, and this estimation is the lowest in the history of the data.[7]
The opinions in the finance establishment are various and a huge debate is rising while the phenomenon is increasing.

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[6]" In a recent PwC survey, more than 75% of asset managers predicted that global ETF AUM would rise to USD 5 trillion by 2020", for more details: https://www.pwc.com/us/en/asset-management/investment-management/publications/assets/pwc-mutual-fund-developments.pdf
[7]https://www.ml.com/articles/market-updates.html

If someone wants to find a starting point, it can be the 1976, when John Bogle and Vanguard launched the first index mutual fund. In forty years, the number of ETFs and mutual funds that tracks an index has increased every years, giving life to a new market and to new tendencies.[8]
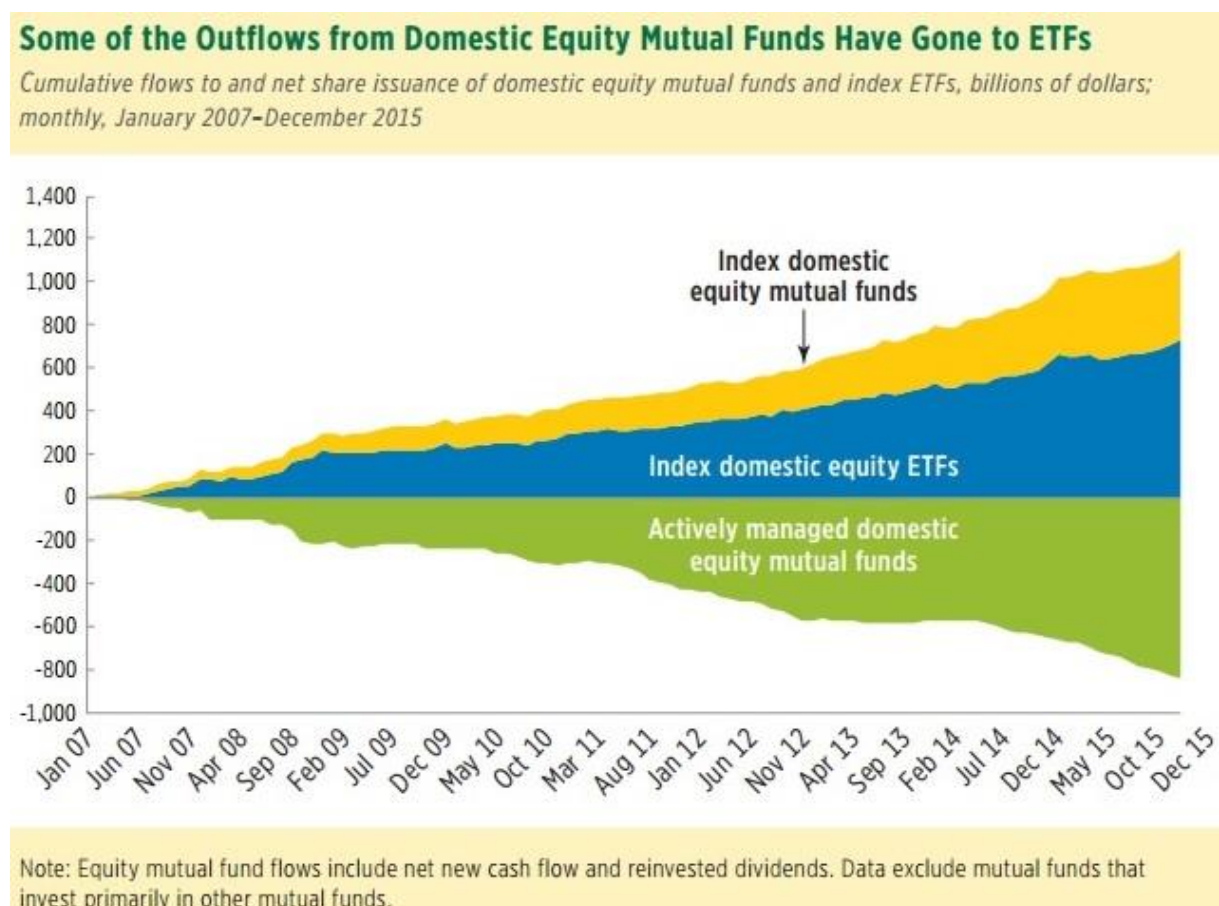
"Democratising finance: How passive funds changed investing" is the tough title of an article of the Financial Times, written by Judith Evans and Jonathan Eley, in which the two journalists reconstruct the history of the passive investing.

They.write : "The process of bringing diversified, affordable investment products to the masses started with investment trusts, which first appeared in the UK in the 1860s and afforded the investor of moderate means the same advantages as large capitalists.

Open-ended mutual funds followed in the 1920s, and were boosted in the 1990s by fund supermarkets, which made them more popular by removing the initial charges for investing. By contrast, passive investing is a fairly recent arrival."[9].

Figure1 Source: Investment Company Institute

Some of the Outflows from Domestic Equity Mutual Funds Have Gone to ETFs

Cumulative flows to and net share issuance of domestic equity mutual funds and index ETFs, billions of dollars; monthly, January 2007-December 2015

Note: Equity mutual fund flows include net new cash flow and reinvested dividends. Data exclude mutual funds that invest primarily in other mutual funds.

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[8] Recently, on the CFA Istitute's Financial Analyst Journal, Bogle writes:" The fundamental principles established by that first index fund are simple: Buy virtually the entire US stock market and hold it intact 'forever,' eliminate advisory fees, and minimize both operating costs and portfolio turnover." for more information, we suggest http://www.cfapubs.org/doi/pdf/10.2469/faj.v72.n1.5
[9] https://www.ft.com/content/b3c0c960-a56c-11e4-bf11-00144feab7de

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The Figure 1 shows the constant shifting in the investors' choices. The amount of dollars invested in index funds and ETFs is raising year by year.

Not all the opinion are enthusiastic.

In a note entitled "The Silent Road to Serfdom: Why Passive Investing is Worse Than Marxism," Inigo Fraser-Jenkins[10], a Bernstein Investment analyst writes according to his group of research:

"A supposedly capitalist economy where the only investment is passive is worse than either a centrally planned economy or an economy with active market led capital management."[11]

This fear is shared by many asset managers across the world.

The possible causes are many. Citing the words of two experts working at Citi Investment Management, Robert Jasminski, Head of Global Equities, and Corey Gallagher, Senior Portfolio Manager: "One possibility is that technology and the availability of timely information are making markets ever more efficient. The more efficient a market is, the harder it becomes to outperform via active management. While there may be an element of this, an even more important factor could be Quantitative Easing (QE). The world's leading central banks own data show they have created trillions of dollars of new money."[12]

The Federal Reserve, followed by the European Central Bank and the Bank of England, started, in different years and in different ways, strong purchasing programs of Government and Corporate bonds. The effects of these operations are ambiguous.

The U.S. economy has come back to its pre-crisis level, while the European Economy is struggling to reach its goals, especially in terms of Inflation.

In the global financial markets, basically these mechanisms have pumped up the stock markets with unrealistic stamina and, like a drug or a doping system, it helped the performances creating an addiction.

Even the CEO and Chairman of BlackRock[13], the world's largest asset manager firm, Larry Fink, during an interview to "Squawk Box", the ultimate pre-market news program of the CNBC, outlined the historical momentum of asset allocation, warning investors about the incorrect use of passive approaches to realize higher returns, usually referred to active strategies.

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[10]Inigo Fraser-Jenkins is Head of Global Quantitative and European Equity Strategy, Bernstein Investment Research and Management.

[11] http://www.bloomberg.com/news/articles/2016-08-23/bernstein-passive-investing-is-worse-for-society-than-marxism

[12] https://www.privatebank.citibank.com/home/opinions/the-age-of-active-investment-isnt-over.html

[13] BlackRock is the most important firm in the asset management world, with more than 5 billion of money invested.

He said that: "We're a believer in active, ... and we're continuing to invest in our active portfolios, [but] you're looking a migration from active to passive."[14]

Is active dead?

It is very difficult to say. What we can say is that the investors are changing their preference, in line with the new context.

The complexity of the modern world does not allow us to make correct prediction on the evolution of investing, but it is clear that the Experts and the Regulators have to analyze this ethos evaluating the possible risks.

The increasing correlation between the asset classes can lead to unseen systematic risks and the market over-reactions, caused by the similar movements of the majority of funds, may damage the stability and the resilience of the system.

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[14] http://www.cnbc.com/2016/10/18/blackrock-ceo-fink-its-a-tough-time-for-our-clients-in-low-rate-environment.html

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2.2 Active and Passive management.

To better understand the theoretical framework in which the work is included, we have to do a step back and explain some important concepts, that are the pillars of Investing.

The task of any fund manager is to choose the right allocation of the resources in order to achieve the goals imposed by his/her contract. This mission is difficult due to the complexity of the financial markets, to their unpredictability and to their depth.

Citing the words of J.E. Beasley, N. Meade, and T.J. Chang (2003) : "The objective for fund managers is to provide a combination of capital growth and income over the medium to long-term."

Two are the main investment strategies investors can use to realize profits with the stock market: active portfolio management and passive portfolio management.

Obviously, a compromise is always possible and a mixed strategy occurs when a part of the resources are invested in active way and the other part in passive way.

These approaches differ in how the account manager interact with the specific benchmark. The managers that use the active approach aim to outperform it, paying attention to market trends, fundamental values, news or macroeconomics influences.

The purpose is to pick the "*winners*" (i.e. the stocks that will outperform the others), taking into account the buy/sell timing and the fundamental analysis.

Viceversa the passive managers try to mimic[15] the trend of the index.

This strategy has the main advantage to be simpler and to lower the transaction cost due to re-allocations, typical of active m.

We can find an interesting and brutal sentence in an old classic book of 1970[16], written by Burton Malkiel, a Finance professor at the Princeton University, who said that: "Even a dart-throwing chimpanzee can select a portfolio that performs as well as one chosen by experts"[17].

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[15] The word "mimic" is usually utilized when referring to the achievement of the same return in the long-term or maybe a pretty-similar return in a shorter timespan, certified by different measures such as tracking error or tracking error variance etc.

[16] The book was adapted and reprinted different times over years and the last version achieved was dated 2007.

[17] This example is taken from the book "A Random Walk Down Wall Street" and it is an arrangement of the more famous "Infinite Monkey Theorem", firstly introduced by Emile Borel.

In an article of the 2005, Malkiel gives a validation of his theory reporting the data of the mutual funds over 30 years and the history confirms its initial raw ideas.

From 1970 to 2003, only a little percentage has substantially over-performed the referred indices, and only for a small amount of years. The large part of the funds have under-performed or, obviously, they ceased to exist, introducing a *"survivor bias"* problem[18].

Using his words "the strongest evidence suggesting that markets are generally quite efficient is that professional investors do not beat the market. Indeed, the evidence accumulated over the past 30-plus years makes me more convinced than ever that our stock markets are remarkably efficient at adjusting correctly to new information."

As showed in the graph below, the number of funds, that have had worse results than the respective Index, is higher than the "winning ones". And these data confirm the hypothesis that, even if the markets suffered bubbles and inefficiencies, it is still very difficult to beat it with a pure active strategy.
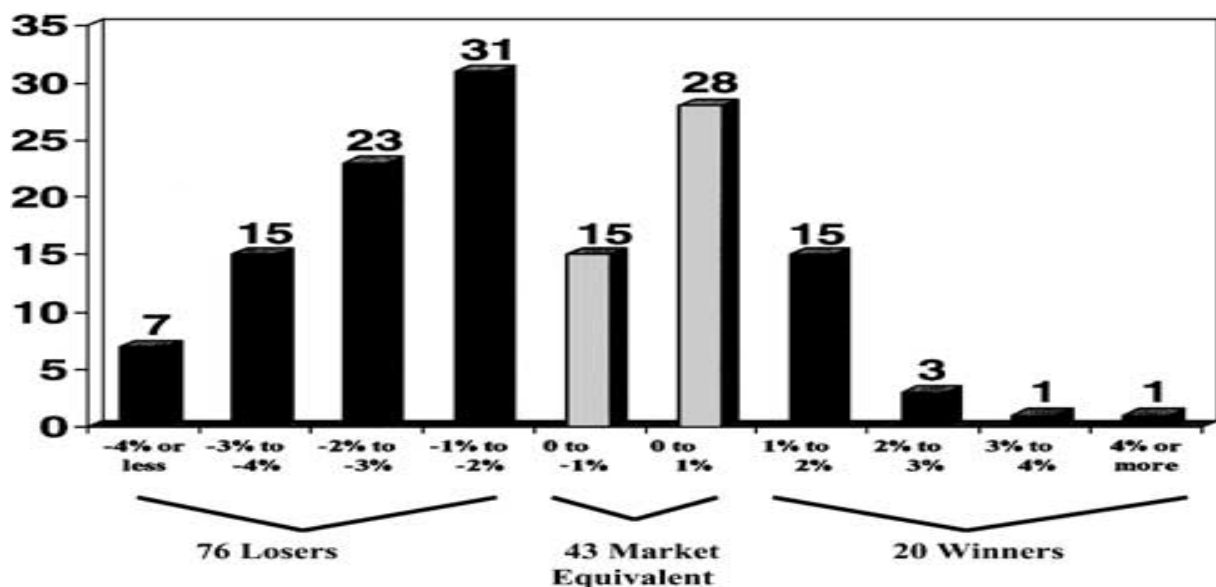
2The odds of success: Returns of surviving mutual funds 1/1970–12/2003
source: Malkiel,B: "Reflections on the efficient market hypothesis: 30 years later"2005

Fuller et al.(2010) ask some important questions about the definition of pure passive management and identify the passive indexing as an active strategy, where some important decisions are taken by the creators of the Index itself.

This theorem asserts that if someone will put a large number of monkeys in a room with a large number of typewriters and give them an infinite length of time, it is *"almost sure"* that at the end they will come out with a copy of the Hamlet of William Shakespear

[18] For a further explanation of this phenomenon, we suggest the work of Elton,Gruber and Blake (1996) "Survivor bias and mutual fund performance."

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This point of view is supported by other studies and gives an interesting formulation of the problem, assessing the predominant importance of the arbitrary changes in the index used as benchmark.

In their work, they write that: "by choosing to index to a particular benchmark, this investor has merely delegated a large number of very active decisions to the people who construct the paper-index, such as Dow Jones & Co., Standard & Poors, Frank Russell, etc. This is because the indexing firm will follow the rules for the construction of the paper-index very closely in order to mimic the particular benchmark as closely as possible. In addition, this investor has delegated the trading decisions for her portfolio to the indexing firm."

Rompotis (2009) makes an extensive comparison of the performances of active and passive mutual funds, assessing the primacy of the latter. The main sources of this link can be find on the lack of market timing ability and on the difficult selection process of the securities.

The mutual funds are rated using Sharpe Ratio, Treynor ratio and total returns and there is no evidence that the active funds over performed the passive ones.

On the contrary, the active funds show riskier and more volatile profiles, indicating a lack in the market timing ability.

These results are also confirmed by Frino,Gallagher and Oetomo (2005), which assert the impossibility for active found managers to consistently outperform the benchmark, when the transaction costs are taken into consideration.

The value of asset managed in passive way, utilizing the S&P500 as benchmark, has overcome the 1 trillion dollars, only considering the U.S. financial market, and the amount is expected to increase. In their work, they outline the possibility for index funds to achieve abnormal returns when the index is rebalancing, using a less aggressive buy/sell strategy.

This methodology is used to keep the funds in line with the index capitalization and is usually done on the rebalancing day, but "the excess demand/supply pressures associated with index revisions represents an index fund manager's trade-off decision between incurring higher trading costs and minimizing the fund's tracking error."

On average, expect for some short-run movements of the indices, all the papers read agree upon the superior of the passive management, especially when the fees and the expenses of the reallocations are considered.

In his work, Gruber (1996) raises a question about the preference of the investors.

Studying the results of the performance of the active mutual funds and the index funds, he shows that the active management is subjected to greater expenses and lower profits, but it is still preferred by a part of the clientele that has no access to important information and is not able to evaluate the future performances.

Choosing a benchmark and try with some methodology to achieve the same return sounds like a simple answer to a difficult question.

Mimic perfectly an index is a demanding goal to achieve and the literature is not coherent in indicating a single model or approach that performs better than others. On the other hand, picking stocks that would outperform the others and choosing the correct composition of the portfolio is a complicated task and, as we have seen, it can lead to terrible valuation errors.

An active manager believes on his/her ability to outperform an index, choosing the stocks that will have superior returns, compared with others. Hence, in this view, the active management is a zero sum game, in which the most skillful managers will beat the market and the others managers.

On the contrary, a manager who believe in the efficiency of the financial market and has not enough confidence on his/her skills uses passive management. If the markets are efficient, it is impossible to beat them for different long periods, so the target of a manager is to obtain the same risk/return ratio of a given benchmark.

When the benchmark is an index, for example the Standard & Poors 500 or the EuroStoxx50, this investment strategy is called "index tracking" or "indexing", and clearly, a passive management fund who follows this strategy is called index fund.

The right definition of "indexing" is given by the glossary of the NASDAQ, that says:
"A passive instrument strategy calling for construction of a portfolio of stocks designed to track the total return performance of an index of stocks"[19]

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[19] http://www.nasdaq.com/investing/glossary/i/indexing
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The number of possible indices to be tracked is raising during the last ten-twenty years, leading to the pursuing of diversified and more complex strategies. Now it is also possible to track a worldwide index[20] or even a regional stock market index[21].

To replicate the performance of an index, using the stocks of it, there are two different approaches:

1) "Full replication"; which consists in including every constituents of the index, simply copying their proportion. The main advantages of this technique are the simplicity and the easy result in a perfect match, but the disadvantages are also very important. High transaction costs are necessary to rebalance the portfolio in a frequent manner, in order to keep it in line with the returns of the benchmark. Moreover, the achieve the proper matching, the portfolio needs to be well capitalized, if the benchmark is an Index with a lot of Constituents.

2) "Partial replication"; in which the manager chooses a small portion of the stocks, contained in the index, trying to replicate the performance of the entire index. Hence, it will be easier to rebalance the weights of the portfolio and the manager incurs in lower transaction costs. The weakness of this approach is the difficulty to reduce the variability between the portfolio and the benchmark. Different methods are been developed during the years and a survey of the main used ones is the topic of the Section 5.

A possible alternative is the synthetic replication, in which the benchmark is tracked by the use of derivatives contracts, such as the futures, options or swaps.[22]

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[20] Such as the "FTSE All-World Index", the "Russell3000" or others.
[21] Like the "S&P Europe 350".
[22] The content of this work is not related to this kind of investment strategy and we suggest the reading of Waring and Attwood (1999) and Cano, Feldman and Smith (2009) for more information.

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2.3 Market Efficiency

From the point of view of the economic theory, the two approach starts from opposite belief of the market efficiency.

In order to understand the motivation of the work it is necessary to recall these concepts.

The paper "The Structure of Stock Market Price" by Eugene Fama (1965) is considered as the fundamental pillar of the "Efficient Market Hypothesis"[23], a theory that influenced the financial thinking and changed the way we take into account the financial markets and their analysis.

In his research, the ideal market gives all the information about the allocation of capital in the stock market.

Fama (1965) describe a market as a situation "in which firms can make production-investment decisions, and investors can choose among the securities that represent ownership of firms' activities under the assumption that security prices at any time "fully reflect" all available information. A market in which prices always fully reflect available information is called efficient."

The expression "*fully reflect*" is the most important and, also the most controversial, due to the difficult demonstration.

