Methods: Game Theory 1

Fall 2015

Problem Set 3: Dynamic Games, Repeated Games

Problem 1: Consider the game in Figure 1 in extensive form in which first player 1, then player 2, and then again player 1 moves. In the payoff vector, the upper (lower) entry in the payoff of player 1 (2). The payoff x for player 1 after his move R is some real number. For each of the two possible

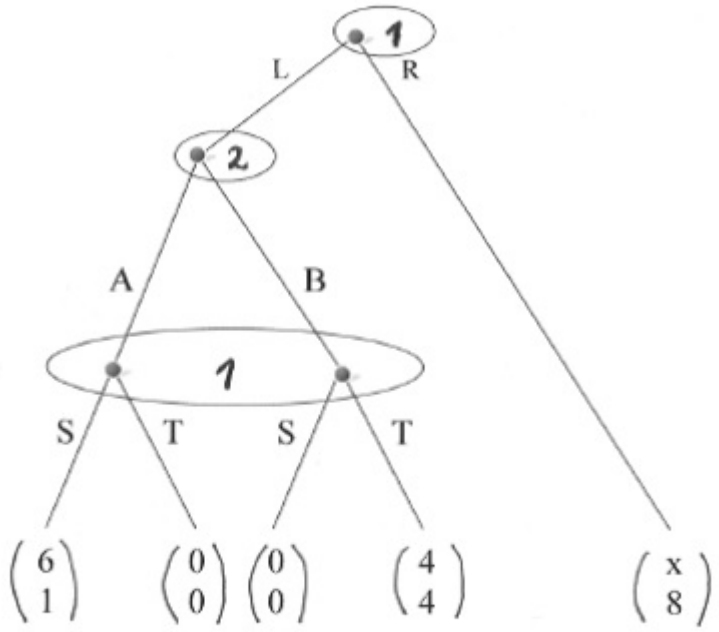

Figure 1: The Game for Problem 1

values $x = 1$ and $x = 5$ find all (both pure and mixed) subgame perfect Nash equilibria of the game. Make sure you describe the equilibria fully.

Solution: The subgame of the game (starting with player 2's move) has the following strategic form:

Player 2

	A	B
S	6,1	0,0
T	0,0	4,4

This game two pure equilibria and one mixed equilibrium.

Problem 2: Consider the following situation: Two upstream suppliers U_1 and U_2 of widgets compete for sales to downstream manufacturers D_1 and D_2 who in turn sell to final consumers. Widgets are identical and constitute a homogenous good. The upstream firms U_1 and U_2 have respectively unit costs of $c_1 = 0$ and $0 < c_2 < 0.5$. Downstream firms D_1 and D_2 use a one-to-one production technology to turn one widget into one gadget. Gadgets are identical and constitute a homogenous good. The downstream firms have no other costs than the costs for the input (widgets). The downstream firms face final consumer demand for gadgets with inverse demand given by $p = 1 - q_1 - q_2$, where q_i is the individual gadget output by downstream firm $D_i, i = 1, 2$. There are no capacity constrainst.

(a) Consider the following two-stage game. In the first stage, the two upstream firms Q_1 and U_2 in prices (Betrand competition), independently and simultaneously choosing prices w_1 and w_2 for their widgets. After observing w_1 and w_2, the two downstream firms D_1 and D_2 buy widgets and then indepedently and simultaneously choose quantities q_1 and q_2 of gadgets to be sold in the final consumer markets. (That is, the two downstream firms compete la Cournot). Determine the subgame perfect Nash equilibrium of this game! [**Hint**: Assume that if both upstream firms choose the same price, every downstream firm will buy from upstream firm U_1]. Determine also the final consumer price and the profits of all firms in the market!

Solution: Each of the downstream firms will receive the input from firm U_1 at price c_2 (or slightly below). Since the upstream firms are playing a Bertrand game with asymmetric costs. That is, U_1 will choose a price of c_2 (or slightly below) and U_2 would choose a price of c_2. The downstreat firms play a standard Cournot game with costst c_2. That is, their profit function would read $\pi D_i = (1 - c_2 - q_1 - q_2)q_i$ and the equilibrium quantities for the downstream firms in the Nash equilibrium in the subgame are given by

$$q_1^{vs} = q_2^{vs} = (1 - c_2)/3,$$

where the label "vs" stands for vertically seperated". The final consumer

price is $p^{vs} = (1 + 2c_2)/3$. Firm U_1's profits are given by

$$\pi_{U1} = 2c_2(1 - c_2)/3,$$

whereas each downstream firm earns

$$\pi_{Di} = (1 - c_2)^2/9.$$