In his theory, the prices of the stocks reflect all the information that are available in the market, no one of them is overvalued or undervalued and they adjust according to the news randomly announced.

Moreover, all increments caused by the news appears to be randomly assigned and the prices are considered as random walks, showing similar characteristics to a brownian motion.

A Brownian motion is a stochastic process that has no memory of its past and evolves with no correlation with it, (i.e. it is impossible to forecast.)

Another feature that influence the level of efficiency in the market is the cost of acquiring information about the stocks.[24]

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[23] From now : EMH.

[24] The costs may be too high to allow a profit or maybe it cannot be possible to acquire some information.
The time spent in achieving the right evaluation of the stocks has to be considered as an important discriminating factor when choosing an investment strategy.

2.2.1 Efficient Market Hypothesis

The EMH can be divided in three forms, differentiated by the current level of efficiency in the market. The three shapes are weak, semi-strong and strong form of efficiency.

The most flexible version of the efficiency is the **weak form**, which affirms the prices reproduce all information in the past prices. In this view, it is not possible to beat the market just looking at the past and, for this reason, the technical analysis is useless[25].

A various number of studies confirm this organization and underline the presence of some discrepancies. For example, the presence of "January Effect" is a well-known phenomenon that affects all the stock prices during the end of the year, in which the stocks show significant downturns and usually have a rebound in the first part of the next year.

The main explanation is the fiscal advantage due to the off-setting of realized capital gains, done by selling the stocks, in order to lower the amount of taxes to be paid, which are calculated at the end of the year.

This mechanism is one of the most important and most utilized to reduce the short-term capital gains, which are taxed at higher federal income tax, comparing to long-term capital gains.

Another possible explanation is the increased wealth at the beginning of the year, due to the bonuses that are received at the end of the previous year.

Another well-documented phenomenon in the stock markets is the "Weekend Effect", that is the presence of consistent higher returns on Fridays, compared to the Mondays of the immediately next week. A possible reason may be the habit of the major companies to release bad news on Friday, when the markets are closed, in order to prevent panic situations.

The stock prices reflect the news when the markets re-open on Monday.

The **semi-strong form** of EMH states that the prices reflect all publicly available information and it is impossible to outperform the market using the fundamental analysis.

Only the informed traders that have access to private information can make profit from the market. When referring to public information, we intend all the information that are available just looking at the official documents, such as balance sheet, financial statements or reports.

In order to ensure the transparency and the fairness of the market, there are proper legislation against the insider trading and the use of non-public information, which can damage the efficiency of the whole market.

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[25] And, for the same reason, the statistical analysis and the chart analysis are also futile.

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The most restrictive form of EMH is the **strong version** of market efficiency, in which the prices reflect all the important information, considering also the private information that are not available to the public.

For this reason, even the use of confidential information is useless, because the prices are already adjusted for all information and no one can substantially make profit.

Malkiel and Fama (1970) tested the theory in a mathematical representation concluding that, although there are insights of dependence in the fluctuations of the stock prices, it is impossible to assert the inefficiency of the market.

To evaluate his theory, Fama organized it in the following mathematical model:

Events occur at time $t - 1$ and at $t + \tau$, with *C=0,1,n.*

$\Phi(\tau - 1)$ is the set of available information at time t-1 and has big influence over the price of the securities.

$m\Phi(\tau - 1)$ is a subset of $\Phi(\tau - 1)$.

P(j,t-1) is the price of security j at time t-1, with j=1,2,n.

$F(p, 1 + t + r \dots pn, t + r/\Phi(\tau - 1))$ is the join probability function of the prices of securities at time $t - \tau$, stated by the market at time *t-1* given the subset of information.

The set of information $\Phi(\tau - 1)$ describes the state of the world at the time indicated and all the actors base their choices on the same information.

We assume that the actors exactly know the join distribution function for the future prices.

The process that creates prices acts in this way:

Taking into account the subset $m\Phi(\tau - 1)$, the market assess a level and a distribution for the prices and then fixes the proper current prices.

 Hence, we can assert the efficiency of the market if

 $m\Phi(\tau - 1) = \Phi(\tau - 1)$

In simple words, if the price *"fully reflects"* all the information available in the market.

2.2.2 The Dow Theory

The emphasis on the identification of a benchmark was firstly introduced by Bachelier (1900). He also outlined the difficulty in beating the market, because if the market is efficient and all the rational investors act in a proper manner, a trader can make profit only anticipating the trend of the market, taking more risks and exposing his portfolio to the unpredictable fluctuations of the prices.

An important contribution, previous respect to the Fama's papers, is the well-known "Dow Theory" which is considered as the milestone of the technical analysis.

Firstly formulated by Charles Dow, at the beginning of the last century, and organized in a series of editorials published on the Wall Street Journal, the theory assessed that the stock market is a measure of the overall status of the economy.

Analyzing it, one trader could easily understand the directions of the market and the presence of major market trends. The death of Charles Dow in the 1902, impeded him to give us an extensive representation of his theory and his work was continued by other authors[26].

The theory is organized in six steps.

The first important proposition is that all the information are discounted into the prices[27] and even the possibility of unpredictable events is included in the prices. One investor has to look at the major indices to understand the possible trends, not looking at the official balance sheets or other documents.

The study of the different trends themselves is the second pillar of the theory. Dow divided the trends into three types. The primary trend is the trend of the whole economy, also cited as long-term trend, and its main disadvantage is the correct identification of the time span to utilize, in order to follow it and not betting against its direction. The secondary or intermediate trend is often a correction of the primary and its fluctuations are usually more volatile. An upward primary trend can be composed by various downward secondary trends, even maintaining an

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[26] See for instance : William P. Hamilton's *"The Stock Market Barometer*" (1922), Robert Rhea's *"The Dow Theory"* (1932), E. George Schaefer's *"How I Helped More Than 10,000 Investors To Profit In Stocks"* (1960) and Richard Russell's "*The Dow Theory Today"* (1985).
[27] Past, current and even future information.

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higher direction. The last trend is the minor trend, which can be considered as the noise of the market and is often ignored by the traders.

The third step is the identification of the different phases of the primary trend, which are accumulation phase, public participation phase and excess phase.

The first one of a bull market is usually timed at the end of a down trend, when the prices show stability and informed investors enter in the market to anticipating the uptrend, thinking that the worst is passed and hoping in a raising in the value of prices. The public participation is when all the other investors choose to follow the trend and they feed it, raising the demand and pushing the prices even higher. The last phase is when the late entrants start to buy the securities, hoping not to be the last, but going to lose their gains mismatching the timing of the market[28].

The fourth step of the Dow Theory states that a reversal from a bull to a bear market is happening only if the major indices are evolving in the same directions. If there is no confirmation, the reversal is unpredictable and the direction of the market cannot be forecast.

The explanation of this point is easy and to be understood it is important to take in mind that the primary trend represents the whole business cycle, and only if the most important charts are concordant the status of the economy can be evaluated.

The Dow Theory is based on the prices as fundamental indicators, but used also the volume to increase the validity of the findings. The volume of the market must confirm the trend. When the trend is upward and the volume is also reaching its peak, it is a signal of the persistency of the trend, because more buyers are willing to a further increasing in value.

On the contrary, if the trend is upward but the volume is decreasing, it means that the trend is expiring and there are little chances to continuation. The last step of the theory indicates that a trend is always valid until important events occur. Only if all the signs are in the same directions, one trader can assert the end of a bull/bear market and the beginning of the opposite one.

For this reason, to look only at the primary trend can be difficult because a transitory sell-off can fool the unskilled trader and lead to an incorrect identification of the main direction.

Even if the Dow Theory can be identified as a fundamental pillar of the technical analysis, it is dated and the evolution of the academic world, and of the financial markets themselves, has lowered its importance in the investing framework.

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[28] The phases of a bear are different, but similar and specular are their features.

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Nowadays the academic world has assessed the ineffectiveness of the technical analysis, which is only sponsored by some trading platform.

The major contribution of Charles Dow remains the indices created and the fundamental idea on which they are based (i.e. the representation of the status of the economy).

"If you torture the data long enough, it will confess" Bernard Coase, "How should Economist choose?"(1981)

Section 3. Cointegration

This section is dedicated to the explanation of the concept of Cointegration, as exposed by Engle and Granger in 1987, and to the recall of the principal methodologies to verify its presence, and the tests able to perform the unit root analysis.

3.1 The Nature of Cointegration

Since the revolutionary work of Clive W.J. Granger (1981), implemented and bettered in 1987 with the help of Robert Engle, Cointegration became an important tool for the econometrics and for all the academics of the world.

The achievement of the Nobel Prize in the 2003 was the most recognition to the importance of the work of the Authors, but also to the importance of the theory itself.

But, how did the idea of Cointegration come out?

In his talk, during the awards ceremony for the Nobel Price, in front of the Royal Swedish Academy, Granger told this story: "A colleague, David Hendry, stated that the difference between a pair of integrated series could be stationary. My response was that it could be proved the he was wrong, but in attempting to do so, I showed that he was correct, and generalized it to Cointegration, and proved the consequences such as the error-correction representation."

From that moment to now, the academic context is changed and many notions have evolved, according to the theoretical progress, but the intuition remains.

The work dated in 1987 has also an interesting story that can be found in the paper written by Ewa Marta Syczewska in 2011, for the thirtieth anniversary of the first one.

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Citing her words: "The first version of the paper, submitted by Granger to *Econometrica*, was rejected for several reasons: lack of empirical application, need of rewriting proof of theorem, etc. Granger then started to work on improved version of his Representation Theorem, and accepted help of Robert Engle in empirical work.

New version "first became Granger and Engle, next Engle and Granger" during his leave on a sabbatical. Second version was again rejected by *Econometrica* so they contemplated sending it to other publishers when *Econometrica* asked them to publish it because "they get so many papers on cointegration that they needed this one for a reference"."

This story was firstly exposed by Granger himself in the 2010[29], who concludes saying that : "A few citations and twenty years later and here we are, although I still believe that the paper would have been successful wherever it was published!"

3.1.1 One theory, many fields

Nowadays, the Cointegration's approach is applied in many fields and is accepted as a fundamental technique for investigating relationships between multivariate non-stationary time series, describing their relationship and both long and short run deviations. Furthermore, it provides to the Macroeconomic study a fundamental tool to link the theory to the empirical estimation, in order to identify a possible temporary equilibrium.

Many are the fields in which the theory assess the presence of long term equilibrium and short term deviations.

Just to city, Cointegrated variables can be identify in:

- Permanent income Hypothesis[30],
- Money demand model[31],
- Growth theory models[32],
- Purchase power parity[33],
- The Fisher equation[34],
- The Expectation Hypothesis of the term structure[35],

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[29] See Granger (2010)." Some thoughts on the development of cointegration"
[30] Consumption and Income.
[31] Money, Nominal Income, Prices and Interest Rates,
[32] Income, Consumption and Investments.
[33] Nominal Exchange rate and foreign and domestic prices.
[34] Nominal interest rate and inflation.
[35] Short term and long term interest rates.

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A various number of procedures have been developed through years and the most famous are the Engle-Granger method and the Johansen's one, introduced by Johansen (1988, 1992) and Johansen and Juselius (1990).

To explain the concept of Cointegration, we must recall the definitions of stationary, non-stationary and integrated process.

A process is stationary if its expected value and variance are constant through time, and its covariance depends only on time.

$$C_\tau = \frac{1}{N - \tau} \sum_{t=1}^{N-\tau} (x_t - \bar{x})(x_{t-\tau} - \bar{x})$$

On the other hand, many economic time series evolve according to a trend, with cyclical fluctuations, or show erratic unpredictable movements, with no mean or constant.

The macroeconomic time series are usually included in the first type, while the financial prices of many products may be modelled using the second specification.

Moreover, analysing a non-stationary process, we can notice the presence of a great dependence of the current observations from their past.

This important feature can lead to a misspecification of the process and involve serious problems in the estimation of the coefficients of the regressions, because the empirical estimators do not converge in probability, invalidating the asymptotic theory.

Expressed with a formula, a general decomposition of a non-stationary time series can be defined as:

$$x_t = TD_t + x_t^*$$

Where:

TD_t is a deterministic function of time and can be:

- A constant, $TD_t = k$,
- A linear trend, $TD_t = k + \delta t$,
- A quadratic trend, $TD_t = k + \delta t + \gamma t^2$,

x_t^* is the stochastic part of the process x_t and, obviously, its characteristics influence the method by which we analysis x_t.

Going a step further, x_t^* can be divided in two parts: an autoregressive component of order p and a moving average of order q:

$$\phi(L)x_t^* = \theta(L)\varepsilon_t$$

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With $\varepsilon_t = IID(0; \sigma_\varepsilon^2)$[36].

We have to distinguish between **trend stationarity** and **difference stationarity**.

A series is stationary around a trend, if the process x_t^* is covariance stationary, i.e. if the roots of the equation $z^p - \phi_1 z^{p-1} - \cdots - \phi_p = 0$ are less than 1 in modulus.

For this reason, the process x_t^* can be decomposed using the Wold decomposition:

$$x_t^* = \frac{\theta(L)}{\phi(L)} \varepsilon_t = \sum_{j=0}^{\infty} \psi_j \varepsilon_{t-j} = \psi(L)\varepsilon_t$$

And the de-trended series $(x_t - TD_t)$ can be expressed by:

$$(x_t - TD_t) = x_t^* = \psi(L)\varepsilon_t = \frac{\theta(L)}{\phi(L)} \varepsilon_t$$

Which follows a stationary ARMA*(p,q)* process:

$$\phi(L)(x_t - TD_t) = \theta(L)\varepsilon_t$$

The trend stationarity implies that the series tends to follows TD_t, and the stochastic part is only transitory.

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[36] The two polynomial can be formalized as follows:
$$\phi(L) = 1 - \phi_1 L - \phi_2 L^2 - \cdots - \phi_p L^p$$
$$\theta(L) = 1 + \theta_1 L + \theta_2 L^2 + \cdots + \theta_q L^q$$
For simplicity, from now on, we suppose that the roots of the following equation:
$$z^q - \theta_1 z^{q-1} - \cdots - \theta_q = 0$$
are less than 1 in modulus. This condition allows us to assume the invertibility of the process.

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The situation radically changes if we introduce a unit root in x_t^*.

In this case, the series is difference stationary if the equation

$$z^p - \phi_1 z^{p-1} - \cdots - \phi_p = 0$$

Which is referred to x_t^*, has one root equal to 1 and the others less than 1 in modulus.

Hence, we can decomposed the autoregressive part as

$$\phi(L) = \phi^*(L)(1 - L)$$

For this reason, the first difference transformation of x_t^* can be expressed by a stationary ARMA*(p-1,q)* process as follows:

$$\phi^*(L)(1 - L)x_t^* = \phi^*(L)(x_t^* - x_{t-1}^*) = \phi^*(L)\Delta x_t^* = \theta(L)\varepsilon_t$$

Henceforth, assuming that the process is covariance stationary, we can decompose it in :

$$\Delta x_t^* = \frac{\theta(L)}{\phi(L)}\varepsilon_t = \psi^*(L)\varepsilon_t$$

With $\psi^*(L) = \sum_{J=0}^{\infty} \psi_j^j L^j$, $\psi^*(1) = \sum_{J=0}^{\infty} \psi_j \neq 0, \psi_0^* = 1$.

If the process x_t^* is difference stationary, it can be said that the series is integrated of order 1 (i.e. $x_t^* \sim I(1)$).

Moreover, the process can be written as the sum of the error term u_t defined as :

$$u_t \stackrel{\text{def}}{=} \psi^*(L)\varepsilon_t = \sum_{J=0}^{\infty} \psi_j^* \varepsilon_{t-j}$$

Making recursive substitution, we obtain $x_t^* = x_0^* + \sum_{J=1}^{t} u_j$.

Where x_0^* is the initial condition and, if u_t is IID, or more simply serial uncorrelated, x_t^* is defined as a random walk process. In this formulation Δx_t^* is a process integrated of order 0.

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Giving a general definition, a process is integrated of order *d, I(d)*, if its *dth*-difference is *I(0)*. A *I(1)* time series tends to diverge as *T* tends to infinite $(T \longrightarrow \infty)$ because its variance is proportional to T and increases as the number of observations increases.

For this reason someone can easily asserts that it is impossible to achieve a long run equilibrium for two series with these features, Cointegration provides a methodology to bypass this problems, utilizing the common (stochastic) trend of the series.

To introduce the argument, we have to make an example.

Considering two random walk series, expressed by:

$$x_t = x_{t-1} + \varepsilon_{tx}$$
$$y_t = y_{t-1} + \varepsilon_{ty}$$
$$\begin{pmatrix} \varepsilon_{tx} \\ \varepsilon_{ty} \end{pmatrix} \sim IID(0, \Sigma). \Sigma = \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix}$$

Linking the first difference of the two variables, we can easily obtain :

$$(1 - L) \begin{pmatrix} x_t \\ y_t \end{pmatrix} = \begin{pmatrix} \varepsilon_{tx} \\ \varepsilon_{ty} \end{pmatrix} \sim IID(0, \Sigma)$$

Multiplying the previous regression by the vector $\beta' = (\beta_1 \ \beta_2)$ with $\beta_1, \beta_2 \neq 0$, we can write it as follows:

$$(1 - L)(\beta_1 x_t + \beta_2 y_t) = \beta_1 \varepsilon_{tx} + \beta_2 \varepsilon_{ty}.$$

For this reason, the linear combination of two random walks can be written as :

$$\xi_t = (\beta_1 x_t + \beta_2 y_t) \sim I(1)$$

The only case that allows us to state the presence of cointegration is when:

$$Prob(\beta_1 \varepsilon_{tx} + \beta_2 \varepsilon_{ty} = 0) = 1$$

Hence, the variance of the process equal to 0 and can be written as :

$$Var(\xi_t) = Var(\beta_1\varepsilon_{tx} + \beta_2\varepsilon_{ty}) = \beta_1^2\sigma_{11} + \beta_2^2\sigma_{22} + 2\beta_1\beta_2\sigma_{21}$$

$$= (\beta_1 \; \beta_2)\begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix}\begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix} = \beta'\Sigma\beta$$

The cointegrating vector between the first and the second process can so be expressed as :
$-\frac{\beta_2}{\beta_1}$ and, in similar way, the linear combination between them is :

$$x_t = -(\beta_2/\beta_1)y_t \quad \text{or} \quad y_t = -(\beta_1/\beta_2)x_t$$

Hence, the linear combination ensures the removing of the shared trend.

$$\beta_1 x_t + \beta_2 y_t = \beta_1(\sum_{J=1}^{t} \varepsilon_{jx}) + \beta_2\left(\sum_{j=1}^{t} \varepsilon_{jy}\right) = \sum_{j=1}^{t}(\beta_1\varepsilon_{jx} + \beta_2\varepsilon_{jy}) = 0$$

Finally, we can report the general definition of **Cointegration**, as stated by Engle and Granger (1987):

Being x_t a vector of n time series, the components of x_t are said cointegrated of order *(d,b)* ,$x_t \sim CI(d,b), d \geq b$ if :

- All the components of x_t have the same order of integration, $x_t \sim I(d)$,
- It can be find a vector β, different from 0, by which the process $\xi_t = \beta' x_t$ has a lower order of integration (i.e. $\xi_t \sim I(d-b)$ with $d \geq b > 0$)

For simplicity, we will consider only the case when the variables are integrated of order 1,*(i.e. d=b=1)*.

This case is the more common in the empirical analysis, but we suggest the reading of Johansen (1995) for information about greater order of integration.

3.1.2 The Engle-Granger Method

In general, the **Engle-Granger method** uses an **OLS regression** and tests the residuals of this regression for stationarity. If the residuals are stationary, it can be said the presence of cointegration between the two series.

Using the words of Rachev et al (2007), in their book *"Financial Econometrics: From Basics to Advanced Modelling Techniques"*:

"Two or more processes are said to be co-integrated if they stay close to each other even if they "drift about" as individual processes. A colorful illustration is that of the drunken man and his dog: both stumble about aimlessly but never drift too far apart."[37]

Hence, the structure of the method is based on the following regression:

$$y_t = \beta x_t + \xi_t$$

Where:

- y_t is the first time series,
- x_t is the second time series,
- ξ_t are the residuals, used to test for stationarity,
- β is the cointegrating vector.

To this formulation, Phillips (1991) adds the specification of the process $x_t : x_t = x_{t-1} + u_t$, in order to achieve a triangular representation.

The estimation of β is given by:

$$\hat{\beta} = \left(\sum_{t=1}^{T} x_t^2 \right)^{-1} \left(\sum_{t=1}^{T} x_t y_t \right)$$

Its behavior depends on the specification of the error terms.

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[37] Actually, this representation was firstly introduce by Murray(1994) in order to simplify the concept. The drunken man is commonly referred to random walk series.

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The distortion of the estimation is given by :

$$\hat{\beta} - \beta = \left(\sum_{t=1}^{T} x_t^2\right)^{-1} \left(\sum_{t=1}^{T} x_{t-1}\xi_t\right) + \left(\sum_{t=1}^{T} x_t^2\right)^{-1} \left(\sum_{t=1}^{T} u_t\xi_t\right)$$

And its asymptotic behavior is influenced by the hypothesis made on the vector $(\xi_t u_t)'$

We have to make a distinction between two main cases:

- Errors IID
- Errors correlated and mutually dependent.

In the first case, the assumption of serial independence and mutual uncorrelation, allow us to assess that the explanatory variable is linear independent from the past of the error terms.

As indicated by Maddala and Kim (1998), the properties of the cointegrating vectors are numerous:

- There is only one cointegrating vector between two variables[38].
- OLS Estimation of Beta is super consistent, (The rate of convergence of β is $\sqrt{T}$).
- The Engle-Granger procedure gives the same asymptotic distribution for ECM parameters as if β was known.
- The asymptotic distribution may be represented as a weighted sum of normal variables and it is included in the Local Asymptotic Multivariate Normal Family (LAMN).

The second case is more common in the empirical analysis and also is important for the specific case of this work.

The error term of the regression is serially correlated.

The error terms of the triangular representation are correlated, (i.e. $E(u_t\xi_s) \neq 0, for\ t, s.$)

The properties in this case change:

- The OLS estimator $\hat{\beta}$ is still super consistent[39].
- The variables converge in distribution to a random variable that does not show the characteristics of a normal
- The t_β statistics, calculated by the OLS estimator, converges in distribution to a random variable, influenced by unknown parameters.

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[38] This is not true for more variables.

[39] For an extensive explanation, we suggest once again the reading of Watson (1994) and Hamilton (1994).

In this case, the expected value of the error term at time t is given by :

$$E[\xi_t | \ldots, u_1, \ldots u_{t-1}, \ u_t, \ u_{t+1}, \ldots u_T] =$$

$$= \sum_{j=-\infty}^{-1} \gamma_j u_{t+j} + \gamma_0 u_t + \sum_{j=1}^{\infty} \gamma_j u_{t+j}.$$

For this reason, Stock (1987) and Stock (1993) introduces a semi-parametric approach, introducing leads and lags of the explanatory variables.

It can be expressed by the following equation:

$$y_t = \beta x_t + \sum_{j=-\infty}^{-1} \gamma_j \Delta x_{t+j} + \gamma_0 \Delta x_t + \sum_{j=1}^{\infty} \gamma_j \Delta x_{t+j} + v_t$$

Where :

- $\sum_{j=-\infty}^{-1} \gamma_j \Delta x_{t+j}$ are the lags.
- $\gamma_0 \Delta x_t$ is the contemporaneous specification of the explanatory variable.
- $\sum_{j=1}^{\infty} \gamma_j \Delta x_{t+j}$ are the leads.
- v_t is the new error term.

The introduction of leads and lags help us to clean the coefficients from the correlation with the error term and to achieve more consistent estimation of the coefficients.

Obviously, in this form is impossible to make inference, due to the infinite number of regressors. In the empirical section, we recall this expression and use it to make inference on the data.

3.1.3 Other models

Engle and Granger (1987) extended their theory in order to show short run and long run deviations. In this way, the **Granger Representation Theorem** shows that Cointegration can be considered to an Error Correction Mechanism (E.C.M.).

Two cointegrated variables can be expressed by the following equation:

$$\Delta y_t = c_0 + \Delta x_t + \alpha(y_{t-1} - \beta x_{t-1}) + \xi_t$$

Where:

- $\varepsilon_{t-1} = y_{t-1} - \beta x_{t-1}$ is the error from *(t-1.)*
- $[1, -\beta]$ is a cointegrating vector.

This type of formulation is not useful for our analysis and we suggest the reading of Engle and Granger (1987) for a more analytic presentation.

Giving just an overview, the other famous procedure is the **Johansen model**[40], that utilized the maximum likelihood approach in a VAR model, by assumption the errors are Gaussian.

The variables: $Y_t = [Y_{1t}, Y_{2t}, ..., Y_{mT}]'$ can be expressed by the following model:

$$Y_t = A_1 Y_{t-1} + \cdots + A_k Y_{t-k} + u_t, \text{ for t} = 1,2,...,\text{T},$$

In this way, the VECM model is :

$$\Delta Y_t = \Pi Y_{t-1} + B_2 \Delta Y_{y-1} + \cdots + B_k \Delta Y_{t-k+1} + \xi_t, \qquad t = 1,2,..,T$$

Where:

- $\Pi = -I + \sum_{i=1}^{k} A_i,$
- $B_j = -\sum_{j}^{k} A_{i=j} \ j = 1,2,...k.$

Π is not full rank, *(r < k)* and can be expressed by:

$$\Pi = \alpha\beta'$$

---

[40] Smith&Harrison (1995) extended the previous example of the drunken man introducing a third variable (the boyfriend) and used it to explain the Johansen procedure.

Where:

- $r = rank(\Pi)$,
- $\beta' Y_{t-1}$ indicates r cointegranting vectors,
- α is the error correction matrix.

A particular case that is worth to be mention is the one exposed by Granger and Joyeux (1980), called **Fractional Cointegration**.

Also cited in Hosking (1981), the Fractional Difference is formulate as follows:

$$(1-L)^d = \sum_{k=0}^{\infty} \binom{d}{k}(-L)^k = \sum_{k=0}^{\infty} \frac{\Pi_{a=0}^{k-1}(d-a)(-L)^k}{k!} = 1 - dL + \frac{d(d-1)}{2}L^2 + \cdots$$

Where:

- d is a frational integration parameter,
- L is the lag operator.

Their autocorrelation function decreases very slowly and it indicates a long-term memory. The fractional process are a middle version between order 1 and 0.

One great contribution can be considered the work of MacKinnon(1990,2010). Through computer simulation, he gives the correct value of the estimation of parameters and adapts the tests to the particular features of non-stationary series.

Moreover, Phillips and Ouliaris (1990) outline that unit root tests applied to the residuals do not show the usual Dickey-Fuller distributions, under the null hypothesis of no-cointegration.

The distribution of these tests have asymptotic distributions that depend on the regressors and the number of parameters included.

The distributions, called "Phillips-Ouliaris distributions" have different critical values.

3.1.4 Correlation versus Cointegration

Cointegration methods look directly at financial asset prices, rather than returns. This is the first important feature and the main reason why co-integration approach spent several years to be accepted and implemented by the academic world.

On the other hand, this is also one of the main strength of this theory because the asset price maintain long-term memory of their trends.

The work of Markowitz (1952) and all the others that are based on it, start their analysis from the return point of view, assessing the basic importance of the correlation between returns. Correlation and Cointegration are similar but distant ideas.

High correlation does not involve high cointegration and, in the same way, high cointegration is not linked to high correlation.

Correlation reflects co-movements in returns, which are stable only in the short-run. It is intrinsically a short run measure, and correlation-based strategies usually require frequent rebalancing.

On the other hand, cointegration measures long run co-movements in prices, which may occur even through periods when correlations appear low. (Alexander 1999)

Hence, cointegration provides a method that take into account the common long-term trends of two series and deviations between the portfolio and the index are possible but only in the short term, even if they can be substantial.

3.2 Principal tests of Stationarity

The first important analysis to do is to understand the nature and the movements of the series, starting from the presence of stationarity.

Through years, many test had developed to achieve significant results. The tests used are divided in two categories: the unit root tests and the stationarity tests.

3.3 Unit Root Test

3.3.1 Dickey-Fuller Test.

The Dickey-Fuller Test allows us to value the presence of a trend or of unit roots, in the series analyzed. We build a classic auto-regression model as:

$$X_t = \alpha X_{t-1} + \varepsilon_t$$

The second step is to subtract the first difference (X_{t-1}) from both sides, in order to obtain:

$$\Delta X_t = (\alpha - 1)X_{t-1} + \varepsilon_t = \delta X_{t-1} + \varepsilon_t$$

If $\delta = 1$, there is the presence of a trend or a unitary root in the serie. So, the Hypothesis system is :

$$H_0 = \delta = 0$$
$$H_1 = \delta < 0$$

If we accept the null hypothesis, we are stating the absence of Unit Root. Usually, there are three model specifications, which are differently applied, taking into account the features of the series analyzed :

- Unit Root Test (Random Walk) : $\Delta X_t = \delta X_{t-1} + \varepsilon_t$
- Unit Root Test with Drift: $\Delta X_t = \alpha_0 + \delta X_{t-1} + \varepsilon_t$
- Unit Root Test with Drift and deterministic time trend : $\Delta X_t = \alpha_0 + \beta t + \delta X_{t-1} + \varepsilon_t$

3.3.2 Augmented Dickey-Fuller Test.

The ADF test follows the same methodology of the simple one, expect for the regression that is changed into:

$$\Delta X_t = \alpha_0 + \beta t + \gamma X_{t-1} + \delta_1 \Delta X_{t-1} + \ldots + \delta_p \Delta X_{t-p} + \varepsilon_t$$

Where: α is a constant, β is a time trend and ρ is the number of autoregressive lags. The variable *p* is determinant to ensure a right estimation. A too high number of lags can damage the validity of the test, while a too low number can not be sufficient to eliminate the heteroscedasticity of the error term. The errors are considered homoscedastic and serially uncorrelated.

Schwerz (1989), through Montecarlo simulations, suggest a simple empirical model:

$$p_{max} = [12(\frac{T}{100})^{\frac{1}{4}}]$$

where *T* is the number of observations.

3.3.3 Phillips-Perron Test

Another important test is the PP Test, which is based on the regression:

$$y_t = \beta c_t + \varphi y_{t-1} + u_t$$

Where u_t is *I(0)* and may be affected by heteroskedasticity, with no restrictions imposed.. Introduced by Phillips and Perron (1988), the test does not take into consideration any analysis of the correlation, and this feature is the main difference with the ADF test.

3.4 Stationariety Test.

3.4.1 KPSS Test

Sometimes is also important to check the results from another point of view. The test invented by Kwiatkowski, D., Phillips, P. C. B., Schmidt, P., and Shin, Y Test (1992)[41] allows us to consider the presence of stationarity as the null hypothesis.

The model structure is organized as follow:

$$Y_t = \xi_t + \varepsilon_t$$

Where Y_t is the data generating process and is formed by ε_t, which is stationary, and ξ_t, which is a random walk.

$$\xi_t = \xi_{t-1} + u_t \qquad\qquad u_t \sim IID(0, \sigma_u^2)$$

If the variance of the process is zero, i.e. $\sigma_u^2 = 0$, ξ_t is equal to the initial condition for every *t* and Y_t is stationary.

Under the null hypothesis, Y_t is stationary.

The test statistic is given by:

$$KPSS = \frac{1}{T^2} \cdot \frac{\sum_1^T S_t^2}{\widehat{\sigma_\infty^2}}$$

The regression can also be augmented including a linear trend.

"Computer are useless, they only give answers" Pablo Picasso.

Section 4. Financial Applications

In this section, we want to go a step further and give a survey of the possible practical applications of the Cointegration in the financial and investing system.

The first paragraph identifies the use of the Cointegration in different contexts,form the Permanent Income Hypothesis to the hedging problem, while the second paragraph shows various methods for index tracking, making a comparison between them and the the Engle-Granger method.

4.1 Use of Cointegration in Finance

The applications of Co-integration in the financial are various and, only in the recent years, the focus is on the possible use in a trading strategy. Starting from the economic theories, Cointegration was applied for the analysis of all the models in which the presence of an equilibrium mechanism is stated.

A possible application is the relationship that exist between spot and future prices in the futures' market. Brenner and Kroner (1995) argue that it is driven by the cost-of-carry dynamics. This relationship is more stable in the currency markets than in the commodity ones and, taking consideration of this ratio, it is possible to make an inference on the unbiasedness hypothesis.

In hedging prospective, the use of cointegration is really useful when the two time series share a common trend and their deviations are considered as temporary. In this view, the cointegration approach ensure a correct specification of the ratio between the two assets.

Ahmed (2015) finds a strong stability and more efficiency using time-varying hedging with equities, bonds and commodities.

The benefits of the Dynamic Regression and Exponential-weighted moving averages are recorded in superior hedge ratios, which outperform the static one.

De Prado&Leinweber (2012) go a step further and introduce a new hedging algorithm called Dickey-Fuller-Optimal (DFO), which is based on a multi-period methodology whose aim is to give the estimated coefficients that lead to a unit root error.

Studying their results, it is clear the predominance of the long-run equilibrium and the estimations are less volatile than the ones obtained by ECM.

In his work, utilizing the Gonzalo & Granger (1995) decomposition, Yang (2012) proposed the use of data adjusted with risk free rate, in order to assess the cointegration between different stock markets. Identifying two main financial stock markets, he demonstrate the presence of strong integration with the global financial system.

Another possible application is the one utilized by Fofana and Seyte (2012), in which the non linear cointegration of international stock markets is analyzed during the financial crisis of the 2007.Their results suggest a contagion mechanism from the S&P500 to the CAC40 and FTSE100. The Engle-Granger and the Johansen tests validate their assumptions showing a strong pair cointegration between the indexes.

Taking into account the relationship between the conditional heteroscedasticity and the cointegration, Wong et al.(2005) utilize a cointegrated vector AR-GARCH model and find some results on their empirical analysis.

Dealing with the stock financial markets, when cointegration is stated, the conditional heteroscedasticity appears to be low and the opposite situation can be observed in the exchange rate markets, where the variability is higher.

Chancharoenchai (2014) employs dynamic ordinary least square and three test of cointegration to demonstrate the relationship between Gross Domestic Product and Inflation Ratio in the Thailand economy. He finds important confirmations about the long-run equilibrium of the two variables and on their Error Connection Mechanism, explicating the different speed of adjustment.

Chamalwa and Bakari (2016) investigate the connections among the Economic Growth, the money supply and credit to the private sector, in the Nigerian economy using the Johansen procedure and the findings are coherent with the theory in assessing a strong cointegration with at most one cointegrating vector.

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Investing their relationship, they find bi-directional influences among the variables, which means that the previous observation of one of them causes a change in the other two.

Thinking at a shift from active management to passive management, the enhanced index strategy can be considered a good step. Using a rigorous method of administration of the stocks and the assets, it can combine an active view and analysis of the fundamentals with a quantitative testing of the variables.

Bansal and Kiku (2011),for example, criticize the standard VAR approach in choosing the optimal portfolio and define a model that takes into account both the short-run and the long-run deviations. The so-called EC-VAR helps the manager in picking out the correct stocks, valuing the returns' dynamics, and the use of an Error Correction Mechanism between cash flow and consumption gives substantial improvements when considering a long-term investment's horizon.

Another possible application of cointegration is its use in the pair trading, a typical situation where a stability condition is often violated in favour of temporary disequilibrium.
Chiu and Wong (2012) identify couple of cointegrated derivatives and use a mean-variance approach to take advantage of their spread. Ho, Ernst and Zhang (2011) employ the Johansen's procedure to investigate the relationship between small and large capitalization stocks.
Taking into account a large number of years, from 1926 to 2006, the data show negative and consistent correlation between the two classes of stocks.
Probably, the effect is amplified by the so-called "size effect", by which a period when the small cap stock prices outperform the large cap ones is followed by years of underperformances and viceversa.

Huang et al (2012) show how the cointegration approach can be applied to the study of the uncovered interest parity. Using a time varying model, they outlined the connection between the long-term interest rates between Malaysia and Singapore.
Their relationship is not stable but the findings of the authors are interesting from the correlation of the shocks. The financial market of Singapore, more developed, leads the Malaysia's one, showing a great dependence in the decisions of the two central banks, especially after the 1997 Asian Financial Crisis.

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As previously exposed, the spread between two cointegrated series is mean reverting and, after short run deviations, it comes back to long run equilibrium. This important feature can be use in trading strategy, choosing the right time to enter in the market, and taking advantage of the possible movements when the temporary disequilibrium is recognized.

4.2 Methods of creation of portfolios

The theory about how is possible to track the index has grown in the recent years.
This chapter starts providing a review or the most important methods found in the literature and we try to adapt one of these at this specific case, in order to verify the power of this method and compare results in term of co-integration, cumulative return and risk-return trade-off.
Speaking about partial replication, the first step to analyze is the selection process of the assets. The easiest method in capitalization-weighted indices is to include in the basket only the stocks with the largest market capitalization.
The same phenomenon can be observed in the price-weighted indices for the stocks that have higher prices comparing with the others. Their influence on the movements of the index are proportional to their price-value.

An important empirical study, which tests the impact of capitalization, industry stratification and weighting is the one proposed by Larsen and Resnik (1998).
The authors demonstrated the bigger effect of the high-capitalized stocks over the low capitalized ones, finding out the advantages in terms of tracking error and standard deviations. Especially, talking about low capitalization indices, the industry stratification plays a less important role that the simple value of the shares.

Dunis and Ho (2005) figure out the possibility to use cointegration, instead of correlation, to create a replica portfolio. They started their analysis choosing in a random way 20 of 50 stocks included in the EuroStoxx50 Index and created different portfolios varying the frequency of rebalancing and the stocks included in the basket. The results are quite impressing and the replica portfolios show higher Sharpe ratios, higher Information ratios and, basically, a good management of the turnover, compared to the correlation-based ones.

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Another method, very useful when dealing with indices with a large numbers of stocks, can be the sampling using the stratification of the index itself. The stocks can be divided exploiting different common features and the choosing process is usually implemented by various optimization techniques.

A good combination could be the matching of information taken by the capitalization structure of the index and the sector membership, taking the representative stocks of each group identified. An extensive survey of this methodology can be find in the work of Maginn et al (2007).

An important and relatively recent method is the one based on Principal Component Analysis (PCA). As showed in Corielli and Marcellino (2006), PCA is based on the assumption that there are some components (the trend and the economic cycle for example) which are common among different stocks and affect them in the same way. Once isolated the components, it is possible to compare the correlation between them and the assets in order to track the index.

Alexander and Dumitru (2003) explained a similar methodology to replicate an index, using the trend in the stock market. They utilized the stock returns to construct a matrix, picking only the first component.

Gilli and Kellezi (2001), instead, use a threshold accepting algorithm and search for sub-optimal results, taking into account the improvements due to the reiteration of the procedure. Citing their words: "The Threshold Accepting Algorithm is a refined local search procedure which escapes local minima by accepting solutions which are not worse by more than a given threshold. The algorithm is deterministic as it does not depend on some probability. The number of steps where we explore the neighborhood for improving the solution is fixed. The threshold is decreased successively and reaches the value of zero after a given number of steps."
In their approach, they included the transaction costs and different constraints, and the results were encouraging.

Frino et al.(2004) compare the differences in terms of returns and trading costs between enhanced index funds and pure index funds, demonstrating the advantage of a smooth rebalancing strategy. Especially during index reversions, a rigid strategy can easily lead to incorrect market timing and result in an increasing of the overall costs.

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An important empirical analysis is given by Thirimanna et al (2010), in which the authors compares the Modern Portfolio Theory against the Cointegration method.

The data are taken from the Colombo Stock Exchange (CSE), the main stock Exchange of the Sri Lanka, during the period after the civil war.

The results are pretty encouraging in terms of Correlation with the Index, Max Sharpe Ratio and Information Ratio, but the portfolios created using the Cointegration technique, underperform the Capital Market Theory's ones, due to the excessive diversification which is usual in a tracking portfolio.

El Hassan and Kofman (2003) define a model that takes into account the relationship between historical return analysis and forecasting of risk factors.

The model is a mixed strategy that starts from the calculation of the efficient frontier and continues inserting constraints on the weights (no short selling) and on tracking error variability. Testing the Jorionis methodology with real data, they create step by step a portfolio of the 30 major stocks of the Australian market, and then they evaluate the possible improvements in a trade-off between forecasts analysis and constrained allocation.

The portfolio tracks the benchmark in a proper manner for half of the period used, but starts to substantially diverge when, in June 2000, the tech bubble collapses damaging the tracking capability. The work makes conclusions about the importance of the period (bull or bear market) and of the previsions about the return variability.

Rudolf et al (1999) give a survey of different tracking error minimization, investigating four model of tracking errors.

The empirical part is performed building a portfolio with six different national stock market index and it is compared with the MSCI world index.

"Rule No 1: Never lose money. Rule No 2: Never forget rule no 1." Warren Buffet.

Section 5. Empirical Part.

This section is dedicated to the empirical test of the Engle-Granger Method using 1500 ca. daily data of the Dow Jones Industrial Average and of 25 of its Constituents.The idea is to construct a stable portfolio in order to track the index.

In the first paragraph, we discuss the history of the Index and the characteristics of the raw data while in the second paragraph we can find the problem formulation and the explanation of the procedures utilized.

The empirical part is performed with the use of Gretl, for the preliminar and static analysis, and with Matlab v.2016, for the dynamic framework and the asset allocation problem[42].

5.1 DJIA

The first important decision to make is to choose what index analyze and what type of data we want to manage. Obviously, the importance of this step is crucial for the whole thesis, because of the different features of the indices.
The chosen index is the Dow Jones Industrial Average[43], the second oldest index of the United States and globally considered as "the pulse of the U.S. stock market".

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[42] All the results are own contributions and all the possible errors reported are own fault.
[43] Henceforth simply DJIA.

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5.1.1 History

Formally established on May 26, 1896 by Charles H. Dow, as a derivation of a previous-built index (The Dow Jones Transportation Index), the DJIA is now one of the most known and quoted of the world.

Simply called "The Dow", in its first composition it included only 12 stocks, representing the principal companies of U.S. economy at the time. Nowadays, the DJIA is a benchmark that tracks the most important American stocks, which are usually considered as leaders of the respective sector. It includes 30 large-cap blue-chip companies, arbitrarily chosen by the editors of the Wall Street Journal. From the 2012, it is owned by the S&P Dow Jones Indices.
Except of General Electric, which was included in the initial list and dropped off only for some years, all the other companies have been changed over time to guarantee the validity of the index in representing the status of the U.S. economy.

Many authors through years have criticized the importance of the DJIA as the barometer of the U.S. financial markets because of its small number of stocks and the discretionary method to choose them. Compared with the Russell 3000 Index, which includes the most important 3000 U.S. companies, the DJIA gives a very restrictive view of the entire system but maintains its importance due to its brand and its significance.

During the years, the DJIA lost its industrial characterization and stocks from different sectors were included, such as for example Goldman Sachs Group Inc., for the financial sector, Wal-Mart Stores Incorporated, for the grocery sector or Apple Inc. that replaced AT&T in 2015.
Unlike other important indices, the DJIA is a price weighted and does not imply the weighting of the market capitalization.
In the initial formulation of Charles H. Dow, the index was a simple arithmetic average of the stock prices. This configuration was abandoned in 1928, when the sum of the prices starts to be divided by a divisor, the Dow Divisor. Currently it is less than a unit[44] and actually works as a multiplier,

$$DJIA = \frac{\sum p}{d}$$

Where *d* is the Dow Divisor and *p* are the prices of the stocks.

---

[44] 0.14602128057775 in late September 2016.
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Through years, the divisor was adjusted in order to ensure the continuity of the value, not taking into account the distortions caused by the changes in the companies listed, stock splits or spinoffs.

When they happen, the Dow Divisor is updated to ensure the equality of quotations before and after the event.

$$DJIA = \frac{\sum p_{old}}{d_{old}} = \frac{\sum p_{new}}{d_{new}}$$

During the last years, the DJIA breaks several records, getting back to the pre-crisis level only in March 2013 and overcoming the psychological threshold of 18.000 points during December 2014.

When the writing of this work comes to the end, in the Autumn[45] of 2016, the DJIA is stable around the 18.100-18.300 points, waiting for the important results of the U.S. elections and for the *"Fed rate hike"* (i.e. the day when finally the Federal Reserve will increase the interest rates).

As showed in the graph below, after the summer, the DJIA is more stationary then before.

Figura 3 Source: Bloomberg.com

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[45] The last version of the Thesis is dated 31.10.2016.

5.1.2. Data

The main difficult is to identify the kind, the frequency and the depth of the data. Indeed the data are drivers of different and complementary possible analysis and can lead to different results.

The opinion of the literature is not coherent in the entire academic world. We choose to carry out the analysis with daily data. The dataset is obtained downloading 1503 daily observations of the DJIA and of the 25 constituents, stable in the Index from the October,01 2009 to the July 31, 2015. from datastream.com

The presence of Volatility Clustering[46] and Heteroscedasticity in this kind of data is known but it does not affect the consistency of the estimation. We try to identify some important features such as the mean, the variance, the kurtosis and the standard deviation[47].

One important feature of the Index is the influence that some companies have on it. As showed in the figure below, the changes in the price of the single stocks affect the DJIA in a sensible manner.

Figura 4 Fonte: http://siblisresearch.com/data/dow-jones-30-weightings/

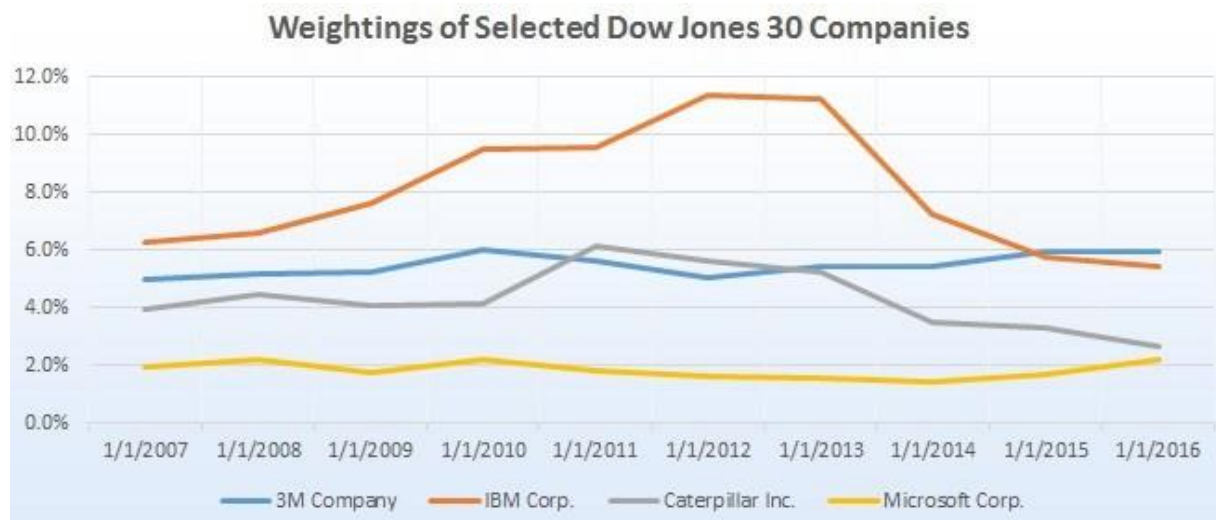[46] A period of high volatility is followed by another period of high volatility, on the other hand small changes are followed by small changes.
[47] The results are exposed in the APPENDIX.

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During the last years, predominant was the effect of the financial crisis, which damaged the Citigroup Inc. quotation causing its expulsion from the constituents' list.

The story of General Motors is similar; the stock was kicked off just before its default.

Last addiction to the Index was Apple Inc., added after the death of Steve Jobs and especially after the splits of its shares, that was the principal motivation over its outstanding.

A price-weighted scheme suffered the excessive value of some stocks, which has too much influence on the movements of the index. For this reason, all the stocks included in this type of index have to be in a defined range.

The enormous value reached by the AAPL stock price, before the split, would have corrupted the validity of the DJIA, because of its excessive influence in the price-weighting scheme.

The same problem can be identified in a capitalization scheme, where the companies that have the greater capitalization on the stock market, have the greater influence on the index.

Hence, it is possible to observe a few number of stocks that lead the index, invalidating the validity of it.

The weighting of the Index is not stable over time and its composition can substantially differs from one year to another. For more information, we suggest the Table1 in the Appendix .

The DJIA is broadly diversified, as showed by the figure below.

Source: http://www.spa-etf.com/best-dow-jones-index-funds-djia-etf-and-short-dow-etf-list/

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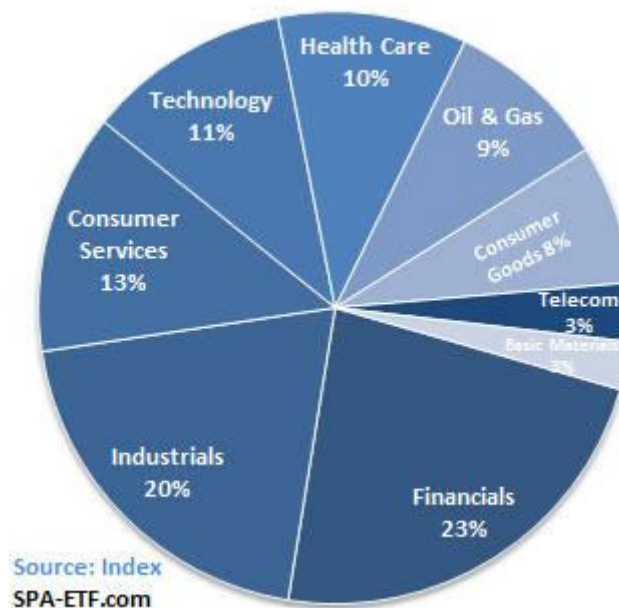
Exploring the features and the limits of the Cointegration is also important to understand how it deals with changes of weightings and different stochastic trends.

Based on the common (stochastic) trend assumed by the stocks, the procedure can also be stable in this complicated situation.

5.2 Problem Formulation

This paragraph is dedicated to the analysis of the relationship between the portfolios and the index, in order to identify which is the best in tracking it.

During the study of the literature dedicated to the index tracking, we choose to follow the work of Alexander & Dumitru (2002), in which they introduce and implement a linear optimization problem. The same methodology is used in several papers, we suggest the reading of Stancu and Radu (2009).

The procedure is simple and can be divided in a number of consequential steps to ensure the fairness of the results.

We have to divide the sample in two period. The first one is used to calibrate the portfolios weights, the second one is used to test the investment method, changing the reallocation timeframe.
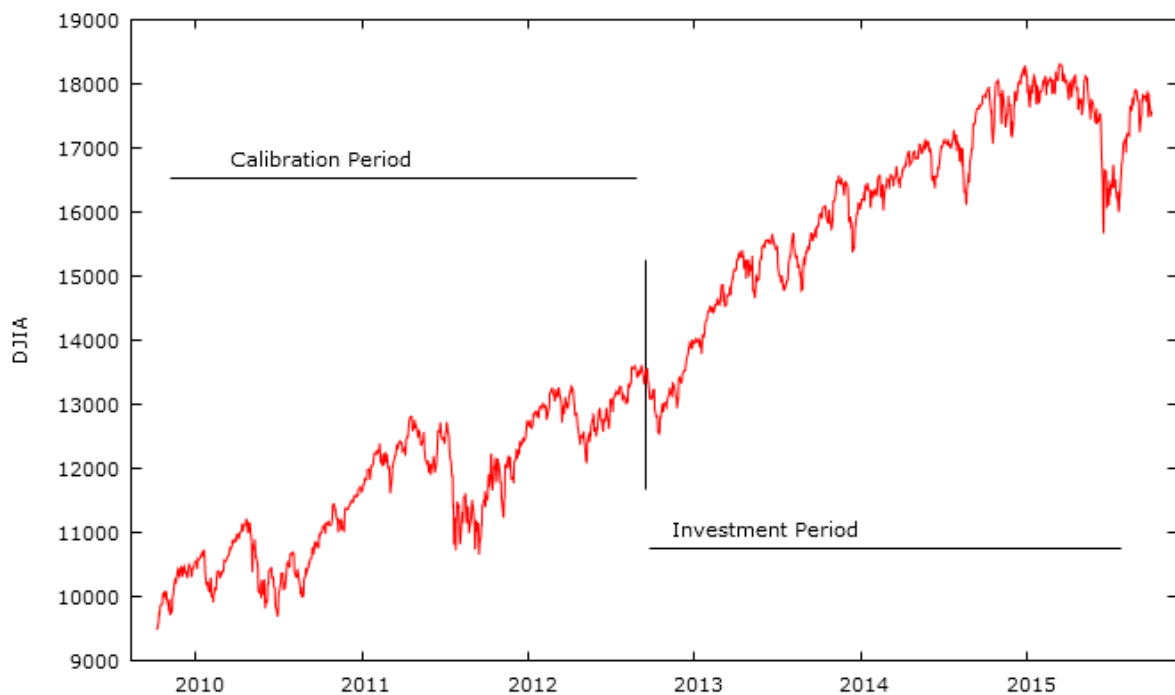
The first important step is to certify the non-stationarity condition of the series used in this empirical test.

Below are exposed, as example, the results for the logarithmic transformation of DJIA:

```
Augmented Dickey-Fuller test for l_DJIA
including 5 lags of (1-L)l_DJIA
(max was 10, criterion AIC)
sample size 744
unit-root null hypothesis: a = 1

  test without constant
  model: (1-L)y = (a-1)*y(-1) + ... + e
  estimated value of (a - 1): 5.68699e-005
  test statistic: tau_nc(1) = 1.36239
  asymptotic p-value 0.9571
  1st-order autocorrelation coeff. for e: -0.004
  lagged differences: F(5, 738) = 3.904 [0.0017]

  test with constant
  model: (1-L)y = b0 + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): -0.00612388
  test statistic: tau_c(1) = -1.40615
  asymptotic p-value 0.581
  1st-order autocorrelation coeff. for e: -0.003
  lagged differences: F(5, 737) = 3.746 [0.0023]

  with constant and trend
  model: (1-L)y = b0 + b1*t + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): -0.0274497
  test statistic: tau_ct(1) = -2.92715
  asymptotic p-value 0.1537
  1st-order autocorrelation coeff. for e: -0.002
  lagged differences: F(5, 736) = 3.242 [0.0067]
```

And these are the results for the differences:

```
Augmented Dickey-Fuller test for d_l_DJIA
including 4 lags of (1-L)d_l_DJIA
(max was 10, criterion AIC)
sample size 744
unit-root null hypothesis: a = 1

  test without constant
  model: (1-L)y = (a-1)*y(-1) + ... + e
  estimated value of (a - 1): -1.1914
  test statistic: tau_nc(1) = -13.8938
  asymptotic p-value 1.271e-029
  1st-order autocorrelation coeff. for e: -0.003
  lagged differences: F(4, 739) = 3.572 [0.0068]

  test with constant
  model: (1-L)y = b0 + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): -1.20352
  test statistic: tau_c(1) = -13.9701
  asymptotic p-value 5.086e-032
  1st-order autocorrelation coeff. for e: -0.004
  lagged differences: F(4, 738) = 3.672 [0.0057]

  with constant and trend
  model: (1-L)y = b0 + b1*t + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): -1.20358
  test statistic: tau_ct(1) = -13.9604
  asymptotic p-value 8.918e-038
  1st-order autocorrelation coeff. for e: -0.004
  lagged differences: F(4, 737) = 3.668 [0.0057]
```

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As expected, and well documented in literature, financial prices are non stationary and the tests confirm our hypothesis. On the contrary, their differences, i.e. the corresponding returns of the stocks, are stationary, having oscillations around the zero.

These are the patterns of the log-prices and the differences of log-prices of the DJIA.
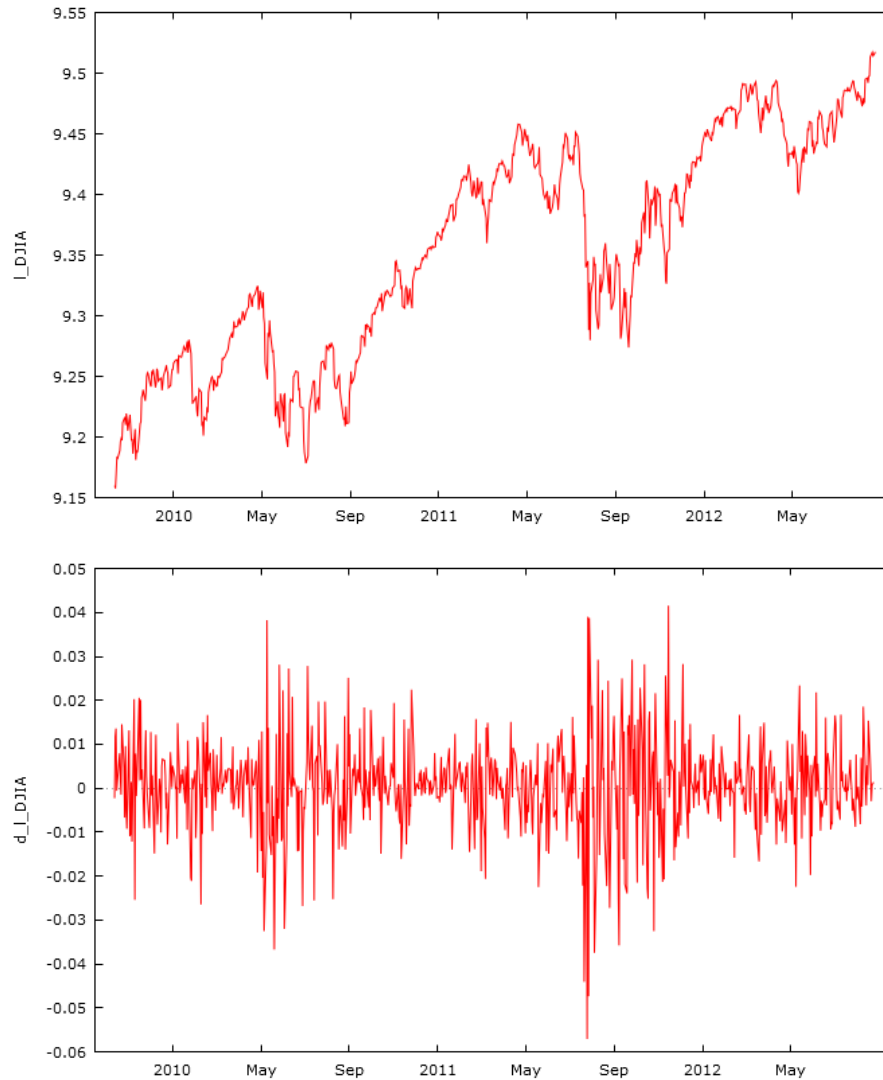
The other analysis and patterns are showed in the Appendix.

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The next step is to procede in the estimation of the cointegrating vector.

More simply, we use an OLS estimation on the following expression:

$$Log(P_{I,t}) = \alpha + \sum_{i}^{N} c_k Log(P_{k,t}) + \varepsilon_t$$

Where:

- α is the constant.
- $Log(P_{I,t})$ is the dependent variable, i.e. logarithmic transformation of the Price Index at time t.
- $Log(P_{k,t})$ are the independent variables, i.e. the logarithmic transformation of the Prices of the Stocks at time t.
- c_k are the estimated coefficients and, after normalization, the weights of the shares in the portfolio.
- ε_t are the residuals, i.e. the tracking error.

The main advantages of this method are its speed of calculation, its flexibility and its adaptability to the context.. The short selling is allowed and the only constraint is on the sum of the weights *(=1)* in order to ensure the comparability between the portfolios and the index. If the residuals of the regression are stationary, the series are cointegrated and this condition is analyzed by the use of stationarity tests

The most cointegrated portfolio will be the one with most stable tracking error.

Using this type of formulation allows us not to take into consideration the nature of the series and to make the most of their long-term memory, but let us prone to the distortion caused by the variability and autocorrelations of the daily data.

For this reason, as previously exposed and following the work of Stock (1987), we introduce an implementation of the model using the Dynamic OLS (DOLS).

In his paper, he writes: "inference based on the standard least squares output can be misleading in time series regressions with both lagged differences and levels of the dependent variable appearing as explanatory variables. In this case, the moment matrix of the levels regressors converges to a limiting random variable, and the distribution of certain regression coefficients will not be well approximated by normality."

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The new procedure can be expressed by the following equation:

$$\text{Log}(P_{I,t}) = \alpha + \sum_{i}^{N} c_k \text{Log}(P_{k,t}) + \sum_{i}^{N} \gamma_k \Delta_{t-3} \text{Log}(P_{k,t}) + \cdots + \sum_{i}^{N} \gamma_k \Delta_{t+3} \text{Log}(P_{k,t}) + \varepsilon_t$$

The DOLS estimator is consistent, asymptotically normal distributed, and efficient.

The addiction of the differenced variables, with 3 lags of the previous observations and 3 lags of the following observations, improves the validity of the regression.

Moreover, in this modification the presence of Cointegration is guaranteed by the stationarity of the tracking error.

The second important step is to choose the stocks that will part of our replicating portfolio.

The stock selection problem is a crucial point that influences all the following work.

In order to ensure the validity of the test, we only use the stocks stable in the basket of the DJIA during the period analyzed and we exclude the others, included and dropped out from the index.

To evaluate the two procedure, we choose to make an initial comparison, identifying the one that gives back the better value of estimation in sample.

As well explained, the DJIA is a price-weighted index; for this reason, we choose to two different asset allocation of the portfolio:

1) Portfolio1 that includes the 25 stocks, using the weights obtained by the Dynamic OLS regression.
2) Portfolio2 that includes the 25 stocks, using the weights obtained by the OLS regression.

 The third step is to select the correct calibration period, in order to choose the correct allocation of the weights. The opinions given by the papers analyzed are many.

Among others, Corielli and Marcellino, (2006), utilizing daily data, suggest the use of approximately 1 years and half. In another important paper, Dunis and Ho (2005) indicate 2 years as the minimum, extendible up to 4.5 years.

Alexander and Dimitru, (2004b) and Alexander and Dumitru (2002) indicate 3 years as the minimum to ensure the stability of weights.

Hence, we choose this length to compare the two formulations.

The procedure is performed on a determined window of 750 observations, which approximately corresponds to 3 years of daily data.

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5.2.1 Evaluation of Cointegration

The portfolios are compared by their in-sample estimation (from observation number 1 to observation number 750) of the stationarity of the error term.

In the images below, we can see the plot of the Residuals of DOLS and OLS method.
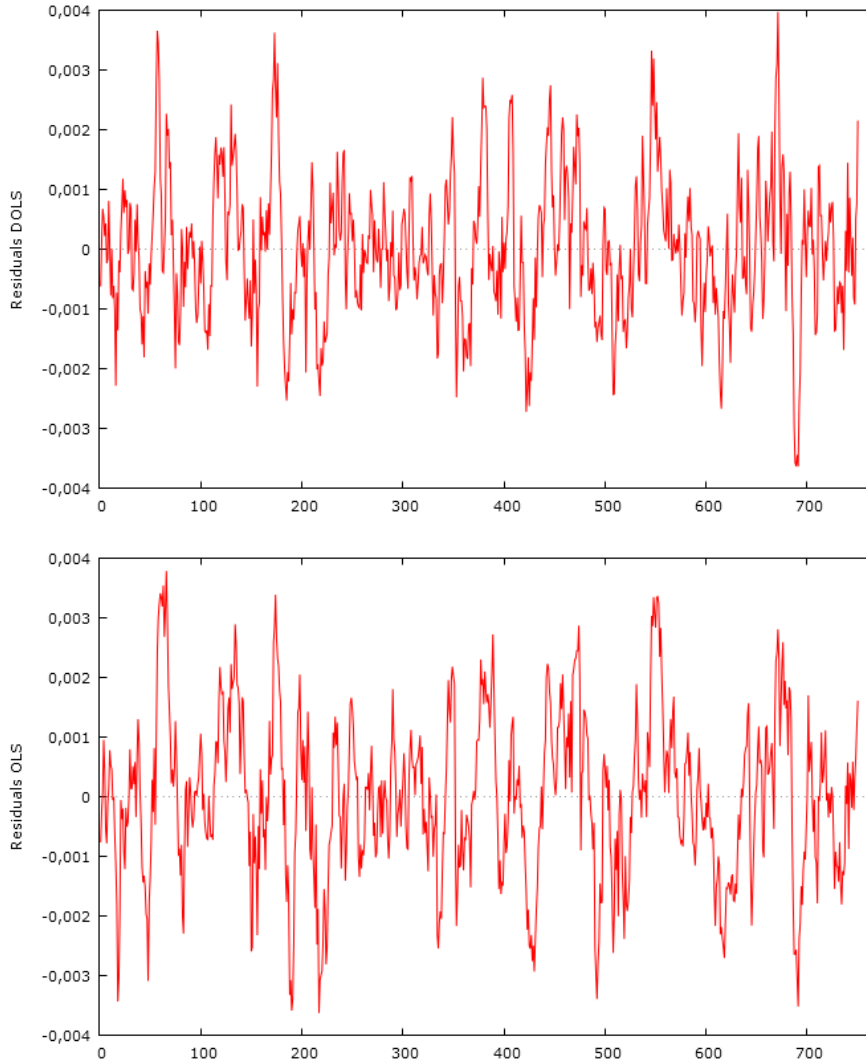
The two procedure give similar estimations, resulting in similar deviations in the graph exposed. The series appear stationary.Among all the papers examined, no one can give us the right critical value for the ADF test that certifies the presence of Cointegration.

The procedure used in the cited papers on similar dataset and number of variables, as Alexander and Dumitru(2001) or Dunis and Ho(2005), utilized the standard ADF critical value, no taking into account the increased number of variables.

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The work of MacKinnon(2010) indicated the right value for no more than twelve explanatory variables, and so we choose the asymptotic value for the specific regression (i.e. constant, 1% -6.63790, 5% -6.11279, 10% -5.83724)[48].

We observe that the ADF statistics is significant for both methods, even if we use the OLS specification to test for stationarity of the residuals, and the DOLS specification to achieve the right coefficients.

Method	ADF test
P1	*-9,12836****
P2	*-7,40611****

The same results are obtained shifting the time window[49].

For this reason, we choose to go on utilizing only this formulation, changing the time of rebalancing and commenting the results.

The first two portfolios, which are used to compare the estimations, are left unchanged during the entire period, giving the same result of a buy and hold strategy.

Instead, the other two portfolios are rebalanced after 1 year and 6 months respectively.

Summing up:

- P1 = 25 Stocks, DOLS estimation, no rebalancing.
- P2 = 25 Stocks, OLS estimation, no rebalancing.
- P3 = 25 Stocks, DOLS estimation, rebalancing after 1 year.
- P4 = 25 Stocks, DOLS estimation, rebalancing after 6 months.

The trade-off between the transaction costs, due to the rebalancing, and the tracking error is predominant in the analysis of the methodology.

A more frequent rebalancing is strictly correlated with an increasing in the expenses and can damage the performances of the portfolios.

Focusing on the empirical method, we choose to neglect the transaction cost. The decision is taken observing the rare impact that this type of costs has considering these reallocation timing.

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[48] The critical value of the ADF is valid in the empirical use, to compare the portfolios and to analyze how the possible manipulation of the procedure influence the error term and its ADF value.
From the pure theoretic point of view, the fact that the procedure was never utilized with similar dataset caused a lack of the specific indication.
Hence, we have to considered the asymptotic value of the formulation with 12 variables, notice that is also the highest in the response surface. We suggest the reading of MacKinnon (2010) for a wider explanation on the critical β .
[49] For more data, we suggest the reading of the table and the charts in the APPENDIX.

For a comparison, we indicate the reading of Grobys (2010).

The tables above shows the evolution of the weights in the four portfolios.

The first table exposes the weights of the buy-and-hold strategy and we can notice that they change for a very small amounts.

The second and third tables show the impact of reallocation on the weights.

At the beginning, the portfolios are well distributed, indicating a good cointegration between the single variables and the Index.

During the investment period, the relationship between the index and some of the constituents changes and, for this reason, we can find a strong concentration in some assets (for example MMM, IBM, HD), while we can observe some negative coefficients, which force us to go short.

Companies	P1	P2
MMM	0,069264	0,063119
AXP	0,021812	0,028781
BA	0,06396	0,061438
CAT	0,067845	0,063301
CVX	0,069371	0,055346
CSCOO	0,018491	0,017838
KO	0,044271	0,031253
DD	0,009763	0,01756
XOM	0,056464	0,066457
GE	0,024265	0,022179
HD	0,014057	0,019063
INTCO	0,006168	0,008778
IBM	0,119256	0,105896
JNJ	0,080345	0,073295
JPM	0,034379	0,035405
MCD	0,014953	0,019231
MRK	0,023228	0,024266
MSFTO	0,04158	0,036096
PFE	0,004731	0,010621
PG	0,014865	0,022589
TRV	0,060082	0,052122
UTX	0,062658	0,073763
VZ	0,006661	0,024851
WMT	0,037574	0,037322
DIS	0,033957	0,029429

Companies	P3_Period1	P3_Period2	P3_Period3
MMM	0,069264	0,078993	0,112838
AXP	0,021812	0,021445	0,059251
BA	0,06396	0,058199	0,028751
CAT	0,067845	0,045859	0,011953
CVX	0,069371	0,03585	0,067033
CSCOO	0,018491	0,015304	-0,01482
KO	0,044271	0,017128	-0,07863
DD	0,009763	0,074367	0,075347
XOM	0,056464	0,071366	0,077608
GE	0,024265	0,008612	-0,03039
HD	0,014057	0,016114	0,072564
INTCO	0,006168	0,004797	0,065238
IBM	0,119256	0,103752	0,089253
JNJ	0,080345	0,050378	0,009517
JPM	0,034379	0,030865	0,045012
MCD	0,014953	0,037454	0,067455
MRK	0,023228	0,016855	0,034711
MSFTO	0,04158	0,027887	-0,01015
PFE	0,004731	0,007534	0,023173
PG	0,014865	0,057865	0,099425
TRV	0,060082	0,041777	0,012829
UTX	0,062658	0,060713	0,07083
VZ	0,006661	0,051227	0,08864
WMT	0,037574	0,045211	0,013432
DIS	0,033957	0,020445	0,009141

Companies	P4_Period1	P4_Period2	P4_Period3	P4_Period4	P4_Period5	P4_Period6.
MMM	0,069264	0,077858	0,078993	0,097156	0,112838	0,19815
AXP	0,021812	0,028132	0,021445	0,025469	0,059251	0,045495
BA	0,06396	0,070228	0,058199	0,039769	0,028751	-0,00195
CAT	0,067845	0,045642	0,045859	0,029778	0,011953	0,024461
CVX	0,069371	0,079984	0,03585	0,014673	0,067033	0,01474
CSCOO	0,018491	0,011521	0,015304	0,016785	-0,01482	0,000498
KO	0,044271	0,019877	0,017128	0,017256	-0,07863	0,022173
DD	0,009763	0,049162	0,074367	0,07114	0,075347	0,073406
XOM	0,056464	0,047987	0,071366	0,115181	0,077608	0,007378
GE	0,024265	0,03721	0,008612	-0,00837	-0,03039	0,023674
HD	0,014057	0,016342	0,016114	0,01798	0,072564	0,108274
INTCO	0,006168	0,011355	0,004797	0,013024	0,065238	0,054628
IBM	0,119256	0,114751	0,103752	0,090221	0,089253	0,046136
JNJ	0,080345	0,056939	0,050378	0,028119	0,009517	-0,05348
JPM	0,034379	0,030241	0,030865	0,035256	0,045012	0,075688
MCD	0,014953	0,030575	0,037454	0,073052	0,067455	0,097613
MRK	0,023228	0,01444	0,016855	0,02898	0,034711	0,028178
MSFTO	0,04158	0,038663	0,027887	0,006404	-0,01015	0,003705
PFE	0,004731	-0,01652	0,007534	-0,02696	0,023173	0,017724
PG	0,014865	0,048723	0,057865	0,066897	0,099425	0,006347
TRV	0,060082	0,06728	0,041777	0,031379	0,012829	0,000688
UTX	0,062658	0,045565	0,060713	0,075066	0,07083	0,06332
VZ	0,006661	0,00579	0,051227	0,053854	0,08864	0,09142
WMT	0,037574	0,045763	0,045211	0,027921	0,013432	0,058522
DIS	0,033957	0,022488	0,020445	0,059966	0,009141	-0,00678

5.3 Portfolio's Analysis

In this paragraph, we analyze what are the features of the different portfolios, with the aim to compare and rank them.

5.3.1 Performance
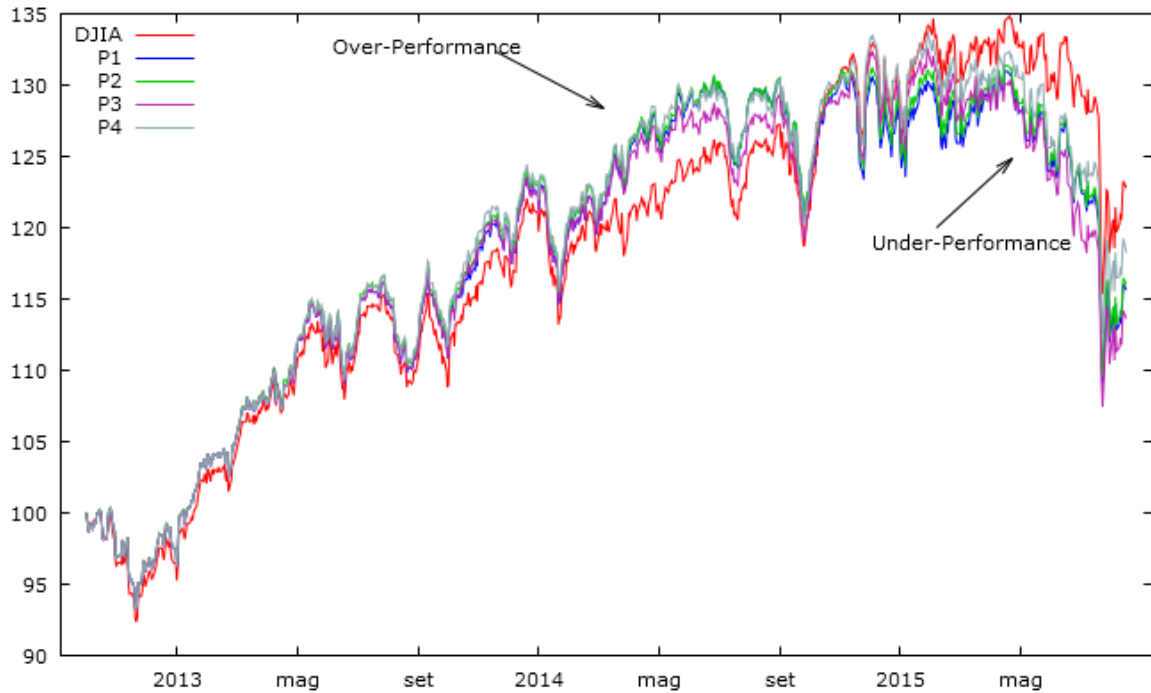
Analyzing the entire timeframe, all the portfolios underperform the DJIA.

We have to notice that for the first and the second years, the Portfolios achieve better performance than the index, losing their over-return in the last year.

This characteristic is also outlined in the table below :

Cumulative Returns	DJIA	P1	P2	P3	P4
0-6 M	6,87 %	7,80 %	7,89 %	7,80 %	7,80 %
6-12 M	7,75 %	8,38 %	8,61 %	8,38 %	9,07 %
12-18 M	4,44 %	4,04 %	4,25 %	3,98%	4,10 %
18-24 M	5,05 %	6,54 %	6,07 %	5,49%	5,66 %
24-30 M	4,97%	-1,12%	-0,74%	0,05%	0,94%
30-36 M	-7,42%	-9,69%	-9,80%	-11,38%	-9,41%

The last period is characterized by a huge volatility in the market and probably the increasing noise of the data damages the estimation of the coefficients.

The use of a more frequent rebalancing does not seem to be effective, but helps to recover a bit.

On the other hand, we can observe the constant over performance in the first two years, when the index was constant up-trending.

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5.3.2 Tracking Error Measures

The tracking quality of a portfolio is calculated on measures, based on simple returns or log returns .Simple returns are expressed by the following formula:

$$R_{i,t} = \frac{P_{i,t} - P_{i,t-1}}{P_{i,t-1}}$$

Where $P_{i,t}$ is the price of the financial asset at time.

Moreover, simple returns give the possibility to manage them across stocks, aggregating and evaluating them as the weighted sum of the constituents of the tracking portfolios.

An initial analogy can be done through the Tracking Error, the MAD and the MSE.

Tracking Error is calculated using the following formula[50]:

$$TE_t = R_{I,t} - R_{p,t} = R_{I,t} - \sum_{i=1}^{N} w_i\, r_{i,t}$$

Where:

- $R_{I,t}$ is the return of the index at time t,
- $R_{p,t}$ is the return of the portfolio at time t,
- $\sum_{i=1}^{N} w_i$ are the weights of N stocks, and is equal to *1*.
- $r_{i,t}$ is the return of the stock i at time t.

Recalling the Regression, the TE is simply the error term "ε_t".

Based on TE, the Mean Absolute Deviation (MAD) measure is formalized through the following expression:

$$MAD_t = \frac{1}{T}\sum_{t=1}^{T}\left|R_{I,t} - R_{p,t}\right| = \frac{1}{T}\sum_{t=1}^{T}|TE_t|$$

The MAD shows the mean absolute difference between the returns.

On the contrary, the Mean Square Error is the mean of the squared differences between the returns.

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[50] For a further explanation of this part, we suggest the reading of the paper of Roßbach and Karlow (2011).

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It can be expressed through the following formulation:

$$MSE = \frac{1}{T}\sum_{t=1}^{T}\left(R_{I.t} - R_{p,t}\right)^2 = \frac{1}{T}\sum_{t=1}^{T}(TE_t)^2$$

These two measures are used for compare different portfolios, but being based on TE, they suffered misspecification of the characteristics of the portfolios.

For this reason, we introduce another measure, the ex-post tracking quality (ETQ), which measure the absolute difference between the values of the portfolios.

It measures the deviations between the values of the tracking portfolios and the benchmark and can be calculate following the expression below:

$$ETQ = \frac{1}{T}\sum_{t=1}^{T}|R_{I,t}^c - R_{P,t}|$$

ETQ is not affected by the negative autocorrelation of the returns even if frequent data are used, because it is based on the values of the tracking portfolio and the index.

The Cumulative Returns can be calculated using the formula:

$$R_t^c = \frac{V_t}{V_0} - 1 = \prod_{k=1}^{t}(1 + R_k) - 1$$

Since ETQ is based on the compounded returns it represents a useful alternative to compare the tracking quality of a portfolio in the long term.

The use of the ETQ demonstrates the difference between the values of the estimated tracking portfolios and of the index.

On the contrary, MAD and MSE are better for evaluating short-term deviations, since they do not take into account the compounding effect of the returns over the value of the portfolios.

Portfolio	MAD	MSE	ETQ
P1	0,102 %	0,000191686	2,55%
P2	0,092 %	0,000163504	2,58%
P3	0,114 %	0,000240779	2,29%
P4	0,116 %	0,000256258	2,23%

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The results show the predominance in the tracking performance of the portfolio 4 in the investment period and this is confirmed by the small difference in the final compounded returns. The other two methodology analyze indicate as the best tracker the P2, the buy-and-hold based on the OLS estimation.

Even if by a very small amount, both the methodology confirm the conclusions

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	Average	Median	Minimum	Maximum
TE_P1	- 0,0017861	- 0,0096683	- 0,044375	0,062269
TE_P2	- 0,0045798	- 0,011229	- 0,046207	0,058235
TE_P3	- 0,00024335	- 0,0094092	- 0,037122	0,082018
TE_P4	- 0,0088525	- 0,011102	- 0,047922	0,053431

The idea is clearer looking at the graph below.

Cumulative Ex Post Tracking Error
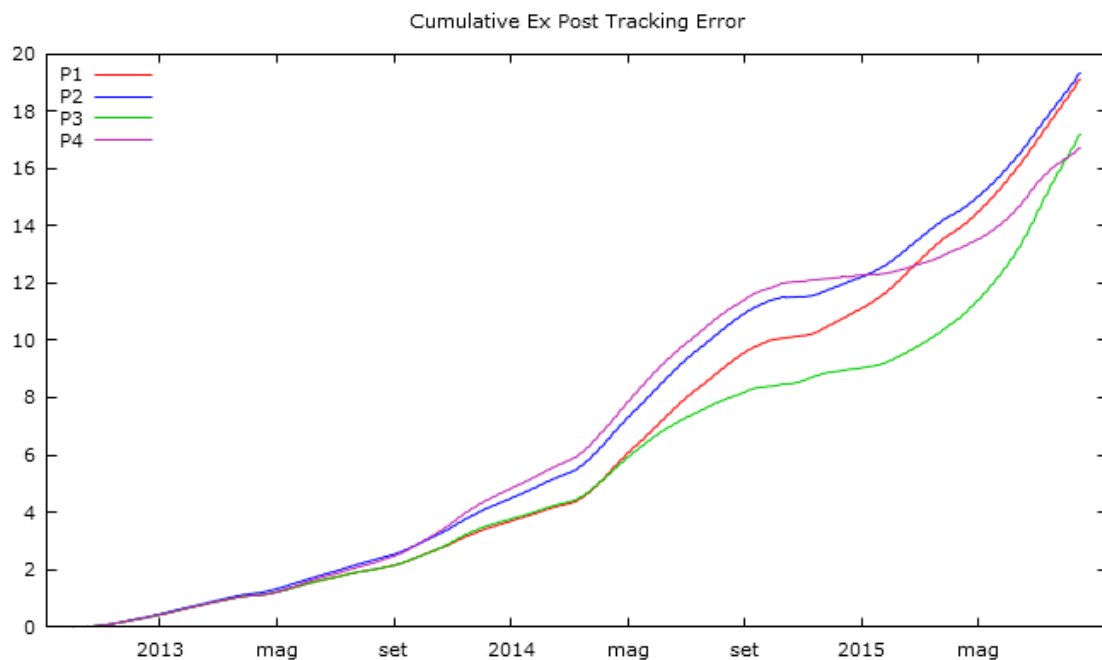
The Portfolio3 confirms to be the more stable in tracking the value of the index, even if it concludes with a lower return.

As exposed in the central part of the investment period, the other 3 portfolios (P1,P2,P4) raise their Cumulative ETQ, while the P3 shows less volatility.

The increasing of reallocation timeframe help us to restore a part of the compounded returns lost, probably offset by the consequential transaction costs, that are not considered in this work.

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The last graph shows the simple cumulative TEs of the different portfolios.

We can assert that the best portfolio is the P3, that remains closer to the index for almost all the period analyzed, even if it substantially diverges in the final part of the investing strategy.

The movements of the cumulative TE of the P4 is interesting: in more than half of the sample, P4 is the more volatile and the less efficient, recovering its ability in the final part, and ending with the better cumulative results, from the point of view of the returns and of the ex-post tracking errors.

Cumulative Tracking Errors
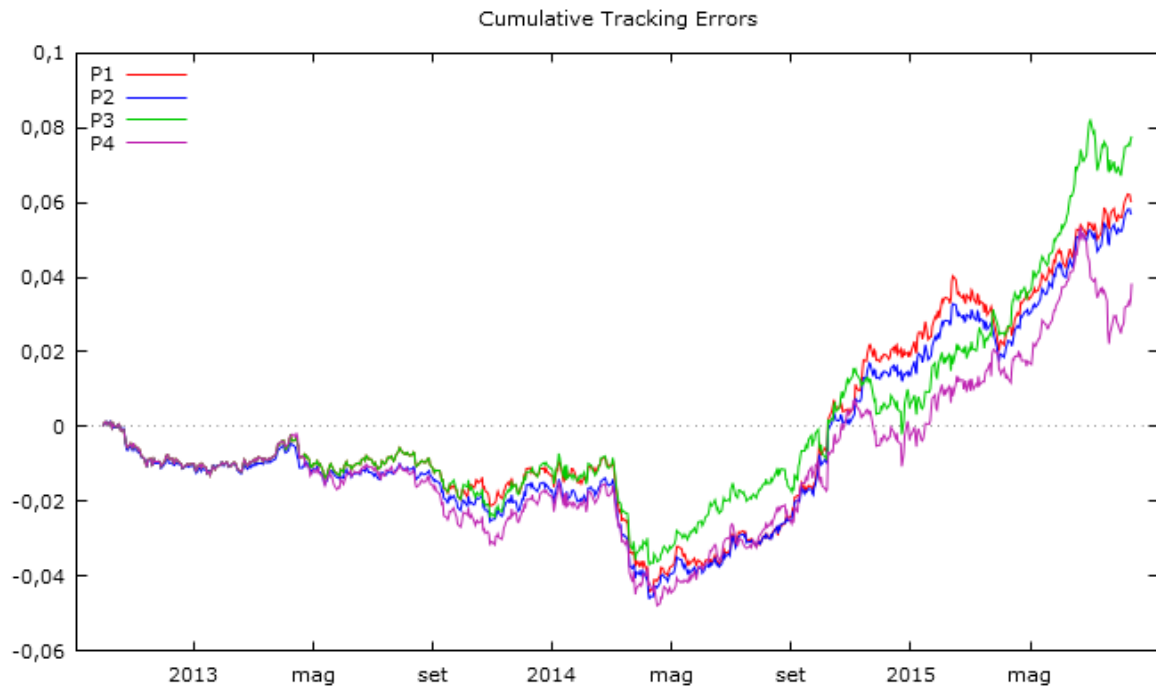
"This is just the beginning" Nathan Hale, "Resistance: Fall of Man".

Section 6. Final Part

6.1 Conclusions

The purpose of this work was to analyze the use of the Cointegration in a passive investment strategy.

To reach this goal, we decide to use a theoretical issue invented by Engle and Granger in the late 80s and adapt it to the Dow Jones Industrial Average studying different periods and identifying the long-term relationship between the Components and the Index. The results show how it is possible to track an index, with only a subset of the stocks included. The initial comparison is made between the two formulations:

- the one used by Alexander and Dumitru (2002), based on the OLS regression,
- the correction suggested by Stock (1987), which uses the Dynamic OLS regression, in order to correct the bias of the data.

Hence, we observe the incorrect specification and sub-optimal allocation due to this kind of phenomenon.

It is surprising that the procedure is basically applied in several papers read, not taking into account the bias and the heteroscedasticity of daily data.

The analysis moves on considering different time of reallocation, changing the time step from a buy and hold strategy to 1 year and 6 months rebalance.

We show that the tracking performances of all portfolios remain valid and significant until the last year, where a period of high volatility damages the tracking error of all portfolios.

A more frequent rebalancing does not improve the tracking ability, since the Cointegration outlines a long-term relationship, but helps to recover a part of the unrealized returns (comparing with the Index).

We can conclude that the practical application of the theory is valid and useful when the Index is trending, but suffers several drawbacks when the oscillations are closer to the mean of the period. Hence, this is one of the first feature to be investigated.

6.2 Further Research

The method and the procedure developed in this work have to be seen as an experimental test to value an important econometric theory, and obviously, the possible implementations are pretty infinite.

Dealing with the testing data, a future research approach is to test the method in different environments, changing the benchmark index and using different timing. The use of an index with a larger number of constituents can be important to study how the methodology deal with increasing number of data.

The new dataset can be an index of a single country (as the S&P500, which is the index of the 500 major public companies of the US market), or maybe a general index with stocks taken by different countries (as the MSCI World Index, which provides the statistics of large companies from 23 developed countries).

For this point of view, the choosing of a correct selection procedure assume an increasing importance and a clustering approach, using the fundamental value or some technical indicators, can lead to significant improvements in the results.

Also from the point of view of a mixture investment strategy, the input of an active framework combined to the passive cointegration approach can increase the profitability and improve the features of the portfolios. For example, the presence of some stocks, chosen to beat the market index, in the cointegration portfolio, can damage his tracking ability but also increase the profits, if the analysis is correct.

As exposed by Alexander (2002), using the cointegration approach in two different portfolios, one with alpha plus, and another with alpha minus is possible to achieve in a constant manner the spread between the two portfolios.

From another point of view, if the ability of the portfolio is sufficient to track the index without frequent rebalancing, the creation of a protective put strategy can be easily implement, giving a strategy less profitable but less risky, eliminating or limiting the possibility of unexpected downturn of the market.

Instead of purchasing long put options for each single stocks, the choosing of a single index put option contract can improve the strategy, considering the fast execution and the reduced costs.

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As well defined by the Chicago Board Option Exchange (CBOE), the Index puts "can be a very useful hedge to protect the value of a portfolio of mixed stocks in case of a market decline. Just as the way protective equity puts work, long index puts can increase in value with a declining underlying index, the degree to which depending on the put strike price chosen.

Potential profits on the puts can be realized by either selling the contracts or exercising them if in-the-money, with these gains at least partially offsetting any decline in portfolio value. The puts limit the portfolio loss to a specific level depending on their strike price in relation to the underlying index level when the protective option position is established. On the upside the portfolio's profit potential is unlimited, but any profits are at least partially reduced by the initial cost of the puts. The break-even point on the upside will be the current portfolio value when it is insured plus the cost of the puts."[51]

The difficult to back test this type of strategy, that was one of the most fascinating implementations analyzed during the work, is given by the data of the option contracts, very volatile and subjected to different evaluating methods.

Just to give a graphic representation, we suggest the view of the image below.
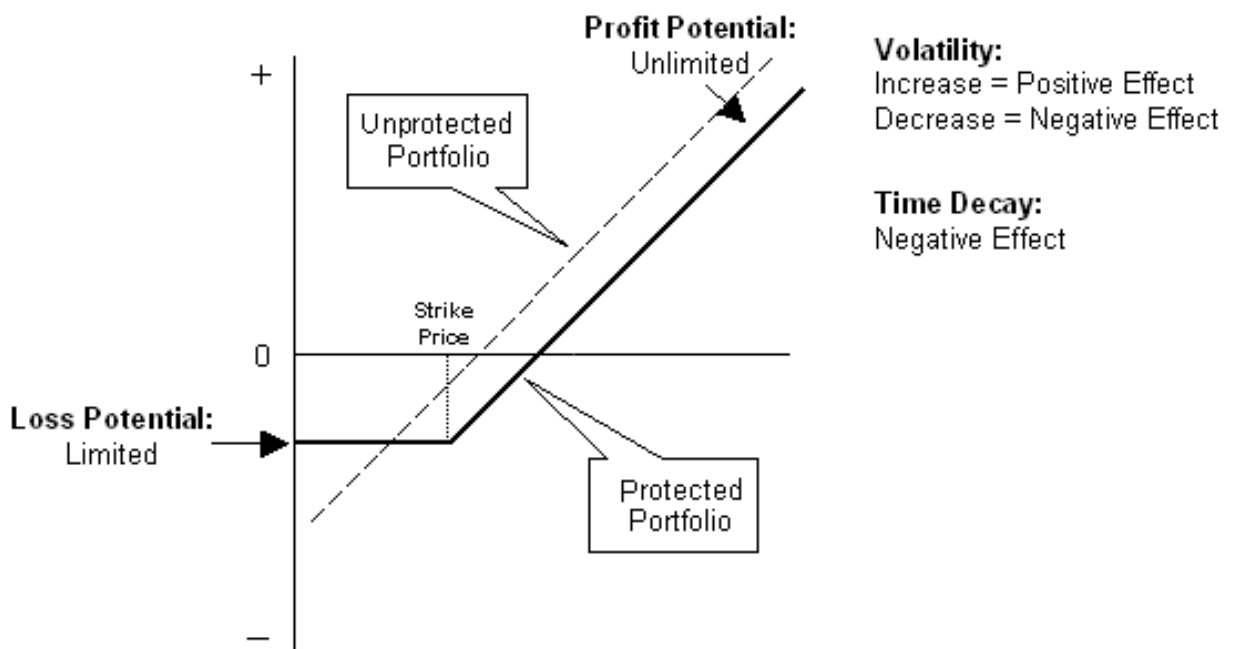
Figura 6 Protected and Unprotected Portfolios. Source: www.cboe.com

The dot line is the Unprotected Portfolio, while the bold line is the Protected Portfolio.

Considering a strong beta coefficient, the possible downturn of the market can damage the overall value of the portfolio. Using Index Puts, we sacrifice a part of the unrealized profits (the

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[51] http://www.cboe.com/strategies/indexoptions/buyindexputstohedge/part1.aspx

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limited loss potential, i.e. the starting point of the bold line) but we limit the possible losses, leaving unlimited the potential profits.

The number of Put Options needed to a full hedge can be acquired following the formula below:

$$N^o of\ Index\ Put = \frac{(Value\ of\ Holding)}{(Index\ Level) * (Contract\ Multiplier)}$$

The option contracts are a wide family of different contract, with various possible maturities and features. American put options may be exercised at any time, while the European options have fixed day of expiration[52].

Another possible implementation can be the use of a forecast analysis, comparing the expected future movements of the index and the constituents, connecting the knowledge of the past with the prevision of the future[53].

The theoretical starting points are various and can lead to different possible improvements as many as the number of critical point of the methodology.

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[52] For an extensive review on this argument, we suggest the reading of "Dynamic hedging: managing vanilla and exotic options", an old but still valid book written by Nassim Taleb in 1997.
[53] For more information, Engle and Yoo (1987) give a general review of the properties of forecasts in a cointegrated system, outlining the inefficiency of vector autoregressive models.

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APPENDIX

Table1.

Ticker	Company Name	12/31/2015	12/31/2014	12/31/2013	12/31/2012	12/31/2011	12/31/2010	12/31/2009	12/31/2008
AA	Alcoa Inc.	-	-	-	0.51%	0.54%	1.01%	1.17%	1.02%
AAPL	Apple Inc.	4.14%	-	-	-	-	-	-	-
AXP	American Express Co	2.73%	3.35%	3.52%	3.37%	2.92%	2.81%	2.94%	1.68%
BA	Boeing Co.	5.68%	4.68%	5.29%	4.42%	4.54%	4.27%	3.92%	3.87%
BAC	Bank of America Corp.	-	-	-	0.68%	0.34%	0.87%	1.09%	1.28%
C	Citigroup Inc.	-	-	-	-	-	-	-	0.61%
CAT	Caterpillar Inc.	2.67%	3.30%	3.52%	5.25%	5.61%	6.12%	4.13%	4.05%
CSCO	Cisco Systems	1.07%	1.00%	0.87%	1.15%	1.12%	1.32%	1.74%	-
CVX	Chevron Corp.	3.54%	4.04%	4.84%	6.34%	6.59%	5.97%	5.58%	6.71%
DD	DuPont	2.62%	2.66%	2.52%	2.64%	2.84%	3.26%	2.44%	2.30%
DIS	The Walt Disney Co.	4.13%	3.39%	2.96%	2.92%	2.32%	2.45%	2.34%	2.06%
GE	General Electric	1.22%	0.91%	1.09%	1.23%	1.11%	1.20%	1.10%	1.47%
GM	General Motors	-	-	-	-	-	-	-	0.29%
GS	Goldman Sachs Group	7.08%	6.98%	6.87%	-	-	-	-	-
HD	Home Depot	5.20%	3.78%	3.19%	3.62%	2.60%	2.29%	2.10%	2.09%
HPQ	Hewlett-Packard Co	-	-	-	0.84%	1.60%	2.75%	3.73%	3.29%
IBM	IBM Corp.	5.41%	5.78%	7.27%	11.23%	11.39%	9.59%	9.49%	7.64%
INTC	Intel Corp.	1.35%	1.31%	1.01%	1.21%	1.50%	1.37%	1.48%	1.33%
JNJ	Johnson & Johnson	4.04%	3.77%	3.55%	4.11%	4.06%	4.04%	4.67%	5.43%
JPM	JPMorgan Chase & Co.	2.60%	2.25%	2.27%	2.58%	2.06%	2.77%	3.02%	2.86%

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KO	The Coca Cola Co.	1.69%	1.52%	1.60%	2.12%	4.33%	4.30%	4.13%	4.11%
MCD	McDonald's Corp.	4.64%	3.38%	3.76%	5.17%	6.22%	5.02%	4.53%	5.64%
MDLZ	Mondelez Int'l	-	-	-	-	2.31%	2.06%	1.97%	2.44%
MMM	3M Company	5.92%	5.92%	5.43%	5.44%	5.06%	5.64%	5.99%	5.22%
MRK	Merck & Co.	2.08%	2.05%	1.94%	2.40%	2.34%	2.36%	2.65%	2.76%
MSFT	Microsoft Corp.	2.18%	1.67%	1.45%	1.57%	1.61%	1.82%	2.21%	1.76%
NKE	Nike, Inc.	2.46%	3.46%	3.05%	-	-	-	-	-
PFE	Pfizer Inc.	1.27%	1.12%	1.19%	1.47%	1.34%	1.14%	1.32%	1.61%
PG	Procter & Gamble	3.12%	3.28%	3.15%	3.98%	4.13%	4.21%	4.39%	5.61%
T	AT&T Inc.	-	1.21%	1.36%	1.98%	1.87%	1.92%	2.03%	2.59%
TRV	The Travelers Companies	4.44%	3.81%	3.51%	4.21%	3.67%	3.64%	3.61%	-
UNH	United Health Group Inc.	4.62%	3.64%	2.92%	3.18%	-	-	-	-
UTX	United Technologies	3.78%	4.14%	4.41%	4.81%	4.53%	5.15%	5.03%	4.86%
V	Visa Inc.	3.05%	9.45%	8.63%	-	-	-	-	-
VZ	Verizon Communications	1.82%	1.69%	1.90%	2.54%	2.49%	2.34%	2.40%	3.08%
WMT	Wal-Mart Stores	2.41%	3.09%	3.05%	4.00%	3.70%	3.53%	3.87%	5.09%
XOM	Exxon Mobil Corp.	3.06%	3.33%	3.92%	5.07%	5.25%	4.78%	4.94%	7.24%

Source: http://siblisresearch.com/data/dow-jones-30-weightings/

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Table2

Descriptive analysis of raw daily prices:

Variables	Average	Median	Minimum	Maximum
DJIA	14340,9	14483,8	9487,67	18636,0
MMM	115,995	105,110	70,9300	181,420
AXP	63,0966	59,8700	32,4600	95,8400
BA	98,7277	84,7850	47,2200	158,310
CAT	85,2355	85,1850	48,8300	116,200
CVX	102,207	103,990	67,3100	134,850
CSCOO	22,9038	23,1075	13,7300	31,5800
KO	37,1496	38,4700	25,0150	46,8900
DD	52,4378	51,0224	29,0118	76,4719
XOM	83,4224	85,3800	56,5700	104,380
GE	22,5334	23,0050	13,8800	32,9300
HD	70,9679	70,1500	24,9600	138,770
INTCO	25,9667	24,5900	17,6650	37,6700
IBM	167,691	168,570	117,850	215,800
JNJ	82,3267	79,1450	57,0200	125,400
JPM	49,8282	48,6700	28,3800	70,0800
MCD	92,1860	94,6300	56,6100	131,600
MRK	45,8695	45,4850	29,8100	63,8600
MSFTO	35,7414	31,6375	23,0100	58,3000
PFE	25,8567	27,9350	14,1400	37,3100
PG	72,6527	73,9350	56,6200	93,4600
TRV	78,3787	80,3650	46,8000	119,040
UTX	91,2558	89,5750	59,6300	124,110
VZ	42,5462	45,1300	25,1723	56,5300
WMT	66,3291	68,5750	48,0000	90,4700
DIS	64,5692	56,8150	27,2100	121,690
Variables	Std. Dev.	Coeff. di variazione	Asimmetry	Kurtosis
DJIA	2724,85	0,190005	-0,0600538	-1,43993
MMM	32,7828	0,282622	0,394081	-1,38438
AXP	17,2879	0,273991	0,218313	-1,23586
BA	32,1273	0,325413	0,199516	-1,60648
CAT	14,8214	0,173887	-0,160327	-0,588061
CVX	16,0859	0,157386	-0,292536	-0,883046
CSCOO	4,04530	0,176621	-0,0895441	-0,913104
KO	5,42384	0,146000	-0,541765	-0,731310
DD	11,2786	0,215085	-0,0627680	-0,939080
XOM	10,5357	0,126294	-0,495783	-0,317385
GE	4,82995	0,214346	0,0809483	-1,09931
HD	34,9320	0,492223	0,398056	-1,14248
INTCO	5,15804	0,198641	0,509841	-1,02401
IBM	25,9071	0,154493	-0,211292	-1,16257
JNJ	19,0156	0,230977	0,298988	-1,35707
JPM	10,5784	0,212297	0,0491452	-1,26032
MCD	15,4847	0,167972	0,0129372	0,0146458

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MRK	9,64100	0,210183	0,123281	-1,43800
MSFTO	9,73961	0,272502	0,646358	-0,910314
PFE	6,41104	0,247945	-0,138349	-1,40201
PG	9,20737	0,126731	0,101020	-1,41110
TRV	22,4271	0,286138	0,210257	-1,38574
UTX	16,7704	0,183774	0,160956	-1,27907
VZ	7,98779	0,187744	-0,574570	-0,853919
WMT	10,6586	0,160692	-0,0356795	-1,33253
DIS	28,0506	0,434427	0,367341	-1,34139

Variables	5% Perc.	95% Perc.	Interquantile Range	Missing Observations
DJIA	10269,6	18041,4	4927,40	0
MMM	78,1460	168,312	58,3075	0
AXP	39,5530	91,1640	30,9950	0
BA	59,8070	146,941	59,6225	0
CAT	58,5700	109,042	20,3350	0
CVX	74,1715	125,325	25,2475	0
CSCOO	16,1115	29,0500	6,38250	0
KO	26,9773	44,5400	7,74500	0
DD	32,5851	69,1937	18,1369	0
XOM	62,4590	99,6295	13,4100	0
GE	15,4957	30,7055	7,74500	0
HD	28,2830	133,095	62,9950	0
INTCO	19,5673	34,9185	9,09750	0
IBM	126,083	205,447	44,3550	0
JNJ	59,6960	112,690	35,9300	0
JPM	33,9000	66,2970	18,5225	0
MCD	63,8560	121,344	16,3425	0
MRK	32,6800	59,7885	19,1575	0
MSFTO	24,7915	54,1210	16,5425	0
PFE	16,4875	34,8700	11,7000	0
PG	60,6030	85,9870	17,1325	0
TRV	49,5860	114,372	42,2075	0
UTX	67,5415	117,555	30,5400	0
VZ	27,2301	52,5255	12,6800	0
WMT	51,2200	82,0445	20,4950	0
DIS	31,2315	110,364	53,3875	0

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Table 3

	ADF P1	ADF P2
Period 1 (1-750)	-9,12836***	-7,40611***
Period 2 (250-1000)	-8,71129***	-7,84118***
Period 3 (500-1250)	-7,5601***	-5,6328

Errors of DOLS regression

Errors of OLS Regression

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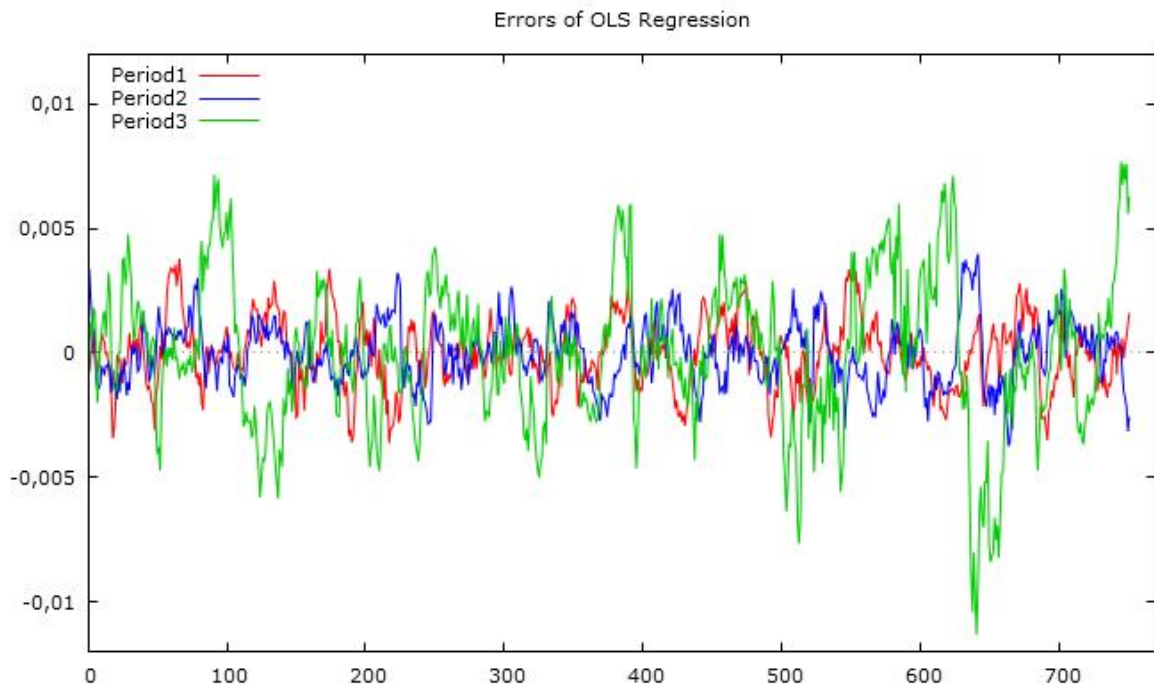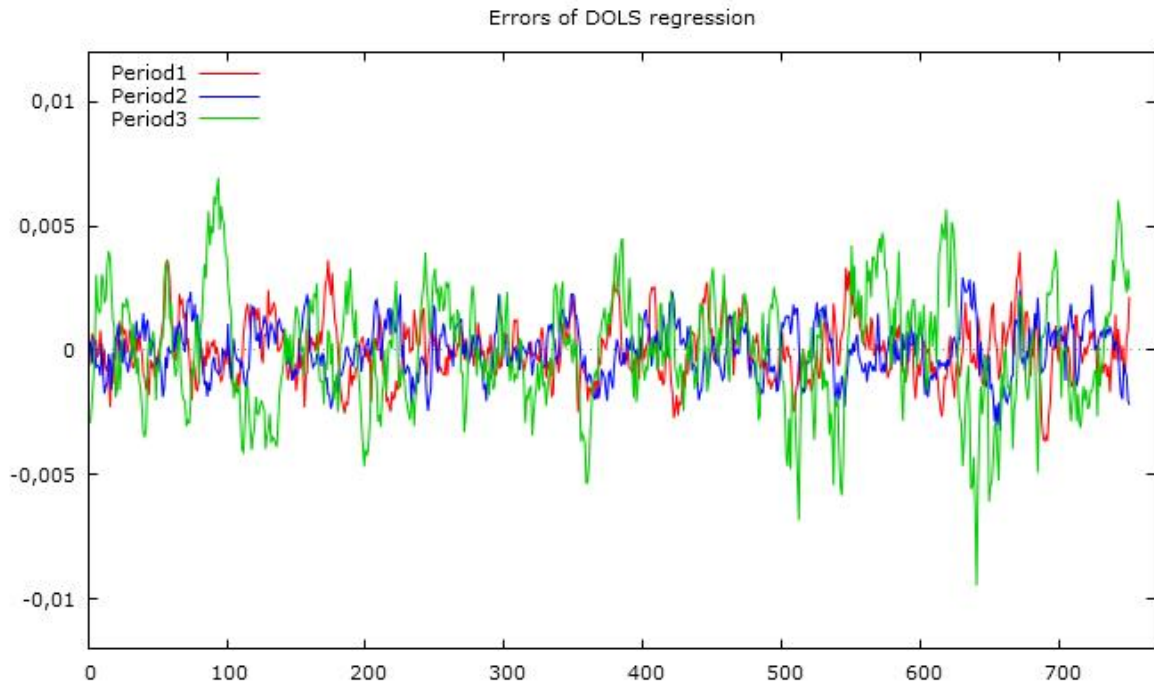
Table 4.

Statistics				
Variable	Average	Median	Min	Max
P2_period1	5,59126e-014	2,73157e-005	-0,00362533	0,00378500
P2_period2	-1,24442e-013	-1,85430e-005	-0,00374842	0,00397585
P2_period3	1,95377e-014	2,16536e-005	-0,0113229	0,00768816
P1_period1	2,77427e-013	-3,60367e-005	-0,00363531	0,00397476
P1_period2	2,75205e-013	-3,82820e-005	-0,00321645	0,00293817
P1_period3	1,03254e-013	-4,41003e-007	-0,00945914	0,00694885
Variable	Std Dev..	Variance	Asimmetry	Kurtosis
P2_period1	0,00136548	2,44216e+010	0,0230187	-0,0214254
P2_period2	0,00125965	1,01224e+010	0,165236	0,103625
P2_period3	0,00298433	1,52747e+011	-0,193799	0,586881
P1_period1	0,00116774	4,20918e+009	0,227292	0,487021
P1_period2	0,00105070	3,81788e+009	0,0539771	-0,281797
P1_period3	0,00234090	2,26713e+010	-0,0312467	0,195513
Variable	5% Perc.	95% Perc.	Interquartile Range	Missing Observations
P2_period1	-0,00240312	0,00229960	0,00168278	0
P2_period2	-0,00196096	0,00207391	0,00171410	0
P2_period3	-0,00478475	0,00514626	0,00363919	0
P1_period1	-0,00187576	0,00203724	0,00141828	0
P1_period2	-0,00176042	0,00173661	0,00143585	0
P1_period3	-0,00382872	0,00395433	0,00312309	0

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Stationarity test for the log-trasformation of DJIA and of the 25 Constituents.

```
Step 1: testing for a unit root in l_DJI

Augmented Dickey-Fuller test for l_DJI
including 10 lags of (1-L)l_DJI
sample size 739
unit-root null hypothesis: a = 1

  test with constant
  model: (1-L)y = b0 + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): -0.00528777
  test statistic: tau_c(1) = -1.19454
  asymptotic p-value 0.6792
  1st-order autocorrelation coeff. for e: -0.003
  lagged differences: F(10, 727) = 2.632 [0.0037]

Step 2: testing for a unit root in l_MMM

Augmented Dickey-Fuller test for l_MMM
including 10 lags of (1-L)l_MMM
sample size 739
unit-root null hypothesis: a = 1

  test with constant
  model: (1-L)y = b0 + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): -0.0193286
  test statistic: tau_c(1) = -2.38347
  asymptotic p-value 0.1465
  1st-order autocorrelation coeff. for e: -0.005
  lagged differences: F(10, 727) = 2.398 [0.0084]

Step 3: testing for a unit root in l_AXP

Augmented Dickey-Fuller test for l_AXP
including 10 lags of (1-L)l_AXP
sample size 739
unit-root null hypothesis: a = 1

  test with constant
  model: (1-L)y = b0 + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): -0.00956714
  test statistic: tau_c(1) = -1.86177
  asymptotic p-value 0.3508
  1st-order autocorrelation coeff. for e: 0.000
  lagged differences: F(10, 727) = 2.962 [0.0012]

Step 4: testing for a unit root in l_BA

Augmented Dickey-Fuller test for l_BA
including 10 lags of (1-L)l_BA
sample size 739
unit-root null hypothesis: a = 1

  test with constant
  model: (1-L)y = b0 + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): -0.0177388
  test statistic: tau_c(1) = -2.82069
  asymptotic p-value 0.05534
  1st-order autocorrelation coeff. for e: 0.001
  lagged differences: F(10, 727) = 0.979 [0.4606]

Step 5: testing for a unit root in l_CAT

Augmented Dickey-Fuller test for l_CAT
including 10 lags of (1-L)l_CAT
sample size 739
```

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```
unit-root null hypothesis: a = 1

  test with constant
  model: (1-L)y = b0 + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): -0.00627115
  test statistic: tau_c(1) = -1.86196
  asymptotic p-value 0.3507
  1st-order autocorrelation coeff. for e: -0.004
  lagged differences: F(10, 727) = 1.573 [0.1101]


Step 6: testing for a unit root in l_CVX

Augmented Dickey-Fuller test for l_CVX
including 10 lags of (1-L)l_CVX
sample size 739
unit-root null hypothesis: a = 1

  test with constant
  model: (1-L)y = b0 + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): -0.00320271
  test statistic: tau_c(1) = -0.918842
  asymptotic p-value 0.7829
  1st-order autocorrelation coeff. for e: 0.001
  lagged differences: F(10, 727) = 2.019 [0.0290]


Step 7: testing for a unit root in l_CSCOO

Augmented Dickey-Fuller test for l_CSCOO
including 10 lags of (1-L)l_CSCOO
sample size 739
unit-root null hypothesis: a = 1

  test with constant
  model: (1-L)y = b0 + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): -0.00756668
  test statistic: tau_c(1) = -1.67325
  asymptotic p-value 0.445
  1st-order autocorrelation coeff. for e: 0.000
  lagged differences: F(10, 727) = 0.680 [0.7433]


Step 8: testing for a unit root in l_KO

Augmented Dickey-Fuller test for l_KO
including 10 lags of (1-L)l_KO
sample size 739
unit-root null hypothesis: a = 1

  test with constant
  model: (1-L)y = b0 + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): -0.001733
  test statistic: tau_c(1) = -0.58725
  asymptotic p-value 0.8711
  1st-order autocorrelation coeff. for e: -0.003
  lagged differences: F(10, 727) = 1.625 [0.0953]


Step 9: testing for a unit root in l_DD

Augmented Dickey-Fuller test for l_DD
including 10 lags of (1-L)l_DD
sample size 739
unit-root null hypothesis: a = 1

  test with constant
  model: (1-L)y = b0 + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): -0.00600525
  test statistic: tau_c(1) = -1.61438
  asymptotic p-value 0.4752
  1st-order autocorrelation coeff. for e: -0.001
```

```
  lagged differences: F(10, 727) = 1.122 [0.3422]

Step 10: testing for a unit root in l_XOM

Augmented Dickey-Fuller test for l_XOM
including 10 lags of (1-L)l_XOM
sample size 739
unit-root null hypothesis: a = 1

  test with constant
  model: (1-L)y = b0 + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): -0.00338837
  test statistic: tau_c(1) = -0.882506
  asymptotic p-value 0.7944
  1st-order autocorrelation coeff. for e: 0.000
  lagged differences: F(10, 727) = 2.480 [0.0063]

Step 11: testing for a unit root in l_GE

Augmented Dickey-Fuller test for l_GE
including 10 lags of (1-L)l_GE
sample size 739
unit-root null hypothesis: a = 1

  test with constant
  model: (1-L)y = b0 + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): -0.0083132
  test statistic: tau_c(1) = -1.44072
  asymptotic p-value 0.5638
  1st-order autocorrelation coeff. for e: -0.003
  lagged differences: F(10, 727) = 2.565 [0.0047]

Step 12: testing for a unit root in l_HD

Augmented Dickey-Fuller test for l_HD
including 10 lags of (1-L)l_HD
sample size 739
unit-root null hypothesis: a = 1

  test with constant
  model: (1-L)y = b0 + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): 0.000122761
  test statistic: tau_c(1) = 0.0472998
  asymptotic p-value 0.9617
  1st-order autocorrelation coeff. for e: 0.000
  lagged differences: F(10, 727) = 0.147 [0.9990]

Step 13: testing for a unit root in l_INTCO

Augmented Dickey-Fuller test for l_INTCO
including 10 lags of (1-L)l_INTCO
sample size 739
unit-root null hypothesis: a = 1

  test with constant
  model: (1-L)y = b0 + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): -0.00889093
  test statistic: tau_c(1) = -1.77103
  asymptotic p-value 0.3954
  1st-order autocorrelation coeff. for e: -0.003
  lagged differences: F(10, 727) = 1.269 [0.2437]

Step 14: testing for a unit root in l_IBM

Augmented Dickey-Fuller test for l_IBM
including 10 lags of (1-L)l_IBM
sample size 739
unit-root null hypothesis: a = 1
```

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```
  test with constant
  model: (1-L)y = b0 + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): -0.00102004
  test statistic: tau_c(1) = -0.409498
  asymptotic p-value 0.9053
  1st-order autocorrelation coeff. for e: -0.002
  lagged differences: F(10, 727) = 2.470 [0.0066]
```

Step 15: testing for a unit root in l_JNJ

```
Augmented Dickey-Fuller test for l_JNJ
including 10 lags of (1-L)l_JNJ
sample size 739
unit-root null hypothesis: a = 1

  test with constant
  model: (1-L)y = b0 + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): -0.0185499
  test statistic: tau_c(1) = -2.36575
  asymptotic p-value 0.1516
  1st-order autocorrelation coeff. for e: -0.001
  lagged differences: F(10, 727) = 1.822 [0.0534]
```

Step 16: testing for a unit root in l_JPM

```
Augmented Dickey-Fuller test for l_JPM
including 10 lags of (1-L)l_JPM
sample size 739
unit-root null hypothesis: a = 1

  test with constant
  model: (1-L)y = b0 + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): -0.0201495
  test statistic: tau_c(1) = -2.77321
  asymptotic p-value 0.06215
  1st-order autocorrelation coeff. for e: 0.002
  lagged differences: F(10, 727) = 2.230 [0.0147]
```

Step 17: testing for a unit root in l_MCD

```
Augmented Dickey-Fuller test for l_MCD
including 10 lags of (1-L)l_MCD
sample size 739
unit-root null hypothesis: a = 1

  test with constant
  model: (1-L)y = b0 + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): -0.00464283
  test statistic: tau_c(1) = -1.9253
  asymptotic p-value 0.3208
  1st-order autocorrelation coeff. for e: -0.001
  lagged differences: F(10, 727) = 1.681 [0.0810]
```

Step 18: testing for a unit root in l_MRK

```
Augmented Dickey-Fuller test for l_MRK
including 10 lags of (1-L)l_MRK
sample size 739
unit-root null hypothesis: a = 1

  test with constant
  model: (1-L)y = b0 + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): -0.00891541
  test statistic: tau_c(1) = -1.4654
  asymptotic p-value 0.5514
  1st-order autocorrelation coeff. for e: -0.005
  lagged differences: F(10, 727) = 2.338 [0.0102]
```

```
Step 19: testing for a unit root in l_MSFTO

Augmented Dickey-Fuller test for l_MSFTO
including 10 lags of (1-L)l_MSFTO
sample size 739
unit-root null hypothesis: a = 1

  test with constant
  model: (1-L)y = b0 + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): -0.0121445
  test statistic: tau_c(1) = -1.96822
  asymptotic p-value 0.3012
  1st-order autocorrelation coeff. for e: -0.000
  lagged differences: F(10, 727) = 1.017 [0.4269]

Step 20: testing for a unit root in l_PFE

Augmented Dickey-Fuller test for l_PFE
including 10 lags of (1-L)l_PFE
sample size 739
unit-root null hypothesis: a = 1

  test with constant
  model: (1-L)y = b0 + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): -0.00190792
  test statistic: tau_c(1) = -0.500264
  asymptotic p-value 0.8889
  1st-order autocorrelation coeff. for e: -0.005
  lagged differences: F(10, 727) = 1.705 [0.0756]

Step 21: testing for a unit root in l_PG

Augmented Dickey-Fuller test for l_PG
including 10 lags of (1-L)l_PG
sample size 739
unit-root null hypothesis: a = 1

  test with constant
  model: (1-L)y = b0 + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): -0.0312049
  test statistic: tau_c(1) = -3.20487
  asymptotic p-value 0.01974
  1st-order autocorrelation coeff. for e: -0.001
  lagged differences: F(10, 727) = 0.900 [0.5323]

Step 22: testing for a unit root in l_TRV

Augmented Dickey-Fuller test for l_TRV
including 10 lags of (1-L)l_TRV
sample size 739
unit-root null hypothesis: a = 1

  test with constant
  model: (1-L)y = b0 + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): -0.00763289
  test statistic: tau_c(1) = -1.32892
  asymptotic p-value 0.6183
  1st-order autocorrelation coeff. for e: -0.006
  lagged differences: F(10, 727) = 2.613 [0.0040]

Step 23: testing for a unit root in l_UTX

Augmented Dickey-Fuller test for l_UTX
including 10 lags of (1-L)l_UTX
sample size 739
unit-root null hypothesis: a = 1
```

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```
   test with constant
   model: (1-L)y = b0 + (a-1)*y(-1) + ... + e
   estimated value of (a - 1): -0.0165733
   test statistic: tau_c(1) = -2.55024
   asymptotic p-value 0.1037
   1st-order autocorrelation coeff. for e: -0.003
   lagged differences: F(10, 727) = 0.725 [0.7015]


Step 24: testing for a unit root in l_VZ

Augmented Dickey-Fuller test for l_VZ
including 10 lags of (1-L)l_VZ
sample size 739
unit-root null hypothesis: a = 1

   test with constant
   model: (1-L)y = b0 + (a-1)*y(-1) + ... + e
   estimated value of (a - 1): -0.00202083
   test statistic: tau_c(1) = -0.7875
   asymptotic p-value 0.8221
   1st-order autocorrelation coeff. for e: 0.001
   lagged differences: F(10, 727) = 1.246 [0.2580]


Step 25: testing for a unit root in l_WMT

Augmented Dickey-Fuller test for l_WMT
including 10 lags of (1-L)l_WMT
sample size 739
unit-root null hypothesis: a = 1

   test with constant
   model: (1-L)y = b0 + (a-1)*y(-1) + ... + e
   estimated value of (a - 1): 0.000724749
   test statistic: tau_c(1) = 0.192293
   asymptotic p-value 0.9722
   1st-order autocorrelation coeff. for e: -0.002
   lagged differences: F(10, 727) = 0.707 [0.7185]


Step 26: testing for a unit root in l_DIS

Augmented Dickey-Fuller test for l_DIS
including 10 lags of (1-L)l_DIS
sample size 739
unit-root null hypothesis: a = 1
   test with constant
   model: (1-L)y = b0 + (a-1)*y(-1) + ... + e
   estimated value of (a - 1): -0.00275257
   test statistic: tau_c(1) = -0.683192
   asymptotic p-value 0.8491
   1st-order autocorrelation coeff. for e: 0.000
```

lagged differences: $F(10, 727) = 1.063$ [0.3887]

ADF Test of the first differences of the log-variables.

```
Step 1: testing for a unit root in d_l_DJI

Augmented Dickey-Fuller test for d_l_DJI
including 10 lags of (1-L)d_l_DJI
sample size 738
unit-root null hypothesis: a = 1

  test with constant
  model: (1-L)y = b0 + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): -1.16112
  test statistic: tau_c(1) = -8.19967
  asymptotic p-value 1.288e-013
  1st-order autocorrelation coeff. for e: 0.001
  lagged differences: F(10, 726) = 2.362 [0.0095]

Step 2: testing for a unit root in d_l_MMM

Augmented Dickey-Fuller test for d_l_MMM
including 10 lags of (1-L)d_l_MMM
sample size 738
unit-root null hypothesis: a = 1

  test with constant
  model: (1-L)y = b0 + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): -1.18399
  test statistic: tau_c(1) = -8.06028
  asymptotic p-value 3.325e-013
  1st-order autocorrelation coeff. for e: 0.001
  lagged differences: F(10, 726) = 2.555 [0.0049]

Step 3: testing for a unit root in d_l_AXP

Augmented Dickey-Fuller test for d_l_AXP
including 10 lags of (1-L)d_l_AXP
sample size 738
unit-root null hypothesis: a = 1

  test with constant
  model: (1-L)y = b0 + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): -1.16117
  test statistic: tau_c(1) = -8.81311
  asymptotic p-value 1.802e-015
  1st-order autocorrelation coeff. for e: 0.000
  lagged differences: F(10, 726) = 1.872 [0.0459]

Step 4: testing for a unit root in d_l_BA

Augmented Dickey-Fuller test for d_l_BA
including 10 lags of (1-L)d_l_BA
sample size 738
unit-root null hypothesis: a = 1

  test with constant
  model: (1-L)y = b0 + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): -1.16289
  test statistic: tau_c(1) = -8.46548
  asymptotic p-value 2.062e-014
  1st-order autocorrelation coeff. for e: -0.001
  lagged differences: F(10, 726) = 1.033 [0.4134]

Step 5: testing for a unit root in d_l_CAT
```

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```
Augmented Dickey-Fuller test for d_l_CAT
including 10 lags of (1-L)d_l_CAT
sample size 738
unit-root null hypothesis: a = 1

  test with constant
  model: (1-L)y = b0 + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): -0.899838
  test statistic: tau_c(1) = -7.31751
  asymptotic p-value 4.462e-011
  1st-order autocorrelation coeff. for e: 0.002
  lagged differences: F(10, 726) = 1.748 [0.0666]


Step 6: testing for a unit root in d_l_CVX

Augmented Dickey-Fuller test for d_l_CVX
including 10 lags of (1-L)d_l_CVX
sample size 738
unit-root null hypothesis: a = 1

  test with constant
  model: (1-L)y = b0 + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): -1.09844
  test statistic: tau_c(1) = -8.69286
  asymptotic p-value 4.208e-015
  1st-order autocorrelation coeff. for e: -0.001
  lagged differences: F(10, 726) = 2.053 [0.0261]


Step 7: testing for a unit root in d_l_CSCOO

Augmented Dickey-Fuller test for d_l_CSCOO
including 10 lags of (1-L)d_l_CSCOO
sample size 738
unit-root null hypothesis: a = 1

  test with constant
  model: (1-L)y = b0 + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): -1.16397
  test statistic: tau_c(1) = -8.71349
  asymptotic p-value 3.639e-015
  1st-order autocorrelation coeff. for e: 0.000
  lagged differences: F(10, 726) = 0.718 [0.7077]


Step 8: testing for a unit root in d_l_KO

Augmented Dickey-Fuller test for d_l_KO
including 10 lags of (1-L)d_l_KO
sample size 738
unit-root null hypothesis: a = 1

  test with constant
  model: (1-L)y = b0 + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): -1.18811
  test statistic: tau_c(1) = -8.35249
  asymptotic p-value 4.508e-014
  1st-order autocorrelation coeff. for e: 0.002
  lagged differences: F(10, 726) = 1.698 [0.0772]


Step 9: testing for a unit root in d_l_DD

Augmented Dickey-Fuller test for d_l_DD
including 10 lags of (1-L)d_l_DD
sample size 738
unit-root null hypothesis: a = 1

  test with constant
  model: (1-L)y = b0 + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): -1.05731
```

```
   test statistic: tau_c(1) = -8.21322
   asymptotic p-value 1.174e-013
   1st-order autocorrelation coeff. for e: 0.001
   lagged differences: F(10, 726) = 1.140 [0.3291]
```

Step 10: testing for a unit root in d_l_XOM

```
Augmented Dickey-Fuller test for d_l_XOM
including 10 lags of (1-L)d_l_XOM
sample size 738
unit-root null hypothesis: a = 1

   test with constant
   model: (1-L)y = b0 + (a-1)*y(-1) + ... + e
   estimated value of (a - 1): -1.04934
   test statistic: tau_c(1) = -8.38294
   asymptotic p-value 3.653e-014
   1st-order autocorrelation coeff. for e: -0.001
   lagged differences: F(10, 726) = 2.359 [0.0096]
```

Step 11: testing for a unit root in d_l_GE

```
Augmented Dickey-Fuller test for d_l_GE
including 10 lags of (1-L)d_l_GE
sample size 738
unit-root null hypothesis: a = 1

   test with constant
   model: (1-L)y = b0 + (a-1)*y(-1) + ... + e
   estimated value of (a - 1): -1.00477
   test statistic: tau_c(1) = -8.16064
   asymptotic p-value 1.681e-013
   1st-order autocorrelation coeff. for e: -0.004
   lagged differences: F(10, 726) = 2.570 [0.0047]
```

Step 12: testing for a unit root in d_l_HD

```
Augmented Dickey-Fuller test for d_l_HD
including 10 lags of (1-L)d_l_HD
sample size 738
unit-root null hypothesis: a = 1

   test with constant
   model: (1-L)y = b0 + (a-1)*y(-1) + ... + e
   estimated value of (a - 1): -1.03974
   test statistic: tau_c(1) = -8.3751
   asymptotic p-value 3.856e-014
   1st-order autocorrelation coeff. for e: 0.000
   lagged differences: F(10, 726) = 0.131 [0.9994]
```

Step 13: testing for a unit root in d_l_INTCO

```
Augmented Dickey-Fuller test for d_l_INTCO
including 10 lags of (1-L)d_l_INTCO
sample size 738
unit-root null hypothesis: a = 1

   test with constant
   model: (1-L)y = b0 + (a-1)*y(-1) + ... + e
   estimated value of (a - 1): -1.02266
   test statistic: tau_c(1) = -8.14087
   asymptotic p-value 1.923e-013
   1st-order autocorrelation coeff. for e: 0.000
   lagged differences: F(10, 726) = 1.373 [0.1883]
```

Step 14: testing for a unit root in d_l_IBM

Augmented Dickey-Fuller test for d_l_IBM

```
including 10 lags of (1-L)d_l_IBM
sample size 738
unit-root null hypothesis: a = 1

  test with constant
  model: (1-L)y = b0 + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): -1.26768
  test statistic: tau_c(1) = -8.8266
  asymptotic p-value 1.638e-015
  1st-order autocorrelation coeff. for e: 0.007
  lagged differences: F(10, 726) = 2.345 [0.0100]
```

Step 15: testing for a unit root in d_l_JNJ

```
Augmented Dickey-Fuller test for d_l_JNJ
including 10 lags of (1-L)d_l_JNJ
sample size 738
unit-root null hypothesis: a = 1

  test with constant
  model: (1-L)y = b0 + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): -1.05741
  test statistic: tau_c(1) = -7.97382
  asymptotic p-value 5.964e-013
  1st-order autocorrelation coeff. for e: 0.001
  lagged differences: F(10, 726) = 1.844 [0.0499]
```

Step 16: testing for a unit root in d_l_JPM

```
Augmented Dickey-Fuller test for d_l_JPM
including 10 lags of (1-L)d_l_JPM
sample size 738
unit-root null hypothesis: a = 1

  test with constant
  model: (1-L)y = b0 + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): -1.08381
  test statistic: tau_c(1) = -8.10592
  asymptotic p-value 2.44e-013
  1st-order autocorrelation coeff. for e: 0.000
  lagged differences: F(10, 726) = 1.061 [0.3899]
```

Step 17: testing for a unit root in d_l_MCD

```
Augmented Dickey-Fuller test for d_l_MCD
including 10 lags of (1-L)d_l_MCD
sample size 738
unit-root null hypothesis: a = 1

  test with constant
  model: (1-L)y = b0 + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): -1.37393
  test statistic: tau_c(1) = -9.53795
  asymptotic p-value 9.855e-018
  1st-order autocorrelation coeff. for e: -0.004
  lagged differences: F(10, 726) = 0.783 [0.6452]
```

Step 18: testing for a unit root in d_l_MRK

```
Augmented Dickey-Fuller test for d_l_MRK
including 10 lags of (1-L)d_l_MRK
sample size 738
unit-root null hypothesis: a = 1

  test with constant
  model: (1-L)y = b0 + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): -0.945758
  test statistic: tau_c(1) = -7.27244
```

```
    asymptotic p-value 5.95e-011
    1st-order autocorrelation coeff. for e: 0.002
    lagged differences: F(10, 726) = 2.295 [0.0118]


Step 19: testing for a unit root in d_l_MSFTO


Augmented Dickey-Fuller test for d_l_MSFTO
including 10 lags of (1-L)d_l_MSFTO
sample size 738
unit-root null hypothesis: a = 1

  test with constant
  model: (1-L)y = b0 + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): -0.976735
  test statistic: tau_c(1) = -7.84965
  asymptotic p-value 1.372e-012
  1st-order autocorrelation coeff. for e: -0.001
  lagged differences: F(10, 726) = 1.046 [0.4025]


Step 20: testing for a unit root in d_l_PFE


Augmented Dickey-Fuller test for d_l_PFE
including 10 lags of (1-L)d_l_PFE
sample size 738
unit-root null hypothesis: a = 1

  test with constant
  model: (1-L)y = b0 + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): -1.1006
  test statistic: tau_c(1) = -8.02452
  asymptotic p-value 4.236e-013
  1st-order autocorrelation coeff. for e: 0.002
  lagged differences: F(10, 726) = 1.144 [0.3263]


Step 21: testing for a unit root in d_l_PG


Augmented Dickey-Fuller test for d_l_PG
including 10 lags of (1-L)d_l_PG
sample size 738
unit-root null hypothesis: a = 1

  test with constant
  model: (1-L)y = b0 + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): -1.14842
  test statistic: tau_c(1) = -8.47915
  asymptotic p-value 1.875e-014
  1st-order autocorrelation coeff. for e: -0.000
  lagged differences: F(10, 726) = 0.868 [0.5631]


Step 22: testing for a unit root in d_l_TRV


Augmented Dickey-Fuller test for d_l_TRV
including 10 lags of (1-L)d_l_TRV
sample size 738
unit-root null hypothesis: a = 1

  test with constant
  model: (1-L)y = b0 + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): -1.07002
  test statistic: tau_c(1) = -7.39418
  asymptotic p-value 2.726e-011
  1st-order autocorrelation coeff. for e: -0.002
  lagged differences: F(10, 726) = 1.282 [0.2364]


Step 23: testing for a unit root in d_l_UTX


Augmented Dickey-Fuller test for d_l_UTX
including 10 lags of (1-L)d_l_UTX
```

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```
sample size 738
unit-root null hypothesis: a = 1

  test with constant
  model: (1-L)y = b0 + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): -1.02088
  test statistic: tau_c(1) = -7.79542
  asymptotic p-value 1.969e-012
  1st-order autocorrelation coeff. for e: 0.000
  lagged differences: F(10, 726) = 1.026 [0.4195]

Step 24: testing for a unit root in d_l_VZ

Augmented Dickey-Fuller test for d_l_VZ
including 10 lags of (1-L)d_l_VZ
sample size 738
unit-root null hypothesis: a = 1

  test with constant
  model: (1-L)y = b0 + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): -0.914497
  test statistic: tau_c(1) = -7.52749
  asymptotic p-value 1.149e-011
  1st-order autocorrelation coeff. for e: -0.001
  lagged differences: F(10, 726) = 1.190 [0.2942]

Step 25: testing for a unit root in d_l_WMT

Augmented Dickey-Fuller test for d_l_WMT
including 10 lags of (1-L)d_l_WMT
sample size 738
unit-root null hypothesis: a = 1

  test with constant
  model: (1-L)y = b0 + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): -1.04487
  test statistic: tau_c(1) = -8.04371
  asymptotic p-value 3.72e-013
  1st-order autocorrelation coeff. for e: -0.001
  lagged differences: F(10, 726) = 0.298 [0.9818]

Step 26: testing for a unit root in d_l_DIS

Augmented Dickey-Fuller test for d_l_DIS
including 10 lags of (1-L)d_l_DIS
sample size 738
unit-root null hypothesis: a = 1

  test with constant
  model: (1-L)y = b0 + (a-1)*y(-1) + ... + e
  estimated value of (a - 1): -1.14279
  test statistic: tau_c(1) = -8.37541
  asymptotic p-value 3.848e-014
  1st-order autocorrelation coeff. for e: -0.001
  lagged differences: F(10, 726) = 1.287 [0.2336]
```

The patterns of the DJIA and of the 25 Constituents:

l_MMM l_AXP l_BA l_CAT l_CVX l_CSCOO l_KO l_DD

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